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Macro Theory III
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Lecture 3: Convex Preference and Type-Identical Competitive Equilibria

Definition: A ***competitive equilibrium*** is an allocation $\{\{\hat{x}^i(k) \text{ for } k \in [0,\lambda^i]\}_{i\in I},\ \ \{\hat{y}^j\}_{j\in J}\}$ and price system ϕ such that

(1) for all *i* and all $k \in [0,\lambda^i]$, $\qquad \hat{x}^i(k) \in \arg\max\{u_i(x) \,|\, x \in X^i, \phi(x) \le \phi(\hat{x}^i(k))\}$;

(2) for all *j*, $\qquad \hat{y}^j \in \arg\max\{\phi(y) \,|\, y \in Y^j\}$;

(3) $\sum_i \int \hat{x}^i(k)\,dk = \sum_j \hat{y}^j$.

Definition: A competitive equilibrium is ***type-identical*** if for all *i* and for all $k, k' \in [0,\lambda^i]$ $\hat{x}^i(k) = \hat{x}^i(k')$.

Definition: Two competitive equilibria are ***equivalent*** if

(i) For every type *i*, the *average* $x^i(k)$ are the same;

(ii) For every *j*, the y^j are the same;

(iii) The price system ϕ is the same.

Definition: A function *f* defined on a convex set *X* is (weakly) ***quasi-concave*** if $f(x) \ge f(x')$ and $\theta \in [0,1]$ implies $f(\theta x + (1-\theta)x') \ge f(x')$.

Definition: We will say that the preferences of agents of type *i* are ***convex*** if the X^i is convex and the u_i is quasi-concave.

Proposition: If the u_i are quasi-concave, any competitive equilibrium is equivalent to a type-identical equilibrium.

Note: By the proposition, if preferences are convex the space of competitive equilibria can be partitioned based upon the type-identical equilibrium to which elements of a partition are equivalent.

Proof Outline:

It is immediate that the candidate type-identical allocation has profit maximization and resource balance. These results do not use the convexity of preferences.

For a competitive equilibrium all agents of the same type face the same maximization problem and therefore realize the same utility, which is denoted by $\overline{u}^i$. The convexity of the X^i insures that mean type i consumption $\overline{x}^i$ belongs to X^i. Further, $\overline{x}^i$ satisfies the budget constraint as the budget constraint is convex. This implies $u^i(\overline{x}^i) \leq \overline{u}^i$. However, the quasi-convexity of preferences implies $u^i(\overline{x}^i) \geq \overline{u}^i$. Thus $\overline{x}^i$ maximizes utility. This proves the equivalence.

Expected Utility Maximization

Under some appealing axioms, people's preferences are ordered by the expected value of a utility function. Let C, a subset of a normed vector space, be the underlying space of consumption vectors and Π be the set of probability measures on the Borel sigma algebra of C, denoted ***B***(C), with bounded support. The utility function is denoted *U(c)*. This utility function is unique up to a transformation of the type $\alpha + \beta\, U(c)$ where $\beta > 0$.

Axiom 1 (Independence Axiom): If π_1, π_2, and $\pi \in \Pi$ and $0 < \theta < 1$, then

$$\pi_1 \prec \pi_2 \quad \text{if and only if} \quad \theta\pi_1 + (1-\theta)\pi \prec \theta\pi_2 + (1-\theta)\pi.$$

Axiom 2: If π_1, π_2, and $\pi \in \Pi$ and $\pi_1 \prec \pi \prec \pi_2$, then there exists $\alpha, \beta \in (0,1)$ such that

$$\pi \prec \alpha\pi_2 + (1-\alpha)\pi_1 \quad \text{and} \quad \pi \succ \beta\pi_2 + (1-\beta)\pi_1 .$$

Note: Axiom 2 is satisfied if preferences over random prospects are continuous.

Definition: Probability measure $\delta(c)$ is ***degenerate*** if it places probability 1 on a single point $c \in C$. Let D denote the set of degenerate probability measures.

Axiom 3 (Measurability): If δ_1, δ_2, and $\delta_3 \in$ D, and $\alpha, \beta \in [0,1]$, then the set

$$\{c \in C \mid \alpha\delta(c) + (1-\alpha)\delta_1 \preceq \beta\delta_2 + (1-\beta)\delta_3\} \in \boldsymbol{B}(C)$$

Axioms 1-3 insure the existence of a measurable utility function. Measurability is needed for integration to be meaningful.

An additional assumption is needed to insure continuity of U. The utility function will be continuous if the following assumption is satisfied.

Axiom 4: If $\delta', \delta'', \delta(c), \{\delta(c_n)\}_{n=1}^{\infty} \in$ D, where $c_n \to c$, and $\alpha \in [0,1]$, then

$$\alpha\delta(c_n) + (1-\alpha)\delta' \preceq \delta'' \quad \forall n \quad \Rightarrow \quad \alpha\delta(c) + (1-\alpha)\delta' \preceq \delta'',$$

and

$$\alpha\delta(c_n) + (1-\alpha)\delta' \succeq \delta'' \quad \forall n \quad \Rightarrow \quad \alpha\delta(c) + (1-\alpha)\delta' \succeq \delta''.$$

Other material covered:

1. Aggregate production possibility set for growth model with firms owning capital.
2. Aggregate production possibility set with household owning and renting the capital to the firms.
3. Interior point to Y.
4. Convexity of Y.
5. Using second welfare theorem to find the competitive equilibrium.