Macro Theory III.
Spring (1), 2000.
Edward C. Prescott.

Lecture 5

Social Planner's Problem:

Let $z = \{z_s\}$ where $s=(c,h,k)$. The element z_s is the measure of people assigned c consumption, work an h hour workweek, and use k units of capital. We continue with the assumptions that proportional feasible changes to the arguments of f lead to the same proportional change in the value of the function, that $n = 1$ is feasible, and that all $k \geq 0$ are feasible. With these assumptions, restricting the plant size to one employee does not reduce the aggregate production possibility set Y.

In terms of z, the type-identical social planner's problem is

$$\text{(P1)} \qquad \begin{aligned} &\max_{z \geq 0} \sum_{c,h,k} z_{chk}\, U(c,1-h) \\ &s.t. \quad \sum_s z_s = \lambda \\ &\qquad \sum_s [c - h\, f(k,1)]\, z_s \leq 0 \\ &\qquad \sum_s k\, z_s \leq \bar{k}\, \lambda \end{aligned}$$

This is a well-behaved linear program. Let $\hat{z}$ denote an optimum.

Stand-in Household:

The commodity space that we will use is $L = \Re^{\#H+2}$. A typical element is $x = \{x_c, \{x_h\}_{h \in H}, x_k\} \in L$. The advantage of this commodity space is that the commodities have empirical counterparts. The consumption set is

$$X = \{x \in L_+ \mid \sum_h x_h = 1; \quad x_k \leq \bar{k}\}.$$

The utility function $u: X \to \Re$ is the value of the program

(P2)

$$
\begin{aligned}
u(x) = \max_{v \geq 0} \quad & \sum_{c,h} U(c,h)\, v_{ch} \\
s.t. \quad & \sum_{c,h} v_{ch} = 1 \\
& \sum_{c,h} c\, v_{ch} = x_c \\
& \sum_{c} v_{ch} = x_h \quad \forall\, h
\end{aligned}
$$

This utility function u is continuous and concave.

Finding an Equilibrium Allocation:

A candidate equilibrium allocation $\{x', y'\}$ is defined based on an optimal $\hat{z}$:

$$
\begin{aligned}
y'_c &= \lambda\, x'_c = \sum_{c,h,k} c\, \hat{z}_{chk} \\
y'_h &= \lambda\, x'_h = \sum_{c,k} \hat{z}_{chk} \quad \forall\, h \\
y'_k &= \lambda\, x'_k = \lambda\, \bar{k}
\end{aligned}
$$

Finding Equilibrium Prices:

Equilibrium prices of the consumption good and the workweeks of different lengths can be obtained from (P2). Substitute the equilibrium x' for x in (P2) and solve for the LaGrange multipliers, namely $\phi, \lambda_c, \{\lambda_h\}_{h \in H}$. Prices are proportional to the marginal utilities. Consequently

$$
\begin{aligned}
p_c &= \lambda_c \\
p_h &= -\lambda_h \quad h \in H
\end{aligned}
$$

The reason for the negative sign in front of λ_h is that we are following National Income Accounts conventions with factor rental prices positive.

The budget constraint can be used to find

$$
p_k = (p_c x_c - \sum_h p_h x_h) / \bar{k} \,.
$$

These prices are the demand reservation prices. The technology side can be used to determine the supply reservation prices. They are the prices that result in zero profit for a plant operated *h* hours with one worker and the optimal amount of capital. Thus,

$$\tilde{p}_h = \max_k \{p_c \, h \, f(k,1) - p_k \, k\}.$$

For commodities that clear at positive quantities, the demand and the supply reservation prices are the same.

Application:

There is an old tradition to refer to the wage as payment per hour. For people working at McDonalds this is a reasonable thing to do. Often, however, this is not a reasonable thing to do. Two 20-hour workweeks are not the same as one 40-hour workweek from the point of view of the operators of technology. There may be fixed time costs of keeping current. A researcher spends 40 hours a week just keeping current, another 20 on Mickey Mouse administration work, and if the person is lucky still another 10 hours on research. Splitting this seventy hours between two people working 35 hours results in zero output.

An interesting fact is that those that work short workweeks are typically paid less per hour than those that work long workweeks. For hours between 20 and 60, compensation is about 1 + 1.5 (h – 40) / 40. The pay for a 40-hour workweek has been normalized to 1. If someone works 60 hours a week, compensation is 1.75. If someone works 20 hours a week, compensation is 0.25.

Problem: Develop a model that displays this fact.

See Fitzgerald (1996) article for a very interesting application.