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Macro Theory III.
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Lecture 6: Calibrating the Hansen Economy

Preferences:

People either work $h = 0.5$ or not at all. Labor is indivisible. Their period utility function is

$$\log c + \alpha \log(1-h).$$

The measure that work is n and the total measure of people is 1. The period utility function for this group is $\log c + \alpha \log(0.5)\, n$. Assume the household owns the capital and rents it to the firm. The date t commodities are (c,x,k,n). The initial capital stock owned by each individual is $k_0 > 0$. The utility function of the stand-in household is

$$E \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} [\log c_t + \alpha\, n_t \log(0.5)]$$

Constraints are $n_t \in [0,1]$ and $c_t \geq 0$ for all t.

Technology: $c + x \leq k^{\theta} n^{1-\theta}$

$k' = (1-\delta)k + x$

Steady State Observations:

$c/y = .75$; $k/y = 15$; $(w\, n) / y = .70$; $n = .80$

This is a quarterly model. This is important in interpreting discount rate ρ and depreciation rate δ. It is important for interpreting the capital-output ratio as well.

A Set of Necessary Conditions for Steady State:

(1) $\delta\, k = x$

(2) $r = i + \delta$

(3) $r = \theta\, k^{\theta-1} n^{1-\theta}$

(4) $w = (1-\theta)\, k^{\theta} n^{-\theta}$

(5) $c + x = k^{\theta} n^{1-\theta}$

(6) $i = \rho$

(7) $\dfrac{1/c}{\alpha \log(1/2)} = \dfrac{1}{-w}$

An **algorithm** to solve these equations for the three prices (*i,r,w*) and four quantities (*c,k,n,x*) is as follows:

Step 1: Solve (6) for i.

Step 2: Solve (2) for r.

Step 3: Solve (3) for k/n.

Step 4: Solve (4) for w.

Step 5: Solve (7) for c.

Step 6: Use (1) to eliminate x in (5) and solve the resulting equation for k.

Step 7: Solve (1) for x.

Step 8: Given k/n and k, compute n.

Note: In an algorithm, only numbers that have already been computed in an early step can be used in the given step.

Calibration to Steady-State Observations:

A good principle is to list the observations to which the measurement instrument, that is the artificial economy, is being calibrated. Here there are four parameters, (α, ρ, θ, δ) and four observations.

$$\delta\, k/y = x/y = 1 - c/y \quad \Rightarrow \quad \delta = .0167$$

$$(\rho + \delta)\, k/y = .30 \quad \Rightarrow \quad \rho = .0200$$

$$n\, w/y = .70 \quad \Rightarrow \quad \theta = .300$$

$$\alpha = -\frac{1}{\log(1/2)} \frac{w/y}{c/y} \quad \Rightarrow \quad \alpha = -\frac{7/8}{\log(1/2)\, .75} = 2.33$$

The measurement instrument (the model economy) has been calibrated to a set of growth facts. I emphasize that the model has ***not*** been estimated. Estimating or measuring how big a model is makes no sense.

The Theory of the Aggregate Production Function

See Prescott, "Business Cycle Theory: Methods and Problems," pp. 8-11.

See Chapter 1 in Cooley (ed.), *Frontiers of Business Cycle Research.*