Macro Theory III.
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Lecture 7

A General Equilibrium Model with Continuing Participation Constraints

(Kehoe and Levine)

People: Measure λ^i for *i* belonging to finite set *I*.

Commodity Vector: $\{x_t(z^t)\}$ for $z^t \in Z^t$, $t \in \{0, 1, 2, ...\}$, where the z^t are event histories. The sets Z^t are finite. The elements $x_t(z^t) \in \Re$. The commodity space is $S = \ell_\infty$.

Utility Functions: The $u^i : S_+ \rightarrow \Re$ are concave and strictly increasing.

$$u^i(x) = \sum_{t=0}^{\infty} \sum_{z^t} \beta^t \pi(z^t)\, u^i[x_t(z^t)]$$

Endowments: $e^i \in S_+$.

Continuing Participation Constraints: People can opt out of the system and just consume their endowment at any time. Let

$$v_t^i(x, z^t) = \sum_{s=t}^{\infty} \sum_{z^s} \beta^s \pi(z^s \mid z^t)\, u^i[x_s(z^s)].$$

This is just the expected discounted future utility of *x* if the event history is z^t. The continuing participation constraints are $v_t^i(x, z^t) \geq v_t^i(e, z^t)$.

Consumption Sets: The consumption sets must incorporate these continuing participation constraints. Thus,

$$X^i = \{x \in S_+ \mid v_t^i(x, z^t) \geq v_t^i(e, z^t) \quad \forall z^t\}.$$

Resource Balance Constraint: $\sum_i \lambda^i \,(x^i - e^i) = 0.$

Convexity of Preferences: The sets X^i are convex given the concavity of u^i.

Extension:

With a single consumption good, not honoring a contract results in an individual being excluded from all future markets. Suppose now there is not a single consumption good but that $x_t(z^t) \in \Re^n$.

If a person has been excluded from borrowing and lending markets, or equivalently the legal system is such that creditors have claim to any asset that the individual might accumulate, that individual can still enter into spot markets. Let $\{p_{tj}(z^t)\}$, $j \in J$ be the relative spot prices at date t if the event history is z^t. These prices enter into the consumption sets.

If people have **identical, convex, homothetic preferences**, these relative spot prices depend only on the **aggregate endowment** $e(z^t) = \sum_i \lambda^i e_t^i(z^t)$.

A Review of Homothetic Preferences

Definition: Preferences of type i are **homothetic** if X^i is a cone, and whenever $u^i(x) = u^i(x')$ for some $x, x' \in X^i$, then $u^i(a\,x) = u^i(a\,x')$ for all $a > 0$.

Comment: If preferences are homothetic and convex, and continuous linear functional $\phi : S \to \Re$ supports the set $X_+^i(\hat{x}) := \{x \in X^i \mid u^i(x) \geq u^i(\hat{x})\}$ at $\hat{x}$, then ϕ supports the sets $X_+^i(a\,\hat{x})$ at $a\,\hat{x}$ for all $a \in \Re_+$.

Note: ϕ will in general depend upon the ray from the origin considered, which is determined by $\hat{x}$. The functions ϕ are price systems.

Comment: With identical, convex, homothetic preferences wealth redistribution has no consequence for prices and aggregate equilibrium quantities. Redistribution just results in individuals scaling their consumption vectors. Thus, a single stand-in person can be used in the analysis to find prices and aggregate quantities.