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Macro Theory III
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Lecture 8

Recursive Competitive Equilibrium

Review stochastic processes (Chapters 7 and 8 in Stokey and Lucas). Review Chapters 1 and 2 in Cooley, *Frontiers of Business Cycle Research*.

Recursive means that there is some variable that completely describes the 'position' of the economy. We refer to this variable as the 'state' of the economy. Everything that happens at date t is a function of the state only. In addition, the distribution of the state next period is a function of the current state and the actions of the economic actors.

To find a suitable state variable, one selects a candidate and determines whether it satisfies the conditions necessary to be a state variable. Often, it will be necessary to add components to initial candidate. It is generally best to have as low a dimensional state variable as possible.

An Arrow-Debreu economy is a ***stationary-recursive in state variable s*** if the following are true:

1. For all type i, the consumption possibility continuations from event-history z^t belong to some correspondence $X_i(s_t)$.

2. For all i, preferences are ordered by

$$E\{\sum_{t=0}^{\infty} \beta^t u_i(x_t, z_t)\}.$$

In order to justify restricting attention to type-identical allocations, the correspondences $X_i(s)$ are convex and the functions u_i are concave in x and are continuous.

3. Production possibility continuations from event-history z^t belong to some set $Y_j(s_t)$ for all j.

4. The stochastic process $\{z_t\}$ is Markov with stationary transition probabilities.

5. The distribution of s_{t+1} depends upon s_t and the period commodity vectors chosen by the individuals and the operators of the technologies.

The Growth Model – A Stationary Recursive Economy

There is measure one of people with preferences ordered by the expected value of

$$\sum_{t=0}^{\infty} \beta^t \, u(c_t, n_t) \,.$$

Here $c \geq 0$ is consumption and $n \in [0,1]$ is labor services. Function u is concave and strictly increasing in c. One unit of capital provides one unit of capital services. The law of motion for an individual's capital stock is

$$k_{t+1} = (1-\delta)\, k_t + i_t$$

Here $i_t \geq -(1-\delta)\, k_t$ is investment. All individuals at date 0 own $K_0 > 0$ units of capital.

The technology is standard and is described by aggregate production function

$$C_t + I_t \leq z_t F(K_t, N_t)\,.$$

Stochastic process $\{z_t \in Z \subset \Re_{++}\}_{t=0}^{\infty}$ is Markov with transition function $Q(z_t, dz_{t+1})$. Production is non-reversible so K_t, $N_t \geq 0$ and $C_t + I_t \leq z_t F(K_t, N_t)$.

The ***state of this economy*** is (K,z). The problem is to find the recursive competitive equilibrium price functions $\{r(K,z),\ w(K,z)\}$, market clearing aggregate functions $\{C(K,z),\ I(K,Z),\ K(K,z),\ N(K,z)\}$, and law of motion for the endogenous state $K' = G(K,z)$. To determine whether a particular set of functions is an equilibrium it is necessary to find the optimal policy function of an individual with $k \in \Re_+$ units of capital. The ***state of this individual*** is (k,K,z). This individual's dynamic program is

$$v(k,K,z) = \max_{c,i,k,n} \left\{ u(c,n) + \beta \int v\big((1-\delta)k + i,\, G(K,z),\, z'\big)\, Q(z,dz') \right\}$$

s.t. $c \geq 0$; $0 \leq n \leq 1$; $(1 - \delta)\, k + i \geq 0$; $k_s \leq k$; and the period budget constraint

$$c + i \leq r(K,z)\; k_s + w(K,z)\; n\,.$$

Note that k_s is the quantity of capital services supplied and k is the individual's stock of capital.

Assuming u is continuous, strictly increasing in c, strictly concave, and bounded, this is a well-behaved dynamic program. Let $\{c(k,K,z), i(c,K,z), n(k,K,z), k_s(k,K,z)\}$ be the optimal policy functions given functions r, w, and G. The equilibrium conditions are as follows:

Market clearing (remember the measure of people is one):

1. $c(K,K,z) = C(K,z)$
2. $i(K,K,z) = I(K,z)$
3. $k_s(K,K,z) = K(K,z)$
4. $n(K,K,z) = N(K,z)$

Profit Maximization:

5. $w(K,z) = z\, F_N\, (K(K,z), N(K,z))$
6. $r(K,z) = z\, F_K\, (K(K,z), N(K,z))$

The final equilibrium condition is that the law of motion of the endogenous, economy-wide state variable must be consistent with the behavior of the optimizing individual with $k = K$.

7. $G(K,z) = (1 - \delta)\, K + I(K,z)$.

In fact there is the additional condition that $C(K,z) + I(K,z) = z\ F(K(K,z), N(K,z))$, but it is satisfied if conditions (1) – (6) hold. In specifying the profit maximizing conditions, constant returns to scale are assumed.