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Macro Theory III.
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Edward C. Prescott.

Lecture 9

Recursive Competitive Equilibrium with Risk Allocation

Growth Economy with Multiple Types of Individuals:

Stochastic process $\{z_t \in Z \subset \Re_{++}\}_{t=0}^{\infty}$ is Markov with transition function $Q(z_t, dz_{t+1})$.

There are measures λ_i of people of type $i \in \{1, \ldots, I\}$ with preferences ordered by the expected value of

$$\sum_{t=0}^{\infty} \beta^t\, u_i(c_t, n_t, z_t).$$

Here $c \geq 0$ is consumption and $n \in [0,1]$ is labor services. Functions u_i are concave and strictly increasing in c. A unit of capital provides one unit of capital services. The law of motion for an individual's capital stock is

$$k_{t+1} = (1-\delta)\, k_t + x_t.$$

Here $x_t \geq -(1-\delta)\, k_t$ is investment. Individuals of type i own $K_{i0} > 0$ units of capital.

An individual of type i working n produces $n\, \pi_i(z)$ units of labor services.

The technology is standard and is described by the aggregate production function

$$c + x \leq F(k, n, z)$$

Production is non-reversible so $k, n \geq 0$ and $c, x \leq F(k, n, z)$. Here n is units of labor services.

The state of this economy is $(K,z) = (K_1, K_2, \ldots, K_I, z)$. Note K without a subscript is a vector that specifies the capital stocks of each of the types.

Securities are introduced to allocate risk. A security of type $s_{z'}$ is a promise of delivery of one unit of the capital good next period conditional on the exogenous state having value z'. A prime is used to denote the next period value of a variable in the recursive language.

The problem is to find the elements that define a recursive competitive equilibrium. The elements that define a RCE are

1. Price functions $\{r(K,z), w(K,z), \{q_{z'}(K,z)\}_{z'\in Z}\}$;
2. Individual type functions $\{c_i(k,K,z), x_i(k,K,z), k_i(k,K,z), n_i(k,K,z), \{s_{z'i}(k,K,z)\}_{z'\in Z}\}$ for each type $i = 1,\ldots,I$;
3. Functions for the firm $\{c(K,z), x(K,Z), k(K,z), n(K,z)\}$;
4. The law of motion for each type's endogenous state $\{K_i' = G_i(K,z,z')\}_{i=1,\ldots,I}$

To determine whether a particular set of functions is an equilibrium it is necessary to find the optimal policy function of an individual of type i with $k \in \Re_+$ units of capital for each type i. The state of this individual is (k,K,z). This individual's dynamic program is

$$v_i(k,K,z) = \max\left\{u(c,n) + \beta \int v_i\left((1-\delta)k + x + s_{z'},\, G(K,z,z'),\, z'\right) Q(z,dz')\right\}$$

s.t. (1) $c \geq 0$; (2) $0 \leq n \leq 1$; (3) $(1-\delta)k + x + s_{z'} \geq 0$ all z'; (4) $k_i \leq k$;

and (5) the period budget constraint

$$c + x + \sum_{z'} q_{z'}(K,z)\, s_{z'} \leq r(K,z)\, k_i + w(rK,z)\, n\, \pi_i(z)\,.$$

The maximization is over c, x, n, k, and the $\{s_{z'}\}_{z'\in Z}$

Assuming u is continuous, strictly increasing in c, strictly concave, and bounded, this is a well-behaved dynamic program. Let $\{c_i(k,K,z), x_i(c,K,z), n_i(k,K,z), k_i(k,K,z), \{s_{z'i}\}_{z'\in Z}\}$ be the optimal policy functions for a type i given functions r, w, and the $\{Q_{z'}\}$.

The equilibrium conditions are as follows:

Factor and Good Markets clear:

1. $\sum_i \lambda_i c_i(K_i, K, z) = c(K, z)$
2. $\sum_i \lambda_i x_i(K_i, K, z) = x(K, z)$
3. $\sum_i \lambda_i k_i(K_i, K, z) = k(K, z)$
4. $\sum_i \lambda_i \pi(z)\, n_i(K_i, K, z) = n(K, z)$

Profit Maximization:

5. $w(K,z) = z\, F_N(k(K,z), n(K,z))$
6. $r(K,z) = z\, F_K(k(K,z), n(K,z))$

Consistency of the law of motion of the endogenous, economy-wide state variable with the behavior of the optimizing individual with $k = K_i$ for each type i:

7. For all i, $G_i(K,z,z') = (1 - \delta)\, K_i + x_i(K_i,K,z) + s_{z'i}(K_i, K, z)$

Security markets clear:

8. For all z', $\sum_i \lambda_i\, s_{z'i}(K, K, z) = 0$

Condition 8 follows because the net supplies of these securities are zero.