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Lecture 10

Infrastructure and the Aggregate Production Relation

If infrastructure is priced, as is the case for toll roads, then there is no problem with standard aggregation theory. Provided economies of scale are exhausted at a level small relative to the size of the economy, profit maximization along with competitive factor markets results in output being maximized given the inputs. There is no need to make a distinction between the *maximum possible* output given inputs and the *actual* output given inputs. They are equal.

As the following example illustrates, this is not the case if there is an externality associated with congestion of a not priced input. Consequently, defining the aggregate production function to be the maximum output that can be produced given the quantities of the inputs, and assuming equilibrium quantities satisfy the aggregate production function, is inconsistent if there is an externality or a not priced input.

An alternative, that is followed here, is to define an **aggregate production relation** to be aggregate output if there are competitive factor markets and profit maximization. This is equivalent to the standard definition absent externalities.

Let G denote the aggregate stock of infrastructure, K the aggregate stock of capital, and N the aggregate labor input. Assume the production unit technologies are

$$y \leq h(G/K)\, g(n)\, k^{\theta}.$$

Let $n^* = \arg\max g(n)\, n^{\theta-1}$. Note restrictions on the function g are needed to insure that the maximum exists. This n^* is the equilibrium number of workers at production plants that are operated.

The rational for the $h(G/K)$ term is that resources used up in transportation are smaller if there are more and better roads given the number of vehicles using the roads. Alternatively, G

may be the quantity of court services for enforcing commercial contracts. The function h is increasing and bounded.

The aggregate production relation is

$$Y \le A\,h(G/K)K^{\theta}N^{1-\theta}, \qquad \text{where } A = \max g(n)\,n^{\theta-1}.$$

This function displays constant returns to scale in (G,K,N). It does not display constant returns to scale in the market inputs (K,N). If h were not homogenous of degree one, the aggregate production relation would not be homogenous of degree one.

Another type of public good is one where there are no congestion externalities. Suppose, for example, the plant technology is

$$y \le h(G)g(k,n)$$

In this pure public good case, the concept of an aggregate production function makes sense. The aggregate production function will have the structure

$$\begin{aligned} &Y = h(G)\,F(K,N) \quad \textit{where} \\ &F(K,N) = \max_{z\ge 0} \int g(k,n)\,dz \\ &\qquad\qquad s.t. \quad \int k\,dz \le K \quad \textit{and} \quad \int n\,dz \le N \end{aligned}$$

Here G is a pure public good and its price should be zero as this results in efficient production. What are some examples of pure public good? The standard example is a weather forecast. The interesting examples of public goods, I think, are new knowledge. Economists typically model this as exogenous growth in G that gives rise to technology change over time.

Some people have argued that there are increasing aggregate returns and that this is crucial for understanding business cycle fluctuations. What are these externalities giving rise to these increasing returns? My position is that until they come up with some foundations for their aggregate assumptions, these people should be ignored. I know of no externalities that are greater when output is above trend and smaller when output is below trend. Congestion works the other way. In peak seasons, there are more, not less, delays at airports.

In the new growth and development field spawned by Paul Romer and Bob Lucas in the mid-eighties, externalities play an important role. Lucas' current work on cities relies on externalities associated with proximity of people has explicit foundations and leads to ways to *measure* this externality. His early work introducing a human capital externality has foundations. If the people you interact with at the work place know more, you have access to more knowledge that you can exploit to provide more units of labor services.