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Macro Theory III
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Lecture 11

Solving for Pareto Optima in the Kehoe-Levine Debt-Constraint Economy

See Lecture 7 for a specification of the economy. Let $\{\varphi_i\}$ be a set of positive weights that sum to one. The φ-social planner problem is

$$\max \sum_i \varphi_i \sum_{t=0}^{\infty} \sum_{z^t} \beta^t \pi(z^t)\, u^i[x_t^i(z^t)]$$

subject to the resource constraints, the continuing participation constraints, and non-negativity constraints.

Assumption: The process $\{z_t\}$ is a finite state Markov Chain with strictly positive transition probabilities $q(z, z')$.

This is not a discounted dynamic programming problem, but discounted dynamic programming methods can be used to find the solution. The analysis first requires finding the set of feasible utilities $W(z) \subset \Re^{\#I}$ for each z. Note that the set of feasible utilities depends upon the current value of the exogenous state variable.

The **resource constraints** are

(1) $$\sum_i \lambda_i (x_i - e_i(z)) \leq 0\,;$$

The **promise keeping constraints** are

(2) $$w_i = u_i(x_i) + \beta \sum_{z'} q(z,z') w_i'(z')\,;$$

The **continuing participation constraints** are

(3) $$w_i \geq \underline{v}_i(z)\,,$$

where $\underline{v}_i(z)$ is the expected discounted utility if an individual chooses not to participate and consume his(her) endowment in the current and future periods.

The **feasible utility constraints** are

(4) $$w'(z') \in W(z') \subset \Re^{\#I} \qquad \forall z'.$$

Finding *W(z)* Sets.

The *W(z)* sets must be known before discounted dynamic programming methods can be used to find the solution.

A mapping *T* from sets $W = \{W(z')\}$ into itself is first defined as follows:

$$T_z(W) = \{w \in \Re^{\#I} \mid \text{s.t. } \exists \ \{x_i\} \text{ and } \{w'(z')\} \text{ that satisfy (1)} - \text{(4) given } w\}$$

The mapping *T* is monotonic. It maps bigger sets into bigger sets. Further the economics of the problem can be used to find a closed, bounded and convex set that contains all the feasible utility vectors as well as some infeasible vector. This set is denoted W_0.

Proposition 1: *T* maps convex compact sets into convex compact sets.

Proposition 2: If $W \supseteq W''$, then $T(W) \supseteq T(W'')$.

Proposition 3: The sets of feasible utility vectors (there is one set for each *z*) is $\lim T^n(W_0)$.

An appropriate W_0 is the one defined by the following set of inequalities:

$$\underline{v}_i(z) \le w_i \le \overline{v}_i(z) \qquad \forall \ i,$$

where $\overline{v}_i(z)$ is type *i* expected utility if type *i* consumes the entire endowment.

Dynamic Program

The state variables are the vector $w \in W(z)$ and z. The decision variables are the state-contingent utility continuations $w'(z') = \{w_i'(z')\}$ for all z' and the consumptions $\{x_i\}$. The period return function is

$$R(x) = \sum_i \varphi_i \, u^i(x_i) \,.$$

The functional equation is

$$v(w,z) = \max_{x,\{w'(z')\}} \{ \, R(x) + \beta \sum_{z'} q(z,z') \, v[w'(z'), z'] \, \}$$

subject to constraints (1) − (4). Observe that the objective is bounded, continuous, and concave. Observe that the constraint set is convex in $(w,\ x,\ \{w'(z')\})$. This is a well-behaved concave, discounted, dynamic program.