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Macro Theory III
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Lecture 12

Recursive Competitive Equilibrium with Endogenous Government Policy

Notation:

Capital letters will be for aggregates and the corresponding lower case letter for an individual. Capital letters will also be used to represent the value of a variable for all other individuals.

People:

There is measure one of individuals with preferences ordered by the expected discounted value of *u(c)*, where consumption $c \geq 0$. The period utility function has the standard properties. People are endowed with one unit of labor at every date. All individuals have the same initial capital stock $K_0 \geq 0$.

Technology:

There is a standard neoclassical production function, *F(K,N)* with capital *K* and labor *N* as inputs.

Government:

The government is benevolent and *maximizes* the welfare of the representative individual. Government consumption is governed by the Markov chain with transition function *Q*. Abusing the notation slightly, $Q(z, z')$ is shorthand for $Q(z,\{z'\})$. It is the probability that $z_{t+1} = z'$ given that $z_t = z$. The government taxes gross income or equivalently gross output at rate $\tau \in [0,1]$. The government issues one period real bills *D*, whose price is denoted by *q*.

Notation:

D is aggregate or per capita real government debt.

$S = (D, K)$.

(S, z) is the economy wide state.

$s = (d, k)$.

(s, S, z) is the individual state variable.

$V(S)$ is the government's value function *prior* to the selection of τ.

$v(s, S, k)$ is an individual's value function *prior* to the selection of τ.

$M(S, \tau)$ is the government's value function *subsequen*t to the selection of τ.

$m(s, S, \tau)$ is an individual's value function *subsequent* to the selection of τ.

Equilibrium Prices:

$q(S, z, \tau)$ is the price of a real bill (subsequent to the selection of τ).

$w(S, z, \tau)$ is the real wage (subsequent to the selection of τ).

$r(S, z, \tau)$ is capital's rental price (subsequent to the selection of τ).

Equilibrium Law of Motion:

$S' = G(S, z, \tau)$

Individual Policy Functions:

$n(s, S, z, \tau)$

$s' = g(s, S, z, \tau)$

$c(s, S, z, \tau)$

Equilibrium at the Second Stage of the Period:

The first problem is that government debt can not get so large that no competitive equilibrium exists that honors this debt. We deal with this by using the return function

$$R(c, D) = u(c) \text{ if } D \leq B \qquad \text{and} \qquad R(c, D) = -\infty \text{ if } D \geq B,$$

for B sufficiently large. This imposes an indebtedness constraint on the government.

First let $\overline{K}$ be the maximal technically possible K given K_0. Let $B(K_0,z_0)$ be the maximal possible initial debt consistent with there being an equilibrium with debt being honored given initial conditions (K_0,z_0). In the above, any $B > \sup\{B(K,z) \mid K \leq \overline{K}, \quad z \in Z\}$ is sufficiently large. No equilibrium will exist if $D_0>B$. Any B bigger than a sufficiently large B gives the same answer. If debt could go to infinity, τ could be set to 0 for all t and effectively there would be lump sum taxes. But, if D went to infinity a transversality condition would be violated and individuals would not be maximizing given prices.

$$m(s, S, z) = \max \{ u(c) + \beta \Sigma_{z'} Q(z, z') v[s', G(S, z, \tau), z'] \}$$

subject to the budget constraint

$$c + k' - (1-\delta) k + q(S, z, \tau) d' \leq r(S, z, \tau) k + w(S, z, \tau) n + d,$$

$$c \geq 0, \quad n \in [0,1], \qquad \text{and } k' \geq (1-\delta) k.$$

The optimal policy functions are

$$c = c(s, S, z, \tau), \qquad n = n(s, S, z, \tau), \qquad d' = g_{d'}(s, S, z, \tau), \qquad k' = g_{k'}(s, S, z, \tau).$$

Profit maximization requires

$$w(S, z, \tau) = F_2(K, n(S, S, \tau)) \qquad \text{and} \qquad r(S, z, \tau) = F_1(K, n(S, S, z, \tau)).$$

Government *period* budget constraint is

$$z + D = q(S, z, \tau)\, g_d(S, S, z, \tau) + \tau F(K, n(S, S, z, \tau)).$$

Consistency requires

$$G_{d'}(S, z, \tau) = g_{d'}(S, S, z, \tau) \qquad G_{k'}(S, z, \tau) = g_{k'}(S, S, z, \tau)$$

Government's value function must satisfy

$$M(S, z, \tau) = R[c(S, S, z, \tau)] + \beta \Sigma_{z'} Q(z, z') V[G(S, z, \tau), z']$$

Equilibrium at the First Stage in the Period:

Let $\tau(S, z)$ be government's tax policy. Equilibrium requires $\tau(S, z)$ maximize $M(S, z, \tau)$. Further

$$V(S, z) = M[S, z, \tau(S, z)]$$

$$v(s, S, z) = v[s, S, z, \tau(S, z)]$$

Comments:

The equilibrium is not the equilibrium that would be obtained if the government could commit to an event-contingent tax policy $\{\tau_t(z^t)\}_{t=0}^{\infty}$. This is the time inconsistency of optimal-policy problem.

Whether the equilibrium concept used above is a good one is subject to debate. There are other sustainable equilibria (Chari and Kehoe) which are better. This equilibrium concept is related to Kreps and Wilson sequential equilibrium (which is for games). When there are many equilibria, which is the appropriate one is an issue that must be addressed.