February 12, 2000

Macro Theory III.
Spring (1), 2000.
Edward C. Prescott.

Lecture 14

A Steady-State Model with Organizations

Economy:

Measure one identical people are born every period and people live for two periods. Their preferences are ordered by $u(c) + \beta\, u(c')$, where u is strictly concave, strictly increasing, and differentiable. Further, $u'(\infty) = \infty$. Here $c \geq 0$ is consumption when young and $c' \geq 0$ is consumption when old.

A person either becomes a worker or joins an organization. A person who chooses to be a worker supplies one unit of labor when young and one unit when old.

An organization with x old members and x' young members produces output

$$f(x, x', n),$$

if it employs n workers. Some restrictions on f will be made that insure a unique steady-state organization size.

The state of the economy is the measure of organizations as indexed by x. The steady state will be characterized by one type of organization existing. The steady state prices are interest rate factor r and wage rate w.

Problem of young worker:

(1) $$u_w(r, w) = \max\{u(c_w) + \beta\, u(c'_w)\}$$

$$\text{s.t.}\quad c_w + r\, c'_w \leq w + rw.$$

Problem of young organization person:

(2) $$u_o(y, z'/x') = \max\{u(c_o) + \beta\, u(c'_o)\}$$

$$\text{s.t.}\qquad c_o + r\, c'_o \leq y + r\, z'/x'.$$

Here z' is next period profit of the organization, which is split equally among the old members.

Problem of the organization:

Profits of the organization are

(3) $$z(x) = \max\{f(x,x',n) - w\,n - y\,x'\}$$

The maximization is over the number of workers *n*, the number of young organization members x', and the compensation to the latter *y*. The maximization is subject to a young participation constraint $u_o(y, z(x')/x') \geq u_w(r,w)$. Here *n* is the number of workers hired, *x* is the number of old organization members, x' the number of new organization members, and *y* is the compensation of young organization members.

A steady-state is a list of prices (r,w), consumptions (c_w, c'_w, c_o, c'_o), optimal organization policy functions $\{x' = g(x), n = n(x), y = y(x)\}$, profit function *z(x)*, measure of organization μ, and organization characteristics $(\hat{x}, \hat{n}, \hat{y}, \hat{z})$ such that

a. Consumptions are optimal given (*r*,*w*), $\hat{x}$, and $\hat{z}$;

b. Policy function *g*, employment function *n(x)* and compensation function *y(x)* are optimal given (*r*,*w*), and profit function *z*;

c. Profit function *z* satisfies functional equation (3);

d. $\mu\, f(\hat{x},\hat{x},\hat{n}) = \mu\, c_w \frac{\hat{n}}{2} + \mu\, c'_w \frac{\hat{n}}{2} + \mu\, c_o \hat{x} + \mu\, c_o{}' \hat{x}$;

e. $\mu\,(\frac{\hat{n}}{2} + \hat{x}) = 1$;

f. $\hat{n} = n(\hat{x})$, $\hat{x} = g(\hat{x})$, $\hat{y} = y(\hat{x})$, and $\hat{z} = z(\hat{x})$;

Parts d. and e. of this definition represent goods and labor market clearing. The organizations' employment of workers *n* is made up of both young and old agents (who are distinct from young and old organization members). In the steady state, there are equal numbers of each, and hence we pick up the $\frac{\hat{n}}{2}$ terms.

An outline of an algorithm to fine a steady state:

1. Given prices (r,w), find the utility of a worker.

2. Solve the functional equation and find optimal policy functions.

3. Check whether goods markets clear (condition d.) and whether credit markets clear, $(c_w - w)\,\hat{n}/2 + (c_o - \hat{y})\,\hat{x} = 0$. The only borrowing and lending is between the young.

4. Given steady-state prices, (r,w), all the other variables can be calculated.

Comment:

Given prices, (r,w), the operator T mapping the space of functions Z into itself is monotonic; that is if $z \geq z'$ for all $x \geq 0$, then $Tz \geq Tz'$. There are theorems that guarantee the existence of a fixed point of monotonic mappings (see Stokey and Lucas, pp 528-29).

If there are minimal and maximal elements in Z, these theorems give a minimal fixed point, z_{min}, and maximal fixed point, z_{max}. Sometimes it can be shown that $||Tz_{min} - Tz_{max}|| \leq ||z_{min} - z_{max}||$. If so, this implies the fixed point is unique.