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THE DEPRESSING EFFECT OF AGRICULTURE INSTITUTIONS ON THE PREWAR JAPANESE ECONOMY

by

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Abstract

In sharp contrast to its fabulous postwar growth, the Japanese economy stagnated for a long time before World War II: perwar Japanese real GNP per worker remained about 40% of that of the leader country, the U.S., at least since 1885, with no capital deepening. This paper identifies as the main cause of the prewar stagnation a barrier that forced the number of persons employed in agriculture to be constant at about 14 million throughout the prewar period. Our two-sector growth model shows that the barrier-induced sectoral misallocation of labor accounts well for a virtual lack of capital deepening and the depressed output level. Were it not for the barrier, the model predicts that Japan's prewar GNP per worker would have been about 50% to 60% of the U.S. level, roughly where prewar Western Europe was. This higher output level comes about because an efficient use of labor otherwise locked up in agriculture raises the economy's overall production efficiency and sparks a rapid capital deepening.

1. Introduction

The Japanese miracle, which lifted the Japanese economy from the ashes of the World War II destruction to the present-day prosperity, is well known. Also well known is the "lost decade" of the 1990s during which growth languished. Much less well known is the decades-long stagnation before World War II: Japanese real GNP per worker remained far below — about 60% below — that of the leader country, the U.S., at least since 1885 (the first year of our dataset) until the war. This paper addresses the question of why the Japanese miracle didn't occur before World War II.

An amazing fact about employment in agriculture in prewar Japan is that it was virtually constant at 14 million persons (more than 60% of total employment in 1885) throughout the entire prewar period. The constancy strongly suggests that there was a powerful barrier that prevented people from moving out of agriculture.

This leads us to examine whether the barrier had a quantitatively important effect on the economic development of prewar Japan. We do so by using a two-sector growth model with agriculture. Our two-sector model builds on the long tradition of modelling the "dual economy" starting with Jorgenson (1961). Its more recent renditions are Echeverria (1997), Laitner (2000), Gollin-Parente-Rogerson (2002), and others. They feature either non-homothetic preferences to accommodate Engel's Law or a decreasing returns to scale in the primary sector's production technology or both.

On this two-sector model we superimpose the labor barrier to see to what extent the resulting sectoral misallocation of labor can account for the prewar Japanese stagnation. It accounts well for the low output level and the slow capital deepening. We then lift the labor barrier to predict what would have happened to Japan's GNP. We find that prewar GNP per worker would have been substantially higher, about 50 to 65%, not 40%, of the U.S. level.

The plan of the paper is as follows. The next section, Section 2, describes those facts about Japan's economic development in more detail. Section 3 advances our sectoral misallocation hypothesis and summarizes our main results. This is followed by five sections of elaboration: a presentation of the two-sector growth model in Section 4, asymptotic properties of the model in Section 5, a calibration of the model in Section 6, and simulation results in Section 7. Section 8 (still incomplete and not included in this version) will take up the issue of why there existed the barrier. Section 9 concludes.

2. Accounting for Japan's Economic Development Since 1885

The Postwar Miracle and Prewar Stagnation

We start out with a look at aggregate output since 1885. Figure 1 shows detrended GNP per worker (GNP divided by working-age population) for Japan and the U.S.[1] Detrending is done as follows. We know from the data on the U.S. output per worker that the long-run growth rate is about 2.0% after World War II and about 1.8% prewar. We set the trend level in period t, denoted $TREND_t$, growing at 1.8% in the prewar period, 4% during the war, and 2.0% after the war. Thus

$$\frac{TREND_{t+1}}{TREND_t} = \begin{cases} 1.018 \ \text{ for } t = 1885, ..., 1939, \\ 1.040 \ \text{ for } t = 1940, ..., 1944, \\ 1.020 \ \text{ for } t = 1945, ... \end{cases} \tag{2.1}$$

Without a wartime growth rate of as high as 4%, detrended GNP per worker would be higher in the postwar period than in the prewar period. The *same* trend is used to detrend both Japanese and U.S. GNP. In the figure, detrended U.S. GNP per worker for 1885-2000 is normalized to unity.

There are three features that would catch anyone's eye. The first is the fabulous growth in the post World War II era, known as the Japanese miracle. There was a five-fold increase in Japan's GNP per worker in 25 years since 1947. The second is the prewar stagnation of several decades: between 1885 and 1940, Japan's GNP per worker remained about 40% of the U.S. GNP per worker and about 50% of Japan's own detrended GNP for 1985-88. The third feature is the stagnation in the 1990s. We have dealt with Japan's 1990s elsewhere (Hayashi and Prescott (2002)). The question we address in this paper is why the Japanese miracle didn't take place until after World War II.

Growth and Level Accounting

A standard way to account for a country's growth is to define the TFP (total factor productivity) as

$$TFP_t \equiv \frac{Y_t}{K_t^{\theta}\ (h_t\, E_t)^{1-\theta}}, \tag{2.2}$$

where Y_t is aggregate output in period t, K_t is aggregate capital stock, E_t is employment, h_t is average hours worked per employed person (so $h_t E_t$ equals total hours worked), and θ is capital's share of aggregate

[1] See Appendix 1 for data sources and how real GNP was constructed from the data.

income. It easily follows from this definition that GNP per worker can be decomposed into four factors:

$$\frac{Y_t}{N_t} = TFP_t^{\frac{1}{1-\theta}} \times \left(\frac{K_t}{Y_t}\right)^{\frac{\theta}{1-\theta}} \times \left(\frac{E_t}{N_t}\right) \times h_t, \tag{2.3}$$

where N_t is working-age population.[2] This formula shows that, in the long run where the capital-output ratio ($\frac{K_t}{Y_t}$), the employment rate ($\frac{E_t}{N_t}$), and hours worked per employed person (h_t) are constant, the trend in GNP per worker ($\frac{Y_t}{N_t}$) is given by $TFP_t^{\frac{1}{1-\theta}}$.

For present purposes, it is more convenient to work with the detrended version of the formula. Dividing both sides of (2.3) by $TREND_t$, we obtain

$$\frac{\frac{Y_t}{N_t}}{TREND_t} = \left(\frac{TFP_t^{\frac{1}{1-\theta}}}{TREND_t}\right) \times \left(\frac{K_t}{Y_t}\right)^{\frac{\theta}{1-\theta}} \times \left(\frac{E_t}{N_t}\right) \times h_t, \tag{2.4}$$

The left-side, GNP per worker relative to the trend, has been shown in Figure 1. Figure 2 plots the detrended TFP factor ($\frac{TFP_t^{\frac{1}{1-\theta}}}{TREND_t}$) with $\theta = 1/3$ (for now, ignore the series labelled "without barrier" in the figure).[3] Table 1 reports the average annual growth rate of detrended per-worker GNP and its four factors shown in (2.4) for prewar and postwar Japan.[4] For the high-growth period of 1960-73, despite a 1.4% (=-0.8%-0.6%) decline in the average hours worked per worker, a high per-worker GNP growth rate of 6.9% (or 4.9% when detrended) is brought about by a high TFP growth of 7.4% and, less importantly, by capital deepening (a 0.8% growth in $\left(\frac{K_t}{Y_t}\right)^{\frac{\theta}{1-\theta}}$). For the prewar period, the TFP growth rate is much lower and, surprisingly, there was no capital deepening: between 1885 and 1940, the capital-output ratio actually declined.

The prewar figures in Table 1 show that the factors on the right hand side of (2.4), including the detrended TFP factor, are more or less stationary. It is therefore meaningful to take the averages of their levels over the whole prewar period and compare the averages for 1985-88, the period during which the Japanese economy was in a steady state (as argued in Hayashi and Prescott (2002)). The ratio of the two averages for each factor as well as for detrended GNP per worker is shown in Table 2. Thus a ratio that is greater than unity means that the factor's value for 1985-88 is greater than its prewar average. It is clear from the table that the huge difference in the standard of living represented by the ratio of 2.13 for detrended per-worker GNP is mainly due to the difference in productivity.

To summarize, Japan's prewar stagnation can be accounted for by the low level of overall TFP and absence of capital deepening.

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[2] This formula has been adopted by King and Levine (1994), Hayashi and Prescott (2002) and others.

[3] We assume that farmers during the off season work outside agriculture and those off-season working hours are not recorded in the data. Hours worked here includes our estimate of those unrecorded hours. See footnote 5 for more details.

[4] Here the initial year for the postwar period is 1960 because the capital stock data for the early 1950s seems unreliable. The average annual rate of change of the capital intensity factor $\left(\frac{K_t}{Y_t}\right)^{\frac{\theta}{1-\theta}}$ for 1950-1973 is -0.5%.

Table 1: Growth Accounting

	Average Annual Growth Rate (in percents) of				
	$\frac{\frac{Y_t}{N_t}}{TREND_t}$	$\frac{TFP_t^{\frac{1}{1-\theta}}}{TREND_t}$	$\left(\frac{K_t}{Y_t}\right)^{\frac{\theta}{1-\theta}}$	$\frac{E_t}{N_t}$	h_t
1885-1940	$2.1\% - 1.8\% = 0.3\%$	$2.5\% - 1.8\% = 0.7\%$	-0.3%	-0.4%	0.3%
1960-1973	$6.9\% - 2.0\% = 4.9\%$	$7.4\% - 2.0\% = 5.4\%$	0.8%	-0.8%	-0.6%

Note: Y_t = GNP, K_t = capital stock, E_t = employment, N_t = working-age population, h_t = average hours per employed person. See (2.2) for the definition of TFP_t. $\theta = 1/3$. The growth rate of $TREND_t$ is 1.8% for 1885-1940 and 2.0% for the postwar period (see (2.1)). The average annual growth rate of a variable X taking a value of X_1 in year 1 and X_T in year T is calculated as $\frac{1}{T-1}\ln(X_T/X_1)$. Therefore, the sum of the growth rates of the factors add up to the growth rate of detrended GNP per worker.

Table 2: Level Accounting, 1885-1940 vs. 1985-88

	$\frac{\frac{Y_t}{N_t}}{TREND_t}$	$\frac{TFP_t^{\frac{1}{1-\theta}}}{TREND_t}$	$\left(\frac{K_t}{Y_t}\right)^{\frac{\theta}{1-\theta}}$	$\frac{E_t}{N_t}$	h_t
$\frac{\text{average over 1985-1988}}{\text{average over 1885-1940}}$	2.13	2.72	1.06	0.94	0.78

Note: See footnote to Table 1 for definition of symbols. By definition, the product of the ratios for the four factors equals the ratio for detrended GNP per worker $\frac{\frac{Y_t}{N_t}}{TREND_t}$.

3. The Basic Idea and Summary of Results

The Sectoral Misallocation Hypothesis

The thesis of this paper is that the labor barrier was an impediment to the economy's overall production efficiency as measured by the TFP. We were led to this thesis by the following observations. Figure 3 shows that employment in agriculture (here and elsewhere excluding forestry and fishery) was essentially constant at 14 million persons in prewar Japan. The figure also shows that, in sharp contrast to the prewar era, postwar Japan witnessed a steep decline in agricultural employment. As the labor force expands, a constant level of employment means a slowly declining employment share. This is shown in Figure 4, where the employment share of agriculture declined only gradually before the war and very sharply postwar. It took Japan 50 years to reduce the agricultural employment share from 60% to 40%. In most other developing and developed countries, the decline is faster.

We hypothesize that there was a barrier in prewar Japan that prevented people from moving out of agriculture. Thanks to this barrier, there was too much labor tied up in the decreasing-returns-to-scale technology called agriculture. For reasons to be speculated on in Section 8, this barrier ceased to operate after World War II. The sectoral misallocation of labor became much less onerous, which must have contributed at least in part to the rapid increase in the overall TFP in postwar Japan. Hansen and Prescott (1999) described the industrial revolution as a switch from a decreasing-returns-to-scale technology (the Malthus technology) to a constant-returns-to-scale technology (the Solow technology). Our hypothesis can be rephrased as saying that the transition to Malthus to Solow was inhibited by the barrier to labor mobility.

Main Results

The rest of this paper is to formalize our sectoral misallocation hypothesis in a two-sector growth model with agriculture and see to what extent the model can account for the prewar stagnation characterized by the low overall TFP and the lack of capital deepening. The paper's main results are the following.

- Our two-sector model can account for the prewar stagnation. The solution path of the model, which takes the actual sectoral TFPs and the labor barrier as given but treats capital accumulation as well as sectoral allocation of capital and labor as endogenous, tracks the prewar data closely. This is shown for GNP in Figure 5: the solution path represented by the dotted line does not depart substantially from the actual except toward the end of the prewar period.

- This sanguine result, however, is due partly to the fact that the model takes the sectoral TFPs as given. Those sectoral TFPs, one for agriculture and the other for the rest of the economy, here deflated by the common trend function $(TREND_t)^{1-\theta}$ (where $TREND_t$ is defined in (2.1)), are shown in Figure 6, which indicates that the non-agricultural TFP rose sharply precisely when the overall TFP did in the postwar period. It follows that our sectoral misallocation hypothesis alone would not be able to explain why TFP in prewar Japan is not as high as its 1985-88 level after detrending. The solution path of the model that does *not* impose the labor barrier but still takes the sectoral TFPs as given will be referred to as the counter-factual simulation. The overall detrended TFP calculated from the counter-factual simulation is shown in Figure 2 by the line labelled "without barrier". The removal of the labor barrier, which improves sectoral allocation of capital and labor, does raise the overall TFP substantially. But it is not enough to lift it to the 1985-88 level.

- The substantially higher overall TFP is not the only feature of the counter-factual simulation. Going back to Figure 5, the uppermost line is the GNP per worker implied by the counter-factual simulation. Japan's GNP per worker, which (as seen in Figure 1) remained about 40% of the U.S. level, would have been far higher, about 50% to 60% of the U.S. level were it not for the barrier. For example, for 1940, the model predicts that per-worker GNP would have been higher than actual by 41%. The improvement in overall TFP is not the only contributing factor. Table 3 reports the level accounting comparing the counter-factual simulation and the actual economy for 1940 (one gets similar results for other prewar years except for the earliest years). It shows that the 41% output gain comes from three sources: the improvement in overall efficiency (already shown in Figure 2), a capital deepening resulting in a higher capital-output ratio, and an 10% increase in average hours worked h_t. This last factor comes about because the employment share of agriculture (where hours worked are lower) is much lower in the simulated economy.[5] The higher capital-output ratio in the face of the 51% higher output for 1940 implies that there would have been a huge investment boom in the prewar period were it not for the labor barrier. That this is so in the simulated economy is shown in Figure 7 which plots $K_t/(h_tE_t)$.

Table 3: Level Accounting for 1940, actual vs. counter-factual

	$\frac{\frac{Y_t}{N_t}}{TREND_t}$	$\frac{TFP_t^{\frac{1}{1-\theta}}}{TREND_t}$	$\left(\frac{K_t}{Y_t}\right)^{\frac{\theta}{1-\theta}}$	$\frac{E_t}{N_t}$	h_t
$\frac{\text{value for 1940 without barrier}}{\text{its actual value for 1940}}$	1.41	1.13	1.13	1	1.10

Note: See footnote to Table 1 for definition of symbols. By definition, the product of the ratios for the four factors equals the ratio for detrended GNP per worker $\frac{\frac{Y_t}{N_t}}{TREND_t}$. Because the counter-factual simulation uses actual E_t and N_t, the ratio for $\frac{E_t}{N_t}$ is unity by construction.

4. The Two-Sector Model

In this section we present the two-sector model with agriculture. For expositional ease, we present a model without intermediate inputs, so that output and value added can be equated. The version of the model we actually solve for the solution path allows sector 1 (agriculture) to use the sector 2 good as an intermediate input. Appendix 3 explains how the intermediate input can be incorporated into the model presented in the text.

[5] Average hours worked in 1940 is 160 hours per month in agricuture and 276 hours in non-agriculture. In all the calculation in the paper, it is assumed that a person employed in agriculture spends 10% of his time in non-agriculture and this extra hours are not recorded in data. Therefore, for 1940 for example, a farmer works 27.6 hours outside agriculture during the off-season, in addition to 160 hours tilling his plot. To state this more formally, let E_{jt} and h_{jt} be employment and hours worked in sector j ($j = 1, 2$), with sector 1 being agriculture. If a farmer spends fraction ξ of his time in sector 2, then aggregate hours worked is $h_{1t}E_{1t} + \xi h_{2t}E_{1t} + h_{2t}E_{2t}$. Thus average hours worked per employed person, h_t, equals $(h_{1t} + \xi h_{2t})s_{Et} + h_{2t}(1 - s_{Et})$, where $s_{Et} \equiv E_{1t}/(E_{1t} + E_{2t})$ is agriculture's employment share. If $h_{1t} + \xi h_{2t} < h_{2t}$, then h_t increases as agriculture's employment share (s_{Et}) declines.

Households

There is a stand-in household with N_t working-age members at date t. The size of the household evolves over time exogenously. The stand-in household's utility function is

$$\sum_{t=0}^{\infty} \beta^t N_t u(c_{1t}, c_{2t}), \tag{4.1}$$

where c_{jt} is per-member consumption of good j ($j = 1, 2$).

Measure E_t of the household work. The household takes total employment E_t as given and decides how it is divided between employment in sector 1 (E_{1t}) and in sector 2 (E_{2t}) (subject to the labor barrier to be introduced shortly). If employed in sector 2, the member works for h_{2t} hours per unit period. On the other hand, if employed in sector 1 (agriculture), the member works not only h_{1t} hours in sector 1 but also ξh_{2t} hours in sector 2. This extra work outside sector 1 is to allow for the fact that during winter many farmers temporarily leave the farm to find work in manufacturing and services (a fact often referred to as *dekasegi* in Japanese). Hours worked (h_{1t}, h_{2t}) are exogenously given to the household. If w_{jt} is the wage rate in sector j and $s_{Et} \equiv E_{1t}/E_t$ is the fraction of employment in sector 1, the household's total labor income is

$$(w_{1t}h_{1t} + \xi w_{2t}h_{2t})E_{1t} + w_{2t}h_{2t}E_{2t} = \left\{w_{2t}h_{2t} + [w_{1t}h_{1t} - (1-\xi)w_{2t}h_{2t}]s_{Et}\right\}E_t. \tag{4.2}$$

There is a barrier to labor mobility requiring employment in sector 1 to be at least $\bar{E}_{1t}$:

$$\text{(the labor barrier)} \qquad E_{1t} \geq \bar{E}_{1t} \ \text{ i.e., } \ E_{1t}/E_t \equiv s_{Et} \geq \bar{s}_{Et} \equiv \bar{E}_{1t}/E_t. \tag{4.3}$$

Looking at the expression (4.2) for labor income, we can easily see that the household would set $s_{Et} \equiv E_{1t}/E_t$ to the maximum possible value of unity if the income differential $w_{1t}h_{1t} - (1-\xi)w_{2t}h_{2t}$ is positive, to the minimum possible value of $\bar{s}_{Et}$ if it is negative, and any value between the minimum and the maximum if there is no income differential. Thus the household's choice of s_{Et} is the following correspondence (set-valued function):

$$s_{Et} = \begin{cases} \bar{s}_{Et} \ \text{ if } \frac{w_{1t}h_{1t}}{w_{2t}h_{2t}} < 1-\xi, \\ 1 \ \text{ if } \frac{w_{1t}h_{1t}}{w_{2t}h_{2t}} > 1-\xi, \\ [\bar{s}_{Et}, 1] \ \text{ if } \frac{w_{1t}h_{1t}}{w_{2t}h_{2t}} = 1-\xi. \end{cases} \tag{4.4}$$

There are two other sources of income for the household. First, if $N_t k_t$ is the capital stock owned by the household (so k_t is the capital stock per worker), its rental income is $r_t N_t k_t$. Unlike labor, we assume no barrier to capital mobility between sectors, so the rental rate r_t does not depend on which sector capital is rented out to.[6] Second, there is a rent earned from land, which is an input to sector 1's production. The period-budget constraint for the household, then, is

$$\begin{aligned} & q_t N_t c_{1t} + N_t c_{2t} + N_{t+1} k_{t+1} - (1-\delta) N_t k_t \\ & \qquad = \left\{ w_{2t} h_{2t} + [w_{1t} h_{1t} - (1-\xi) w_{2t} h_{2t}] s_{Et} \right\} E_t + r_t N_t k_t - \tau (r_t - \delta) N_t k_t - \pi_t, \end{aligned} \tag{4.5}$$

where δ is the depreciation rate, q_t is the relative price of good 1 in terms of good 2, τ is the tax rate on net capital income, and π_t is taxes other than the capital income tax less land rent. The second good is the numeraire. So, for example, w_{1t} is the sector 1 wage rate in terms of good 2. Since hours worked as well as total employment E_t are exogenous, the tax on labor income is not distortionary and is included in π_t.

With s_{Et} $(\equiv E_{1t}/E_t)$ determined according to (4.4) for each t, the stand-in household chooses a sequence $\{c_{1t}, c_{2t}, k_{t+1}\}_{t=0}^{\infty}$ so as to maximize its utility (4.1) subject to the sequence of period-budget constraints (4.5) for $t = 0, 1, 2, ...$ If $\beta^t \lambda_t^{-1}$ is the Lagrange multiplier for the period t budget constraint (i.e., if λ_t is the ratio of β^t to the Lagrange multiplier), the first-order conditions with respect to $(c_{1t}, c_{2t}, k_{t+1})$ are given by

$$\frac{\partial u(c_{1t}, c_{2t})}{\partial c_{1t}} = \frac{q_t}{\lambda_t}, \tag{4.6}$$

$$\frac{\partial u(c_{1t}, c_{2t})}{\partial c_{2t}} = \frac{1}{\lambda_t}, \tag{4.7}$$

$$\lambda_{t+1} = \beta \lambda_t [1 + (1-\tau)(r_{t+1} - \delta)]. \tag{4.8}$$

Since λ_t is the reciprocal of the Lagrange multiplier for the budget constraint, it measures how wealthy the consumer is. The first-order conditions for consumption, (4.6) and (4.7), can be solved for consumption as

$$c_{1t} = c_1(q_t, \lambda_t) \ \text{ and } \ c_{2t} = c_2(q_t, \lambda_t). \tag{4.9}$$

Finally, the the transversality condition is

$$\lim_{t \to \infty} \frac{\beta^t \lambda_t k_t}{R_1 \cdot R_2 \cdots R_t} = 0 \ \text{ where } \ R_t \equiv 1 + (1-\tau)(r_t - \delta). \tag{4.10}$$

Firms

The production function for sector 1 is

$$Y_{1t} = TFP_{1t}\, K_{1t}^{\theta_1}\, L_{1t}^{\eta}, \tag{4.11}$$

where TFP_{1t} is the total factor productivity, K_{1t} is capital input (demand for capital services), and L_{1t} is labor input (total hours worked demanded) in sector 1. Land is the third input, but since it is constant, its

[6] The rental rate is net of intermediation costs. See below on firms in sector 2.

contribution is submerged in the TFP. Because of the existence of the fixed factor of production, we have a decreasing returns to scale in capital and labor: $\theta_1 + \eta < 1$. The first-order conditions, which equate marginal productivities to factor prices, for firms in sector 1 are

$$r_t = \theta_1 \, q_t \; TFP_{1t} \, K_{1t}^{\theta_1 - 1} \, L_{1t}^{\eta}, \tag{4.12}$$

$$w_{1t} = \eta \, q_t \; TFP_{1t} \, K_{1t}^{\theta_1} \, L_{1t}^{\eta - 1}. \tag{4.13}$$

Production in sector 2 does not require land and exhibits constant returns to scale:

$$Y_{2t} = \; TFP_{2t} \, K_{2t}^{\theta_2} \, L_{2t}^{1-\theta_2}. \tag{4.14}$$

Unlike in sector 1, capital input in sector 2 involves costly financial intermediation. That is, if the household wishes to rent machines to sector 2, those machines need to be deposited at a bank. The bank then rents out those machines to firms in sector 2. This financial intermediation is costly because the bank incurs a cost of ϕ per machine for this intermediation service. This means that the rental rate faced by firms in sector 2 is $r_t + \phi$, while the rental rate for the household net of the intermediation cost is r_t (as assumed in the household budget constraint (4.5)). Therefore, the first-order conditions for sector 2 is

$$r_t + \phi = \theta_2 \; TFP_{2t} \, \left(\frac{K_{2t}}{L_{2t}} \right)^{\theta_2 - 1}, \tag{4.15}$$

$$w_{2t} = (1 - \theta_2) \; TFP_{2t} \, \left(\frac{K_{2t}}{L_{2t}} \right)^{\theta_2}. \tag{4.16}$$

The reason we need to allow for the intermediation cost for sector 2 is that the rate return from capital (net of depreciation) for sector 2 is substantially higher than the interest rate on government bonds. This issue will be discussed further in Section 6 on calibration.

Market Equilibrium

The second good can be either consumed or invested. We also assume that government purchases G_t is on the second good. Thus the market equilibrium conditions are (with lower case letters k_{1t} and k_{2t} denoting per-worker quantities)

$$\text{(good 1)} \quad N_t c_{1t} = Y_{1t}, \tag{4.17}$$

$$\text{(good 2)} \quad N_t c_{2t} + (N_{t+1} k_{t+1} - (1-\delta) N_t k_t) + G_t + \phi N_t k_{2t} = Y_{2t}, \tag{4.18}$$

$$\text{(capital services)} \quad k_{1t} + k_{2t} = k_t, \tag{4.19}$$

$$\text{(labor in sector 1)} \quad L_{1t} = h_{1t} E_{1t}, \tag{4.20}$$

$$\text{(labor in sector 2)} \quad L_{2t} = h_{2t} E_{2t} + \xi h_{2t} E_{1t}, \tag{4.21}$$

where $k_{1t} \equiv K_{1t}/N_t$ and $k_{2t} \equiv K_{2t}/N_t$. The right side of (4.18) is output of sector 2 net of the cost of financial intermediation.

A *competitive equilibrium* given the initial capital stock k_0 and the sequence of exogenous variables $\{G_t,\, E_t,\, h_{1t},\, h_{2t},\, TFP_{1t},\, TFP_{2t}\}_{t=0}^{\infty}$ is a sequence of prices and quantities, $\{\lambda_t,\, q_t,\, w_{1t}, w_{2t},\, r_t,\, k_{t+1},$ $k_{1t}, k_{2t},\, s_{Et},\, L_{1t}, L_{2t}\}_{t=0}^{\infty}$, satisfying the following conditions:

(i) the household's first-order conditions (4.4), (4.8), and (4.9), and the transversality condition (4.10),

(ii) the firms' first-order conditions (4.12), (4.13), (4.15), and (4.16),

(iii) the market-clearing conditions (4.17)-(4.21).

Three remarks are in order.

- We are assuming that the first good is non-tradable. We also studied the model in which agricultural goods are tradable, with qualitatively similar results. The open-economy model is presented in Appendix 4.

- As in Hayashi and Prescott (2002), we treat claims on foreigners as part of the capital stock, so investment here $(N_{t+1}k_{t+1} - (1-\delta)N_t k_t)$ is the sum of domestic investment and the current account, and $q_t Y_{1t} + Y_{2t}$ is GNP (in terms of good 2), not GDP.

- As is standard in the real business-cycle models with non-distortionary taxes, the sequence of taxes is not included the equilibrium conditions, because the amount of a lump-sum tax is endogenously determined so that the government budget constraint holds period-by-period. By the Ricardian equivalence, any other sequence of taxes with the same present value results in the same competitive equilibrium. This also means that the household budget constraint need not be included as part of the equilibrium conditions because it is implied by the market-clearing conditions, the government budget constraint, and the factor exhaustion condition (that payments to factors of production, including land, sum to output).

Reducing Equilibrium Conditions into a Two-Equation Detrended Dynamical System

Let s_{Kt} be the capital share of sector 1 and ψ_t be the government's share of sector 2 output:

$$s_{Kt} \equiv \frac{k_{1t}}{k_{1t}+k_{2t}}, \;\; \psi_t \equiv \frac{G_t}{Y_{2t}}. \tag{4.22}$$

Also define:

$$\begin{gathered} X_{Yt} \equiv (TFP_{2t})^{\frac{1}{1-\theta_2}}\, h_{2t}E_t/N_t, \;\; X_{Qt} \equiv TFP_{1t}^{-1}\,(h_{1t}E_t)^{-\eta}\, TFP_{2t}^{\frac{1-\theta_1}{1-\theta_2}}\,(h_{2t}E_t)^{1-\theta_1}, \\ \tilde{k}_t \equiv \frac{k_t}{X_{Yt}}, \;\; \tilde{\lambda}_t \equiv \frac{\lambda_t}{X_{Yt}}, \; \tilde{q}_t \equiv \frac{q_t}{X_{Qt}}. \end{gathered} \tag{4.23}$$

To anticipate the discussion in the next section, X_{Yt} will be the trend for both k_t and λ_t, while q_t's trend is X_{Qt}.

It is shown in Appendix 2 (and in Appendix 3 for the case with an intermediate input) that the above equilibrium conditions (i)-(iii) imply the following two nonlinear difference equations:

$$\text{(resource constraint)} \qquad \tilde{k}_{t+1} = \frac{N_t}{N_{t+1}} \frac{X_{Yt}}{X_{Y,t+1}} \left[[1-\delta-(1-s_{Kt})\phi]\tilde{k}_t + (1-\psi_t)\,\tilde{y}_{2t} - \frac{c_2(\tilde{q}_t X_{Qt}, \tilde{\lambda}_t X_{Yt})}{X_{Yt}} \right], \tag{4.24}$$

$$\text{(Euler equation)} \qquad \tilde{\lambda}_{t+1} = \frac{X_{Yt}}{X_{Y,t+1}} \beta\, \tilde{\lambda}_t \left\{ 1 + (1-\tau) \left[\theta_2 \, \frac{\tilde{y}_{2,t+1}}{(1-s_{k,t+1})\tilde{k}_{t+1}} - \phi - \delta \right] \right\}, \tag{4.25}$$

where

$$\tilde{y}_{2t} \equiv \tilde{k}_t^{\theta_2}\,(1-s_{Kt})^{\theta_2}\,[1-(1-\xi)s_{Et}]^{1-\theta_2}. \tag{4.26}$$

This is a dynamical system in two variables, the detrended capital stock $\tilde{k}_t$ and the detrended shadow price $\tilde{\lambda}_t$, because the other endogenous variables appearing in the system, $(s_{Kt}, s_{Et}, \tilde{q}_t)$, are functions of the two states.

Those functions relating $(\tilde{k}_t, \tilde{\lambda}_t)$ to $(s_{Kt}, s_{Et}, \tilde{q}_t)$ can be obtained as follows (see Appendix 2 for more details). The market equilibrium condition for good 1 (4.17) and the equality of the marginal products of capital between two sectors (implied by (4.12) and (4.15)) can be written as

$$\text{(good 1)} \qquad \frac{c_1(\tilde{q}_t X_{Qt}, \tilde{\lambda}_t X_{Yt})}{X_{Yt}/X_{Qt}} = \tilde{y}_{1t}, \tag{4.27}$$

$$\text{(equqlity of marginal products of capital)} \quad \theta_1 \frac{\tilde{q}_t \tilde{y}_{1t}}{s_{Kt} k_t} = \theta_2 \frac{\tilde{y}_{2t}}{(1-s_{Kt})k_t} - \phi, \tag{4.28}$$

where

$$\tilde{y}_{1t} \equiv \tilde{k}_t^{\theta_1}\, s_{Kt}^{\theta_1}\, s_{Et}^{\eta}. \tag{4.29}$$

Furthermore, when $\bar{s}_{Et}$ is low enough so that the labor barrier is not binding, we have $w_{1t}h_{1t} + \xi w_{2t}h_{2t} = w_{2t}h_{2t}$ or $\frac{w_{1t}h_{1t}}{w_{2t}h_{2t}} = 1-\xi$, which can be reduced to

$$\text{(equality of sectoral incomes)} \qquad \frac{\eta \, \frac{\tilde{q}_t \tilde{y}_{1t}}{s_{Et}}}{(1-\theta_2)\frac{\tilde{y}_{2t}}{1-(1-\xi)s_{Et}}} = 1-\xi. \tag{4.30}$$

For each period t, given $(\tilde{k}_t, \tilde{\lambda}_t)$, we can solve (4.27), (4.28), and (4.30) for $(s_{Kt}, s_{Et}, \tilde{q}_t)$. If the s_{Et} thus obtained does not satisfy the labor barrier $s_{Et} \geq \bar{s}_{E1t}$, then we set $s_{Et} = \bar{s}_{Et}$ and use (4.27) and (4.28) to solve for $(s_{Kt}, \tilde{q}_t)$.

By way of summarizing this subsection, let $\mathbf{x}_t \equiv (\tilde{k}_t, \tilde{\lambda}_t)$ and $\mathbf{y}_t \equiv (s_{Kt}, s_{Et}, \tilde{q}_t)$, and write the two-equation dynamical system (4.24) and (4.25) as

$$\mathbf{x}_{t+1} = \mathbf{f}_t(\mathbf{x}_t, \mathbf{y}_t),\ \mathbf{y}_t = \mathbf{g}_t(\mathbf{x}_t). \tag{4.31}$$

Here, $\mathbf{g}$ is the function described in the previous paragraph that determines $\mathbf{y}_t$ subject to the labor barrier. In standard one-sector real business cycle models, the relevant dynamical system governing the capital stock and the shadow price can be made autonomous upon suitable detrending, under the assumption that the exogenous variables (or the growth rates thereof) settle down to constants in the long run. Imposing an appropriate transversality condition is then accomplished by locating the stable saddle path of the autonomous sytem. In contrast, as will be verified in the next section, our two-sector model remain non-autonomous even after suitable detrending, because Engel's law and the time-varying nature of the labor barrier render the $\mathbf{f}$ and $\mathbf{g}$ functions non-stationary (i.e., time-varying, with t subscript). How to locate the stable saddle path for the present *non*-autnomous dynamical system is the subject of the next section.

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5. Existence of An Asymptotic Steady State

We assume that the share of government purchases and hours worked as well as the growth rates of the trending exogenous variables eventually become constant:

$$\begin{gathered}\text{for sufficiently large } t,\ \psi_t = \psi \in (0,1),\ h_{1t} = h_1,\ h_{2t} = h_2,\\ \frac{E_{t+1}}{E_t} = \frac{N_{t+1}}{N_t} = n,\ \frac{TFP_{1,t+1}}{TFP_{1t}} = g_1,\ \frac{TFP_{2,t+1}}{TFP_{2t}} = g_2.\end{gathered} \tag{5.1}$$

We will assume for those constant growth rates that

$$g_2 > 1,\ g_1\, g_2^{\frac{\theta_1}{1-\theta_2}}\, n^{\theta_1+\eta-1} > 1. \tag{5.2}$$

As clear from the definition of the trends (4.23), the first condition guarantees X_{Yt}, which is the trend for sector 2's output per worker (Y_{2t}/N_t), to grow in the long run, while under the latter condition X_{Yt}/X_{Qt} grows in the long run, which is needed for the dynamical system to have a stable saddle path under the Stone-Geary utility function to be specified below in (5.5). These conditions are satisfied by the calibrated parameter values of the next section.

Unlike in standard real business cycle models, an assumption like (5.1) is not enough to render the detrended dynamical system (4.31) autonomous in the long run for two reasons. First, since X_{Yt} and X_{Qt}

are not constant, neither $\frac{c_2(\tilde{q}_t X_{Qt}, \tilde{\lambda}_t X_{Yt})}{X_{Yt}}$ in (4.24) nor $\frac{c_1(\tilde{q}_t X_{Qt}, \tilde{\lambda}_t X_{Yt})}{X_{Yt}/X_{Qt}}$ in (4.27) is a stationary (i.e., time-invariant) function of $(\tilde{k}_t, \tilde{\lambda}_t)$. Second, even if $\frac{c_1(\tilde{q}_t X_{Qt}, \tilde{\lambda}_t X_{Yt})}{X_{Yt}/X_{Qt}}$ in (4.27) is stationary, the $\mathbf{g}$ function is still a non-stationary function of $\mathbf{x}_t = (\tilde{k}_t, \tilde{\lambda}_t)$ when the labor barrier is binding with the time-varying lower bound $\bar{s}_{Et}$.

To understand the long-run properties of our dynamical system (4.31), we *temporarily* assume that $\frac{c_1(\tilde{q}\, X_{Qt}, \tilde{\lambda}\, X_{Yt})}{X_{Yt}/X_{Qt}}$ and $\frac{c_2(\tilde{q}\, X_{Qt}, \tilde{\lambda}\, X_{Yt})}{X_{Yt}}$ are time-invariant function of $(\tilde{k}, \tilde{\lambda})$ when X_{Yt} and X_{Yt}/X_{Qt} grow at constant rates. Clearly, the only demand system having this property is[7]

$$c_1(\tilde{q}\, X_{Qt}, \tilde{\lambda}\, X_{Yt}) = \mu_1 \frac{\tilde{\lambda} X_{Yt}}{\tilde{q} X_{Qt}} \quad \text{or} \quad \frac{c_1(\tilde{q}\, X_{Qt}, \tilde{\lambda}\, X_{Yt})}{X_{Yt}/X_{Qt}} = \mu_1 \frac{\tilde{\lambda}}{\tilde{q}}, \tag{5.3}$$

$$c_2(\tilde{q}\, X_{Qt}, \tilde{\lambda}\, X_{Yt}) = \mu_2 \tilde{\lambda} X_{Yt} \quad \text{or} \quad \frac{c_2(\tilde{q}\, X_{Qt}, \tilde{\lambda}\, X_{Yt})}{X_{Yt}} = \mu_2 \tilde{\lambda}. \tag{5.4}$$

The utility function that generates this demand system is linear logarithmic:

$$u(c_1, c_2) = \mu_1 \log(c_1) + \mu_2 \log(c_2).$$

We can now describe how to find the solution to the dynamical system under (5.3) and (5.4).

(i) There is a set over which the dynamical system is autonomous. As explained in the previous section, the labor barrier does not bind if and only if the s_{Et} that solves (4.27), (4.28), and (4.30) is greater than $\bar{s}_{Et}$. Let $\mathcal{A}_t$ be the set of $(\tilde{k}, \tilde{\lambda})$ such that the labor barrier does not bind. Under (5.3), none of these three equations involves the trends, so the $\mathbf{g}$ function is stationary over $\mathcal{A}_t$. Furthermore, under (5.4), the $\mathbf{f}$ function is stationary because (4.24) no longer involves the time trends. So the dynamical system is autonomous over $\mathcal{A}_t$.

(ii) Since $\bar{s}_{Et}$ declines with time, this set $\mathcal{A}_t$ expands with time. Let $(\tilde{k}_{ss}, \tilde{\lambda}_{ss}, s_{K,ss}, s_{E,ss}, \tilde{q}_{ss})$ be the steady state for this autonomous dynamical system, which is obtained by dropping the time subscript from $(s_{Kt}, s_{Et}, \tilde{q}_t, \tilde{k}_t, \tilde{\lambda}_t)$ in (4.24)-(4.30). The Inada condition ensures that $s_{E,ss} > 0$ (agriculture does not disappear in the long run) so that, with $\bar{s}_{Et}$ approaching 0 as $t \to \infty$, $(\tilde{k}_{ss}, \tilde{\lambda}_{ss})$ is an interior point of $\mathcal{A}_t$ for sufficiently large t.

(iii) It is verified numerically for the calibrated parameter values (see Appendix 3) that the eigenvalues of the two-dimensional linearlized system at the steady state $(\tilde{k}_{ss}, \tilde{\lambda}_{ss})$ consist of one that is greater than unity and the other that is less than unity. Therefore, the steady state is a saddle point for the autonomous system defined over $\mathcal{A}_t$. So for sufficiently large t, the solution of the dynamical system

[7] This is a special case of Theorem 3.6.4 of Eichhorn (1978).

is on the stable saddle path converging to the steady state $(\tilde{k}_{ss}, \tilde{\lambda}_{ss})$. The intermediate path leading to the saddle path from a given initial capital stock $\tilde{k}_0$ can then be determined as follows: pick a sufficiently large T so that the system is on the stable saddle path, and then find $\tilde{\lambda}_0$ such that $(\tilde{k}_T, \tilde{\lambda}_T)$ eminating from $(\tilde{k}_0, \tilde{\lambda}_0)$ is on the stable saddle path (which is a one-dimensional manifold).

It should be noted in passing that the dynamical system starting from $(\tilde{k}_{ss}, \tilde{\lambda}_{ss})$ would not stay there because at $t = 0$ the labor barrier may be binding (that is, $(\tilde{k}_{ss}, \tilde{\lambda}_{ss})$ may not be in $\mathcal{A}_0$). Therefore, $(\tilde{k}_{ss}, \tilde{\lambda}_{ss})$ should be called an *asymptotic* steady state.

The problem with the linear-logarithmic utility function is that the share of food expenditure is constant at μ_1. To accomodate Engel's law, we introduce minimum consumption for food:

$$u(c_1, c_2) = \mu_1 \log(c_1 - d_1) + \mu_2 \log(c_2),\ d_1 > 0. \tag{5.5}$$

Now the food demand is given not by (5.3) but by

$$c_1(\tilde{q}\, X_{Qt}, \tilde{\lambda}\, X_{Yt}) = d_1 + \mu_1 \frac{\tilde{\lambda} X_{Yt}}{\tilde{q} X_{Qt}} \quad \text{or} \quad \frac{c_1(\tilde{q}\, X_{Qt}, \tilde{\lambda}\, X_{Yt})}{X_{Yt}/X_{Qt}} = \frac{d_1}{X_{Yt}/X_{Qt}} + \mu_1 \frac{\tilde{\lambda}}{\tilde{q}}. \tag{5.6}$$

Alhough the $\mathbf{f}$ function remains stationary, the $\mathbf{g}$ function is no longer stationary over $\mathcal{A}_t$ thanks to the detrended minimum consumption $\frac{d_1}{X_{Yt}/X_{Qt}}$, so nowhere in the $(\tilde{k}, \tilde{\lambda})$ plane is the dynamical system (4.31) autonomous. However, this two-dimensional non-autonomous system can be converted into a three-dimensional autonomous system.[8] Define

$$z_t \equiv X_{Qt}/X_{Yt}. \tag{5.7}$$

By (5.2), z_t follows a first-order difference equation

$$z_{t+1} = \frac{1}{g_1\, g_2^{\frac{\theta_1}{1-\theta_2}}\, \eta^{\theta_1+\eta-1}}\, z_t. \tag{5.8}$$

Add this equation to the two-equation dynamical system to form a three-equation dynamical system in $(\tilde{k}_t, \tilde{\lambda}_t, z_t)$. Clearly, this augmented system is autonomous (because $(s_{Kt}, s_{Et}, \tilde{q}_t)$ is a stationary function of $(\tilde{k}_t, \tilde{\lambda}_t, z_t)$) and the steady state is given by $(\tilde{k}, \tilde{\lambda}, 0)$ where $(\tilde{k}, \tilde{\lambda})$ is the steady state for the two-dimensional autonomous system with $d_1 = 0$. For the calibrated parameter values, the eigenvalues for the three-dimensional linearlized system at this steady state consist of two roots that are less than one and one that is greater than one. So this three-dimensional dynamical system has a saddle.

[8]We are grateful to Lars Hansen for suggesting this idea.

6. Calibration for Prewar Japan

We have ignored intermediate inputs to production in sector 1 for expositional clarity. With good 2 used as an intermediate input in sector 1 as indicated by (A3.10) of Appendix 3, the dynamical system (4.24), (4.25), (4.27), (4.28), and (4.30) becomes (A3.12)-(A3.16). The required modifications are: a re-definition of sector 1's parameters θ_1 and η through deflation by $(1-\alpha)$ (where α is the share of the intermediate input in sector 1) and an explicit recognition of resources used up as sector 1's intermediate input in the resource constraint for good 2 (see the last term of (A3.12)).

We calibrate this model with an intermediate input as follows.

θ_1, η, α (the share parameters for capital, labor, intermediate input sector 1): These parameters are taken from Table 2-5 of Hayami (1975), which shows factor shares in agriculture since 1885. Taking the prewar averages of those shares, we obtain: $\theta_1 = 0.1$, $\eta = 0.5$, $\alpha = 0.146$ (so land rent's share is 0.254).

θ_2 (capital's income share in sector 2): There is a capital share estimate for the prewar private nonagricultural sector by Minami and Ono (1978). It rises from 39.4% for 1896 to 54.2% for 1940. Rather than letting θ change over time, we set $\theta_2 = 1/3$.

δ (depreciation rate): its calibrated value is calculated as the average of the ratio of depreciation to the capital stock in the LTES.

τ (tax rate on capital income): the average of the ratio of our estimate of taxes on capital income to the estimate of capital income from Minami and Ono (1978) is about 0.17. Presuming that the Minami-Ono capital income estimate might be overstated, we set $\tau = 0.2$.

β (discounting factor) and ϕ (proportional cost of intermediation): Under the Stone-Geary utility function, we have $c_{2t} = \mu_2 \Lambda_t$. Substituting this and (4.15) into the Euler equation (4.8), we obtain

$$\frac{c_{2,t+1}}{c_{2t}} = \beta \left[1 + (1-\tau) \left(\theta_2 \frac{Y_{2t}}{K_{2t}} - \phi - \delta \right) \right]. \tag{6.1}$$

We set β to the standard value of 0.96. We take the sample average of both sides for 1885-1940 and solve for ϕ. The sample average of $\frac{c_{2,t+1}}{c_{2t}}$ is about 1.042 while that of $1 + (1-\tau)\left(\theta_2 \frac{Y_{2t}}{K_{2t}} - \delta\right)$ (one plus the after-tax net rate of return implied by the Cobb-Douglas technology) is 1.116 with $\tau - 0.2$ and $\delta = 0.04265$. So if we used (6.1) with $\phi = 0$ to pin down β, the calibrated value of β would have been 0.9336.

d_1 (food subsistance level): We set it equal to 0.5 times the 1885 value of sector 1 output per worker. The choice of this parameter value does not change the simulation results greatly.

μ_1, μ_2 (expenditure shares): The Engel coefficient is about 0.15 in recent years for Japan. Thus we set μ_1, which is the food share in the asymptotic steady state, to 0.15. Without loss of generality, we can normalize the sum of μ_1 and μ_2 to be unity. So $\mu_2 = 0.85$.

Table 4: Calibration

parameter	calibrated value
d_1 (minimum subsistence level for good 1)	50% of sector 1 output in 1885
μ_1 (asymptotic consumption share of good 1)	0.15
μ_2 (asymptotic consumption share of good 2)	0.85
θ_1 (capital share in sector 1)	0.1
η (labor share in sector 1)	0.5
α (share of intermediate inputs in sector 1)	0.146
θ_2 (capital share in sector 2)	1/3
δ (depreciation rate)	0.04265
β (discounting factor)	0.96
τ (tax rate on capital income)	0.2
ϕ (proportional intermediation cost)	0.0389

Calibrated values are in Table 4. A point to be noticed is the low calibrated value of β. It comes about because the prewar capital-output ratio for sector 2 (and also for sector 1 for that matter) is low.

7. Findings

We now wish to use the calibrated model to answer two questions: (a) how closely does the model track historical data? (b) what would have happened had there been no labor barrier? The former question is answered by solving the model with the labor barrier in place. The latter question is answered by solving the model *without* the labor barrier, namely, by running the counter-factual simulation.

Initial Conditions and Exogenous Variables

In solving the model with or without the labor barrier, the capital stock in 1885 is taken as the initial capital stock. In both simulations, the exogenous variables are:

h_{1t}, h_{2t} (hours worked in two sectors),

TFP_{1t} and TFP_{2t},

E_t (aggregate employment),

$\bar{E}_{1t}$ (lower bound for sector 1 employment E_{1t}),

ψ_t (share of government expenditure in sector 2's output),

N_t (working-age population).

For these variables except for $\bar{E}_{1t}$, we use their actual values for the sample period (1885-1940). Regarding the lower bound $\bar{E}_{1t}$, we set it equal to the observed employment E_{1t} (so $\bar{E}_{1t} = E_{1t}$).

For periods beyond the sample period, the projected values of those exogenous variables are set as follows.

hours worked (h_1, h_2): their projected values are set equal to the values at the end of the sample period. They are 160 hours and 276 hours per month, respectively.

TFP growth rates for two sectors (g_1, g_2): their projected growth rates are set to their averages for 1885-1940 of 1.045% and 1.768%, respectively. So $g_1 = 1.01045$, $g_2 = 1.01768$.

the growth rate of aggregate employment and working-age population n: it is set to the average over 1885-1940 of the growth rate of working-age population is 1.098%. So $n = 1.01098$.

government share of sector 2 output (ψ): its value is set at 0.174, which is the sample average of ψ_t over 1885-1940.

the lower bound for sector 1 employment ($\bar{E}_{1t}$) : we set it to its 1940 value of about 14 million persons.

Therefore, we are assuming that in the prewar period agents did not anticipate the actual development of the exogenous variables in the postwar period, let alone the war.

Results

Given the initial conditions and the sequences of exogenous variables, we can solve the model and calculate the sequence of endogenous variables $(q_t, s_{Kt}, s_{Et}, k_t, \lambda_t)$. Figure 8a reports the sequence of s_{Et} (sector 1's employment share), one with the labor barrier and one without. The sequence without the labor barrier shows that a far lower fraction of the labor force would have been employed in sector 1. The red line is the employment share with the barrier. Because of the constraint setting the lower bound on sector 1 employment, it is equal to the actual share for all years of the sample period. In this simulation with the barrier, the constraint is binding for more than 100 years. Figure 8b shows the capital share for sector 1. Sector 1's capital share with the labor barrier is lower than that without the barrier. This is to be expected: too much labor in agriculture is compensated for by the level of capital stock that is lower than would have been optimal without the barrier.

Real GNP implied by the simulation in question (with or without barrier) is calculated in the same way actual real GNP is calculated from data for Figure 1. That is, let $\hat{Q}_t$ and $\hat{Y}_{jt}$ ($j = 1, 2$) be the relative price and outputs from the simulation. We construct the chain-type Fisher quantity index using ($\hat{Q}_t, \hat{Y}_{1t}, \hat{Y}_{2t}$) as described in footnote 1. For the base year of 1935, real GNP is calculated using the 1935 prices in data, so it equals the sum $\hat{Y}_{1t} + \hat{Y}_{2t}$. The overall TFP implied by the simulation in question uses the same formula (2.2).

In Section 3 we already have commented on the simulation results about GNP (in Figure 6) and the overal TFP (in Figure 2). Our main finding was that labor barrier had two depressing effects. First, it prevented the economy's factor endowments to be allocated efficiently, thus reducing the overall production efficiency measured by TFP. Second, this distortion in factor allocation was a powerful hindrance to capital accumulation. This second effect can be illustrated visually. Figure 9 is the phase diagram for the detrended two-equation dynamical system (4.24) and (4.25) (or more precisely (A3.12) and (A3.13)) with and without the labor barrier. To emphasize the overall movements, we smooth the sequence of exogenous variables by forcing them grow at constant rates from the beginning. As the figure indicates, the stable saddle path with labor barrier eventually converges to the steady-state, but only after spending many periods shedding capital. Thus the labor barrier is a powerful impediment to investment. This point is illustrated in Figure 10, where the marginal productivity schedules of capital investors in the model would have faced in date 0 (1885) with and without barrier.[9] The Figure indicates that the labor barrier reduced the gross return to

[9] The schedule is a graph of $\theta_2\, \tilde{k}_{t+1}^{\theta_2-1} \left(\frac{1-s_{K,t+1}}{1-(1-\xi)s_{E,t+1}}\right)^{\theta_2-1}$ in (4.25). It depends on $(\tilde{k}_{t+1}, \tilde{\lambda}_{t+1})$ (recall that $s_{E,t+1}$ and $s_{K,t+1}$ are functions of $(\tilde{k}_{t+1}, \tilde{\lambda}_{t+1})$. The graph in Figure 10 is drawn given $\tilde{\lambda}_{t+1}$ equals the value for period $t = 0$ (so $t + 1$ is 1886) in the simulation. The graph is not sensitive to the choice of $\tilde{\lambda}$.

capital by as much as 30%.

8. Why Did the Barrier Exist?

In this section, we explore reasons for the existence of the lower bound for agricultural employment, which we have taken for granted so far.

The Cityward Movement of the Peasant

In prewar Japan, as already emphasized, agricultural employment was virtually constant at 14 million. Related facts about prewar Japan are the following.

- The number of farm households (households whose head's main occupation is a farmer) was constant at 5.5 million[10] and the population in those farm households was constant at 30 million or about 5.5 persons per household throughout the prewar era.[11] Because there was virtually no migration from the city,[12] the constancy of the number of farm households implies that the head of the farm household was almost always succeeded by one of its children upon its death or retirement.[13]

[10]The sources are the tables in the *Annual Department of Agriculture Tabulations* and the censuses at various years.

[11]See the data appendix for the method used in the LTES (the Long-Term Economic Statistics, the macro dataset used in our study) to estimate farm population.

[12]See Takagi (1956) and Taeuber (1957, pp. 126-127, particularly nootnote 8) for census data and various other sources on the population supporting the lack of reverse migration from the city. One major non-census evidence is a classic study on the cityward movement of the peasant by Nojiri (1942). Based on voluminous data collected from field work, he concludes (see Section 1, Chapter 2 of Part 1) that the incidence of a household head (and hence the entire household) leaving the village for the city is negligible when compared to the number of non-head household members doing the same. A survey cited in Namiki (1957, Section 3) reports that in 18 villages between 1899 and 1916 there were only 21 households whose head changed occupations from agriculture. Since, as shown by these studies, there was virtually no attrition of farm households and since the number of farm households was constant, we can conclude that there was no reverse movement of individuals from the city. There was a fair amount of regional variation in the distribution of farm households, with Hokkaido (the northernmost island previously only sparsely populated) gaining at the expense the Kinki area (where Kyoto and Osaka are located). This merely implies that there was migration from one rural area to another.

[13]That the occupation as farmer is inherited from one generation to the next is quite well known in Japanese agricultural economics, but there is no official statistics on social mobility directly documenting it. There is a survey called the "SSM (Social Stratification and Social Mobility) Survey" conducted every ten years since 1955 by the Institute of Social Science of University of Tokyo, which asks the respondent about the father's occupation (among other things). Sato (1998, Appendix 2) reports that about 90% (650 in number) of 732 respondents born between 1896 and 1925 who were currently in agriculture at the times of the survey replied that their father

- Prewar fertility rate in rural areas was about 5.[14] As shown in Honda (1950), it follows by simple algebra from the constancy of the number of farm households and the relatively small household size of 5.5 that all children except for the heir and its spouse left the village.[15]

- Primogeniture was prevalent at least in rural areas. Inheritance was impartible and the entire estate went to one of the sons (usually the eldest son) who also succeeded the father's occupation as a farmer.[16]

- There was a large rural-urban disparity in household income. Ohkawa (1955, Chapter 1) was probably the first to verify this long-alleged fact on a national account basis. The LTES (the prewar macro dataset used in our study) shows that value added per employment in agriculture is only 20% to 1/3 of that in non-agriculture in the prewar era.

- The steep postwar decline in agricultural employment after 1955 (see Figure 3) was initiated by young

was in agriculture. The remaining 10% probably entered agriculture after the war. Between 1945 and 1949, when the majority of the population had difficulty getting enough to eat, agricultural employment increased from slightly below 14 million to nearly 17 million.

[14] See, e.g., the table on the gross reproduction rate by industrial type reported on p. 246 (right below Table 96) in Taeuber (1958). The prewar urban fertility rate was about 4.

[15] The infant mortality rate was such that only 4 of 5 children survive into adulthood. Assuming that a generation is 27 years, a natural increase of 4 adults occurs in about 200 thousand (= 5.5 million/27) households, and in equally numerous households a natural decrease of 2 adults (death of two old parents) occurs every year. That's a natural net annual increase of 400 thousand persons (= 80 thousand less 40 thousand). In each of those 200 thousand households, two of the four adult children succeed their deceased parents and remain in the village. Those two successors are the heir (one of the sons, usually the eldest) and a daughter who came from a different farm household as his wife. The remaining two children leave the household well before the death of their parents but not immediately after finishing primary education.

[16] As far as we know, there is no direct prewar evidence for the impartibility for farm households. The prewar Civil Code stipulates impartible inheritance to be the default mode, namely, the next head inherit the whole property if the current head died without leaving a will. In the early postwar period, out of the concern that the new Civil Code, which stipulates equal division of the estate as the default mode, would result in widespread subdivision of farmland, a quasi-government body conducted a series of surveys on the inheritance of the farmland. The earliest survey, conducted between January 1948 and August 1949 with the help of local governments, collected about 33 thousand cases of the inheritance of the farmland. Matsumura (1957) reports that one child inherited the entire farmland in 84.7% of those cases. A 1978 survey also asked 5,326 farmers over the age of 60 about their intention regarding inheritance. 79% of them said they would leave the entire farmland to the heir, according to a report by *Zenkoku Nogyo Kaigisho* (1979). Therefore, impartible inheritance was the norm even under the new Civil Code. It would have been more prevalent in the prewar era when the Civil Code made it the default mode of inheritance. Regarding the succession by the eldest son, available evidence from non-government surveys shows that it was less prevalent than might have been commonly assumed. For example, Tsuburai (1998, Table 6) finds that, in the 1965 and 1995 waves of the SSM Survey mentioned in footnote 13, about 57% of those respondents born between 1896 and 1925 who succeeded the father's occupation as a farmer (246 persons) were the eldest son. Nojiri (1942) finds that, in his field work data covering 20 villages and about ten thousand farm households, 23% of those males who left the village were eldest sons (see his Table 225).

cohorts. In his perceptive essay on postwar employment in agriculture, Namiki (1957), drawing on data from the 1955 census and other sources, noticed that the decline was concentrated in the 14-19 age bracket. His speculation, which was proved correct by time, was that the youth defecting to the city include eldest sons, who in the prewar tradition are expected to succeed their fathers' occupation as farmer.

Why Did Agricultural Employment Remain Constant?

Given the large rural-urban income disparity in prewar Japan, we are not surprised that household members not designated as heir left agriculture. The mystery is why the son designated as heir (and his wife) stayed in agriculture. An explicit calculation of farm and urban incomes relevant for the peasant contemplating on migration was given by Masui (1969). He notes that for a son and his prospective wife the comparison should be between farm labor income for both the son and his wife combined and urban labor income for the son only because the wife's employment opportunity in the city was severely limited in the prewar era. Furthermore, if the son is the heir, his farm income should also include imputed rent from the farm property he would eventually inherit. Masui's calculation for 1923-39 shows that the heir's farm income, with wife's labor income and imputed rent figured in, is *less* than urban income for 1928-37 but he dismisses this result on the ground that agricultural prices were severely depressed in those years.

Besides the issue of why peasants didn't leave the village during those depressed years, three questions can be raised about Masui's thesis. First, the heir could sell the inherited farmland and live in the city to collect the higher urban income. However, to prevent this, the father could require the son to remain on the farm until he inherits the land. By the time his son inherits the estate, it may be too late for him to start a career in the city.[17]

Second, there may not have been much to inherit for the heir to a sharecropper. According to Momose (1990, p. 149), the right to cultivate had a property value, between 10% and 30% of the value of land. Even if the cultivation right is transferable from father to son (which we don't know is the case), the property value may not have been large enough to make up for the rural-urban income disparity for tenant farmers. To the extent that it was not, it is difficult to explain on purely economic grounds why tenant farmers, comprising about 30% to 40% of all farm households,[18] did not leave agriculture.

Third, as pointed out by Namiki (1957), the land reform instituted by SCAP (the Supreme Commander

[17] This argument was suggested to me by Andrew Foster.

[18] According to the agricultural department tabulations mentioned in footnote 10. The percentage is substantially higher if those owner farmers who also cultivates someone else's land are included.

of the Allied Powers, namely Douglas McCarthur) in 1951, which transferred land ownership to tenant farmers, should have made it more attractive for previous tenant farmers to stay in agriculture in the postwar period. The postwar large-scale defection to the city took place despite the land reform.

Our explanation of the constancy of agricultural employment is an elaboration of Namiki's (1957) conjecture that the economic forces favoring migration to the city, which had been held in check by patriarchy in the prewar era, were let loose in the years following the defeat of Japan. To appreciate his logic, it is necessary to understand the nature of patriarchy that regulated the behavior of individuals in prewar Japan through the Civil Code. The distinguishing feature of the prewar Civil Code (as opposed to the new postwar Civil Code) is its recognition of the institution called the *ie* (sometimes translated "house") and its headship. The *ie* is "a 'stem family' — an organization that transcended its present members through history and spanned generations through the eldest son" (Ramseyer (1996, p. 82)).[19] Vogel (1967, pp. 92-94) describes the *ie* in the context of the cityward migration mentioned above:

> The *ie* is a patrilineal organization with rapid segmentation in each generation. One son, usually the first, inherits all the family property, including land, home, and ancestral treasures. Daughters enter their husband's *ie* upon marriage, and sons who do not succeed in their parents' *ie* can either be adopted as heirs in families with no sons or start relatively independent "branch" lineages of their own *ie*. .... Because one son remained in the rural area and inherited the family property, the *ie* was maintained and remained the basic unit of rural organization. Sometimes it was the first son who migrated to the city, but in any case the head of the *ie* has remained remarkably powerful in deciding who would and would not go to the city.

As we mentioned already, those sons not designated as heir to the headship left the village for a good economic reason, and it would also have been the head's wish anyway, since the size of the farmland the family owns (or the family has the right to cultivate) would not have been enough to support those "excess sons".[20] Why did the heir remain in the village and how was he able to find a bride? The power of the head sanctioned by the prewar Civil Code explains it. According to Oda (1992, p. 232), under the Code "the head of the family had the power to designate the place where family members should live, to control

[19]For a more detailed description of *ie* by a westerner, see Taeuber (1958, Chapter VI).

[20]The annual department of agriculture tabulations mentioned in footnote 10 show that the size distribution of land managed by farm households was remarkably equal and stable, with 50% to 60% concentrated in the 0.5-2.0ha bracket. How to feed second and third sons without subdividing the family plot of land was a major issue in Japanese agricultural economics in the very early postwar period.

their choice of marriage-partner and to expel them from the family when necessary."[21] A very prominent legal scholar Takeyoshi Kawashima notes that the peasant life before the introduction of the Civil Code in 1898 did not quite honor the sort of patriarchy described in the Code (Kawashima (1957, pp. 8-9, 46)). He then notes, however, that the Code, along with the indoctrination by the government (much like the sort practiced in North Korea today) whereby every schoolchild was required to study a textbook (called *Kyoiku Chokugo*, see the attached figure which is the front page of a textbook wide in use in 1890) expounding on obedience to the father and to the emperor, must have made a profound influence particularly on the peasant population as can be surmised, for example, from the decline of peasant rebellions since the end of the 19th century.[22] The heir stayed in the village, and a daughter married into a farm household, because it was what father wished.

This explanation of the constancy of agricultural employment in prewar Japan also explains its steep postwar decline. A new Constitution was adopted in 1947 and there was a wholesale revision of the Civil Code under the direction of SCAP. Article 24 of the new Constitution states that "....With regard to choice of spouse, property rights, inheritance, choice of domicile,... laws shall be enacted from the standpoint of individual dignity ...". The new Civil Code no longer recognizes the *ie* and the dictatorial power of the head. School textbooks were rewritten to fully reflect those changes. It is not surprising at all, then, that the large-scale defection to the city since around 1955 was initiated by young cohorts, who were not subject to the prewar indoctrination of the supremacy of *ie* over individuals. Reinforcing the tide to the city is the fact that the new Civil Code encourages equal division while the old Code facilitated (but did not require) primogeniture. Now the eldest son who inherits the entire estate by the will of his father is obligated to pay the other siblings a minimum claim (half of what they would have received under the default mode of inheritance (i.e., equal division)), which certainly makes it less economically attractive for the eldest son to

[21] Relevant articles of the prewar Civil Code are the following (copied from Ramseyer (1996, Chapter 5)). Article 749: "members of a house may not determine their place of residence, if the head of the house objects to their choice"; article 750(a): "In order for a family member to marry or to enter an adoptive relationship, he or she must obtain the consent of the family head"; article 750(b): "If a family member marries or enters an adoptive relationship in violation of the previous subsection, the head may, within one year of the date of such marriage or adoption, expel such member from the family or refuse his or her reentry into the family"; article 772(a) states that until man and woman reach the statutory age — thirty for men, twenty-five for women — they could marry only if both of their parents consented.

[22] Kawashima (1957, p. 10, and Chapter 1, particularly pp. 46-47). Ramseyer (1996, Sections 2 and 3 of Chapter 5) argues that the prewar Civil Code did not confer the head the power to control residence and marriages of the family members. However, the court cases he cites are not about disputes between the head and the heir, and the census data he cites for his point that family members did not stay in the family (which means the head's threat to expel had no bite) actually reinforces our claim that all the non-heir siblings left the stem household.

stay in the village.

9. Conclusion

Appendix 1: Data Description (incomplete)

Prewar data on Japan are from the *LTES* (*Long Term Economic Statistics*), which is a consistent system of national income accounts compiled by academics at Hitotsubashi University. U.S. GNP per worker for 1998 is assumed to be 1.339 times that for Japan. U.S. real GNP is taken from Table 7.1 of the NIPA, which is a chain-type quantity index using the Fisher formula. Japan's real GNP is calculated as follows: (i) the nominal value added and the associated deflator for agriculture and non-agriculture are taken from the LTES for prewar and from the Japanese National Accounts for postwar; (ii) the difference between GNP and GDP is added to non-agriculture; and (iii) using q_t (the price of agricultural goods in terms of non-agricultural goods, normalized to 1 for 1934-36) and Y_{jt} (real value added for sector j, $j = 1, 2$), which can be calculated from steps (i) and (ii), calculate the chain-type Fisher quantity index. Real GNP for 1935 is calculated as $Y_{1t} + Y_{2t}$ for $t = 1935$. GNP for other years are calculated using the Fisher quantity index. Namely, the Fisher formula for real GNP in year t relative to its value in year $t - 1$ is

$$\sqrt{\frac{q_{t-1}Y_{1t} + Y_{2t}}{q_{t-1}Y_{1,t-1} + Y_{2,t-1}} \times \frac{q_t Y_{1t} + Y_{2t}}{q_t Y_{1,t-1} + Y_{2,t-1}}}.$$

Working-age population for Japan is the population aged 15 and over for prewar and between age 20 and 69 for postwar (in this version, total population is used for the U.S.).

Appendix 2: Reducing Equilibrium Conditions to the Two-Equation Dynamical System

This appendix describes in detail how the equilibrium conditions in Section 4 can be reduced to the two-equation dynamical system.

- We first reduce the equilibrium conditions except the employment arbitrage condition (4.4) and the transversality condition (4.10) into seven equations. The factor market equilibrium conditions ((4.19), (4.20), and (4.21)) and the definition $s_{Kt} \equiv k_{1t}/(k_{1t}+k_{2t})$ and $s_{Et} \equiv E_{1t}/E_t$ imply

$$\begin{aligned} k_{1t} &= s_{Kt}\, k_t,\ k_{2t} = (1-s_{Kt})\, k_t, \\ L_{1t} &= s_{Et}\, h_{1t} E_t,\ L_{2t} = [1-(1-\xi)s_{Et}]\, h_{2t} E_t. \end{aligned} \tag{A2.1}$$

Substituting this, (4.9), and the production functions ((4.11), and (4.14)) into the market equilibrium conditions for good 1 and good 2 ((4.17) and (4.18)), and recalling that $\psi_t \equiv G_t/Y_{2t}$, we obtain

$$N_t c_1(q_t,\lambda_t) = TFP_{1t}\,(h_{1t}E_t)^{\eta}\,(N_t k_t)^{\theta_1}\, s_{Kt}^{\theta_1}\, s_{Et}^{\eta}, \tag{A2.2}$$

$$\begin{aligned} &N_t c_2(q_t,\lambda_t) + N_{t+1}k_{t+1} - (1-\delta)N_t k_t \\ &\quad = (1-\psi_t)\,TFP_{2t}\,(h_{2t}E_t)^{1-\theta_2}\,(N_t k_t)^{\theta_2}(1-s_{Kt})^{\theta_2}[1-(1-\xi)s_{Et}]^{1-\theta_2} - \phi N_t(1-s_{Kt})k_t. \end{aligned} \tag{A2.3}$$

Using (A2.1), the firms' first-order conditions can be written as

$$r_t = \theta_1\, q_t\, TFP_{1t}(h_{1t}E_t)^{\eta}\,(N_t k_t)^{\theta_1-1}\, s_{Kt}^{\theta_1-1} s_{Et}^{\eta}, \tag{A2.4}$$

$$w_{1t} = \eta\, q_t\, TFP_{1t}\,(h_{1t}E_t)^{\eta-1}\,(N_t k_t)^{\theta_1}\, s_{Kt}^{\theta_1}\, s_{Et}^{\eta-1}, \tag{A2.5}$$

$$r_t = \theta_2\ TFP_{2t}\,(h_{2t}E_t)^{1-\theta_2}\,(N_t k_t)^{\theta_2-1}\left(\frac{1-s_{Kt}}{1-(1-\xi)s_{Et}}\right)^{\theta_2-1} - \phi, \tag{A2.6}$$

$$w_{2t} = (1-\theta_2)\,TFP_{2t}\,(h_{2t}E_t)^{-\theta_2}\,(N_t k_t)^{\theta_2}\left(\frac{1-s_{Kt}}{1-(1-\xi)s_{Et}}\right)^{\theta_2}. \tag{A2.7}$$

We have thus reduced the equilibrium conditions given in Section 4 into seven equations — (A2.2)-(A2.7) and (4.8) — along with the employment arbitrage condition (4.4) and the transversality condition (4.10).

- We now reduce these seven equations into five equations written in terms of detrended variables. To reproduce the definition of detrended variables in Section 4,

$$\begin{aligned} X_{Yt} &\equiv (TFP_{2t})^{\frac{1}{1-\theta_2}}\, h_{2t}E_t/N_t,\ \ X_{Qt} \equiv TFP_{1t}^{-1}\,(h_{1t}E_t)^{-\eta}\ TFP_{2t}^{\frac{1-\theta_1}{1-\theta_2}}\,(h_{2t}E_t)^{1-\theta_1}, \\ \tilde{k}_t &\equiv \frac{k_t}{X_{Yt}},\ \ \tilde{\lambda}_t \equiv \frac{\lambda_t}{X_{Yt}},\ \tilde{q}_t \equiv \frac{q_t}{X_{Qt}}. \end{aligned} \tag{A2.8}$$

Substituting $k_t = \tilde{k}_t X_{Yt}$, $\lambda_t = \tilde{\lambda}_t X_{Yt}$, and $q_t = \tilde{q}_t X_{Qt}$ into (A2.2)-(A2.7) and (4.8), observing cancellation of a number of terms, and eliminating r_t and combining the equations for the wage rates

(A2.5) and (A2.7) into their ratio form, we obtain

$$\frac{N_{t+1}}{N_t}\frac{X_{Y,t+1}}{X_{Yt}}\tilde{k}_{t+1} = [1-\delta-(1-s_{Kt})\phi]\tilde{k}_t + (1-\psi_t)\tilde{y}_{2t} - \frac{c_2(\tilde{q}_t X_{Qt}, \tilde{\lambda}_t X_{Yt})}{X_{Yt}}, \tag{A2.9}$$

$$\frac{X_{Y,t+1}}{X_{Yt}}\,\tilde{\lambda}_{t+1} = \beta\,\tilde{\lambda}_t\left\{1+(1-\tau)\left[\theta_2\,\frac{\tilde{y}_{2,t+1}}{(1-s_{K,t+1})\tilde{k}_{t+1}} - \phi - \delta\right]\right\}, \tag{A2.10}$$

$$\frac{c_1(\tilde{q}_t X_{Qt}, \tilde{\lambda}_t X_{Yt})}{X_{Yt}/X_{Qt}} = \tilde{y}_{1t}, \tag{A2.11}$$

$$\theta_1\,\frac{\tilde{q}_t\tilde{y}_{1t}}{s_{Kt}\tilde{k}_t} = \theta_2\,\frac{\tilde{y}_{2t}}{(1-s_{Kt})\tilde{k}_t} - \phi, \tag{A2.12}$$

$$\frac{w_{1t}h_{1t}}{w_{2t}h_{2t}} = \frac{\eta\,\frac{\tilde{q}_t\tilde{y}_{1t}}{s_{Et}}}{(1-\theta_2)\frac{\tilde{y}_{2t}}{1-(1-\xi)s_{Et}}}, \tag{A2.13}$$

where

$$\tilde{y}_{1t} \equiv \tilde{k}_t^{\theta_1}\, s_{Kt}^{\theta_1}\, s_{Et}^{\eta},\ \tilde{y}_{2t} \equiv \tilde{k}_t^{\theta_2}(1-s_{Kt})^{\theta_2}[1-(1-\xi)s_{Et}]^{1-\theta_2}. \tag{A2.14}$$

- To recapitulate, the equilibrium conditions besides the arbitrage condition (4.4) and the transversality condition (4.10) can be reduced to the five equations (A2.9)-(A2.13). We now show that $(s_{Kt}, s_{Et}, \tilde{q}_t)$ given $(\tilde{k}_t, \tilde{\lambda}_t, X_{Yt}, X_{Qt})$ is uniquely determined by (A2.11)-(A2.13) and (4.4), so that the equilibrium conditions reduce to the two-equation dynamical system (A2.9) and (A2.10) in the two states $(\tilde{k}_t, \tilde{\lambda}_t)$. The graph of (4.4), which is a correspondence from $\frac{w_{1t}h_{1t}}{w_{2t}h_{2t}}$ to s_{Et}, is the staircase shown in Appendix Figure 1. This can be interpreted as the supply curve of sector 1 labor. To derive a demand curve, we can use (A2.11) and (A2.12) to solve for $(\tilde{q}_t, s_{Kt})$ given $(s_{Et}, \tilde{k}_t, \tilde{\lambda}_t, X_{Yt}, X_{Qt})$, and then substitute these solved-out values into (A2.13) to obtain $\frac{w_{1t}h_{1t}}{w_{2t}h_{2t}}$ as a function of $(s_{Et}, \tilde{k}_t, \tilde{\lambda}_t, X_{Yt}, X_{Qt})$. It is easy to show that the function is strictly decreasing in s_{Et}, so the graph of $\frac{w_{1t}h_{1t}}{w_{2t}h_{2t}}$ as a function of s_{Et} is downward-sloping. This is the demand curve for sector 1 labor. Three possible cases are drawn in Appendix Figure 1. In Case A, the demand curve intersects with the vertical portion of the staircase (the graph of the arbitrage condition) at $s_{Et} = \bar{s}_{Et}$. In thise case, the labor barrier $s_{Et} \geq \bar{s}_{Et}$ is binding. Case B is that the intersection is on the horizontal portion of the staircase. In this case, $\frac{w_{1t}h_{1t}}{w_{2t}h_{2t}} = 1-\xi$, so from (A2.13) we have

$$1-\xi = \frac{\eta\,\frac{\tilde{q}_t\tilde{y}_{1t}}{s_{Et}}}{(1-\theta_2)\frac{\tilde{y}_{2t}}{1-(1-\xi)s_{Et}}}. \tag{A2.15}$$

  The final case, Case C, is that the demand curve intersects with the staircase at $s_{Et} = 1$. In either case, given $(\tilde{k}_t, \tilde{\lambda}_t, X_{Yt}, X_{Qt})$, s_{Et} is uniquely determined. Given $(s_{Et}, \tilde{k}_t, \tilde{\lambda}_t, X_{Yt}, X_{Qt})$, $(s_{Kt}, \tilde{q}_t)$ is uniquely determined by (A2.11) and (A2.12), as just explained.

- In Case C, sector 1 is so productive that all employment is in agriculture (which means $L_{2t} = \xi h_{2t}E_t$). Under the calibrated parameter values and the initial conditions specified in the text about the model, this case does not arise in any period. Assuming that only Case A or Case B is relevant, the value of s_{Et} satisfying (A2.11)-(A2.13), and (4.4) can be determined as follows. Notice from the previous paragraph that in Case B $(s_{Kt}, s_{Et}, \tilde{q}_t)$ solves (A2.11), (A2.12), and (A2.15). If Case C is impossible, then Case A (where the labor barrier $s_{Et} \geq \bar{s}_{Et}$ is binding) obtains if and only if the s_{Et} that solves these three equations is less than $\bar{s}_{Et}$ (this corresponds to point A' in Appendix Figure 1). Therefore, to find $(s_{Kt}, s_{Et}, \tilde{q}_t)$ that solves (A2.11)-(A2.13) and (4.4) given $(\tilde{k}_t, \tilde{\lambda}_t, X_{Yt}, X_{Qt})$, we can proceed as follows:

1. Solve (A2.11),(A2.12), and (A2.15) for $(s_{Kt}, s_{Et}, \tilde{q}_t)$. If $s_{Et} \geq \bar{s}_{Et}$, then the solution is found.
2. If $s_{Et} < \bar{s}_{Et}$, then set $s_{Et} = \bar{s}_{Et}$ and use (A2.11) and (A2.12) to solve for $(s_{Kt}, \tilde{q}_t)$ given s_{Et} thus determined.

Appendix 3: Incorporating Intermediate Input to Agriculture

In this appendix, we allow an intermediate input to Sector 1. For this more general model, we derive a two-equation dynamical system, compute its steady state, and show that the steady state is a saddle.

The Equilibrium Conditions

With intermediate input to agriculture denoted by M_t, the production function for sector 1 is

$$Y_{1t} = TFP_{1t}\, K_{1t}^{\theta_1}\, L_{1t}^{\eta}\, M_t^{\alpha}. \tag{A3.1}$$

The contribution of land is implicit in this production function, so we have a decreasing returns to scale in capital, labor, and intermediate inputs:

$$\theta_1 + \eta + \alpha < 1. \tag{A3.2}$$

The firms' first-order conditions for sector 1 are:

$$r_t = \theta_1\, q_t\; TFP_{1t}\, K_{1t}^{\theta_1 - 1}\, L_{1t}^{\eta}\, M_t^{\alpha}, \tag{A3.3}$$

$$w_{1t} = \eta\, q_t\; TFP_{1t}\, K_{1t}^{\theta_1}\, L_{1t}^{\eta-1}\, M_t^{\alpha}, \tag{A3.4}$$

$$1 = \alpha\, q_t\; TFP_{1t}\, K_{1t}^{\theta_1}\, L_{1t}^{\eta}\, M_t^{\alpha-1}. \tag{A3.5}$$

With the production function for sector 2 being the same as in the text, the marginal productivity conditions for sector 2 are (4.15) and (4.16).

Solving (A3.5) for M_t and substituting it into (A3.1), (A2.4), and (A2.5), we obtain

$$Y_{1t} = q_t^{\frac{\alpha}{1-\alpha}}\; \widetilde{TFP}_{1t}\, K_{1t}^{\tilde{\theta}_1}\, L_{1t}^{\tilde{\eta}}, \tag{A3.6}$$

$$r_t = (1-\alpha)\, \tilde{\theta}_1\, q_t^{\frac{1}{1-\alpha}}\; \widetilde{TFP}_{1t}\, K_{1t}^{\tilde{\theta}_1 - 1}\, L_{1t}^{\tilde{\eta}}, \tag{A3.7}$$

$$w_{1t} = (1-\alpha)\tilde{\eta}\, q_t^{\frac{1}{1-\alpha}}\; \widetilde{TFP}_{1t}\, K_{1t}^{\tilde{\theta}_1}\, L_{1t}^{\tilde{\eta}-1}, \tag{A3.8}$$

where

$$\widetilde{TFP}_{1t} \equiv \alpha^{\frac{\alpha}{1-\alpha}}\; TFP_{1t}^{\frac{1}{1-\alpha}},\;\; \tilde{\theta}_1 \equiv \frac{\theta_1}{1-\alpha},\;\; \tilde{\eta} \equiv \frac{\eta}{1-\alpha}. \tag{A3.9}$$

(A3.6) replaces (4.11), (A3.7) replaces (4.12), and (A3.8) replaces (4.13). Noting that $M_t = \alpha q_t Y_{1t}$, the equilibrium condition for good 2, (4.18), is now

$$\text{(good 2)} \quad N_t c_{2t} + (N_{t+1}k_{t+1} - (1-\delta)N_t k_t) + G_t + \alpha q_t Y_{1t} = Y_{2t} - \phi(1 - s_{Kt})N_t k_t. \tag{A3.10}$$

The Detrended Dynamical System

The trend X_{Yt} is the same as before, as defined in (4.23), but the trend X_{Qt} with the intermediate input is

$$X_{Qt} \equiv \left[\widetilde{TFP}_{1t}^{-1} (h_{1t}E_t)^{-\tilde{\eta}}\; TFP_{2t}^{\frac{1-\tilde{\theta}_1}{1-\theta_2}} (h_{2t}E_t)^{1-\tilde{\theta}_1} \right]^{1/(1-\alpha)}. \tag{A3.11}$$

It is straightforward to show that the five equations corresponding to (A2.9)-(A2.13) are:

$$\frac{N_{t+1}}{N_t}\frac{X_{Y,t+1}}{X_{Yt}}\tilde{k}_{t+1} = [1-\delta-(1-s_{Kt})\phi]\tilde{k}_t + (1-\psi_t)\tilde{y}_{2t} - \frac{c_2(\tilde{q}_t X_{Qt}, \tilde{\lambda}_t X_{Yt})}{X_{Yt}} - \alpha\,\tilde{q}_t\tilde{y}_{1t}, \quad \text{(A3.12)}$$

$$\frac{X_{Y,t+1}}{X_{Yt}}\tilde{\lambda}_{t+1} = \beta\,\tilde{\lambda}_t\left\{1+(1-\tau)\left[\theta_2\,\frac{\tilde{y}_{2,t+1}}{\tilde{k}_{t+1}(1-s_{K,t+1})} - \phi - \delta\right]\right\}, \quad \text{(A3.13)}$$

$$\frac{c_1(\tilde{q}_t X_{Qt}, \tilde{\lambda}_t X_{Yt})}{X_{Yt}/X_{Qt}} = \tilde{y}_{1t}, \quad \text{(A3.14)}$$

$$(1-\alpha)\,\tilde{\theta}_1\,\frac{\tilde{q}_t\tilde{y}_{1t}}{\tilde{k}_t s_{Kt}} = \theta_2\,\frac{\tilde{y}_{2t}}{\tilde{k}_t(1-s_{Kt})} - \phi, \quad \text{(A3.15)}$$

$$\frac{w_{1t}h_{1t}}{w_{2t}h_{2t}} = \frac{(1-\alpha)\tilde{\eta}\,\tilde{q}_t\tilde{y}_{1t}/s_{Et}}{(1-\theta_2)\tilde{y}_{2t}/[1-(1-\xi)s_{Et}]}. \quad \text{(A3.16)}$$

where

$$\tilde{y}_{1t} \equiv \tilde{q}_t^{\frac{\alpha}{1-\alpha}}\,\tilde{k}_t^{\tilde{\theta}_1}\,s_{Kt}^{\tilde{\theta}_1}\,s_{Et}^{\tilde{\eta}}, \quad \tilde{y}_{2t} \equiv \tilde{k}_t^{\theta_2}\,(1-s_{Kt})^{\theta_2}\,[1-(1-\xi)s_{Et}]^{1-\theta_2}. \quad \text{(A3.17)}$$

The Autonomous System and its Steady State

To study the asymptotic property of the autonomous system, in the rest of this Appendix, assume:

(a) the trends N_t and X_{Yt} grow at constant rates (so $\frac{N_{t+1}}{N_t} = n$ and $\frac{X_{Y,t+1}}{X_{Yt}} = g_2^{\frac{1}{1-\theta_2}}$) and $\psi_t = \psi$,

(b) the demand functions c_1 and c_2 are those given in (5.3) and (5.4), reproduced here:

$$\frac{c_1(\tilde{q}\,X_{Qt}, \tilde{\lambda}\,X_{Yt})}{X_{Yt}/X_{Qt}} = \mu_1\frac{\tilde{\lambda}}{\tilde{q}}, \quad \frac{c_2(\tilde{q}\,X_{Qt}, \tilde{\lambda}\,X_{Yt})}{X_{Yt}} = \mu_2\tilde{\lambda}.$$

Clearly, under these two conditions, the detrended dynamical system (A3.12)-(A3.16), with $\frac{w_{1t}h_{1t}}{w_{2t}h_{2t}}$ set equal to $1-\xi$, is autonomous. In this autonomous system, the path of X_{Qt} can be arbitrary (it does not have to be a constant-growth-rate path, for example) simply because X_{Qt} does not enter the system. However, as mentioned in the text, for the non-autonomous system exhibiting Engel's law to asymptotically behave like the autonomous system to be studied below, we need X_{Yt}/X_{Qt} to grow without limit.

The steady-state $(\tilde{k}, \tilde{\lambda}, \tilde{q}, s_K, s_E)$ of this dynamical system satisfies the following five equations that can be obtained by dropping the time subscript in the system:

$$n\,g_2^{\frac{1}{1-\theta_2}} = [1-\delta-(1-s_K)\phi] + (1-\psi)\frac{\tilde{y}_2}{\tilde{k}} - \mu_2\,\frac{\tilde{\lambda}}{\tilde{k}} - \alpha\tilde{q}\tilde{y}_1 \quad \text{(A3.18)}$$

$$g_2^{\frac{1}{1-\theta_2}} = \beta\left\{1+(1-\tau)\left[\theta_2\,\frac{\tilde{y}_2}{\tilde{k}(1-s_K)} - \phi - \delta\right]\right\}, \quad \text{(A3.19)}$$

$$\mu_1\tilde{\lambda} = \tilde{q}\tilde{y}_1, \quad \text{(A3.20)}$$

$$(1-\alpha)\,\tilde{\theta}_1\,\frac{\tilde{q}\tilde{y}_1}{\tilde{k}s_K} = \theta_2\,\frac{\tilde{y}_2}{\tilde{k}(1-s_K)} - \phi, \quad \text{(A3.21)}$$

$$1-\xi = \frac{(1-\alpha)\tilde{\eta}\,\tilde{q}\tilde{y}_1/s_E}{(1-\theta_2)\tilde{y}_2/[1-(1-\xi)s_E]}, \quad \text{(A3.22)}$$

where

$$\tilde{y}_1 = \tilde{q}^{\frac{\alpha}{1-\alpha}}\, \tilde{k}^{\tilde{\theta}_1}\, s_K^{\tilde{\theta}_1}\, s_E^{\tilde{\eta}}, \quad \tilde{y}_2 = \tilde{k}^{\theta_2}\,(1-s_K)^{\theta_2}\,[1-(1-\xi)s_E]^{1-\theta_2}. \tag{A3.23}$$

There is a closed-form expression for s_K:

$$s_K = \frac{1-\delta-\phi+\frac{(1-\psi)(r+\phi)}{\theta_2} - ng_2^{\frac{1}{1-\theta_2}}}{\frac{(1-\psi)(r+\phi)}{\theta_2} + \frac{(\alpha\mu_1+\mu_2)r}{\mu_1(1-\alpha)\tilde{\theta}_1} - \phi}, \tag{A3.24}$$

where

$$r \equiv \frac{\frac{g_2^{\frac{1}{1-\theta_2}}}{\beta} - 1}{1-\tau} + \delta \tag{A3.25}$$

is the steady-state gross pretax rate of return from capital.

Under the calibrated parameter values, the Jacobian of the mapping from $(\tilde{k}_t, \tilde{\lambda}_t)$ to $(\tilde{k}_{t+1}, \tilde{\lambda}_{t+1})$ for the dynamical system is numerically calculated and its engenvalues are (1.2083, 0.8735). So the steady state is a saddle.

Appendix 4: The Open Economy Version

This appendix describes the model obtained from opening up the model of Appendix 3 to trade in goods and services.

The Detrended Dynamical System

If the country can exchange good 1 for good 2 at a given relative price of q_t, the market equilibrium conditions for good 1 and good 2 can be combined into one. This means that, in the detrended dynamical system of Appendix 3, we can drop (A3.14) and modify the resource constraint (A3.12) as

$$\begin{aligned}\frac{N_{t+1}}{N_t}\frac{X_{Y,t+1}}{X_{Yt}}\,\tilde{k}_{t+1} = [1-\delta-(1-s_{Kt})\phi]\tilde{k}_t + (1-\psi_t)\tilde{y}_{2t} - \frac{c_2(\tilde{q}_t X_{Qt}, \tilde{\lambda}_t X_{Yt})}{X_{Yt}} - \alpha\,\tilde{q}_t\tilde{y}_{1t} \\ + \tilde{q}_t\left[\tilde{y}_{1t} - \frac{c_1(\tilde{q}_t X_{Qt}, \tilde{\lambda}_t X_{Yt})}{X_{Yt}/X_{Qt}}\right].\end{aligned} \tag{A4.1}$$

The additional term on the right side is the net export of good 1, valued in terms of good 2. This is the only change; the dynamical system is made up of (A4.1), (A3.13), (A3.15), and (A3.16) with $\tilde{y}_{1t}$ and $\tilde{y}_{2t}$ still given by (A3.17).

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Figure 1: GNP Per Working-Age Population relative to Trend
(1.8% Prewar, 2.0% Postwar, US average over 1885-2000 = 1.0)

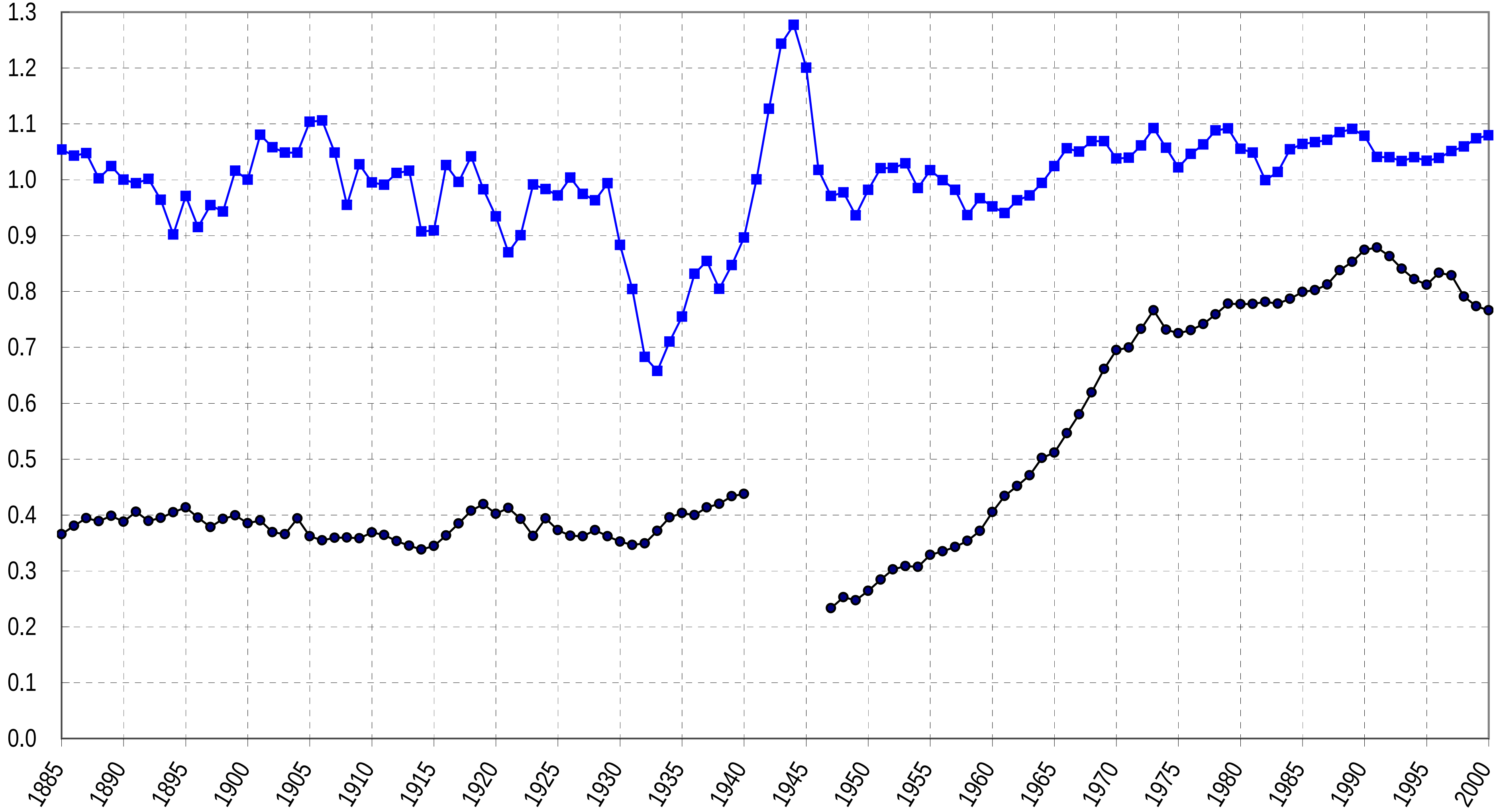


Figure 2: Japan's TFP Factor (TFP^(1/(1-theta))) Relative to Trend
1885-1940 average=1

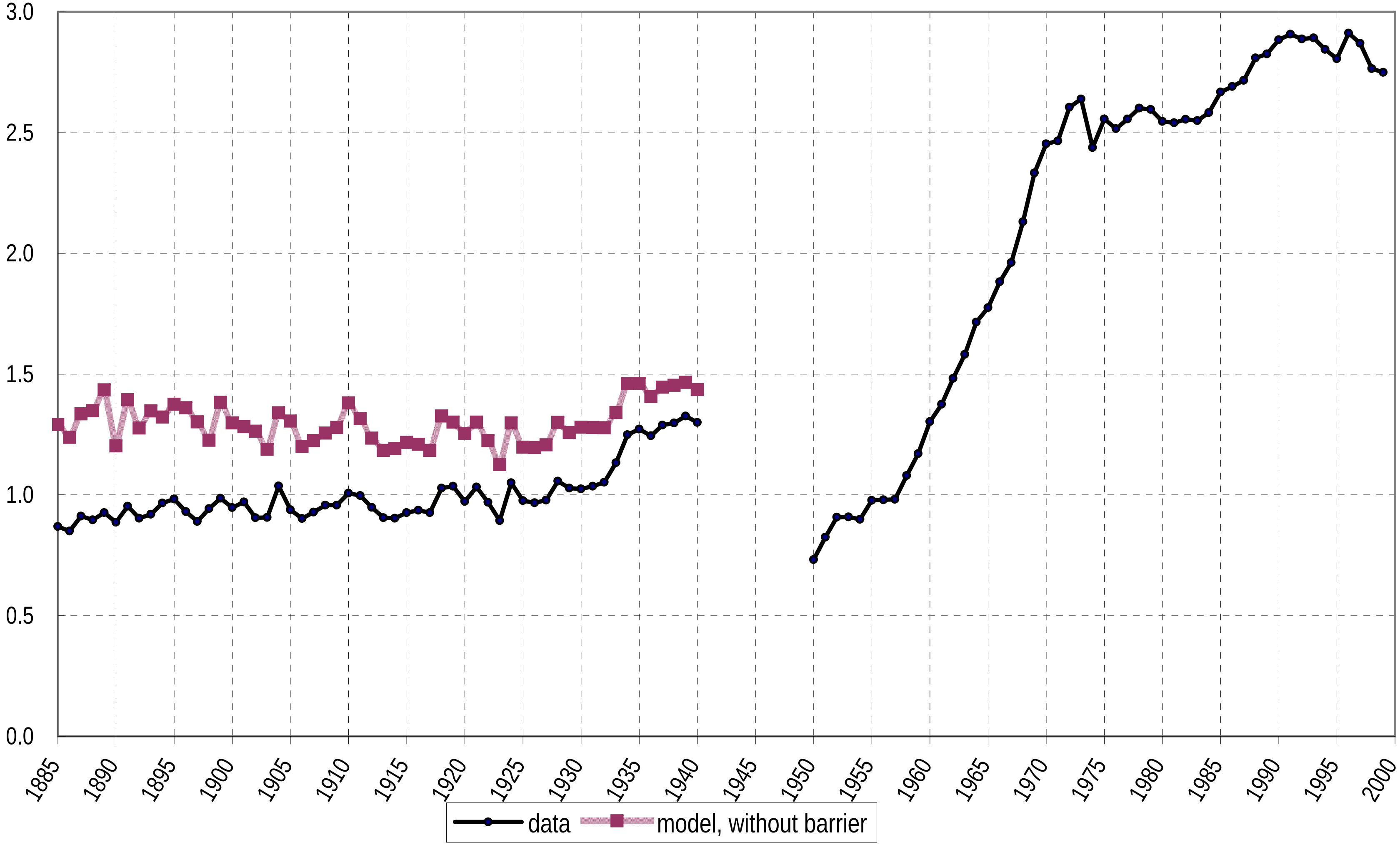


Figure 3: Employment in Agriculture, 1885-2001

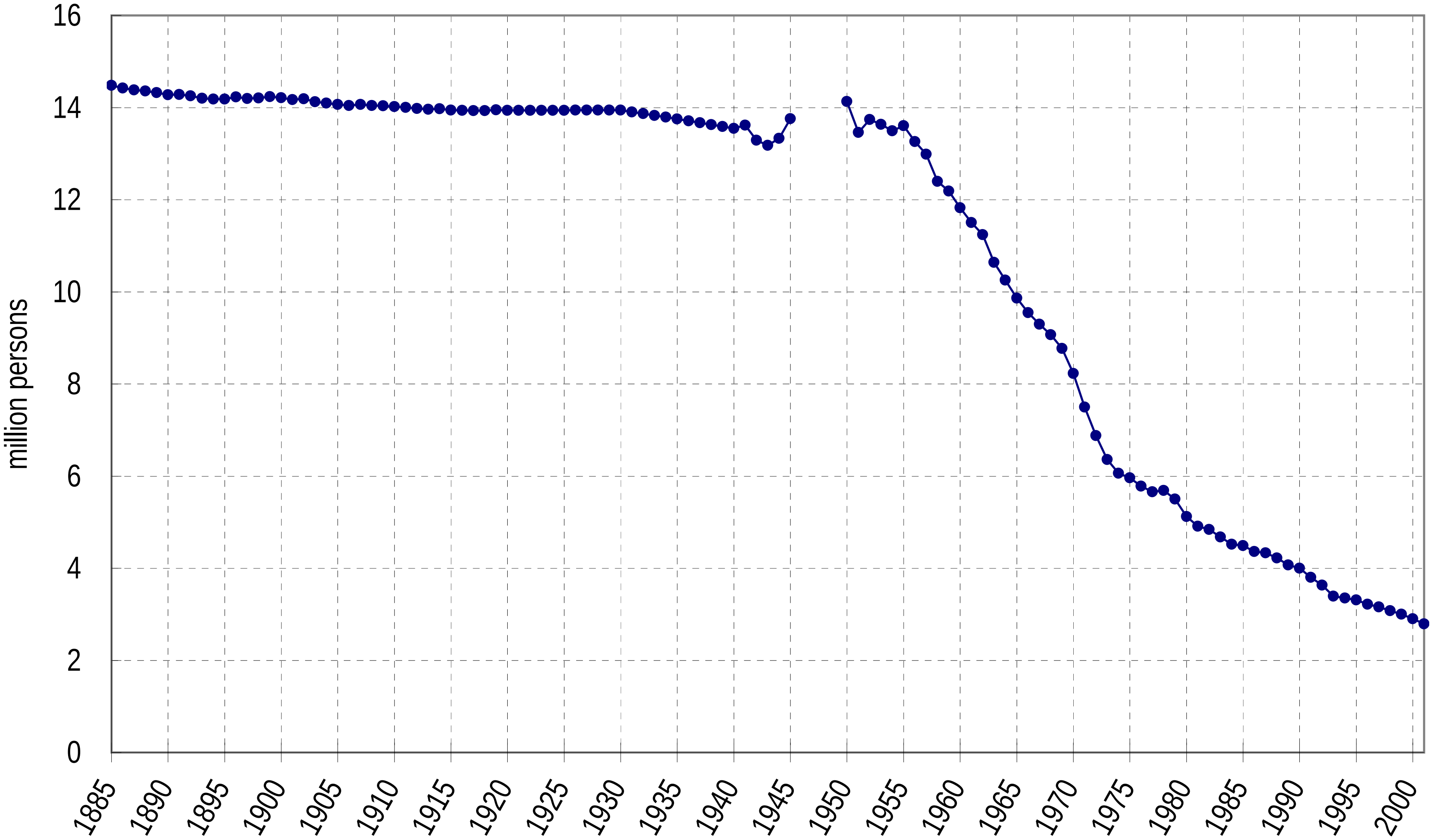


Figure 4: Employment Share of Agriculture, 1885-2000

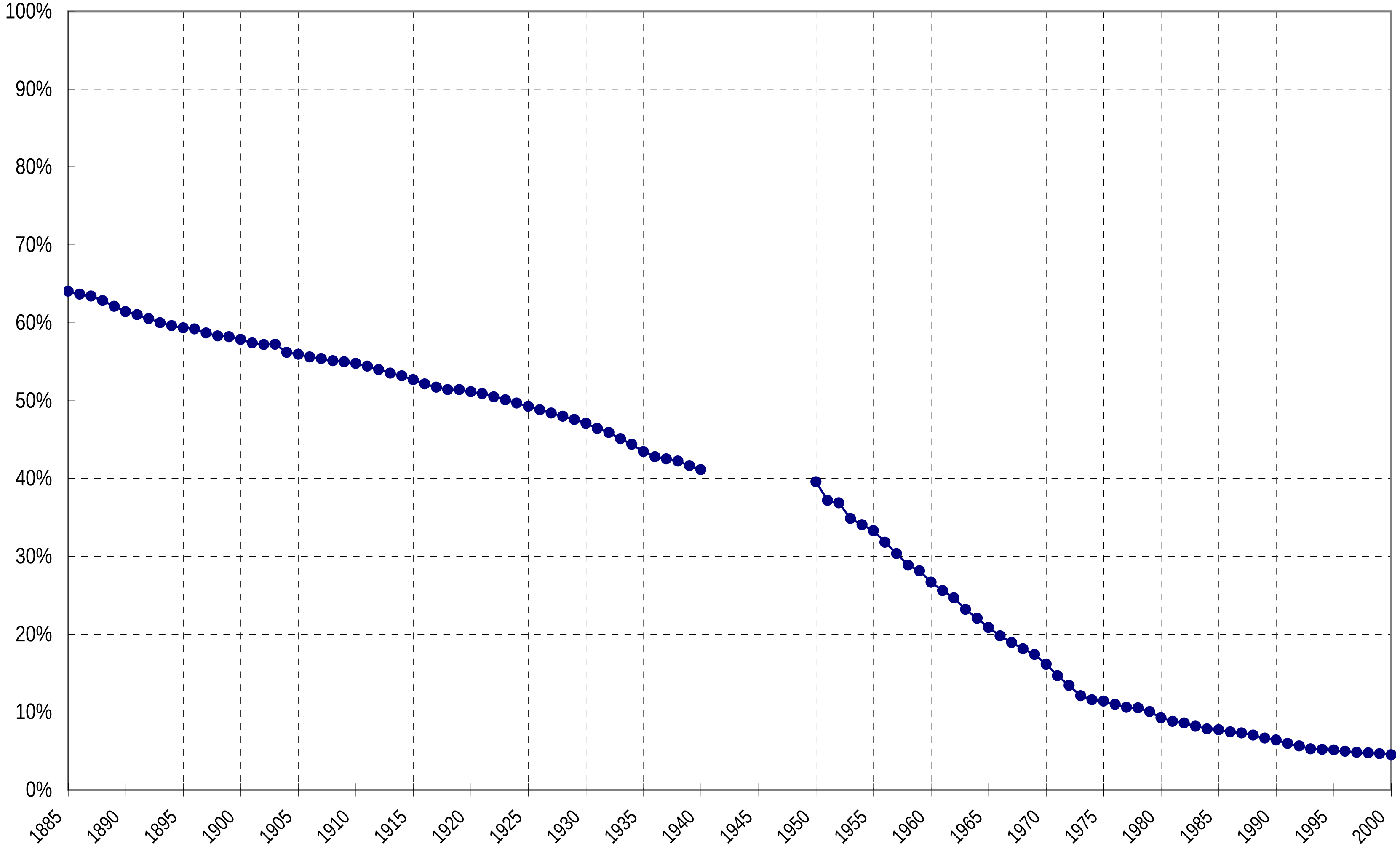
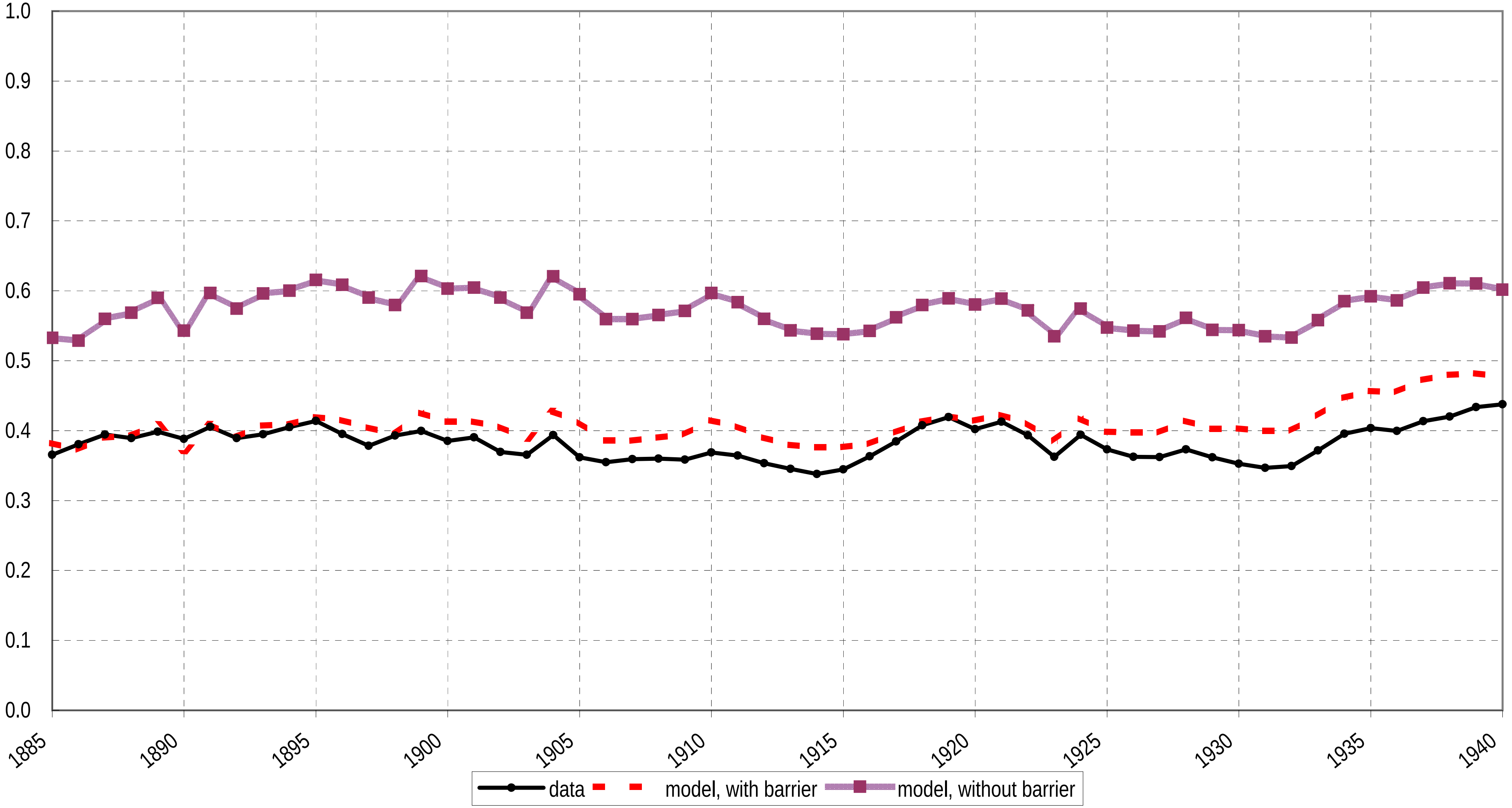


Figure 5: GNP per Working-age Population relative to Trend, 1885-1940
U.S. average over 1885-2000 =1.0



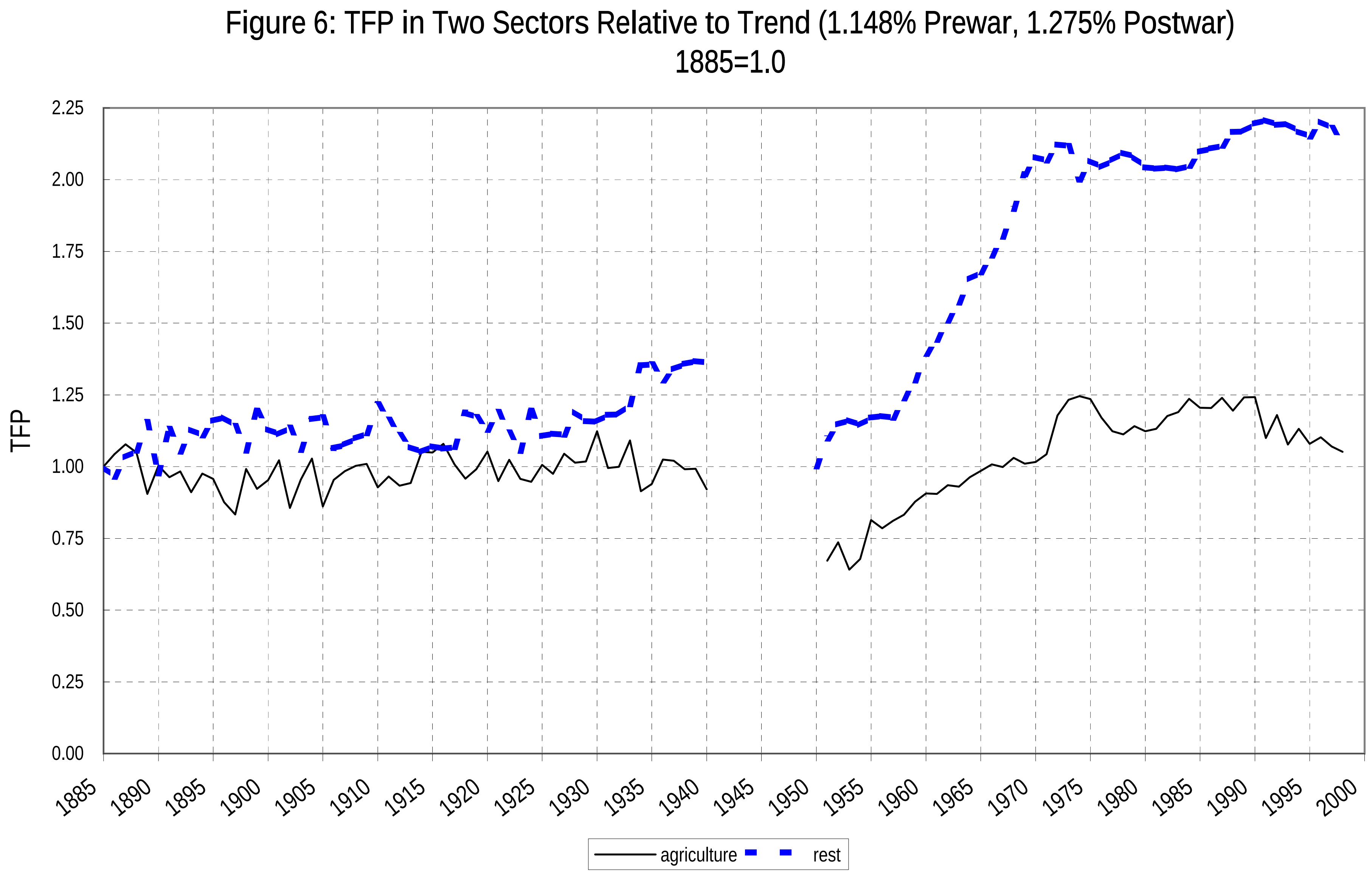


Figure 6: TFP in Two Sectors Relative to Trend (1.148% Prewar, 1.275% Postwar)
1885=1.0
TFP
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
1885
1890
1895
1900
1905
1910
1915
1920
1925
1930
1935
1940
1945
1950
1955
1960
1965
1970
1975
1980
1985
1990
1995
2000
agriculture
rest

Figure 7: Capital-Total Hours Ratio (K/(hE))
actual 1885 value = 1.0

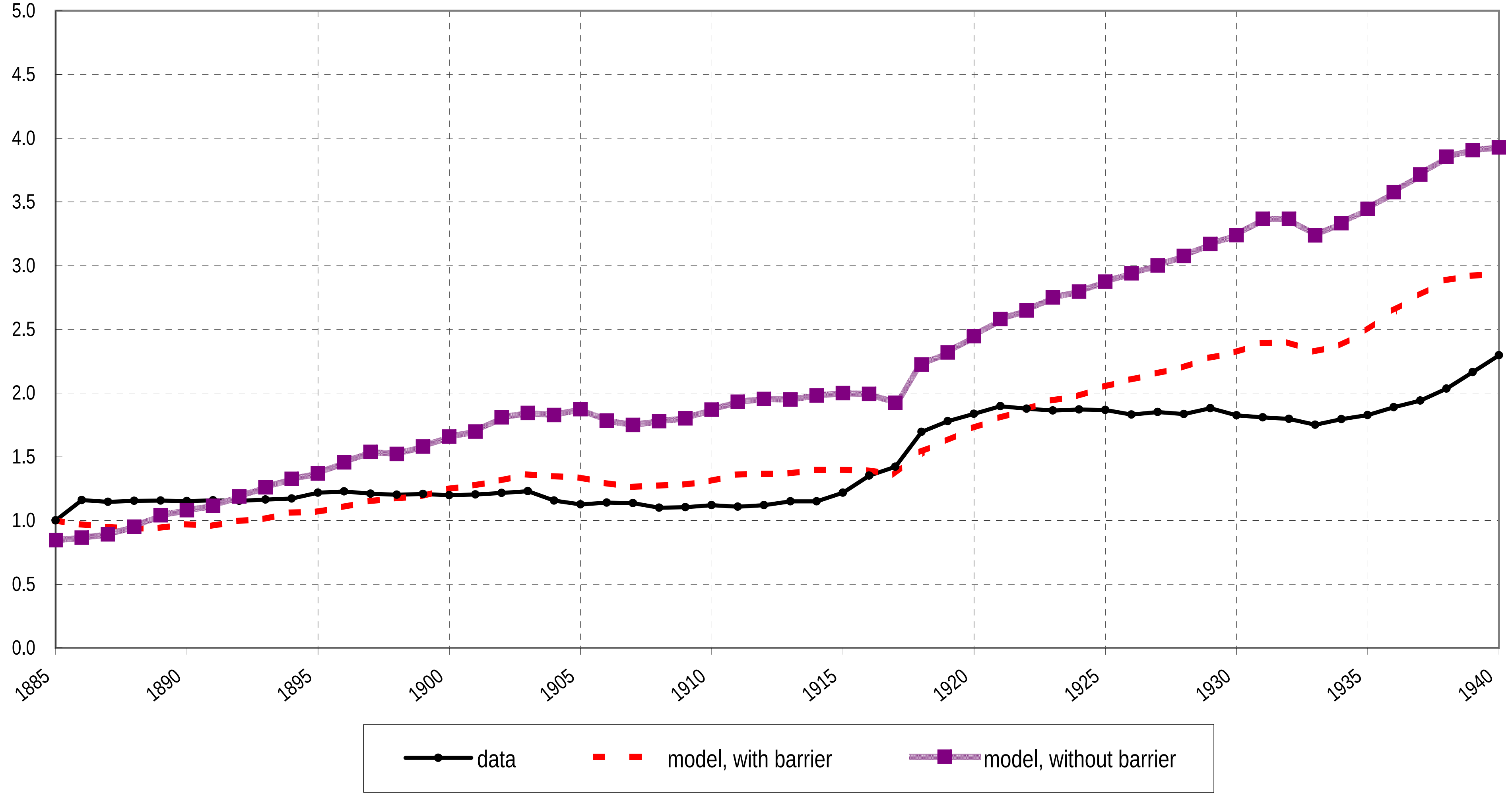
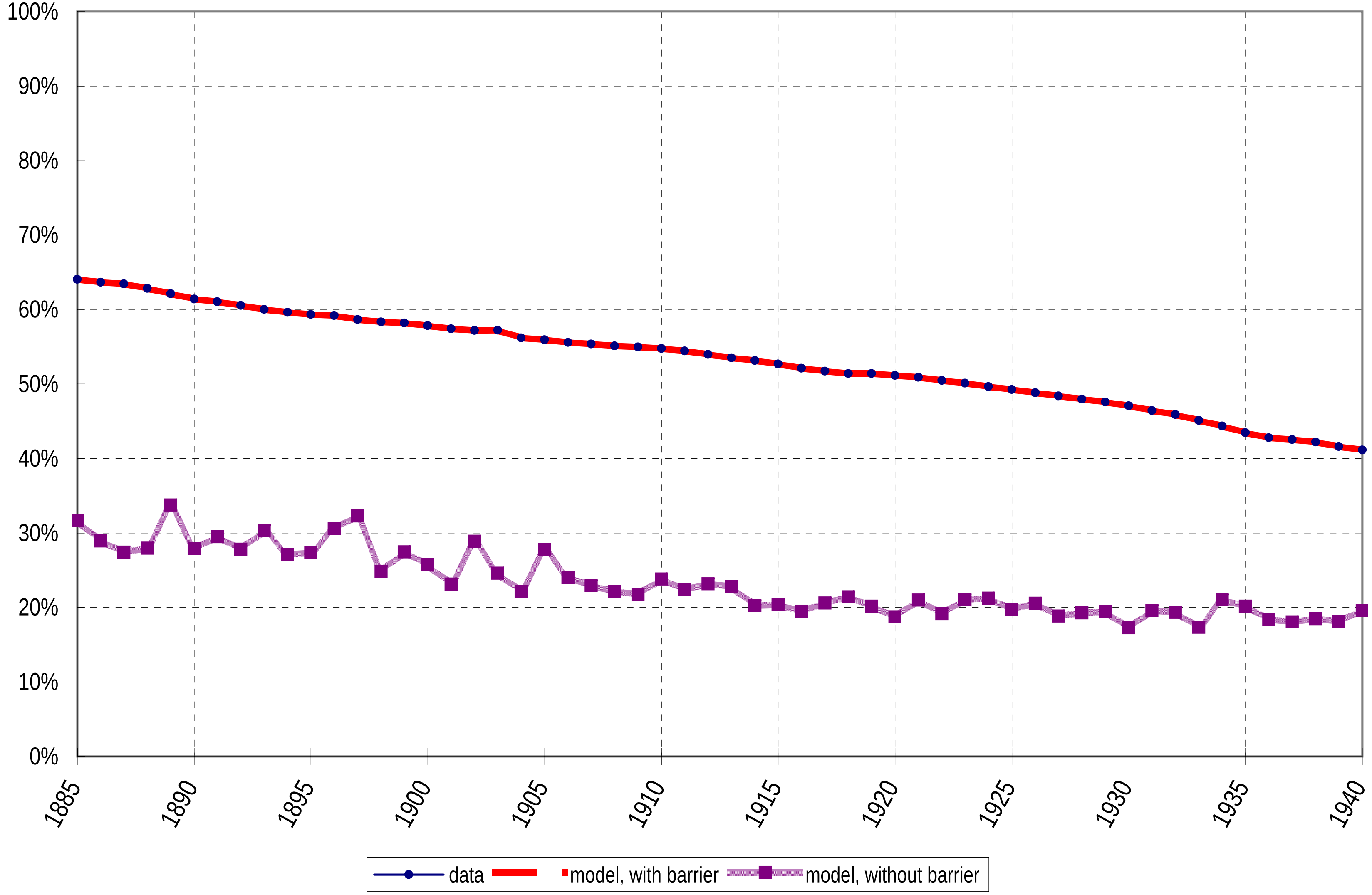


Figure 8a: Employment Share of Agriculture, 1885-1940



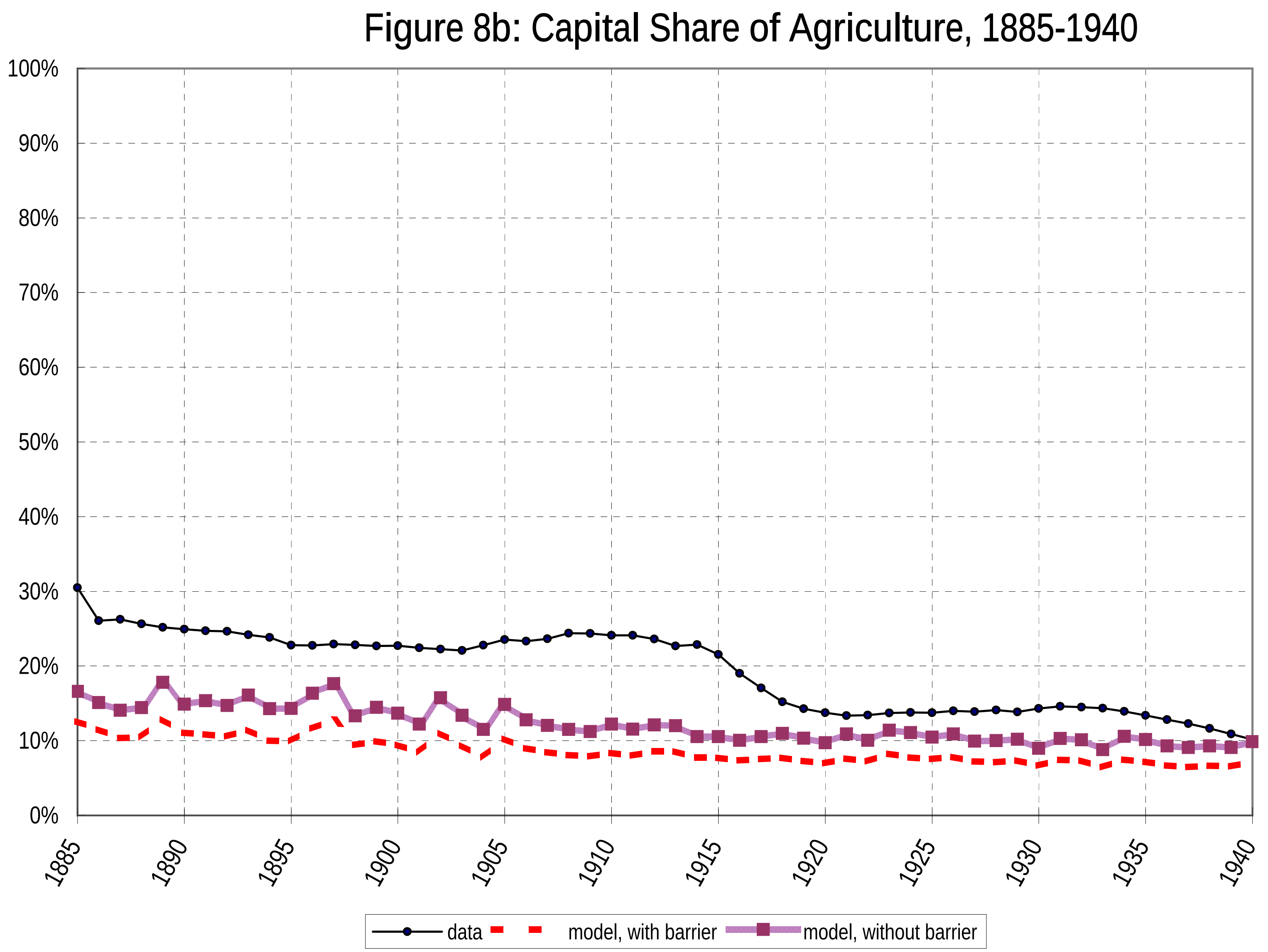


Figure 8b: Capital Share of Agriculture, 1885-1940
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
1885
1890
1895
1900
1905
1910
1915
1920
1925
1930
1935
1940
data
model, with barrier
model, without barrier

FIgure 9: Phase Diagram: Smoothed Exogenous Variables

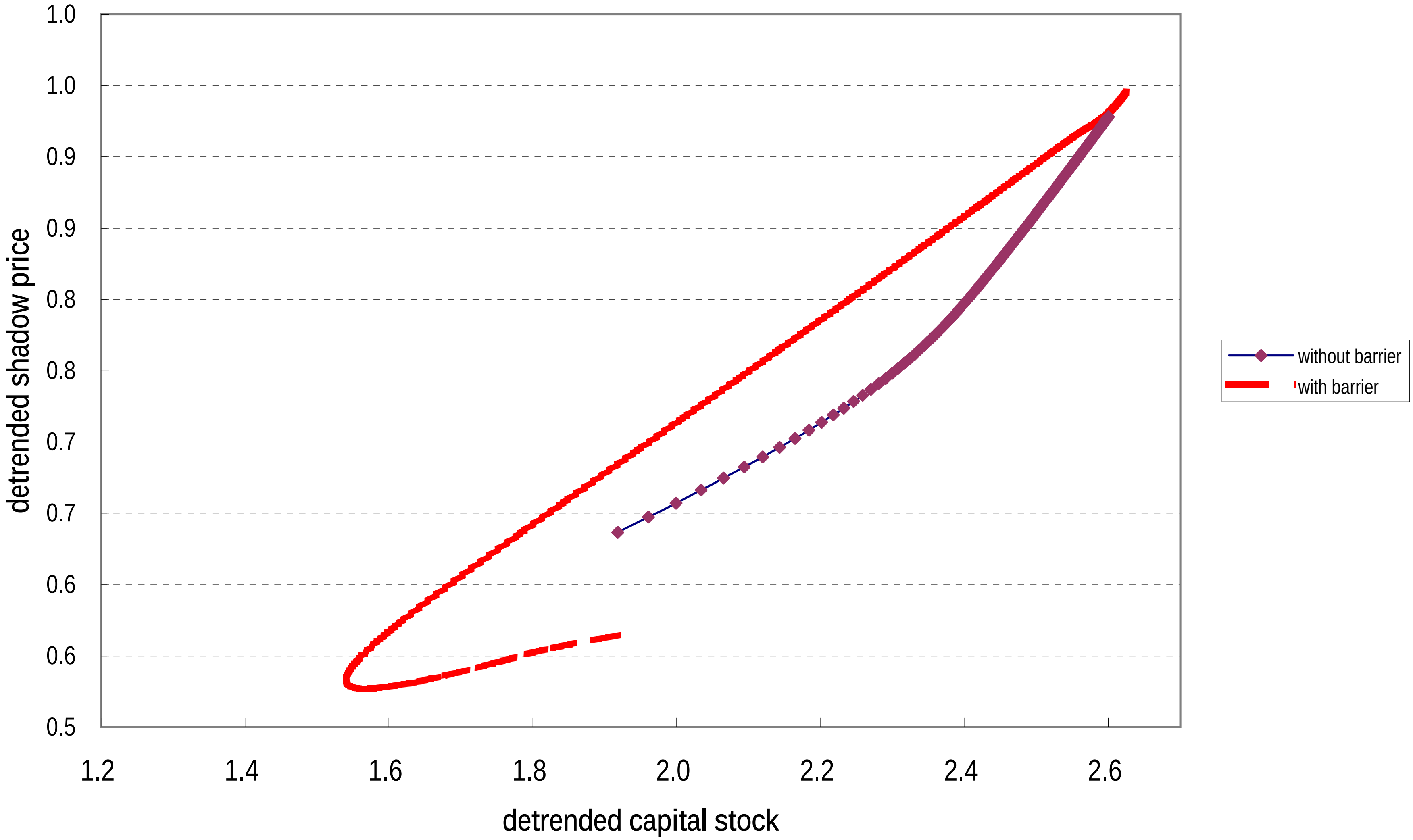
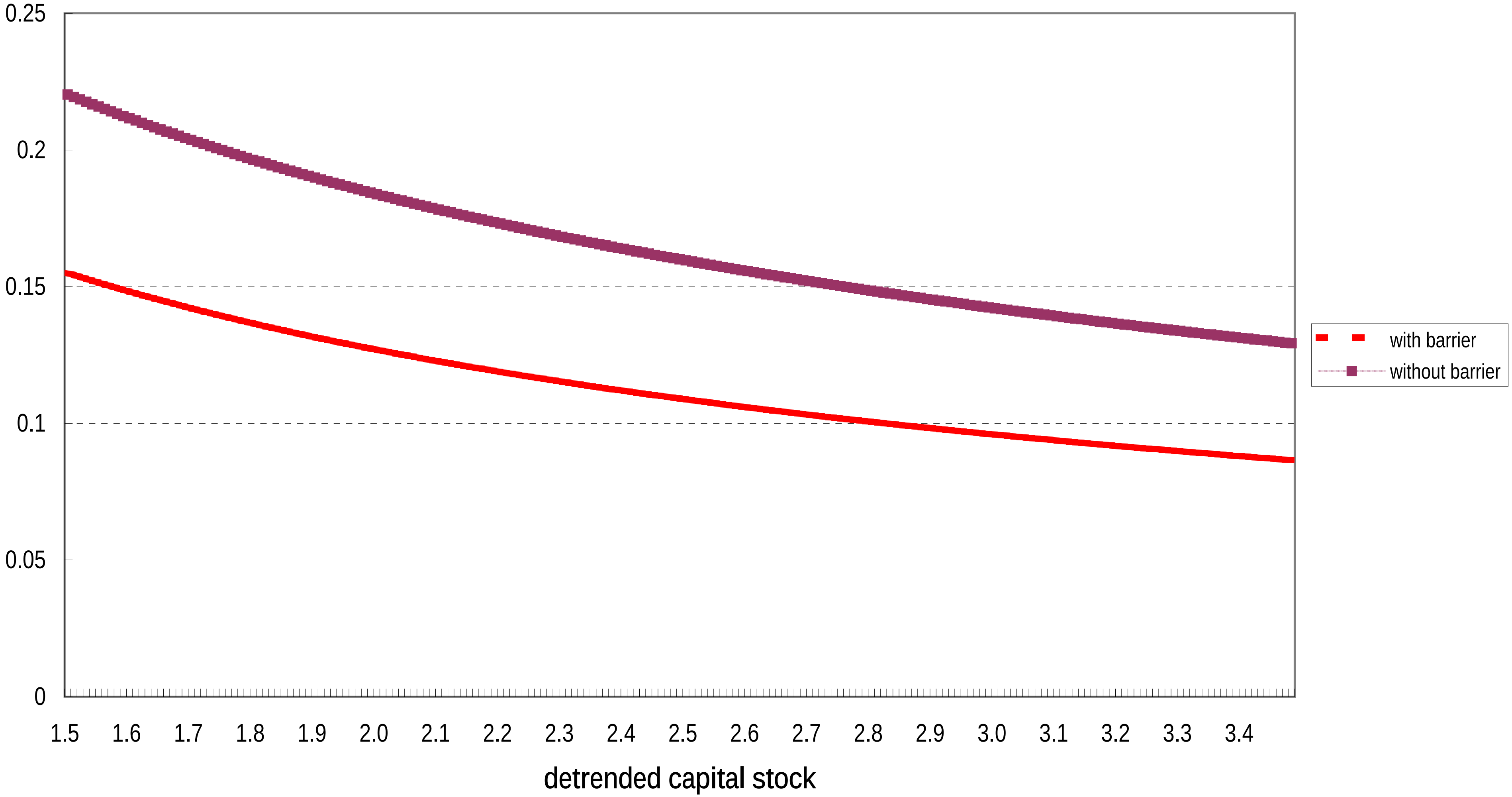


Figure 10: Marginal Productivity of Capital



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