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ABSTRACT

We develop a theory of *sweat equity*—the value of business owners' time and expenses to build customer bases, client lists, and other intangible assets. We discipline the theory using data from U.S. national accounts, business censuses, and brokered sales to estimate a value for sweat equity for the private business sector equal to 1.2 times U.S. GDP, which is roughly the value of fixed assets in use in these businesses. Although latent, the equity values are positively correlated with business incomes, ages, and standard measures of markups based on accounting data, but not with financial assets of owners or standard measures of business total factor productivity (TFP). We use our theory to show that abstracting from sweat activity leads to a significant understatement of the impacts of lowering business income tax rates on both the extensive and intensive margins. We also document large differences in the effective tax rates and the effects of tax changes for owner and employee labor inputs. Lower tax rates on owners result in increased self-employment rates and smaller firm sizes, whereas lower rates on employees have the opposite effect. Allowing for financial constraints and superstar firms does not overturn our main findings.

Keywords: Intangibles, business valuation

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1. Introduction

Private businesses in the United States now account for over 60 percent of yearly business net income.[1] Empirical evidence suggests that a significant part of this income is compensation for time that owners devote to building sweat equity in their business in the form of valuable client lists, customer bases, brands, and other intangible assets. Beyond providing current income to owners, these investments also generate capital gains when the business is eventually sold and intangible capital is transferred. Because standard theories in public finance abstract from sweat equity, they miss potentially important margins of substitution in response to both business and labor tax reforms. In this paper, we develop a theory in which this investment is a central feature and use it, along with U.S. national accounts and business census microdata, to measure net incomes and sweat equity in private business. Once measured, we quantify its role for tax policy reform.

As evidence of the labor content of business income, Smith et al. (2019) use tax filings to show that business income falls substantially after owner retirements and premature deaths, implying that much of the income is a return on the owner's time. Aggregating across pass-through entities, they estimate that three-quarters of the business profits are actually payments to labor.[2] If the payments were purely compensation to nontransferable human capital, at the time a business sells, we would observe that only financial and fixed assets are transferred. Instead, using data from brokered sales compiled by Pratt's Stats, we find that a large fraction of transferred assets are categorized by the Internal Revenue Service (IRS) as Section 197 intangible assets. These assets include customer- and information-based intangibles, trademarks, trade names, franchises, contracts, patents, copyrights, formulae, processes, designs, patterns, noncompete agreements, licenses, permits, and goodwill. We use a sample of 6,855 sales of private businesses over the period 1994–2017 to construct ratios of intangible asset values to the total assets—what we call the *intangible intensity*. We find an average intensity of 58 percent and a median of 64 percent. The remaining value is attributed to cash, trade receivables, inventories, fixed assets, and real

[1] Bhandari et al. (2019) document a 60.6 percent share of taxable net income and a 65.5 percent share of post-audit net income for private U.S. businesses over the period 2004–2014.

[2] Most private businesses are pass-through entities—sole proprietorships, S corporations, or partnerships—that distribute all earnings to owners. They report business net incomes on their individual tax returns and account for 57 percent of post-audit business net income over the period 2004–2014. See also Koh, Santaeulalia-Llopis, and Zheng (2018), who provide evidence of a stable U.S. labor share after recategorizing part of business profits as labor compensation.

estate. These estimates are robust to conditioning on legal structure, industry, and firm size. (See Bhandari and McGrattan 2019 for full details.)

To measure these intangible assets for ongoing concerns, we develop a dynamic general equilibrium lifecycle model with privately and publicly held businesses. We explicitly model the time use of private business owners. Besides leisure, they put time into two activities: production of goods and services and accumulation of sweat capital, which consists of building their client lists, customer bases, goodwill, and so on. Sweat capital is an input of production, along with tangible capital, owner hours, and employee hours. The income generated from sweat capital can be thought of as dividends whose present value is the sweat equity we are interested in measuring. During each period over their life cycle, individuals choose to run their own business or work for another business—either private or public—with this occupational choice driven primarily by stochastic productivities in each activity, accumulated sweat capital, and tax policy. We assume nonsweat capital can be rented and external labor hired, and therefore the main start-up costs are the time and expenses of the owner for accumulating sweat capital, an asset that is not pledgeable.

To parameterize the model, we use aggregate data from the U.S. national income and product accounts (NIPA), panel data of business incomes and wages from the IRS, cross-sectional data on business ages and owner hours from the U.S. Census Survey of Business Owners (SBO), and intangible share of business valuations from Pratt's Stats. Two key parameters in our model that have not been estimated in other studies are the share of sweat capital in private business production and the deterioration of this capital when the current owner is not actively using it. These parameters are identified by matching model and data intangible intensities and business age profiles. Also novel here is the estimation of private business owners' labor income as a share of GDP using data from NIPA and the IRS. This exercise is relatively straightforward for pass-through business owners, for whose income we estimate a share of 9 percent of GDP. Imputations are needed in the case of private C corporation owners, for whose income we estimate a share of roughly 2 percent of GDP. The present value of this labor income is partly a wage payment for time in production and partly the value of the intangible assets in the business.

Conceptually, our measure of sweat equity is the shadow value for a hypothetical mutual fund that passively invests in all potential private business owners, reaping the net returns after paying owners for their labor in producing private goods and services. For all private businesses, we estimate an aggregate sweat equity value of 1.2 times GDP, which is roughly equal to the value of tangible assets in use in these businesses and about 84 percent of the market capitalization of publicly held corporations. This measure includes both transferable wealth in the form of sweat capital and nontransferable wealth in the form of an owner-specific endowment of productivity to run a business. Values reported in brokered sales or business surveys would include only transferable assets. To estimate this value for ongoing concerns, we survey owners in our model at a point in time and ask at what price they would be willing to sell the transferable sweat capital. We find an average value for current owners that is 30 percent of the sweat equity value. We also find that the dispersion in this estimate of wealth is far greater than in that of sweat equity values.

Although we cannot directly observe the sweat capital or the implied sweat equity of ongoing businesses, the latent capital stocks are positively correlated with some observable variables that could serve as useful proxies. For example, we find that the sweat capital is positively correlated with business incomes and ages, since production cannot occur without clients or customers. Sweat capital is also positively correlated with standard measures of markups—sales relative to variable costs—if expenses are incurred when building the client list. This is true even though there are no actual markups in the baseline model. We find a negative correlation with standard measures of TFP that only count the tangible capital stocks and no relation with financial assets even when we allow for working capital constraints.

We investigate the quantitative role of including owner time in production and sweat capital accumulation for the study of business taxation. Specifically, we compare the predictions of lowering taxes on incomes of privately and publicly held businesses in our baseline model to those of a nested model in which owner time is fixed and there is no sweat capital. This nested model is the span-of-control model of Lucas (1978), which has become the standard framework in the literature on entrepreneurial choice. As compared with no-sweat, Lucas type models, we find much larger effects of lowering tax on private businesses in our baseline model. The no-sweat model has a negligible intensive margin effect because a tax on business income ultimately falls on the return to a *fixed* managerial input and hence is not distortive. Introducing sweat, we find a large effect on the intensive margin as owners work longer hours and hire less outside labor. Despite these large effects, we find the short-run responses and implied labor-supply elasticities of most owners in line with empirical estimates. There are two main reasons for this result. First, owner time in our model is a non-traded input, and hence the shadow wage (proxied by the disutility of supplying hours) suffers a larger adverse incidence when taxes are lowered and owners work more. Second, owners can increase production hours by spending less time building sweat capital. Even on the extensive margin, we find larger effects than in Lucas (1978). Allowing for sweat investment leads to increased returns in other factors and amplifies incentives to enter when taxes on businesses incomes are lowered.

If we additionally lower tax rates on corporate profits, then we find larger responses for the effects on the private business sector across the models with and without sweat activity but similar predictions for the effects on the C-corporate sector and aggregated data. Across the business owners, we find that most of the changes are attributed to businesses that have high productivities and large sweat capital stocks. Although true productivities are exogenous and true markups are constant, standard measures of total factor productivities and markups are significantly higher after the tax change.

Since private business income is primarily labor income, we compare the economic effects of taxing owner net incomes versus employee earnings. Lowering tax rates on owner time leads to a shift in the business labor force: owners put in more of their own time producing goods and services and hire fewer paid employees. In effect, the tax change results in more firms that are smaller in scale. Lowering tax rates on employee time leads to fewer owners since owners can make more working for someone else. However, the owners that find paid employment more attractive are not the most productive in business. Even with lower taxes on paid employment, the very highly productive owners would still find it optimal to run a business. With fewer owners and continued demand for the goods they produce, relative prices rise, and more outside labor and capital are used to meet that demand. In effect, the tax change results in fewer firms that are larger in scale.

Our paper is related to studies of small businesses and entrepreneurship. There are now many quantitative theories of entrepreneurship. Most model entrepreneurs as owners of physical capital subject to uninsurable idiosyncratic risk and financing constraints. See, for example, Angeletos and Calvet (2006), Boar and Midrigan (2019), Buera (2009), Cagetti and De Nardi (2006), Dyrda and Pugsley (2017), Li (2002), Meh (2005), Peter (2019), and Quadrini (1999, 2000). These studies focus mainly on the role of financial frictions in accounting for dispersion in survey-based measures of wealth and income.[3] We include working capital constraints disciplined by estimates of available funds to value added but find they have a negligible effect on the results our tax analyses. We also include superstar firms—whose owners earn 10 times the median labor income—and the model can generate large wealth ginis without assuming extreme productivity differences in the distribution. (See Castaneda, Diaz-Gimenez, and Rios-Rull, 2003.)

Another related line of research models entrepreneurial choices as driven by the nonpecuniary benefits of owning a business. See, for example, Hamilton (2000), Moskowitz and Vissing-Jorgensen (2002), Hall and Woodward (2010), and Hurst and Pugsley (2011, 2017). This literature is informed by survey responses of small-business owners and evidence that these owners have lower accumulated earnings over time than they would have had if they had worked for someone else and made fewer risky investments. We find that differences in the effective marginal tax rates of business owners and wage earners can account for almost all differences in pre-tax earnings. Thus, altering preferences to include a role for nonpecuniary benefits does not alter our main quantitative findings.

None of the studies on entrepreneurial choice explicitly model the accumulation of the business owner's sweat in building the business and therefore cannot be used to estimate aggregate or cross-sectional valuations of this key business asset or its role for tax policy reform.[4]

2. Theory

In this section, we develop a theory to measure sweat equity in private businesses and to serve as a tool for the study of tax policy counterfactuals. We start with an overview of the environment

[3] The literature on factor misallocation uses similar theories of entrepreneurs to quantify cross-country differences in aggregate productivity. See, for example, Buera and Shin (2013), Midrigan and Xu (2014), and Restuccia and Rogerson (2008), as well as Hopenhayn's (2014) survey for a complete list of references.

[4] In other literatures, researchers model investments in intangible capital—including brand and customer capital—to study trade patterns, asset pricing, firm dynamics, and business cycles, but they do not model the time-use decisions of private business owners. See, for example, Arkolakis (2010), Atkeson and Kehoe (2005), Belo, Lin, and Vitorino (2014), Drozd and Nosal (2012), Gourio and Rudanko (2014), and McGrattan and Prescott (2010a,2010b).

and then turn to a full description of the dynamic programs solved by different agents in the economy.

2.1. Overview

The economy is populated with individuals who age stochastically and are endowed with skills that govern their productivities in running businesses and paid employment. Over the life cycle, they make occupational choices: they earn either wage income as employees or business income as owners of private firms.

We assume that there are two business sectors: publicly held C corporations and privately held pass-through businesses that sell goods and services.[5] Businesses in the two sectors differ in their technologies, exposure to idiosyncratic risk, and tax treatment. Moreover, the goods produced in the two sectors are imperfectly substitutable.

Publicly held C corporations are assumed have fully diversified ownership. They use fixed assets as well as the time of paid employees as inputs to a constant-returns production technology. Their owners are assumed to be outside shareholders who do not work in the business. In the case of private firms, owners bear idiosyncratic risk and put time into producing goods and services and into building sweat capital—the business customer base, client list, and other non-pledgeable intangible assets.[6] In addition to owner sweat, private firms use fixed assets and time of paid employees.

Business incomes in the two sectors face different tax treatment. C corporations pay corporate income tax on profits, and the shareholders pay individual income tax on any distributions, while pass-through entities distribute all profits to their owners, who pay individual income taxes on the proceeds.

There is a competitive intermediation sector with risk-neutral financial intermediaries, who

[5] In the United States, there are privately-held C corporations for which we have very limited data from the IRS. When calibrating the model, we separate businesses into C corporations and pass-through businesses, but later use the limited data that we have to impute sweat equity valuations for privately-held C corporations.

[6] Much of C-corporate intangible investment does show up in the national accounts as intermediate purchases or employee compensation. A good example of the latter is wage compensation to R&D scientists.

accept deposits and use the funds to purchase equities of publicly held firms and government bonds and fixed assets that they rent to private firms.

Finally, there is a nonbusiness sector that includes production by the government, households, and nonprofit institutions that primarily serve households. Government purchases are financed by taxes on consumption, individual incomes, and business incomes.

We next turn to a formal description of this environment.

2.2. Occupational Choice

At a point in time, the state vector s for any individual—whether an entrepreneur or an employee—includes financial assets a, sweat capital κ, the productivity of working for someone else ϵ, the productivity of running a business z, and age j. The occupational choice of an individual is made to maximize the overall value:

$$V_j(s) = \max\{V_{j,p}(s), V_{j,w}(s)\},$$

where $V_{j,p}(s)$ is the value to running one's own *private* business, and $V_{j,w}(s)$ is the value to *working* for those in age group j. To keep the life cycle problems tractable, we allow for stochastic aging between young ages, $j = y$, and old ages, $j = o$ as in Blanchard (1985). We also assume that individuals spend some fraction of their life in paid employment and another in self-employment; the spells do not overlap. In reality, some individuals do both activities simultaneously: they work for someone else and run a business. However, data on time use show that average hours on the primary job are much higher than on the secondary job.

Individuals who run a private business make decisions related to both their business and personal finance. The dynamic programming problem for young owners is:

$$V_{y,p}(s) = \max_{\substack{c_c, c_p, a', h_y, \\ h_\kappa, k_p, n_p}} \{U_p(c(c_c, c_p), \ell) + \beta \sum_{\epsilon', z'} \pi(\epsilon', z' | \epsilon, z) V(s')\} \tag{2.1}$$

subject to

$$\begin{aligned} a' = [&(1+r)a + py_p - (r+\delta_k)k_p - wn_p - e - (1+\tau_c)(c_c + pc_p) \\ &- T^b(py_p - (r+\delta_k)k_p - wn_p - e)]/(1+\gamma) \end{aligned} \tag{2.2}$$

$$\kappa' = \left[\left(1-\delta_\kappa\right)\kappa + f_\kappa\left(h_\kappa, e\right)\right] / \left(1+\gamma\right) \tag{2.3}$$

$$y_p = z f_y\left(\kappa, k_p, n_p, h_y\right) \tag{2.4}$$

$$\ell = 1 - h_\kappa - h_y \tag{2.5}$$

$$a' \geq \chi p y_p, \tag{2.6}$$

where $V_{y,p}(s)$ is the discounted present value of utility for an individual of type "young private business owner" (y, p) with assets a, sweat capital κ, productivities (ϵ, z), transition probabilities $\pi(\cdot|\cdot)$, and continuation value V. Utility depends on consumption of goods produced in both C corporations, c_c, and private firms, c_p—which could be imperfectly substitutable—and leisure ℓ. Asset holdings carried to next period, a', are equal to incomes from current financial asset holdings that earn an after-tax interest rate of r and business net income after subtracting consumption expenditures and net tax payments. The term $(1+\gamma)$ in (2.2) appears because all nonstationary variables are detrended by the economy-wide growth rate $(1+\gamma)^t$. Business net income before tax is computed as total sales py_p less rental payments for marketable fixed assets, rk_p; depreciation, $\delta_k k_p$; employee wages, wn_p; and expenses, e. Owners take the goods price, p; the interest rate, r; and the wage rate, w, as given when solving the maximization problem (2.1). Business owners use the schedule $T^b(\cdot)$ to determine taxes on business income net of transfer payments. They also pay a tax of τ_c on consumption.

Owners allocate nonleisure hours between growing their businesses, h_κ, and producing goods and services, h_y. These hours enter the production functions for sweat capital (2.3) and goods and services (2.4), respectively. We assume that owners cannot produce without sweat capital—that is, $f_y(0, k_p, n_p, h_y) = 0$. We have in mind that the business needs customer or client lists before producing goods and services for customers and clients. These lists accumulate with owner time and expensing, as in (2.3), and are potentially transferable through inheritance or sales, while productivity z is specific to the owner. This is a key distinction of our model relative to entrepreneurial choice models in the tradition of Lucas (1978) and human capital models in the tradition of Ben Porath (1967). The constraint (2.6) on assets for the business owners depends on the term $\chi p y_p$, which can be interpreted as a working capital constraint for business owners.

Our formulation ignores the transitions of businesses from the private sector to the public sector. This is informed by studies of firm dynamics that use panel data (see Cole and Sokolyk (2018)) and conclude that firms' choice of the legal form of organization is largely set in stone at inception. [7]

The problem of working for someone else is relatively standard. (See, for example, Aiyagari (1994), Imrohoroglu et al. (1995), and Huggett (1996).) In this case, the individuals choose consumption of C-corporate goods and services, c_c, consumption of private firm goods and services, c_p, leisure ℓ, and financial assets next period a'. The problem solved by young workers is given by:

$$V_{y,w}(s) = \max_{c_c, c_p, \ell, a'} \{U_w(c(c_c, c_p), \ell) + \beta \sum_{\epsilon', z'} \pi(\epsilon', z'|\epsilon, z) V(s')\} \tag{2.7}$$

subject to

$$a' = [(1+r)a + w\epsilon n - (1+\tau_c)(c_c + pc_p) - T^w(w\epsilon n)]/(1+\gamma) \tag{2.8}$$

$$\kappa' = (1-\lambda)\kappa, \tag{2.9}$$

$$\ell = 1 - n \tag{2.10}$$

$$a' \geq 0, \tag{2.11}$$

where $V_{y,w}(s)$ is the discounted present value of utility for an individual of type "young worker" (y, w) with state $s = \{a, \kappa, \epsilon, z\}$, transition probabilities $\pi(\cdot|\cdot)$, and continuation value $V(s)$. These individuals earn asset income that pays after-tax interest r and wages that earn w per effective hour regardless of whether they work for a private or public firm. The net tax schedule for wages is given by $T^w(\cdot)$, and consumption expenditures are taxed at rate τ_c. Workers who have previously run or inherited a business may have accumulated sweat capital. If $\lambda > 0$, the value of this capital deteriorates while not in use.

2.3. Continuation values

Productivity levels are permanently lower in old age: the shocks ϵ and z are replaced by $\zeta\epsilon$ and ζz, with $\zeta < 1$. The dynamic programming problems with value functions $V_{o,p}$ and $V_{o,w}$ for older

[7] There are, of course, nonzero public listings (IPOs), and our estimates of aggregate sweat equity will miss the compensation to owners' sweat that is realized in an IPO. However, IPO activity is small in the aggregate and accounts for about 0.2 percent of GDP. Also, pass-through forms such as S corporations are restricted from issuing shares with differential voting rights. Therefore, nearly all IPO activity, including the pre-IPO venture financing or angel investing, occurs via non-pass-through firms. Adding a listing decision is an interesting extension to our framework, but not a focus of this study.

private business owners and workers are formulated in the same way as for younger individuals with modified continuation values.

The continuation values in (2.1) and (2.7) are given by

$$V(s') = \pi_y V_y(s') + (1-\pi_y) V_o(s'), \tag{2.12}$$

where π_y is the probability of remaining in the young state and s' is the state next period. The analogue for older individuals is

$$V(s') = \pi_o V_o(s') + \iota(1-\pi_o) \frac{\pi(\epsilon', z')}{\pi(\epsilon', z'|\epsilon, z)} V_o(s'), \tag{2.13}$$

where π_o is the probability of remaining in the old state and $1-\pi_o$ is the probability of dying. The parameter $\iota \in [0,1]$ is a measure of altruism. Upon death, the next generation receives all assets a and a share $1-\lambda_d$ of the sweat capital but draws new productivity levels—which is why the ratio of probabilities $\pi(\epsilon, z)/\pi(\epsilon', z'|\epsilon, z)$ appears in the formula (2.13). Later, we extend the model to allow for the transfer of sweat capital through brokered sales as well as through inheritance.[8]

2.4. C Corporations

There is a competitive C-corporate sector with firms choosing hours n_c and fixed assets k_c to solve the following dynamic program:

$$v_c(k_c) = \max_{n_c, k_c'} \{(1-\tau_d) d_c + \frac{(1+\gamma)}{(1+r)} v_c(k_c')\}$$

subject to

$$k_c' = [(1-\delta_k) k_c + x_c] / (1+\gamma)$$

$$y_c = AF(k_c, n_c)$$

$$d_c = y_c - w n_c - x_c - \tau_p (y_c - w n_c - \delta_k k_c),$$

where d_c are corporate dividends that are taxed at rate τ_d after paying corporate income taxes at rate τ_p, x_c is C-corporate investment, and y_c is C-corporate output from a constant returns to scale technology F with TFP given by A. Employees working for C corporations earn the same hourly wage, w, as employees in private businesses.

[8] Since few U.S. businesses sell in any given year, our main quantitative results are hardly affected, but adding this feature requires specification of the intermediary's problem, additional notation, and additional parameters. Thus, we include this case with our sensitivity analysis.

2.5. Financial Intermediary

There is a competitive intermediation sector with risk-neutral financial intermediaries that accept deposits and use the funds to invest in C-corporate equities, government bonds, and fixed assets.

At the beginning of each period, the net worth of an intermediary is the value of its equity shares ς, bonds b, and fixed assets k, less the value of deposits owed to households a. During the period, the intermediary receives dividend income from C corporations, interest income from bonds, and rental income on fixed assets. It also pays interest on deposits. The dynamic program in this case is:

$$v_I(x) = \max_{x'}\{d_I + \frac{(1+\gamma)}{(1+r)} v_I(x')\}, \tag{2.14}$$

where the state vector is $x = [\varsigma, b, k, a]'$. The intermediary dividends d_I, income y_I, and net worth nw are as follows:

$$\begin{aligned} d_I &= y_I + (1-\delta_k)\,k + nw - (1+\gamma)\,nw' \\ y_I &= (1-\tau_d)\,d\varsigma + rb + (r+\delta_k)\,k - ra \\ nw &= q\varsigma + b + k - a, \end{aligned}$$

where q is the per-share price of corporate equities. Free entry into the intermediary sector means that the present value $v_I(x)$ is equal to zero.

2.6. Fiscal Policy

The government spends g; borrows b; and collects taxes on consumption at rate τ_c, labor earnings with schedule T^w, private business income with schedule T^b, C-corporation dividends at rate τ_d, and C-corporation profits at rate τ_p. The government budget constraint is given by

$$\begin{aligned} g + (r-\gamma)\,b = \tau_c \left(\int c_c(s)\,ds + \int p c_p(s)\,ds\right) + \int T^w\left(w\epsilon(s)\,n(s)\right) ds \\ + \int T^b\left(p y_p(s) - (r+\delta_k)\,k_p(s) - w n_p(s) - e(s)\right) ds + \tau_p\left(y_c - w n_c - \delta_k k_c\right) \\ + \tau_d\left(y_c - w n_c - (\gamma+\delta_k)\,k_c - \tau_p\left(y_c - w n_c - \delta_k k_c\right)\right). \end{aligned} \tag{2.15}$$

Here again, we assume that all variables are divided by the technological trend growth.

2.7. Equilibrium

A stationary recursive competitive equilibrium is value functions $V_{y,w}$, $V_{y,p}$, $V_{o,w}$, and $V_{o,p}$; policy functions a', κ', c_c, c_p, ℓ, n, k_p, n_p, h_y, h_κ, and e; C corporation choices n_c, k_c; prices r, w, p; and a measure over types indexed by the state s and age j such that

- given prices, the policy functions for employees—namely, a', κ', c_c, c_p, ℓ, and n—solve dynamic programming problems associated with value functions $V_{y,w}$ and $V_{o,w}$;
- given prices, the policy functions for private business owners—namely, a', κ', c_c, c_p, ℓ, k_p, n_p, h_y, h_κ, e—solve dynamic programming problems associated with value functions $V_{y,p}$ and $V_{o,p}$;
- given prices, the policy functions for C corporations—namely, n_c and k_c'—solve the dynamic programming problem associated with v_c;
- given prices, the policy functions for financial intermediaries—namely, $x = [\varsigma, b, k, a]'$—solve the dynamic programming problem associated with v_I;
- the labor market clears: $n_c = \int (n(s)\epsilon(s) - n_p(s))ds$;
- the asset market clears: $\int a(s)ds = b + (1-\tau_d)k_c + \int k_p(s)ds$;
- the private business goods market clears: $\int y_p(s)ds = \int c_p(s)ds$;
- the C-corporate goods market clears:

$$y_c = \int c_c\,(s)\,ds + \int e\,(s)\,ds + (\gamma + \delta_k)\left(k_c + \int k_p\,(s)\,ds\right) + g;$$

- the government budget constraint in (2.15) is satisfied;
- the measure of types over states (a, κ, ϵ, z) and ages (y, o) is invariant.

In specifying the asset market clearing condition, we have used the fact that different tax treatments for corporate and pass-through profits implies a relative price of fixed assets of $1 - \tau_d$.

2.8. National Accounts

When parameterizing the model, we will ensure that the implied shares of national incomes and products are aligned with data from the Bureau of Economic Analysis (BEA). In this section, we mathematically define and summarize these shares.

First, we need to introduce some notation. Let x_c and $\{x_p(s)\}$ be investments in fixed assets used in C corporations and private businesses, respectively. Let $\bar{x}_{nb}$ and $\bar{y}_{nb}$ be investments and outputs of the nonbusiness sector, which includes households, nonprofits, and government. The nonbusiness income less investment is included with household transfers but assumed to be exogenous. We do this to ensure that the model accounts can be directly compared to U.S. accounts.[9] Let y denote GDP, which is the sum of C-corporate output, y_c; private output less intermediate expenses, $\int(py_p(s)-e(s))ds$; and nonbusiness income, $\bar{y}_{nb}$. With these definitions, we can summarize the income and product shares as follows:

Income shares:

Sweat income	$\int(py_p(s) - (r+\delta_k)k_p(s) - wn_p(s) - e(s))ds)/y$
Nonsweat labor income	$w(n_c + \int n_p(s)ds)/y$
C corporations	wn_c/y
Private business	$w\int n_p(s)ds/y$
Business capital income	$((r_c+\delta_k)k_c + (r+\delta_k)\int k_p(s)ds)/y$
C corporations	$(r_c+\delta_k)/y$
Private business	$(r+\delta_k)\int k_p(s)ds/y$
Nonbusiness income	$\bar{y}_{nb}/y$

Product shares:

Private consumption	$(\int c_c(s) + pc_p(s))ds)/y$
Government consumption	g/y
Investment	$(x_c + \int x_p(s)ds + \bar{x}_{nb})/y$
C corporations	x_c/y
Private business	$\int x_p(s)ds/y$

[9] If we were to directly compare fixed assets of the model and data, we would also have to add nonbusiness capital in our measure of total fixed assets.

Nonbusiness $\bar{x}_{nb}/y$

The data analogue of *sweat income* is BEA proprietors' income—which includes incomes of sole proprietors and partners—plus IRS S-corporate compensation and business income from trade. From this, we subtract payments to capital owned by the businesses using information on rents and interest payments in IRS income statements. (See Bhandari and McGrattan (2019) for full details on this construction.) *Nonsweat labor income* is BEA business compensation less S-corporate compensation. *Business capital income* is BEA rental income, net interest, consumption of fixed capital, and corporate profits less IRS S-corporate business income from trade. *Nonbusiness income* is BEA labor and capital income attributed to factors in the household, nonprofit, and government sectors.

Next, consider the product side of the accounts. *Private consumption* is BEA personal consumption expenditures on nondurable goods and services.[10] *Public consumption* is government consumption of goods and services. Finally, data on investments and nonsweat capital stocks by legal entity is available from the BEA fixed asset tables.

3. Model Parameters

In this section, we set parameters of preferences, technologies, stochastic processes, and government policies to match key moments for U.S. aggregate data and microsamples of businesses. Our primary data sources are the BEA for NIPA and fixed asset tables, the Bureau of Labor Statistics (BLS) for time use data, the SBO for characteristics of businesses and owners, the IRS for incomes and tax rates, and Pratt's Stats for data on assets transferred in brokered sales.[11]

3.1. Functional Forms

We start with our functional form choices for the utility function $\{U_w(\cdot), U_p(\cdot)\}$, the production technology $F(\cdot)$ of C corporations, and the production technologies $f_\kappa(\cdot)$ and $f_y(\cdot)$ available to

[10] BEA personal consumption expenditures and capital incomes must be adjusted by adding imputed services for durables and subtracting sales tax.

[11] For compatibility with the SBO microsample on business, we use data for 2007 where possible. We also check that nothing changes if we average over more years.

private businesses—namely,

$$U_w(c,\ell) = \left(c\ell^{\psi}\right)^{1-\sigma} / (1-\sigma)$$

$$U_p(c,\ell) = U_w(c,\ell) + \xi$$

$$c(c_c, c_p) = c_c^{\eta} c_p^{1-\eta}$$

$$F(k_c, n_c) = k_c^{\theta} n_c^{1-\theta}$$

$$f_{\kappa}(h_{\kappa}, e) = h_{\kappa}^{\vartheta} e^{1-\vartheta}$$

$$f_y(\kappa, k_p, h_y) = \kappa^{\phi} k_p^{\alpha} h^{\nu}$$

$$h(h_y, n_p) = \left(\omega h_y^{\rho} + (1-\omega) n_p^{\rho}\right)^{\frac{1}{\rho}},$$

where $\phi + \alpha + \nu = 1$ and ξ captures nonpecuniary benefits from running a business. In addition to the parameters of these functions, we need to set depreciation rates δ_k, δ_κ, the discount rate β, the growth rate γ, and the rate of deterioration of sweat capital λ. Without loss of generality, we set the level of TFP in C-corporate production, A, so that y_c is normalized to 1 in equilibrium.

Baseline parameters are reported in Table 1. We next describe how we set them.

3.2. Preferences

Starting with the preferences reported in panel A of Table 1, we set $\psi = 1.38$ in order to match total hours of work for employees in C corporations, n_c; employees in private pass-through businesses, $\int n_p(s)ds$; and owners in private businesses, $\int (h_y(s) + h_\kappa(s))ds$.[12] The parameter η, which governs the consumption shares of publicly and privately produced goods, is set equal to 0.449 to ensure that noncorporate business income is 9 percent of GDP. For curvature in preferences, we use a standard estimate of the inverse of the intertemporal elasticity of $\sigma = 1.5$. The parameter ξ governs the nonpecuniary benefit of running a business and is typically chosen to deliver an earnings differential of roughly 35 percent between similar individuals—one who is in paid employment, and one who runs a business for at least 10 years. (See Hamilton 2000 and Hurst and Pugsley 2017.) In our case, the differentials in effective taxes on wage and pass-through income come close to

[12] According to data from the BLS, the noninstitutional population ages 16 to 64 was 197 million in 2007, and the average annual hours per person were 1,465. If each person has 5200 hours of annual discretionary time, then 28.2 percent of aggregate available time is allocated to work. Government, nonprofit, and household employees—which we include with nonbusiness activity—contribute roughly 5.8 percentage points, and thus business hours are assumed to be 22.4 percent of aggregate available time.

guaranteeing this difference in pre-tax returns to labor, and therefore we set $\xi = 0$ in our baseline model and check that variations do not affect our quantitative results. To match a 4 percent annual interest rate, we set $\beta = 0.98$.

3.3. Technologies

Next, consider parameters of technologies reported in panel A of Table 1. For aggregate growth in technology, γ, we use the U.S. trend rate of 2 percent. The fixed asset shares in C corporations and private pass-through businesses are set so as to ensure that k_c/y and $\int k_p(s)ds/y$ are roughly 2 times GDP and 1 times GDP, respectively, as is the case for the United States.[13] This disciplines our choices for the C-corporate share $\theta = 0.5$ and the private pass-through share $\alpha = 0.3$. The capital stocks also depend on our choice of the rate of depreciation. We use NIPA fixed asset tables, which include both flows and stocks, and set $\delta_k = 0.041$ to ensure the model values for investment rates x_c/k_c and $\int x_p(s)/k_p(s)ds$ are consistent.

The private business sector has two production technologies: one for accumulating sweat capital, f_κ, and one for producing goods and services, f_y. For f_κ, we do not have direct evidence of the shares and thus use indirect evidence from the BEA's benchmark 2007 input-output table on labor and intermediate shares in the advertising and related services sector (NAICS 5418). Based on these shares, we set $\vartheta = 0.418$. We use the same depreciation rate for fixed assets and sweat capital because there is no empirical analogue for the latter.[14] For f_y, we need to estimate share parameters for sweat capital and labor inputs as well as the elasticity of substitution between owner and employee time. Since NIPA construction of sweat income includes payments for both owners' hours in production h_y and accumulated sweat capital, κ, we need additional information to identify share parameter ϕ (and, residually, ν).

The additional information that we use to identify the sweat capital share, ϕ, is Pratt's Stats' broker data on sales of private businesses. We use our sample of 6,855 sales over the period 1994–2017 with records of the purchase-price allocation across different asset categories. Such records

[13] These estimates are based on an expanded notion of intellectual property product (IPP) investments, which are estimated at 12 percent of GDP, rather than those currently counted in NIPA, which are estimated at 4 percent of GDP. See Bhandari and McGrattan (2019) for details.

[14] In our sensitivity analysis, we check the robustness of this choice as well as our specification for f_κ.

are kept for tax purposes to determine the purchaser's basis in each acquired asset and the seller's gain or loss on the transfer of each asset. (See IRS Form 8594.) The asset categories include cash and deposit accounts, government securities, debt instruments, inventory, fixed assets and land, and intangible assets. The intangibles assets include both goodwill and all Section 197 intangibles, such as customer bases and trademarks. As we noted earlier, the intangible intensity—or ratio of the intangible asset value to total asset value—has a mean of 58 percent. We choose $\phi = 0.15$ in order to generate a comparable average ratio in the model (both here and when we add brokered sales). In the model, we compute the intangible intensity $\text{ii}(s)$ for a business with state s as follows:

$$\text{ii}\left(s\right) = \frac{v_{\kappa}\left(s\right)}{v_{\kappa}\left(s\right) + k_p\left(s\right)}, \tag{3.1}$$

where $v_\kappa(s)$ is the amount of cash needed to leave a business owner indifferent between continuing in business with sweat capital κ and selling it; that is, $v_\kappa(s)$ satisfies

$$V_{j,p}\left(s\right) = V_{j,w}\left(a + v_{\kappa}\left(s\right), 0, \epsilon, z\right). \tag{3.2}$$

In effect, $v_\kappa(s)$ is the value of transferable intangible assets.

One potential issue with using the intangible intensity based on broker data is that we may encounter selection bias, conditioning only on businesses that eventually sold. However, both in the data and in the model, we find that the intangible intensity is not systematically different when we condition on different business characteristics. As we document in Bhandari and McGrattan (2019), the average intangible intensity based on Pratt's Stats does not vary systematically with industry or firm size or indicators of distressed sales. Similarly, estimates in the model are not that different as we vary owner productivities. The reason is that productive owners increase their use of physical capital as they build their sweat capital in the business, and unproductive owners scale both down. Later, we formally model selection by extending the theory to allow for the buying and selling of sweat capital via a competitive broker. Recalibrating the extended model, we find a similar prediction for the average intangible intensity with ϕ equal to 15 percent.

Given this value for ϕ, the share of labor (owner plus employee time) is $\nu = 0.55$, and the predicted hours of work for business owners, $\int (h_y(s) + h_\kappa(s))ds$, are roughly 23 percent of all hours worked in business. This prediction is close to the 25 percent estimate based on data available from the 2007 SBO microsample. The share parameter governing owner and employee hours, ω, is set equal to 0.425 in order to generate a prediction that 33 percent of aggregate employee compensation is paid by pass-through businesses, consistent with NIPA. The parameter ρ governs the elasticity of substitution between owner and employee hours in private business; the more substitutable they are, the greater the opportunity is for an owner to scale up the business if productivity is high. We set $\rho = 0.5$, which implies variation in payroll share per owner hour that is consistent with the 2007 SBO microsample.

Both the income share on sweat capital and the deterioration rate λ of this capital outside of the business play a quantitatively important role for the age profile of businesses because both affect the option value for business owners. From the SBO, we have information about the year business owners started or, in the case of nonfounders, acquired their share of the business. We use this information to compute a profile of business ages and compare it to model predictions. The higher the share ϕ of sweat capital in production (and the lower ν is), the longer the duration of benefits from sweat in the business. The higher the deterioration rate λ, the more costly it is to exit and reenter the business sector. This cost naturally lengthens the age of business. Given a value for ϕ has been identified using information on intangible intensities of businesses, we set the deterioration rate λ equal to 0.5 to ensure consistency of the model and data on the age profile.

3.4. Life Cycle

There are several parameters governing life cycle patterns, which are chosen to match overall U.S. population statistics and those in business. The stochastic aging parameters are set equal to $\pi_y = 0.978$ and $\pi_o = 0.933$ to ensure that one-fourth of the model population is over 65, with the average duration of working years at 45. We set the old-age productivity shock to $\zeta = 0.5$ and the sweat deterioration rate at death to $\lambda_d = 0.9$ in order to match business age profiles for young and old owners. The parameter ι was set to 1, implying that parents are fully altruistic.

3.5. Financing

We have one parameter related to business financing, which is the tightness of the working capital constraint in (2.6). Based on the work of Hurst and Lusardi (2004) and Chari (2014), we set χ in our baseline model to zero. Using surveys of businesses, Hurst and Lusardi (2004) found no relationship between wealth and business entry, with the exception of those at the very top of the wealth distribution. Chari (2014) constructed time series estimates of available funds and investments using business data from NIPA, Compustat, and the IRS and, found that available funds for large and small firms were higher than the total invested for virtually every year. Later, in our sensitivity analysis, we set χ equal to the maximum observed ratio of available funds to value added found in the samples Chari (2014) considers and show that our results are unchanged.

3.6. Productivity processes

The productivity shocks are modeled as uncorrelated Markov chains with the states and transition matrices for ϵ_t and z_t shown in Table 1, panel B. The transition matrix for ϵ_t is consistent with the estimated wage processes of Low, Meghir, and Pistaferri (2010) for U.S. households in U.S. Census Survey of Income and Program Participation (SIPP). To construct this matrix, we take a panel of simulated wages from their estimated quarterly model, annualize the simulated data, and then run a fixed effect regression of log wages on one lag and a set of controls (namely, age, age squared, education, and their interactions). We use the estimate of the coefficient on the log of lagged wages (0.7) and our estimate of the standard deviation of the regression residuals (0.16) and apply the method of Tauchen (1986) to estimate the Markov chain shown in Table 1, panel B.

The transition matrix for z_t is taken from Debacker, Panousi, and Ramnath (2013), who use a panel of businesses in the IRS Statistics of Income subsample to construct transitions for business incomes. We use the same estimates for our productivity transition matrix and find that the implied transition matrix for business income is not significantly different. For the z grid, we face the challenge that the upper income bracket in Debacker, Panousi, and Ramnath (2013) is top-coded to protect privacy. Since we know the income distribution is skewed, we use a squared log-normal autoregressive process with the variance chosen to generate the 90th percentile business income relative to the median wage income as in Debacker, Panousi, and Ramnath (2013). We view our choice of z grid as conservative. Later, in our sensitivity analysis, we introduce a small number (1 percent) of superstar owners whose incomes are 10 times larger than the median wage earner's and show that the differences between model predictions with and without sweat activity are even greater when the skewness of the productivity process is increased.

3.7. Tax Rates

The third set of parameters are the tax rates and schedules reported in panel C of Table 1. We use effective rates based on NIPA government revenues and IRS data. The tax rate on consumption, $\tau_c = 0.065$, is found by dividing total sales and excise taxes in NIPA by personal consumption expenditures. To compute the tax rate on profits, we use information on domestic earnings from IRS federal and state returns and foreign earnings from the U.S. balance of payments. The statutory federal tax rate on domestic C-corporate profits is 35 percent, but there is a 9 percent deduction for firms that qualify for the domestic production deduction. From NIPA data, we know that 15 percent of corporate tax revenues are paid to the states. Using this information, we compute a 40 percent tax rate on domestic profits for 2007. To compute an effective rate on all profits, we use data on global tax rates from KPMG International and BEA balance of payment data on foreign corporate earnings. In 2007, 27 percent of profits were earned abroad with a weighted average tax rate of 25 percent. Thus, our estimate for the effective tax rate on profits, τ_p, is 36 percent.

To construct tax rates on dividends, wages, and sweat income to business owners, we use data on taxable incomes from the IRS individual income tax returns reported in the Statistics of Income (SOI) and available by source of income and by size of adjusted gross income (AGI). Taxable incomes and information on marital status are used to construct weighted federal marginal tax rates for each AGI income bracket, with weights equal to the fraction of returns filed as married filing jointly, married filing separately, single, and heads of households. For the federal tax rate on dividends, we compute an average marginal rate using the same procedure as in Barro and Redlick (2011). Specifically, if households in AGI income group i pay τ_i on an additional dollar of income and earn $y_i/\sum_i y_i$ of the total income, then the average marginal rate is $\hat{\tau} = \sum_i \tau_i y_i / \sum_i y_i$. The tax rates τ_i are themselves weighted averages of rates on ordinary, qualified, and untaxed dividends, with weights equal to the fraction of dividends in each category. We use data for tax year 2007, which is the same year of the SBO microsample. In that year, owners of taxable accounts also paid roughly 5 percent in state and local taxes on dividend income. Untaxed dividends are held in pension funds and retirement accounts, which account for 44 percent of all equities owned by households. Adding federal, state, and local, we estimate a weighted marginal tax rate τ_d of 13.3 percent.

For the tax schedule $T^w(y)$ of employee labor income in (2.8), we compute the federal marginal tax rate on an additional dollar of wages and salaries for each AGI income bracket in the SOI. We add Federal Insurance Contributions Act (FICA) taxes for each bracket; in 2007, those with the lowest incomes paid 15.3 percent for Social Security and Medicare, while those above the Social Security cap paid 2.9 percent for Medicare. Additionally, we add a 4 percent tax rate for state and local taxes. In the model, the income of individual i, y_i, is defined as per working-age person, while the SOI incomes are reported per return. Thus, we divide the SOI incomes per return by the number of adults per return. The number of adults is proxied by total exemptions less exemptions for children at home. The result is a nonlinear function that is well approximated by a continuous piecewise linear curve, with an initial intercept that is set so that aggregate taxes net of transfers relative to GDP is consistent with U.S. accounts. In the last column of Table 1, panel C, we report marginal rates $T^{w\prime}(y)$ for tax year 2007. The income levels reported have been normalized by dividing each by GDP per working-age person. The IRS reports data for 20 AGI brackets, but we find that the tax function is well approximated by a piecewise function with only seven.

To estimate the tax schedule $T^b(y)$ of sweat income to business owners in (2.2), we require tax audit data in addition to taxable incomes reported in the SOI, since there is significant misreporting on business tax returns. To compute the federal marginal rate for a particular AGI interval, we estimate the tax paid on *reported* business income from all sources—namely, sole proprietorships, partnerships, and S corporations—for an additional dollar of *true* business income. To do this, we need estimates from audit data on misreporting for all three business entities. The General Accounting Office (2009, 2014) reviewed confidential findings from tax audits of S corporations and estimated that owners report 82 cents per dollar of business net income.[15] Johns and Slemrod (2010), using data from the National Research Program for tax year 2001, report that sole proprietors report 43 cents per dollar of income. To infer partners' misreporting, we use the BEA

[15] Note that S corporations also have an incentive to report wage income as a distribution to avoid payroll taxes. See Smith et al. (2017).

estimate of total misreporting of unincorporated businesses together with the estimate for sole proprietors from Johns and Slemrod (2010). With this information, we can infer that partners would have reported only 47 percent per dollar of income in the 2007 tax year. For the individual y_i values, we follow the same procedure as above and compute a piecewise linear schedule with per-person estimates normalized by GDP per person. The intercept in the schedule is chosen so that transfers for the median household are the same regardless of whether they earned business or wage income.

In panel C of Table 1, we report our estimates. The marginal tax rates for wages and salaries and business income include federal, state and local, and FICA obligations. What is noteworthy is how much lower the *effective* tax rates on owners of private business are than they are on employees or owners of C corporations, who pay taxes on dividends and corporate profits. On the spending side, we choose g to ensure that the share of spending on goods and services by the government in GDP (g/y) is roughly equal to the NIPA value.

3.8. Validation

With the baseline parameters in Table 1, we compute an equilibrium of the model and check that the implied national accounts and business age profiles are in fact aligned with U.S. data. Using income and product categories of Section 4.1, Table 2 reports the model and data accounts, which are, by our choice of parameters, intended to be close.[16] Figure 1 shows the age profile for businesses in the model and data.[17] The data are taken from the public-use micro-sample of the 2007 SBO. We find that 11 percent of owners started running their business in the year of the survey. For those who started more than four years before, we have only bracketed information and thus report the averages in the interval. By construction, our model is intended to be consistent with these data.

Given our model is now parameterized to match key statistics in U.S. data, we next turn to our main results and policy experiments.

[16] Given the number of moments to be matched, we accomplish this with the help of a Nelder-Mead optimization algorithm.

[17] Here, we display results for all owners but find similar results after conditioning on age, sector, and hours in business.

4. Business Valuations

We use the model to estimate the sweat equity in U.S. private business, which is the present discounted value of a hypothetical mutual fund that holds shares in all private businesses and receives the cash flow from investing in sweat capital.

4.1. Estimates of Private Business Values

We denote the value of sweat capital as $v_b(s)$ for an individual with state $s = (a, \kappa, \epsilon, z, age)$ and compute it as follows:

$$v_b(s) = d(s) + \sum_{\epsilon', z'} \pi(\epsilon', z' | \epsilon, z) M(s, s') v_b(s'), \tag{4.1}$$

where $d(s) = \phi p y_p(s) - e(s)$ is the sweat dividend and $M(s, s')$ is the discount factor. Note that the dividend does not include payments to owner hours in production, h_y, but does include payments to sweat capital κ accruing to all future generations. For example, if we aggregate private businesses, assuming a mutual fund holds shares in all private businesses, then the appropriate discount factor is $(1+\gamma)/(1+r)$. Conceptually, this mutual fund value is comparable to stock market share values of publicly traded firms and thus serves as a useful benchmark when comparing valuations of private and public businesses.

We can compute $v_b(s)$ for all individuals, including those currently working as employees, since employees could run businesses in the future. When we aggregate, we find it to be large: the total sweat equity value for the United States is estimated to be 1 times GDP for pass-through businesses.[18] This estimate is equal to the value of fixed assets used in private pass-through businesses and roughly 70 percent of the market capitalization of publicly traded C corporations. The magnitude is easy to justify if we consider that 9 percent of national income is sweat income paid to owner time in production and building the business. Capitalizing this income with the mutual fund discount factor and multiplying the result by the fraction of time owners put in building sweat capital yields estimates that are on the order of 1 times GDP.

If private C corporations use the same production technologies as pass-through businesses, we can impute a value of sweat equity for all private businesses. Since the sweat equity value is the

[18] If we price the stream of dividends using the owner's discount factor, $M(s, s') = U_c(c', \ell')/U_c(c, \ell)$, then the average sweat equity value is 0.87 times GDP, which is lower since owners face idiosyncratic risk.

present value of pre-tax cash flow to a hypothetical mutual fund, we simply take the result for pass-through businesses (1 GDP) and multiply by the ratio of post-audit incomes of all private businesses relative to pass-through businesses. Bhandari et al. (2019) estimate this ratio to be 1.2 using IRS data from corporate filings of Schedule M-3 and BEA imputations of misreported corporate incomes. The Schedule M-3 links IRS data with the Securities and Exchange Commission 10-K filings for publicly-traded firms, allowing researchers to infer the split of income earned by privately and publicly held C corporations. For 2007, the implied income for privately held C corporations was 1.8 percent of GDP, which, if added to the 9 percent of pass-through income, yields a ratio of private to pass-through net income of 1.2. This, in turn, implies a private business sweat equity value of 1.2 times GDP, which is roughly 84 percent of the market capitalization of publicly traded C corporations.

The incomes being valued in (4.1) are payments to both nontransferable productivity z, which is specific to an owner, and transferable sweat capital κ, which is eventually bequeathed or sold. Thus, the mutual fund shares would be worth more than the cash value v_κ in (3.2) given the latter is the price offered current owners only for their sweat capital. For our baseline parameterization (using data for pass-through businesses only), the average value for $v_b(s)$, if we condition on business owners, is 1.24 times GDP per capita, whereas the average value for $v_\kappa(s)$ is 0.37 times GDP per capita, or 30 percent of the sweat equity value. A value of 0.37 implies a price of goodwill that is roughly equivalent to 2 times annual business income—a finding that is consistent with the Pratt's Stats data where we find this ratio to be about 1.5 for sole proprietors and between 2 and 3 for partnerships and S corporations.

In Table 3, we report average valuations along with cross-sectional information on valuations, intangible intensities, and business returns for the model calibrated to pass-through data. In the first two columns, we report the cross-sectional statistics for the sweat capital sale values, $v_\kappa(s)$, and the associated intangible intensities, $\text{ii}(s)$, defined in (3.1). The sale values are increasing and convex in the level of sweat capital, κ. The intuition for this pattern comes from the feature that owner time is finite and disutility of hours is convex. Therefore, accumulating higher and higher stocks is very costly, given that it takes many years and some luck to build a business, but owners are compensated for their time when they sell the business. Since we computed $v_\kappa(s)$ for all ongoing businesses, we find a wide range of values: from less than 2 percent of GDP per capita for businesses in the bottom 10 percentile that have almost no sweat capital to over 143 percent for the top 1 percent that have large stocks. The associated intangible intensities are on average 58 percent by choice of the income share ϕ but also cover a wide range. Experienced business owners with high productivity can scale up their business by hiring outside labor and capital and have relatively low intensities, whereas new businesses that are just starting out have relatively high intensities.

If we compare the distributions of $v_\kappa(s)$ and $\mathrm{ii}(s)$ to the sweat equity values shown in the third column of Table 3, $v_b(s)$, we see that the latter is significantly less skewed. This follows from the fact that sweat equity is the value of all future cash flows, while the sale price is the value of the current stock of sweat capital if sold today. Future cash flows are not that different across owners given the high frequency of switching in and out of businesses and the assumption of a common productivity process. The ratio of sweat equity at the 75th percentile to the 25th percentile is a little over 2. However, we find a wide range of business net incomes and thus large dispersion in income-to-value ratios and gross returns of the business. The mean gross return is 7.7 percent with a standard deviation of 22 percent. The dividend yield is 1.7 percent, and therefore the mean capital gain is 6 percent. The 10th to 95th percentile range in gross returns is -12 to 55 percent, with most of the difference due to capital gains.

Because of this dispersion, the commonly used procedure of estimating wealth as the ratio of income divided by a common rate of return—sometimes called capitalizing income—would lead to wrong answers. Following such a procedure would lead to the conclusion that there is significant dispersion in valuations.

4.2. Correlates of Private Business Values

While the sales and sweat equity values are meaningful summary statistics in the model, we do not have reliable empirical counterparts of these statistics for ongoing concerns.[19] Even for businesses that sell, we only record the transferable part of the assets, which may not earn the same

[19] Surveys of business owners do ask for self-reports of business valuations. Bhandari et al. (2019) show that the estimates of income-to-value ratios from these surveys are significantly overstated relative to Pratt's Stats and Center for Research in Security Prices (CRSP) data.

future dividends as it would with the former owner. Here, we report statistics for variables that are empirically measurable and potentially correlated with sweat capital and the corresponding business valuations. In doing so, we illustrate how researchers using standard measures of firm markups and TFP can be mislead if they ignore intangible investments made by firms.

Before computing these statistics, we first sort owners by years since the acquisition of their business and then by the sweat capital they have accumulated. Sorting by age is important because there is a clear business life cycle in our model. Evidence of this is shown in Figure 2, where we plot the average sweat capital for owners with different tenures in the business. The figure shows that significant sweat capital building occurs in the first 10 years for a typical business, followed by decumulation. In most cases, the decumulation of capital occurs either because productivity is low and the owner scales back or because productivity is high and the owner substitutes his or her time for outside labor and capital. Because of these life cycle patterns, we limit attention to *observable* characteristics for businesses acquired in the last 10 years, after sorting them into quintiles based on their sweat capital stocks.

In Table 4, we report averages of these potential proxies. The first column shows the average number of years in business, which is positively correlated with sweat capital, as we saw in Figure 2. Owners in the lowest quintile have been in business an average of 2.5 years, while owners in the highest quintile have been in business almost twice as long. We should note that these estimates are well below the 10 year cutoff because there is still significant heterogeneity within the quintiles. For example, in the first few years, new entrants with very high productivity levels spend significant hours to accumulate sweat capital quickly, thus driving down the estimates for years in business in the higher sweat capital quintiles. We should also note that years in business are not correlated with owners' nontransferable productivity z, so this observable statistic serves as a good proxy for sweat capital regardless of the owner's productivity level.

The next two columns of Table 4 show results for business incomes and the fixed asset input, which are highly positively correlated with the overall productivity level $z\kappa^{\phi}$ and thus serve as good proxies for κ. The higher their productivity, the more incentivized owners are to build up κ. If productivity remains persistently high, owners continue to build up sweat capital and scale up production by hiring more outside labor and renting more physical capital. It turns out that as a business scales up, so does the ratio of net income to cost of goods—the standard measure of variable markups, which are computed for a business with state s as follows:

$$\mu(s) = 100\left(\frac{py_p(s) - (r+\delta_k)\,k_p(s) - wn_p(s) - e(s)}{(r+\delta_k)\,k_p(s) + wn_p(s) + e(s)}\right). \tag{4.2}$$

Expensing that occurs when businesses are young drives net incomes down and variable costs up. The opposite is true once the capital has been accumulated, implying a very high correlation with measured markups despite the fact that true markups in the model are equal to zero. As shown in Table 4, we find a wide range of estimates after sorting businesses, with a 43 percent markup in the top quintile. Researchers abstracting from intangible assets might wrongly conclude that large firms in our model were earning significant monopoly rents.[20]

Statistics based on standard calculations of TFP might also mislead researchers faced with data from this model.[21] In Table 4, we report results for the following measure of total factor productivity computed for a business with state s:

$$\text{tfp}(s) = \log y_p(s) - 0.33\log k_p(s) - 0.67\log n_p(s). \tag{4.3}$$

We find TFP is low for businesses with high sweat capital stocks because those businesses are productive and can scale up their hiring of outside factors much more than they can scale up their own time. If we could observe the "true" productivities, z, we would have predicted that high sweat capital businesses are the high productivity firms, the very opposite of what we would conclude by using typical measures of TFP.

In the last column of Table 4, we report financial assets for the businesses, which show no relation to sweat capital and are thus a poor proxy. Note that here we include a but not v_κ, as is sometimes done in studies of business wealth. There is no pattern because firms have access to rental markets and can therefore scale up easily if they have high productivity or high sweat capital. However, even if working capital constraints were included, one would need a high value for χ to see much of a pattern because owners still have access to rental markets, and a key investment when building the business is their own time.

[20] See, for example, Atkeson and Burstein (2008), Edmond, Midrigan, and Xu (2018); and Boar and Midrigan (2019).

[21] Crouzet and Eberly (2018) explore the relationships between TFPs, markups, and intangible investment shares for publicly traded firms in the retail sector. No comparable data exist for private firms.

In summary, we find large sweat equity values for private businesses and significant dispersion in business intangible intensities and returns. Roughly 30 percent of the value is attributable to transferable sweat capital, and 70 percent of the value is due to nontransferable owner-specific productivity. High-value businesses are older, larger in scale, and appear to have high measured markups.

Next, we consider the tax experiments and show that sweat capital and owner time in production play an important role for our quantitative results.

5. Tax Policy Experiments

To quantify the role of owner time in the business when evaluating tax changes, we make two types of comparisons. First, we contrast the effect of lowering tax rates on private pass-through businesses and C corporations in our baseline model with the effect of lowering them in a nested model that has owner time fixed.[22] Second, we contrast the impact of lowering tax rates on labor income earned by owners and by employees.

For comparability across experiments, we lower average marginal tax rates (AMTR) by the same amount:

$$\Delta \log (1 - \tau_{AMTR}) = 0.156,$$

which is on the order of the tax change for corporations in the 2017 U.S. tax reform. The AMTR for private business and wages is computed by taking a weighted average of each filer's marginal rate, with weights given by the taxable net income as in Barro and Redlick (2011). In the case of private business, lowering all marginal rates by 50.6 percent implies a 15.6 percentage point decline in the AMTR. For wage earners, we lower marginal rates by 25.5 percent to achieve the same result. In both cases, we adjust intercepts in the piecewise linear tax schedules to ensure the schedule remains continuous. For C corporations, there is only one rate, so the AMTR is simply the income tax rate τ_p, which is lowered from 36 percent to 26 percent for federal- and state-level

[22] The policy experiments we conduct are not intended to be a careful study of a particular reform in U.S. history but are instead a proof of concept intended to highlight the economic forces at work in our model with sweat activity. For related work that focuses specifically on the recent Tax Cuts and Jobs Act of 2017, see Barro and Furman (2018).

taxes. In each case, we compute the stationary equilibrium associated with the new taxes. Debt levels adjust so that the government budget constraint is satisfied.

5.1. Lower Taxes on Business

We start by comparing aggregate results in the baseline model to those of the nested model that abstracts from owner time in production and building sweat capital. The latter is the standard framework used in the study of entrepreneurial choice, which, as mentioned earlier, is a version of the Lucas (1978) span-of-control model. There are two main takeaways from this exercise. First, the response to lowering private business tax rates is much larger in the baseline model than in Lucas (1978)—both on the intensive and extensive margins—with most of the change due to highly productive owners and not necessarily those with the highest financial assets or sweat capital. Second, lowering the corporate income tax rate has similar effects for most aggregate variables, with the exception of hours.

5.1.1. Aggregate Responses

Consider first a lowering of just the private business tax rates. In the first two columns of Table 5, the main results for this experiment are shown for the baseline model with sweat capital and hours and the nested Lucas (1978) model that has a production function given by

$$y_p = z k_p^{\alpha} n_p^{\nu}. \tag{5.1}$$

In this nested case, we set $\phi = \omega = 0$, and we reparameterize the consumption share η and labor share ν in order to match two statistics: the share of employee hours allocated to private business and the share of pass-through income in total income. The new values are $\eta = 0.50$ and $\nu = 0.39$.

Our first observation is that with a lowering of tax rates, the change in total private sector output is predicted to be quite small in the Lucas (1978) model without sweat activity. In fact, output *falls* by 0.4 percentage points, whereas output in the private sector rises by 1.9 percentage points in the baseline model with sweat activity. To understand these differences, consider first the decisions of very productive owners. These owners are not marginal in terms of their entry or exit decisions irrespective of the tax rates. These owners in the Lucas (1978) environment will expense all the variable factors, and therefore the only taxed factor is the fixed managerial input.

In this case, if we ignore the general equilibrium effects on prices and wages, the model predicts no change in response to a lowering of tax rates for very productive firms.[23] On the other hand, in the baseline model, we predict a quantitatively large response to the lower income tax rates on the intensive margin. As we show in Table 5, owners increase time in production by 17.5 percent and time in building their business by 8.6 percent. As a result, output in the private business sector is *higher* by 1.9 percent, and sweat capital is higher by 4 percent.

In both models, lowering tax rates on business income leads to more entry. In the Lucas (1978) model, the main driver for the size of the extensive margin response is the mass of agents at the exogenous productivity entry threshold. In contrast, when we model sweat activity, the extensive margin changes are more substantial. Post-entry owners work harder, resulting in a larger endogenous productivity $z\kappa^{\phi}$, which amplifies the incentives to enter. In the baseline model, the fraction of owners increases by 7.9 percent and in the no-sweat Lucas (1978) model, the fraction of owners increases by only 3.5 percent.

The large increase in owners' hours is accompanied by a large decrease of 5.8 percent in private business employee hours for the baseline model—much larger than the 1.4 percent decline in the no-sweat model. We find that fixed asset investments also decline in both models but only modestly.

When we compare aggregate statistics for this tax experiment for the models with and without sweat activity, we find stark differences in measured total factor productivity, measured markups, and total hours. For example, if we compare aggregate TFP in the private sector as it is typically measured—the logarithm of $\int y_p(s)ds$ divided by $\int k_p(s)^{.33} n_p(s)^{.67} ds$—we find an increase of 6.3 percent in the baseline model but only 0.8 percent in the no-sweat model. This difference arises because measured TFP picks up changes in the unmeasured factors of production. If we compare the aggregate markup in the private sector as it is typically measured—total business net income divided by total variable costs for capital, labor, and expenses—we find a large increase of 7 percent in the baseline and no change in the no-sweat model. In fact, true markups are zero in both regardless of tax policy. Predictions for aggregate hours are also different, since owner time

[23] To see this formally, consider a simpler case where the tax rate on business is linear and firms maximize: $(1-\tau_b)(py_p - wn_p - (r+\delta_k)k_p)$. In this case, the first-order conditions with respect to labor n_p and fixed assets k_p are independent of τ_b.

is fixed in the no-sweat model. We find an increase of 1.5 percent in the baseline and a decline of 1.4 percent in the no-sweat model. The other aggregate comparisons for C corporations and the overall economy are similar in the two models given the only modification we made was to private business production in (5.1). For example, the differences for aggregate GDP and consumption are on the order of 0.1 percentage points.

In the right panel of Table 5, we show the results for lowering marginal rates on private business net incomes as before and additionally lowering the corporate profits tax rate τ_p comparably by 15.6 percentage points. Lower profits taxes have a significant effect on fixed asset investments, with changes that are roughly the same in the two models. As we show in Table 5, both models predict a fixed asset investment increase of roughly 7.5 percent in private businesses and 23 percent in C corporations. As before, we find similar results for the aggregate data. For example, in both models, wages rise by close to 10 percent, fixed assets rise by 17.5 percent, and GDP and consumption are higher by a little more than 7 percent. The main exceptions are private sector outcomes such as the measured TFP and markup as well as total hours, all of which rise significantly in the model with sweat but not in the one without.

5.1.2. Distributional Responses

To better highlight the mechanisms at work in the baseline model with sweat activity, we next explore the distributional impacts of lowering just the tax on private business, T^b. We first sort businesses into bins using three different sorts: first by their productivity z, second by sweat capital stocks κ, and third by financial asset balances a. We then attribute the total change in a particular variable of interest to the different bins, with contributions summing to the total change. The point of the computation is to show that the method of sorting affects the results: high z businesses are not the same as high κ businesses or high a businesses.

In Table 6, we report the results of the decomposition for changes in sweat capital, κ, owner hours, $h_y + h_\kappa$, and employee hours, n_p, arising from a change in lower tax rates on private business. Since z takes on five values, we sort businesses using the discrete values. Since κ and a are continuous, we assign owners to quintiles. Sorting by productivity, we find that the highest productivity (z) businesses contribute the most to the total changes in the private business production factors. For example, the high-z firms contribute 56 percent to total changes in sweat capital, 92 percent to total changes in owner hours, and 98 percent to total changes in employment. This result is partly due to selection: individuals with low z do not choose to run businesses so there are fewer owners in the lowest categories.

A different picture emerges if we sort businesses by sweat capital or financial assets. Despite an increase in the number of owners, the post-reform distributions of sweat and financial assets shift downward. This result is consistent with the finding that a lowering of taxes on business income causes a shift in labor inputs from employees to owners and a downward shift in the overall scale of private businesses. In the case of sweat capital, most of the changes are attributed to businesses in the fourth quintile—those that are still trying to grow and have high levels of capital. In the case of financial assets, most of the changes in sweat capital and owner hours are attributed to the businesses in the lowest quintile.

Results in Table 6 are predictions of total long-run changes, which are difficult to test. A more interesting and potentially testable prediction of the model is the short-run response to a tax change for different groups of individuals. This is relevant for applied work in public finance concerned with measuring short-run elasticities. To estimate these responses and the implied elasticities, we draw individuals (of measure zero) from the ergodic distribution associated with our baseline and track their decisions in the economy with new taxes and prices. For a given variable and a sequence of shocks, we can compute the percentage differences in time series and obtain a distribution of changes averaged by owner characteristics such as productivity, sweat capital, and financial assets.

Here, we report the findings for total owner hours one year after the tax change, conditioning on their productivity. For most of the distribution, the changes are in the range of 3 to 4 percent, implying a short-run elasticity on the order of 0.2 to 0.25. These estimates are in the range of empirical estimates from micro studies cited by Chetty et al. (2011) but low when compared to the macro estimates of 2 to 3 typically found in the business cycle literature. There are two reasons for the low implied labor supply elasticities in our model. First, a low tax rate on business income increases hours, but it lowers the marginal rate of substitution between leisure and consumption—which is effectively the shadow wage of owners. Thus, unlike in the standard business cycle model, our model features a non-traded input in production—namely, owner time, which faces a higher tax incidence. This in turn implies two offsetting effects for the price of owner time and a lower response of hours. Second, elasticities are lower because there are two uses of owner time in the business—namely, h_y and $h_k appa$. Owners with high levels of sweat capital increase production hours h_y by substituting out of both leisure and sweat building h_κ, thereby attenuating the response of total hours. For such owners, adding more sweat capital is costly, and therefore they exploit the tax cut by scaling up other factors of production such as fixed assets.

Moving beyond the one year change, we summarize the differences in paths by computing a measure of consumption-equivalent change across the baseline economy and the one with lower taxes. Hypothetically, we ask individuals in the baseline economy—before they know their lifetime history of shocks—to compute the increase in consumption bundle c necessary to leave them indifferent between remaining in the baseline economy and moving to one with lower tax rates. We find a gain of 2.3 percent for lowering tax rates on private business income and 4.7 percent for lowering rates on wages. If we sort individuals by gains, we find a small range in the case of lower taxes on private businesses: the lowest decile with high-ϵ/low-z types has an average gain of 1.8 percent, while the highest decile with low-ϵ/high-z types has an average gain of 2.7 percent.

5.2. Lower Taxes on Labor

Private business activity is also affected by the taxation of employees, since the tax schedule T^w is relevant for owners' opportunity cost of time. In this section, we contrast the results in the baseline model of lowering the tax schedule on employee time, T^w, with the results above of lowering the tax schedule on owner time, T^b. We show that changing these schedules has different implications for self-employment rates, private business production, and the organization of firms.

As before, we assume a 15.6 percentage point decline in the average marginal tax rate on T^b and separately on T^w. In Table 7, we report results for key statistics that highlight the differences between these tax policy changes. In the first row, we report the fraction of individuals running their own businesses, which we refer to as the *self-employment rate*. As we noted earlier, there is a 7.9 percent increase in this rate if T^b is lowered. Contrast this with lower taxes on employees, which results in an 18.1 percent decline in the self-employment rate. Lowering taxes on employees leads to a larger response in part because the effective tax rates are much higher on employees than owners and in part because total hours in employment are much higher than in business.

Evidence from these experiments reveals another important difference: lower business taxes result in more businesses that are smaller in scale (for example, mom and pops), while lower employee taxes result in fewer businesses that are larger in scale (for example, businesses with many employees). As we showed earlier, with lower tax rates on private business incomes, owners do more production themselves and hire fewer employees, whose time following the reform is relatively more costly. With lower tax rates on employees, individuals with high (but not too high) productivity levels in business, z—who would have run businesses if employment taxes had not been lowered—choose to work for someone else. In Table 7, we report that total employee hours are higher by 16.8 percent because both publicly and privately held businesses hire more employees. For private businesses, the surge in employment is concentrated at the top, namely, those with the high z, high κ businesses. These businesses take the opportunity of a decline in tax rates on employees to scale up. Their owners put less time into production but more time into building sweat capital. Overall, there is a decline of 3.9 percent in sweat capital building following a lowering of T^w, but this is due to an extensive margin response, since there are fewer mid-κ businesses following the tax reform.

In summary, the tax experiments reveal that including owner time in business and sweat capital changes the economics of tax reform both qualitatively and quantitatively. We next turn to our sensitivity analysis.

6. Sensitivity

In this section, we show that adding features to the model that are common in the literature on entrepreneurial choice does not overturn any of our main findings. Specifically, we include financial constraints on working capital, superstar owners with high productivity levels, and brokered sales of businesses. Differences between the baseline model and extensions with financial constraints and brokered sales turn out to be quantitatively small. Differences between the baseline model and the extension with superstar owners are quantitatively larger for some predictions but strengthen the finding that abstracting from sweat activity leads to understatements of the effect of tax reforms.

6.1. Financial Frictions

In the baseline model, we set the parameter χ governing the severity of borrowing constraints in private businesses to zero. Here, we use evidence reported in Chari (2014) on available funds—which is a measure of funds available for gross investment or financial activities—divided by business value added. We set χ conservatively to the maximum of the estimates he reports.

The most relevant estimate from Chari (2014) is based on U.S. flow of funds data for nonfinancial corporations over the period 1952–2012. At the start of the sample, the ratio of available funds to value added was a little over 15 percent and rose steadily after 2000 to 25 percent, the latter being the estimate we use. In economic terms, this estimate translates into a requirement that roughly a quarter of a year of sales be available for business working capital. We should note that this maximal value is in line with the other evidence in Chari (2014) from large firms in the IRS and Compustat data that have maximal ratios of available funds to sales on the order of 12 to 14 percent. Given aggregate business receipts are roughly twice value added, the maximal value from these other datasets is in line with our choice of $\chi = 0.25$.

In Table 8, we compare the baseline results for $\chi = 0$ with this alternative and find the differences to be negligible. We also experimented with much larger estimates found in the literature and still found negligible effects. High values for χ change the asset distribution but not the main estimates in Table 8 because owners save as a precaution to avoid hitting the constraints.

6.2. Superstar Owners

Next, we change the baseline model by adding an additional productivity state z at the top. Since Debacker, Panousi, and Ramnath (2013) could not publish any information about owners at the top of the income distribution, we add a new point on the productivity grid—that is, z_6—and use published IRS taxable incomes to set it. If we set $z_6 = 1.25 z_5$, the model generates business income at the 95th percentile that is 10 times the median labor income, consistent with the published IRS data. Because no panel data are published by the IRS, our choice of transition probabilities into and out of this state is necessarily somewhat arbitrary. Here, we assume that the only transition in and out of state 6 is via state 5. The average duration in state 6 is taken to be a typical working life of 45 years, which implies that the probability of remaining in state 6 must be equal to $1 - 1/45$. We assume that the probability of transitioning from state 5 to state 6 is equal to the probability of transitioning from state 6 to 5 and choose it in such a way so as to ensure that only 1 percent of owners are in the high productivity state in the stationary distribution. We also recalibrate the deterioration rate λ to 0.7 to generate a consistent acquisition profile as in Figure 1.

We should note this version of the model passes a standard litmus test for researchers studying income and wealth dispersion—namely, that wealth is more dispersed than income. (See Castaneda, Diaz-Giminez, and Rios-Rull, 2003.) Furthermore, Quadrini (2000) reports evidence that wealth-to-income ratios are significantly higher for business owners than for workers. Our model predicts both greater dispersion in wealth than in income and higher wealth-to-income ratios for business owners relative to workers. If we drop individuals in the top and bottom income deciles—those with negative incomes or large inheritances—we find wealth-to-income ratios in the range of 1 to 5.7 for workers and 3.5 to 22.7 for business owners.[24]

In Table 8, we report our main statistics for this case. With higher variability in the productivity process, we find a higher sweat equity value of 1.15 times GDP for mutual fund investors and a higher average gross return of 10.4 percent. The intangible intensity is 56 percent and thus not much different from the baseline. In terms of tax experiments, we find larger average responses. Looking across the distribution of owners, we find that the highest-z superstar owners account for a very small portion of the total change.

6.3. Brokered Sales

The final extension that we consider is to explicitly introduce brokered sales into the baseline model. In this section, we describe the problem of a broker who buys and sells sweat capital and the data used to discipline additional parameters. Because few U.S. businesses sell in any given year, we find that the extended model predictions are close to those of our baseline model without brokered sales.

With sales, individuals choose whether to buy or sell κ simultaneously with occupation. That

[24] We do not attempt to match these statistics to survey data used by the previous literature because of documented measurement issues with business wealth statistics elsewhere. See Bhandari et al. (2019).

is, given their state s and age (y or o), all individuals can decide whether to sell their sweat capital κ and become a worker, keep the κ and become a worker, run a business with the κ they currently have, or run a business after buying additional κ from the broker. If individuals are indifferent between selling and not selling, we break the tie with probability π^f. Sellers exchange κ for cash in the amount $v_\kappa(s)$, which would be the value of the κ after paying any capital gains tax. Brokers own a technology to produce new businesses of size $\bar{\kappa}$. They offer homogeneous price-quantity bundles $(p_{\bar{\kappa}}, \bar{\kappa})$ to all potential buyers. There is free entry in the broker market, and the zero profit condition for the broker is

$$p_{\bar{\kappa}} \int \mathbf{1}_{\text{buy}}\left(s\right) f\left(s\right) ds = \pi^f \int v_\kappa\left(s\right) \mathbf{1}_{\text{sell}}\left(s\right) f\left(s\right) ds,$$

where $f(s)$ is the measure of type s individuals. Our definition of a competitive equilibrium must be amended to include $(v_\kappa(s), p_{\bar{\kappa}})$ and the condition that the sale price is such that sellers are indifferent between continuing to run their businesses and entering employment with additional cash, as in (3.2). Any capital gains taxation would also be included in (2.15).

To discipline π^f and $\bar{\kappa}$, we use microdata from the 2007 SBO microsample. The parameter π^f is set such that the fraction of exiting owners who sold their businesses is roughly 9 percent. The size $\bar{\kappa}$ is set such that we match the fact that 8 percent of SBO businesses with at least one owner reported that they acquired their share of the business over the period 2000–2007 through a purchase. We also recalibrate the deterioration rate λ to 0.7 and ω to 0.478 in order to match the acquisition profile and sweat income share.

Results are reported in Table 5 and show that the main results are robust to including brokered sales. Specifically, we continue to find high estimates for sweat equity values and intangible intensities and our predictions for the tax experiments are close to our baseline model. The main reason we find little change is the small number of sales that occur in the data. Given that we parameterize the model to match this fact, we find little change in the main results.

7. Conclusions

In this paper, we developed a theory of intangible investments made by private business owners and used it to reassess central questions in public finance. We found that abstracting from these investments has quantitatively important consequences for both measurement and policy. We find that the stocks of intangible assets are as large as those of tangible assets. We find that leaving them out of tax analyses leads to significant understatements of the impact of lowering tax rates on private businesses. We view these findings as a proof of concept for future analyses of specific tax reforms.

There are several extensions of our model that should be explored if better data become available. We model individuals choosing between paid and self-employment when in fact there are those that engage in both activities or have family members that do so. In our model, sweat capital cannot be accumulated while in paid employment or by employees, which may have consequences for measuring the stock. The choice of legal form of organization is not included here, but would likely depend on expectations about current and future tax policies. Especially relevant is the choice of becoming a private C corporation that pays corporate income taxes but has few owners with significant exposure to idiosyncratic risk. Finally, more can be done in modeling business sales, especially with regards to the bargaining of the purchase price.

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Figure 1. Business Age Profile

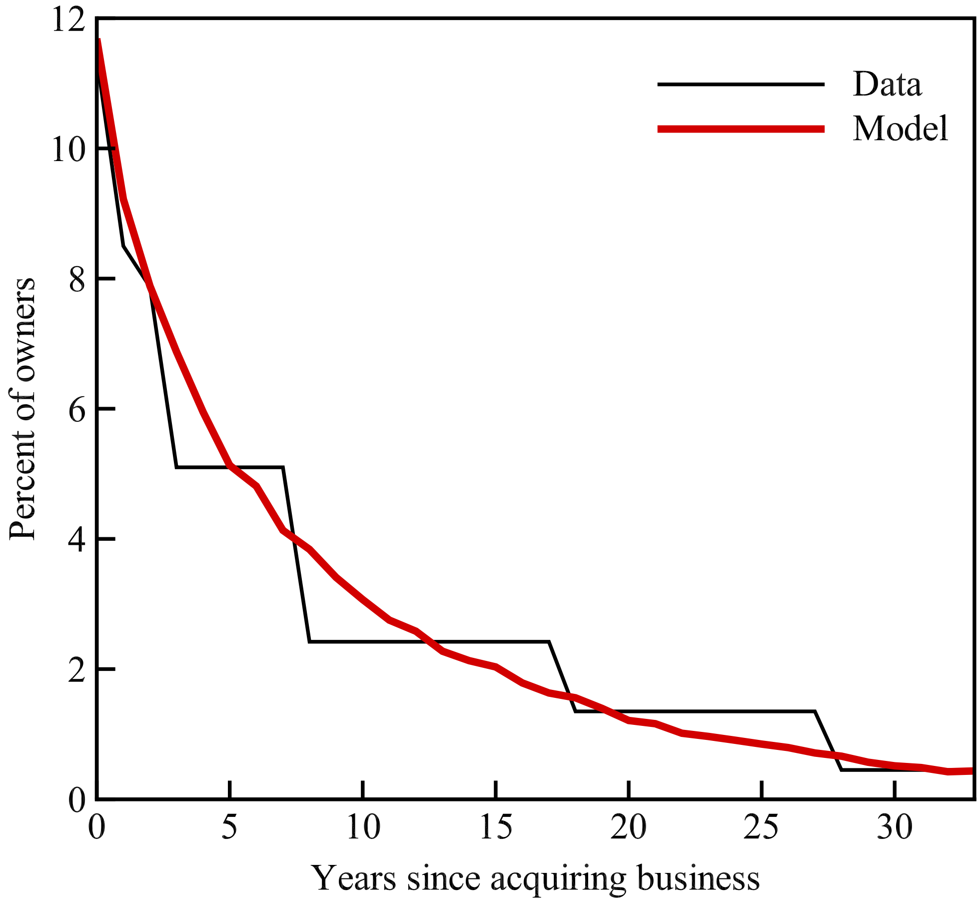

Figure 2. Sweat Capital By Age

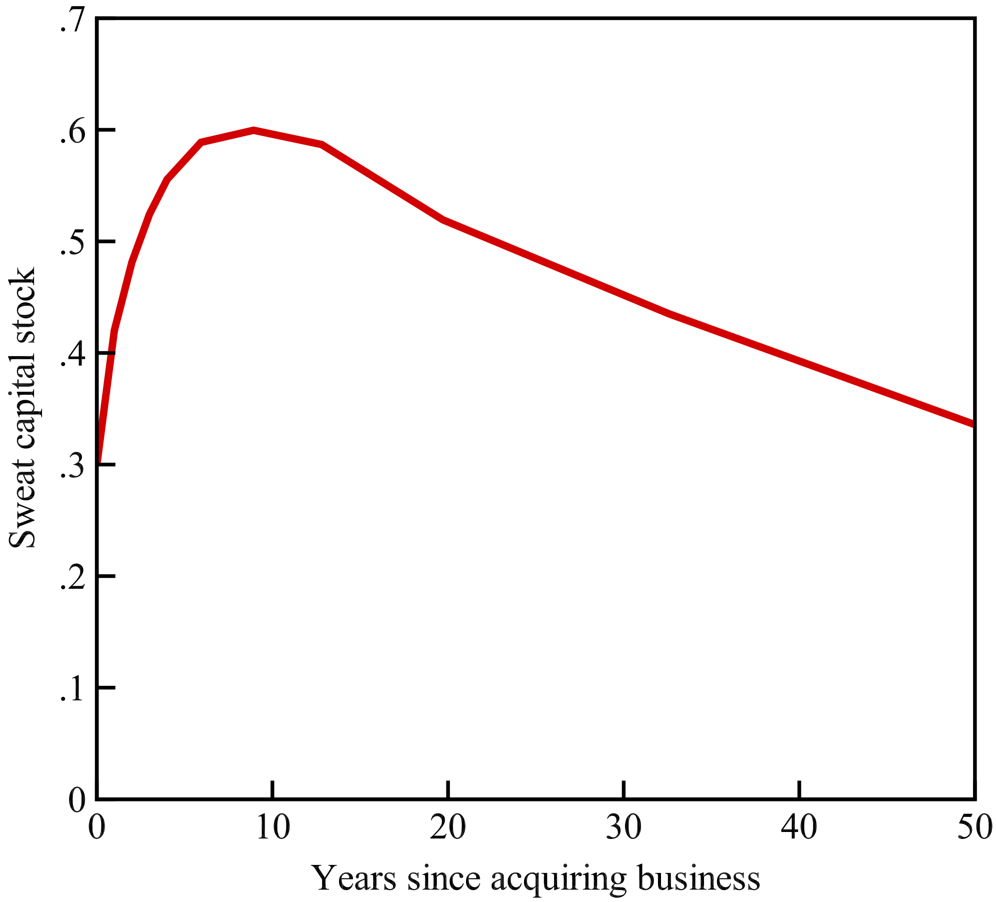

Table 1. Baseline Model Parameters

A. Preferences, Technologies, Life Cycle and Financing

Parameter	Expression	Value
Preferences		
Discount factor	β	0.98
Leisure weight	ψ	1.38
Intertemporal elasticity inverse	σ	1.5
C-corporate good share (%)	η	44.9
Love of business parameter	ξ	0
Technologies		
Technology growth (%)	γ	2.0
C-corporate fixed asset share (%)	θ	50.2
Private business fixed asset share (%)	α	30.0
Fixed asset depreciation (%)	δ_k	4.1
Sweat capital depreciation (%)	δ_κ	4.1
Sweat capital owner hour share (%)	ϑ	41.8
Private business sweat capital share (%)	ϕ	15.0
Private business labor share (%)	ν	55.0
Sweat capital deterioration (%)	λ	50.0
Private business owner hour share (%)	ω	42.5
Private business hours substitution parameter	ρ	0.5
Life cycle		
Old-age productivity (%)	ζ	50.0
Probability of remaining young (%)	π_y	97.8
Probability of remaining old (%)	π_o	93.3
Sweat deterioration at death (%)	λ_d	90.0
Altruism weight	ι	1
Financing		
Working-capital constraint	χ	0

Table 1. Baseline Model Parameters (cont.)

B. Productivity Transition Probabilities

Productivity in t	Productivity in $t+1$				
Employment, ϵ_t	.509	.713	1	1.40	1.97
.509	.424	.549	.027	0	0
.713	.046	.621	.327	.005	0
1	.001	.145	.709	.145	.001
1.40	0	.005	.327	.621	.046
1.97	0	0	.027	.549	.424
Business, z_t	.432	.657	1	1.52	2.32
.432	.612	.170	.098	.065	.055
.657	.172	.551	.187	.064	.025
1	.099	.191	.475	.190	.045
1.52	.060	.055	.164	.558	.164
2.32	.046	.009	.034	.135	.776

Table 1. Baseline Model Parameters (cont.)

C. Government Taxes

Parameter	Expression	Value
Tax rates (%)		
Consumption	τ_c	6.5
Profits	τ_p	36.0
Dividends	τ_d	13.3
Marginal rates, wage schedule	$T^{w\prime}(y_i)$	%
$y_i/y \in [-\infty, \; 0.173]$		29.3
$[0.173, 0.262]$		32.4
$[0.262, 0.404]$		34.3
$[0.404, 0.732]$		39.0
$[0.732, 1.409]$		40.0
$[1.409, 3.138]$		40.8
$[3.138, \infty]$		41.9
Marginal rates, private business schedule	$T^{b\prime}(y_i)$	%
$y_i/y \in [-\infty, \; 0.153]$		14.0
$[0.153, 0.304]$		18.3
$[0.304, 0.912]$		20.1
$[0.912, 2.667]$		23.5
$[2.667, 5.727]$		26.2
$[5.727, 9.104]$		26.9
$[9.104, \infty]$		28.0

Table 2. National Account Shares, Data and Baseline Model

	Data	Model
Income Shares		
Sweat income	0.090	0.090
Nonsweat labor income	0.331	0.319
C corporations	0.220	0.215
Private business	0.111	0.103
Business capital income	0.314	0.309
Nonbusiness incomes	0.265	0.284
Product Shares		
Private nondurable consumption	0.575	0.554
Government consumption	0.133	0.142
Investment	0.292	0.304
C corporations	0.134	0.121
Private business	0.048	0.066
Nonbusiness	0.110	0.117

Note: Three adjustments are made to NIPA income and product: sales taxes are subtracted, consumer durables are classified as investment, and additional IPP categories are included with investment that are not currently included in NIPA investment.

Table 3. Cross-Sectional Statistics for All Private Businesses

Statistics	Sale value	Intangible intensity	Sweat equity	Gross return	Dividend yield
Mean	37.3	57.9	124	7.7	1.7
Std. dev.	47.4	39.5	64.9	22.4	10.0
Percentiles:					
10th	1.9	11.7	59.8	−11.6	−0.1
25th	2.9	13.5	71.5	1.2	0.0
50th	8.4	66.9	99.5	2.3	0.0
75th	74.1	99.8	178	15.6	3.6
95th	132	100	240	55.2	14.6
99th	144	100	251	93.5	16.1

Note: The sale value for a business is the amount of cash to leave the owner indifferent between selling the sweat capital and keeping it and is given by $v_\kappa(s)$ in (3.2). The intangible intensity of a business with state s is the ratio $v_\kappa(s)/(v_\kappa(s)+k_p(s))$. To compute $v_\kappa(s)$ and the intensity, we condition on businesses that have $k_p(s)>0$. The sweat equity for a business is the present discounted value of net dividends, $\phi p y_p(s)-e(s)$, to the owner with state s and is given by $v_b(s)$ in (4.1). The gross return on the business is the sum of the capital gain to sweat equity plus the dividend yield. Both are in percentage terms. The dividend yield is the ratio of net dividends to sweat equity value.

Table 4. Characteristics of Private Businesses Acquired in Last 10 Years, with Businesses Sorted by Sweat Capital Stocks

Quintile	Years in business	Business income	Fixed assets	Measured markup	Measured TFP	Financial assets
1	2.5	0.0	0.0	−46.3	113	508
2	2.9	0.2	2.5	−31.9	113	533
3	3.3	2.9	120	3.8	105	588
4	3.8	24.9	354	19.6	100	441
5	4.4	65.2	724	42.5	91.5	556

Note: The statistics reported are quintile averages. Business age is in years. Business income and assets are normalized by per capita GDP and multiplied by 100. Measured markup is the ratio of business income to cost of goods sold, which is then multiplied by 100. Measured TFP is the ratio of business output to fixed assets raised to the power 1/3 times employee hours raised to power 2/3. The TFP estimates are normalized by the mean value and multiplied by 100.

Table 5. Effects of Lower Business Tax Rates in Models with and without Sweat Activity

	% Changes from lower tax rates on:			
	Private businesses		All businesses	
	Baseline	No sweat	Baseline	No sweat
Private business activity				
Output	1.9	−0.4	4.5	1.6
Relative price	−2.9	−0.7	2.5	5.5
Sales	−1.1	−1.2	7.0	7.1
Owner hours, production	17.5	–	17.0	–
Owner hours, sweat	8.6	–	9.1	–
Sweat capital	4.0	–	6.8	–
Employee hours	−5.8	−1.4	−6.8	−2.3
Fixed asset investment	−0.6	−0.9	7.5	7.4
Measured TFP	6.3	0.8	6.9	0.7
Measured markup	7.0	0.0	7.1	0.0
C-corporate activity				
Output	−0.8	−1.1	12.1	11.8
Consumption	−1.1	−1.2	7.0	7.1
Employee hours	−1.3	−1.4	2.1	2.0
Fixed asset investment	−0.3	−0.8	22.9	22.5
Aggregates				
Wage rate	0.5	0.3	9.8	9.6
Interest rate	−1.1	−0.7	−14.1	−13.9
GDP	−0.7	−0.8	7.2	7.4
Sweat equity	1.8	–	10.8	–
Consumption	−1.1	−1.2	7.0	7.1
Hours	1.5	−1.4	2.9	0.6
Fixed asset investment	−0.4	−0.9	17.5	17.5
Tax revenues	−5.3	−4.5	−1.3	−0.6
Debt	−26.9	−21.0	−13.7	−7.0

Note: Measured TFP in the private sector is computed by first aggregating outputs and inputs. Measured markup is computed as the ratio of aggregate business income to aggregate variable costs.

Table 6. Distributional Effects of Lower Tax Rates on Private Business Sorting by Productivity, Sweat Capital, and Financial Assets

Characteristic	Sweat capital	Owner hours	Employee hours
Sort by productivity (z)			
Group 1	0.3	0.0	0.0
2	0.2	0.0	0.0
3	0.4	0.1	0.0
4	0.8	1.2	−0.2
5	2.2	14.4	−5.7
Total change (%)	3.9	15.7	−5.8
Sort by sweat capital (κ)			
Quintile 1	0.0	0.0	0.0
2	0.6	0.3	0.0
3	4.7	0.6	2.6
4	18.2	36.7	26.5
5	−19.6	−21.9	−35.0
Total change (%)	3.9	15.7	−5.8
Sort by financial assets (a)			
Quintile 1	7.2	11.3	4.7
2	3.2	5.2	0.9
3	1.4	2.8	−1.2
4	−1.6	0.4	−2.9
5	−6.4	−4.0	−7.5
Total change (%)	3.9	15.7	−5.8

Note: The statistics reported are changes contributed by each group of owners, with the sums equal to column totals. There are five productivity (z) levels in the baseline model. For sweat capital (κ) and financial assets (a), we group by quintiles.

Table 7. Effects of Lower Tax Rates on Labor Inputs of Business Owners and Employees

	% Changes from lower tax rates on:	
	Owners	Employees
Self-employment rate	7.9	−18.1
Total employee hours	−3.4	16.8
Private business	−5.8	18.4
C-corporate	−1.3	11.9
Total owner hours	15.7	−11.1
Production	17.5	−13.0
Sweat building	8.6	−3.9
Wage rate	0.5	0.7
Relative price	−2.9	4.5

Table 8. Sensitivity of Main Results in Extensions of Baseline Model

		Extended to include:		
Statistics (%)	Baseline model	Financial frictions	Superstar owners	Brokered sales
Aggregate sweat equity to GDP	101	102	115	103
Private business intangible intensity	57.9	57.9	55.5	52.3
Private business gross return	7.7	7.7	10.4	6.5
Private business effects, lower T^b:				
Sweat capital	3.9	3.9	7.5	4.3
Owner hours	15.7	15.5	24.2	16.7
Superstar hours	14.4	14.2	0.2	15.4
Employee hours	−5.8	−5.8	−10.6	−5.8
Private business effects, lower T^w:				
Sweat capital	2.9	1.7	0.9	0.7
Owner hours	−11.1	−11.4	−28.9	−10.3
Superstar hours	−10.2	−10.5	0.1	−9.1
Employee hours	18.4	17.7	26.8	18.3