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ON THE PERPETUATION OF UNEMPLOYMENT, UNDERNOURISHMENT AND INEQUITABLE LAND OWNERSHIP IN DYNAMIC GENERAL EQUILIBRIUM

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Abstract: We incorporate the consumption-ability relationship of static "efficiency wage" models into a dynamic general equilibrium model. We show that for many aggregate land stocks, there is a continuum of unemployment rates which could persist indefinitely as part of a stationary equilibrium. For many of these aggregate land stocks, both unemployment and full employment are distinct possibilities. Broadly speaking, more unemployment corresponds to more undernourishment and more inequality in land distribution. Thus our results suggest that the market mechanism is less efficacious than land reform in reducing unemployment and undernourishment.

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1. Introduction

We analyze a competitive dynamic model which incorporates unemployment, undernourishment, and asset inequality as its central variables. Our objective is to characterize fully the stationary general equilibria of such a system. In doing so, some insight is gained into the persistence of unemployment and undernourishment in developing countries, and in particular, into the role of market forces in combatting such phenomena.

Widespread undernourishment is a pervasive and undeniable fact of life in many developing countries, though the exact magnitudes are open to debate. It is known that the undernourished are assetless (or near-assetless) and that they have difficulty gaining secure access to the labor market. Economic theory has been applied to understand some of these correlations. In particular, the "efficiency wage" theories of unemployment, pioneered by Leibenstein (1957), have been put forward to address precisely these issues.[1] Indeed, the <u>formal</u> structure has turned out to be broader in scope than the nutrition-related issues they were originally designed to handle.[2]

We now have fairly comprehensive <u>static</u> general equilibrium models of asset inequality, undernourishment and unemployment (Dasgupta and Ray, 1986, 1987a). The focus of inquiry in those studies is on the nature of the causal mechanism "inequality → undernourishment → unemployment". This mechanism is best studied in a static context, for in a dynamic model, the labor market affects asset inequality. In the present paper, we study the entire bidirectional dynamic relationship. Accordingly, we construct a dynamic model and accommodate a number of features that cannot be satisfactorily explored in a static setting. Among these, two bear mentioning at the outset.

First, we incorporate the important observation that a person's <u>current</u> ability to perform productive tasks depends at least as much on his nutritional <u>history</u> as on his current nutritional intake.[3] Indeed, this observation has often been advanced as a criticism of static "efficiency wage" models that view ability as a function of current consumption alone. However, the implications of this observation have not been studied previously in an explicit dynamic model, and our results demonstrate that this observation does not lead to results that are markedly at variance with the "static" predictions.

Second, we permit asset holdings to vary endogenously over time. An individual derives nutrition, not only from consumption out of wage and rental income, but also from consumption financed by the sale of land. Thus, relatively poor individuals may need to decumulate land in order to maintain a nutrition level sufficient for work. On the other hand, relatively rich individuals are in a position to accumulate land without jeopardizing their ability to work. Indeed, these are the fundamental ways in which asset inequality evolves over time in our model.

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1.1. Brief Description of Model

We consider a competitive infinite-horizon economy, where a single output ("food") is produced using land and labor power in a constant-returns-to-scale technology. There is a large number of agents, each endowed with an initial <u>nutrition stock</u> and an initial <u>land holding</u>. Each agent is free to cultivate his own land, to lease in or to lease out land, to hire in or to hire out labor power, as well as to buy or to sell land. All markets are taken to be perfectly competitive. In order to be <u>able</u> to work, however, the individual must possess a minimum nutrition stock.[4] His current nutrition stock is last period's nutrition stock after "depreciation" plus current energy intake minus current energy expenditure. The individual's land holding is updated to reflect purchases and sales of land.

Each individual solves a dynamic optimization problem, while in their role as firms they maximize profits in every period. We study the dynamic general equilibrium of this system, postulating that all markets clear. In particular, we assume that the labor market clears. This particular specification, made primarily to simplify the exposition, is different from the non-Walrasian concept studied in static efficiency wage theories, so it is worth commenting on this difference at the very outset.

Unemployment might conceivably arise in three ways. First, a person may be <u>able</u> to work (in the sense that his nutrition stock exceeds the critical minimum) but is <u>unwilling</u> to do so.

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Second, a person may be unable to work, although he would like to. Finally, a person may be both able and willing to work, but is excluded from the labor market by some rationing mechanism. This last concept is usually referred to as involuntary unemployment. By our postulate that labor markets clear, we do not accommodate the third variety of unemployment. Neither do we study the first. Rather, we focus on the second: the unemployed are unable to work, although they would like to.[5] Whether we call such unemployment involuntary or not,[6] there is no doubt that it represents a serious lack of access to the labor market. In our ethical priorities, it is as unacceptable a state of affairs as involuntary unemployment in the narrower sense of the term.

We could have incorporated the third variety of unemployment as well, using the "extended equilibrium" approach developed in Dasgupta and Ray (1986).[7] We avoid doing so to keep technical complications to a minimum.

1.2. Summary of Results

The Characterization Theorem (Section 3) completely characterizes the stationary equilibria, and reveals that, in general, there is a continuum of stationary equilibria for every aggregate land stock. These equilibria differ in prices, land distribution, and unemployment rate. Broadly speaking (see Section 4 for details), higher unemployment rates correspond to more undernou-

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rishment and greater inequality in land distribution.

Although it is not surprising that different distributions of the same aggregate land stock can perpetuate themselves,[8] it is significant that different unemployment rates can persist indefinitely in stationary equilibrium. For many aggregate land stocks, there are stationary equilibria with unemployment even though there is also a stationary equilibrium with full employment. The perpetually unemployed individuals simply lack the necessary nourishment-in spite of the fact that other equilibria with less unemployment may exist. Our result suggests that there is nothing inherent in the competitive market system which will alter the situation over time, and that the root cause of this difficulty is the initial land distribution.

At one level, it might seem possible to construct, for any aggregate land stock, a stationary equilibrium involving undernourishment and unemployment: simply start some individuals off with extremely low nutrition stocks and zero land holdings. But this is not true: we must also find a price system (wages, rental rates, and land prices) that sustains this configuration as a dyanamic general equilibrium. Indeed, if the aggregate land stock of the economy is "large enough" (in a sense made precise in Section 3), there <u>cannot</u> be a stationary equilibrium involving undernourishment and unemployment. On the other hand, if the aggregate land stock is "small", <u>every</u> stationary equilibrium must involve undernourishment and unemployment. It is the intermediate zone that is of greatest interest. Here, as already

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mentioned, there are equilibria with full employment (the economy is "productive" enough), but there are also equilibria with unemployment. In this case, a careful and progressive land reform will not only improve income distribution-it will also increase employment and output.

Given the multiplicity of stationary equilibria, it is not surprising that no one of them is _globally stable_. (Global stability would mean that starting from any initial land distribution and nutrition distribution, there exists a dynamic equilibrium whose prices and distributions converge over time to those of the stationary equilibrium in question.) Nevertheless, some remarks can be made on the _local stability_ of each stationary equilibrium. Section 5 takes this up.

First, we study _individual stability_, in the sense that we perturb an individual's initial land and nutrition stocks while keeping equilibrium prices fixed (they will indeed remain unaltered in response to a "measure-zero" change). We establish that a small increase in an unemployed individual's initial stocks will _not_ bring about a permanent change in his employment status. Indeed, in one version of our model, the additional stocks are drawn down over time as the individual regresses to his earlier stationary position (Stability Theorem 2). We establish similar stability results for employed individuals.

Finally, we briefly discuss (local) _system_ stability, which analyzes changes in the initial stocks of a set of individuals of _positive_ measure. In general, such changes perturb dynamic equilibrium prices, and thereby complicate the analysis immensely. Nevertheless, individual stability results throw some light on system stability. For instance, we show that a "small" land reform, in the sense of a minor redistribution of land from the employed to the unemployed, has no effect on the equilibrium unemployment rate. Rather, the land reform must exceed a critical minimum size if it is to reduce unemployment.

2. The Model

2.1. Production

We consider an infinite-horizon dynamic economy in which food is the only consumption good. Food is produced using the production function $F(k,e)$, where k denotes land and e denotes labor-power as measured in efficiency units (i.e. tasks accomplished). The production function is taken to be concave, continuously differentiable and constant-returns-to-scale. It is also assumed to increase in k and in e, to display diminishing marginal product in each input, and to satisfy the Inada end-point conditions for each input. The interpretations that $F(k,e)$ is a production technology for one giant <u>competitive</u> firm, or that it is a technology available to all individuals in the economy, are both equally consistent with our analysis, and the reader may adopt either. Let K be the <u>aggregate</u> land stock in the economy.

2.2. Individual Feasibility

There is a continuum of agents, indexed by a and distributed uniformly on $[0,1]$. Each individual begins every time period with two stocks: his <u>land ownership</u> ($k \geq 0$) and his previous <u>nutritional level</u> ($n \geq 0$). In every period the individual may be employed (signalled by a dummy variable e set equal to 1) or unemployed ($e = 0$). He also consumes a certain amount of food ($c \geq 0$). Given the current <u>price of land</u> (p), the <u>rental rate on land</u> (r), and the <u>wage rate per efficiency unit</u>, i.e. piece rate per task accomplished (q), the agent's employment status and consumption level determine his nutritional level and his land ownership for the next time period, and the whole process repeats itself.

We make the following assumption concerning the relationship between employment and nutrition.

<u>Assumption 1</u>: An agent either supplies one (efficiency) unit of labor if he is employed, or zero units if he is unemployed. To be employed, the agent's current nutritional level must be at least $\tilde{n}$, where $\tilde{n}$ is an exogenously given positive number. If employed, the agent expends x units of energy on work, where x is an exogenously given nonnegative number. If unemployed, the agent expends no energy.

We postpone our comments on Assumption 1 until we have completed a formal description of the agent's feasible set.

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Suppose that a vector of prices (p,q,r) is given and is time-stationary.[9] Then consider an agent at time t with stocks (k_{t-1}, n_{t-1}) of land and nutrition at the end of date $t-1$.

If $e = 1$ (the agent is employed), we then have

(1) $$k_t = k_{t-1} + (rk_{t-1} + q - c_t)/p \geq 0 \text{ , and}$$

(2) $$n_t = bn_{t-1} + \lambda(c_t) - x \geq \tilde{n} \text{ .}$$

Equation (1) describes the evolution of land holdings and is self-explanatory. Equation (2) is the nutrition balance equation. Last period's nutrition stock n_{t-1} depreciates by a factor $(1-b)$,[10] so that bn_{t-1} is available at time t. Current consumption c_t adds to the nutrition stock via a function $\lambda(c_t)$. We take it that $\lambda(0) = 0$, that λ is weakly increasing[11] and differentiable, and that the derivative λ' is bounded above by the constant h.[12] Finally, employment requires an energy expenditure $x \geq 0$. The inequality $n_t \geq \tilde{n}$ in (2) captures the stipulation of Assumption 1: in order to work, the agent must have attained a certain threshold level of nutrition.

If $e = 0$ (the agent is unemployed), the stock equations (1) and (2) are modified to read:

(3) $$k_t = k_{t-1} + (rk_{t-1} - c_t)/p \geq 0 \text{ , and}$$

(4) $$n_t = bn_{t-1} + \lambda(c_t) \geq 0 \text{ .}$$

Again, (3) should be self-explanatory. The nutrition balance equation (4) now subtracts no energy due to work,[13] and further-

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more, the constraint $n_t \geq \tilde{n}$ does not appear.

For an examination of nutrition balance equations such as (2) and (4), the reader is referred to Dasgupta and Ray (1987b) and the references contained therein. It should be observed that our formulation places importance on <u>both</u> the agent's nutrition <u>history</u> (captured via n_{t-1}) and his current consumption as determinants of current work ability. This is in contrast to the numerous models of nutrition-efficiency relationships in a static context. While many of these acknowledge the inherently dynamic nature of the relationship, their questions are somewhat different than the ones addressed here and do not necessitate an explicit modelling of this dynamic.[14]

While our model is richer in this respect, it does simplify matters by assuming that labor-power is a simple step-function defined over the nutritional stock.[15] This is the crux of Assumption 1. It would be desirable, but much more difficult analytically, to formulate labor-power as a smoothly increasing function of both nutrition stock and current energy expenditure.

We do not interpret $c = 0$ as complete fasting, $e = 0$ as complete inactivity, and $n = 0$ as death. Rather, we use $(c,e,n) = (0,0,0)$ to represent subsistence on consumption created outside the economy under review, such as begging or gleaning. One could, at the cost of additional complexity, represent such subsistence with positive constants.

2.3. Individual Preferences

Each agent derives a single-period utility $\nu(c)$ from a consumption of c in that period. The function ν is taken to be nonnegative-valued, weakly increasing, differentiable and strictly concave. Moreover, we assume there is some $e^0 < 1$ such that for sufficiently large c, $\nu(c) < c^{e^0}$.[16] Future utilities are discounted, and the agent seeks to maximize the infinite-horizon sum of discounted utilities.

This much is fairly standard. However, we are dealing here with a framework that embodies persistent malnutrition as one of its fundamental variables, and this is likely to have repercussions on behavior via a lowered probability of survival.[17] We incorporate this possibility by assuming that the discount factor applied at date t to the infinite-horizon utility from date $t+1$ is a weakly increasing function of nutrition. Specifically, for any stream of consumption and nutrition $\langle(c_t, n_t)\rangle_{t=1}^{\infty}$, we write the agent's utility function as

$$\Sigma_{t=1}^{\infty}\{\Pi_{s=1}^{t-1}\delta(n_s)\}\nu(c_t) \quad , \tag{5}$$

where $\delta(n) = \sigma(n)/(1+\rho)$, $\rho > 0$ is a fixed rate of impatience, and $0 < \sigma(n) \leq 1$ gives the probability of being alive during the next period given current nutrition n. We assume that σ is continuous, weakly increasing when $n < \tilde{n}$, and constant at s when $n \geq \tilde{n}$. Thus, δ is continuous, weakly increasing when $n < \tilde{n}$, and constant at $d = s/(1+\rho)$ when $n \geq \tilde{n}$. One special

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case is certain survival: σ is constant at one and δ is constant at $d = 1/(1+\rho)$.[18]

The individual seeks to maximize (5) subject to the constraints (1)-(4) given prices (p,q,r) and his initial endowment of land and nutrition (k_0,n_0) . A solution to this problem is a sequence $<(c_t,e_t,k_t,n_t)>_{t=1}^{\infty}$. Such a solution is stationary if the subscript t can be dropped.

2.4. Stationary Equilibria

A stationary equilibrium in this model is an infinite-horizon competitive equilibrium with stationary prices and a stationary solution for each individual. Formally, a stationary equilibrium consists of the following (time-stationary) objects: prices (p,q,r) , a consumption distribution $c : [0,1] \to \mathbb{R}_+$, an employment distribution $e : [0,1] \to \{0,1\}$, a land distribution $k : [0,1] \to \mathbb{R}_+$, and a nutrition distribution $n : [0,1] \to \mathbb{R}_+$. This collection of prices and distributions is a stationary equilibrium if it satisfies the following four properties.[19]

(a) For every $a \in [0,1]$, $(c(a),e(a),k(a),n(a))$ is a stationary solution to the maximization problem (1)-(5) given prices (p,q,r) and initial stocks $(k(a),n(a))$.[20]

(b) The aggregate land holding K is distributed by k : $\int k \, d\mu = K$.

(c) The input markets clear given competitive behavior by the producer(s): $(K,\int e \, d\mu)$ maximizes $F(K',E') - rK' - qE'$

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over $(K',E') \in \mathbb{R}^2_+$.[21]

(d) The food market clears: $\int c\, d\mu = F(K, \int e\, d\mu)$.

3. Characterization of Equilibrium Unemployment Rates

3.1. Characterization Theorem

For most aggregate land stocks, our model admits a <u>continuum</u> of stationary equilibria. These stationary equilibria are substantially different from one another: they involve different equilibrium prices and different unemployment rates. In particular, we shall see that the same economy could possess two stationary equilibria, one with full employment and the other with a positive unemployment rate. Which of the two prevails in the long-run depends on the <u>initial</u> distribution of stocks.

Let U denote the unemployment rate $1 - \int e\, d\mu$. We will characterize the various unemployment rates U that can prevail as the aggregate land stock K is parametrically varied. Accordingly, we shall often equate an economy with its aggregate land endowment K . We define two critical values of the aggregate land endowment K : Define $\underline{K}$ by

$$\lambda[F(\underline{K},1)] = (1-b)\tilde{n} + x ,$$

and define $\bar{K}$ by

$$\lambda[F_E(\bar{K},1)] = \tilde{n} + x .$$

Although we postpone a full interpretation of these critical

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values until Subsection 3.2, note that $\underline{K} < \bar{K}$ follows immediately.[22]

<u>Characterization Theorem</u>: Consider any economy K with $K > 0$. Define $\underline{U}(K) = (\underline{K}-K)/\underline{K}$, and $\bar{U}(K) = (\bar{K}-K)/\bar{K}$. Then the economy K has a stationary equilibrium with unemployment rate U if and only if one of the following three conditions is met:

1) $K \in (0,\underline{K})$ and $U \in [\underline{U}(K),\bar{U}(K))$,
2) $K \in [\underline{K},\bar{K})$ and $U \in [0,\bar{U}(K))$, or
3) $K \geq \bar{K}$ and $U = 0$.

<u>Remark</u>: See Figure 1. The theorem states that for economies such as K_1 , the set of all equilibrium unemployment rates is precisely equal to the interval $[\underline{U}(K_1),\bar{U}(K_1))$. Thus, every equilibrium for these economies exhibits unemployment ($U > 0$) . In contrast, for economies such as K_2 , both full employment ($U = 0$) and unemployment ($U > 0$) are possible in equilibrium. Finally, for economies such as K_3 , all equilibria necessarily exhibit full employment.

Thus, the essential indeterminacy of this model is captured in stark detail: If $K < \bar{K}$, the market mechanism can support a whole range of unemployment rates. This range of unemployment rates corresponds to the range of land distributions which perpetuate themselves in a dynamic general equilibrium (see Section 4): Greater inequality in land distribution leads to more unemployment and more malnutrition.

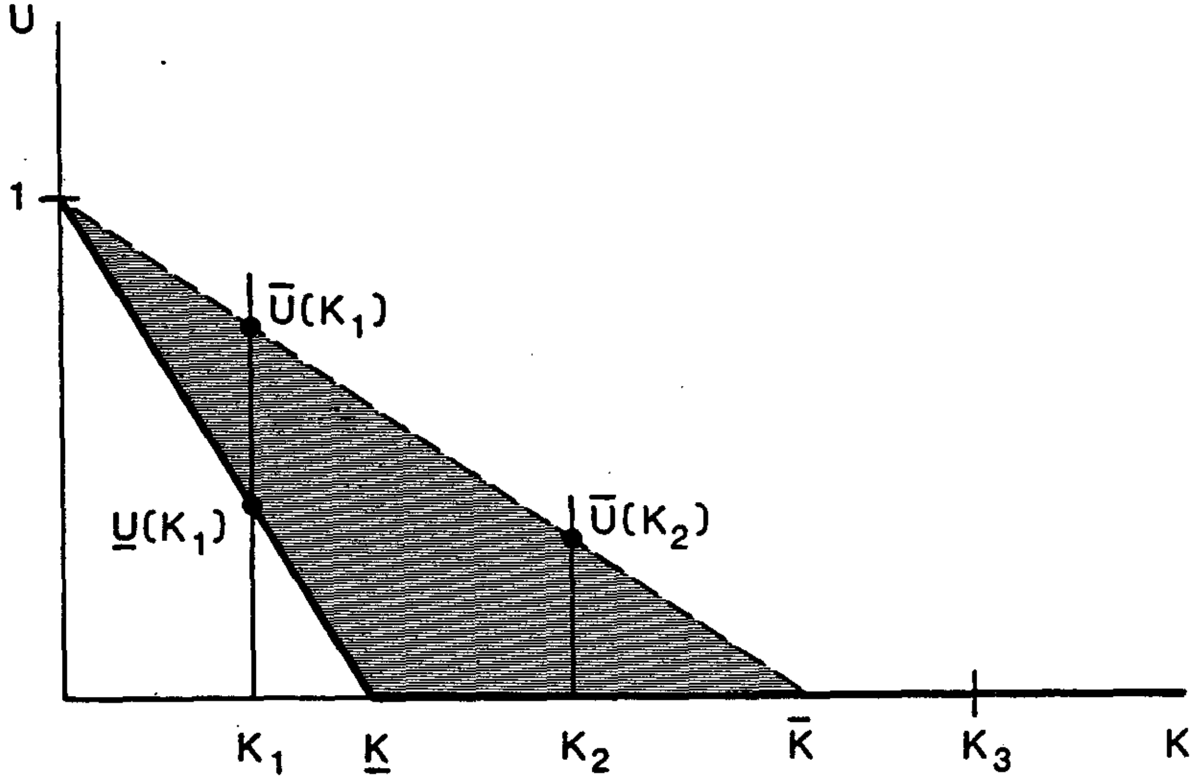

Figure 1: The economy K has a stationary equilibrium with unemployment rate U if and only if the pair (K,U) is on the solid line or within the shaded triangle.

The region $K \in [\underline{K}, \bar{K})$ is of particular interest, for here, unemployment and malnutrition (i.e. nutrition levels below $\tilde{n}$) can exist <u>in equilibrium</u> even when there are other equilibria that can guarantee full employment and adequate nutrition for all. This unattractive outcome is perpetuated by the market mechanism given an initial inequitable distribution of land.

One naturally wonders whether this indeterminacy affects economic efficiency. One measure of efficiency would be aggregate food production: Clearly production increases with employment, and thus, equilibria with relatively high rates of unemployment are inefficient in this sense. Pareto-efficiency provides another criterion: We conjecture that every stationary equilibrium is Pareto-efficient just like the static general equilibrium of Dasgupta and Ray's (1986) malnutrition model is shown to be Pareto-efficient.

<u>Proof of Characterization Theorem</u>: The proof has three main parts.

Part 1. First, suppose K and U satisfy one of the three conditions. We will construct for economy K a stationary equilibrium with unemployment rate U. Define prices by

$$q = F_E(K, 1-U) \;,$$
$$r = F_K(K, 1-U) \;, \text{ and}$$
$$p = [d/(1-d)]r \;.$$

Define the distributions (c,e,k,n) by

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$$(c(a),e(a),k(a),n(a)) = \begin{cases} (\ 0,\ 0,\ 0,\ 0\ ) & \text{if } a \in [0,U) \\ (\ rk(a)+q,\ 1,\ K/(1-U),\ \{\lambda[c(a)]-x\}/(1-b)\ ) & \text{if } a \in [U,1]\ . \end{cases}$$

Properties (b)-(c) follow immediately from the definitions of k , r , and q . Property (d) follows from Euler's Theorem: $c(a) = F(K,1-U)/(1-U)$ for all $a \in [U,1]$.

Property (a) is more involved. First consider the unemployed agents. If $U = 0$, this case is vacuous. If $U > 0$, take $a \in [0,U)$. By the fact that $U < \bar{U}(K)$,

$$\begin{aligned} \lambda(q) - x &= \lambda[F_E(K,1-U)] - x \\ &< \lambda[F_E(K,1-\bar{U}(K))] - x \\ &= \lambda[F_E(\bar{K},1)] - x \\ &= \tilde{n}\ . \end{aligned}$$

Thus it is impossible that agent a can work without first accumulating land or nutrition. It is also impossible for agent a to accumulate either one without labor income. Therefore the stationary stream $(c(a),e(a),k(a),n(a)) = (0,0,0,0)$ is the only feasible stream. (Section 5 makes the distinct and significant observation that this optimum is locally stable.)

Second consider the employed agents. Take $a \in [U,1]$ and consider the following "artificial" maximization problem: Given $(k_0,n_0) = (k(a),n(a))$, choose $\langle(c_t,e_t,k_t,n_t)\rangle_{t=1}^{\infty}$ to maximize

$$\Sigma_{t=1}^{\infty}\ d^{t-1}\nu(c_t) \tag{6}$$

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subject at each t to

$$n_t = bn_{t-1} + \lambda(c_t) - xe \text{ , and} \tag{7}$$

$$k_t = k_{t-1} + (rk_{t-1} + qe - c_t)/p \text{ .} \tag{8}$$

In this "artifical" problem, we deliberately do <u>not</u> impose a nutritional requirement for work. Therefore, the agent always works and nutrition plays no role. It can be shown by standard arguments, using $p = [d/(1-d)]r$ and the strict concavity of ν , that the unique optimum of (6)-(8) is the stationary stream $(c(a),e(a),k(a),n(a))$.

We make an important preliminary observation concerning this optimum: $n(a) \geq \tilde{n}$. If $K < \underline{K}$, this observation follows from the fact that $U \geq \underline{U}(K)$:

$$\begin{aligned} n(a) &= \{\lambda(rk(a) + q) - x\}/(1-b) \\ &= \{\lambda \circ F[K/(1-U),1] - x\}/(1-b) \\ &\geq \{\lambda \circ F[K/(1-\underline{U}(K)),1] - x\}/(1-b) \\ &= \{\lambda \circ F[\underline{K},1] - x\}/(1-b) \\ &= \tilde{n} \text{ .} \end{aligned}$$

If $K \geq \underline{K}$, the observation follows from the definition of $\underline{K}$:

$$\begin{aligned} n(a) &= \{\lambda(rk(a) + q) - x\}/(1-b) \\ &= \{\lambda \circ F[K/(1-U),1] - x\}/(1-b) \\ &\geq \{\lambda \circ F[K,1] - x\}/(1-b) \end{aligned}$$

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$$\geq \{\lambda_o F[\underline{K},1] - x\}/(1-b)$$
$$= \tilde{n} \ .$$

The "artifical" maximization problem (6)-(8) is intimately related with the "true" maximization problem (1)-(5). First, the objective function (5) is bounded above by the objective function (6) because the function δ is bounded above by the constant d . Second, the feasible set defined by (7)-(8) contains the feasible set defined by (1)-(4), for in contrast to (1)-(4), work does not require adequate nutrition in (7)-(8). Third, because of our preliminary observation that $n(a) \geq \tilde{n}$, $(c(a),e(a),k(a),n(a))$ is also feasible in (1)-(4). And fourth, because $n(a) \geq \tilde{n}$, the objective function (5) equals the objective function (6) at $(c(a),e(a),k(a),n(a))$. These four facts prove that $(c(a),e(a),k(a),n(a))$ is the unique optimum of (1)-(5).

Part 2. Suppose $K < \underline{K}$ and $U < \underline{U}(K)$. Assume (contrary to the theorem) that there exists a stationary equilibrium $[p,q,r;c,e,k,n]$ with unemployment rate U . Since K is the aggregate land stock and $1-U$ is the measure of the set of employed people, there must be some employed agent a such that $k(a) \leq K/(1-U)$. This fact, together with (1) and Euler's Theorem, implies

$$c(a) = rk(a) + q$$

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$$\leq rK/(1-U) + q$$
$$= F[K/(1-U),1] \ .$$

Yet, (2) and the definition of $\underline{K}$ require

$$c(a) \geq \lambda^{-1}((1-b)\tilde{n} + x)$$
$$= F[\underline{K},1]$$

These two statements about $c(a)$ require that $K/(1-U) \geq \underline{K}$. This, however, is impossible because $U < \underline{U}(K)$.

Part 3. Consider an economy K and unemployment rate U such that $U > 0$ and $U \geq \bar{U}(K)$. Assume that there is a stationary equilibrium $[p,q,r;c,e,k,n]$ with unemployment rate U . By property (c) of the equilibrium, the fact that $U \geq \bar{U}(K)$, the definition of $\bar{U}(K)$, and the definition of $\bar{K}$,

$$\lambda(q) - x = \lambda(F_E(K,1-U)) - x$$
$$\geq \lambda(F_E(K,1-U)) - x$$
$$= \lambda(F_E(\bar{K},1)) - x$$
$$= \tilde{n} \ .$$

This inequality reveals, via (1) and (2), that any individual can work in every time period, regardless of the land and nutrition he inherits from the previous period. Thus, every agent will necessarily be employed in every time period. Consequently, it is impossible that there is a stationary equilibrium with a positive rate of unemployment greater than or equal to $\bar{U}(K)$.

Q.E.D.

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3.2. Viable and Full-Employment Thresholds

One interesting feature of the Characterization Theorem is the zone $[K,\bar{K})$. Within this region, *both* full employment and unemployment are possibilities, depending on the initial distributions of land and nutrition. Accordingly, it may be worth interpreting the threshold levels $\underline{K}$ and $\bar{K}$.

The latter may be called the *full-employment threshold*. It is a level of the capital stock beyond which *every* stationary equilibrium, no matter how unequal the implied (utility) distribution, must yield full employment and therefore a nutrition level of at least $\tilde{n}$ to all. For such economies, poverty and malnutrition cannot be an outcome of the mechanisms stressed in this paper.[23]

The former threshold may be called the *viable threshold*. We will define this somewhat differently, and then demonstrate its equivalence with $\underline{K}$. Define the viable threshold to be the smallest economy K that can *feasibly employ* all its members (thereby providing each with a nutrition level of at least $\tilde{n}$).[24] More precisely, the viable threshold is the smallest K such that there exist distributions c , e , k , and n (each a function from $[0,1]$ into $\mathbb{R}_+$) satisfying

$$\int c \, d\mu = F(K,1) \ ,$$

$$e(a) = 1 \quad \text{for all} \quad a \in [0,1] \ ,$$

$$\int k \, d\mu = K \ , \text{ and}$$

$$n(a) = [\lambda(c(a))-x]/(1-b) \geq \tilde{n} \quad \text{for all} \quad a \in [0,1] \ .$$

Observe that this definition only deals with feasible allocations, and not with allocations that might arise out of some market equilibrium. However, it can be shown that this allocation for the viable threshold can be supported with prices as a stationary equilibrium.[25] A corollary of this observation is that the viable threshold equals $\underline{K}$.

So $\underline{K}$ is a threshold level where full-employment is just feasible, and $\bar{K}$ is another threshold level where full employment is an inevitable consequence of every stationary equilibrium. We reiterate our simple but important observation: $\underline{K}$ is less that $\bar{K}$. It is this intermediate zone of economies, capable of attaining full employment and an adequate level of nutrition, but locked into competitive equilibria that do not come close to achieving these standards, that is consequently of greatest interest. We feel that this zone is a caricature—but a useful one—of many developing countries, where resources are adequate but nevertheless unemployment and malnutrition prevail. Our analysis reveals that there is nothing inbuilt in the market mechanism that will correct this.

4. Graphical Depiction of Equilibrium Land Distributions

For any economy with aggregate land stock K , there is a wide variety of stationary equilibria. This variety has two dimensions. First, and the more important, the Characterization Theorem states that if $K < \bar{K}$, the economy K can sustain a continuum of equilibrium unemployment rates U . Second, given K , U , and the resulting equilibrium prices (p,q,r) , there is considerable freedom concerning the distribution of land among individuals. This second dimension is explored in a graphical manner here in Section 4.

4.1. Construction

Consider an individual's two stock variables: land k and nutrition n . The constraints (3)-(4) require that every stationary land-nutrition pair with<u>out</u> employment lies within

$$\underline{S} = \{ \ (k,n) \mid n = \lambda(rk)/(1-b) \ \} \ .$$

Similarly, the constraints (1)-(2) require that every stationary land-nutrition pair with employment lies within

$$\bar{S} = \{ \ (k,n) \mid n = \{\lambda(rk + q) - x\}/(1-b) \text{ and } n \geq \tilde{n} \ \} \ .$$

See Figures 2-5. For future reference, we define the point $(\bar{k},\bar{n})$ to be the unique element of $\bar{S}$ which satisfies either $n = \tilde{n}$ or $k = 0$. Both $\underline{S}$ and $\bar{S}$ must be upward-sloping, and

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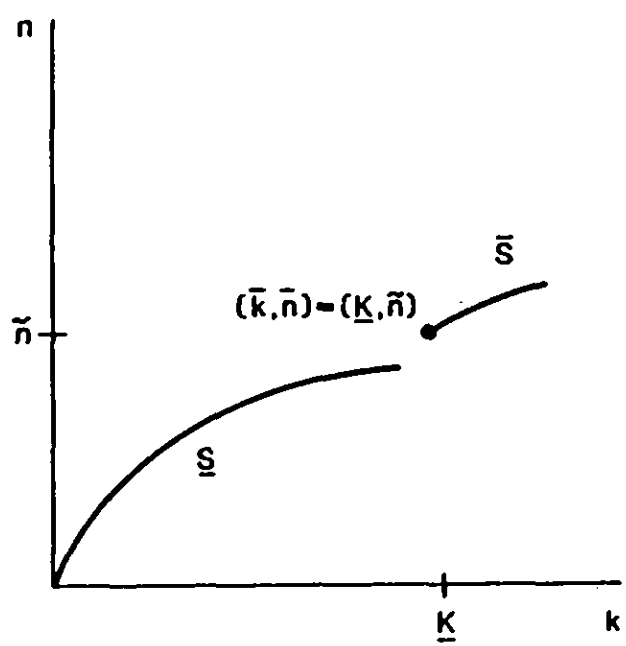
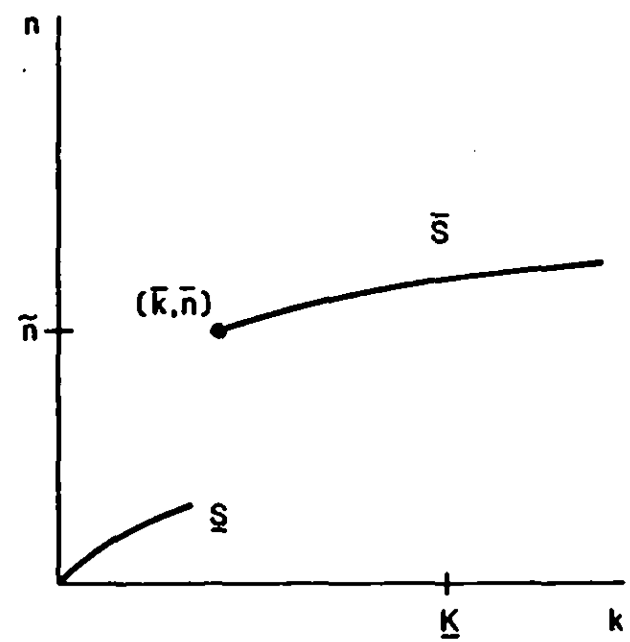
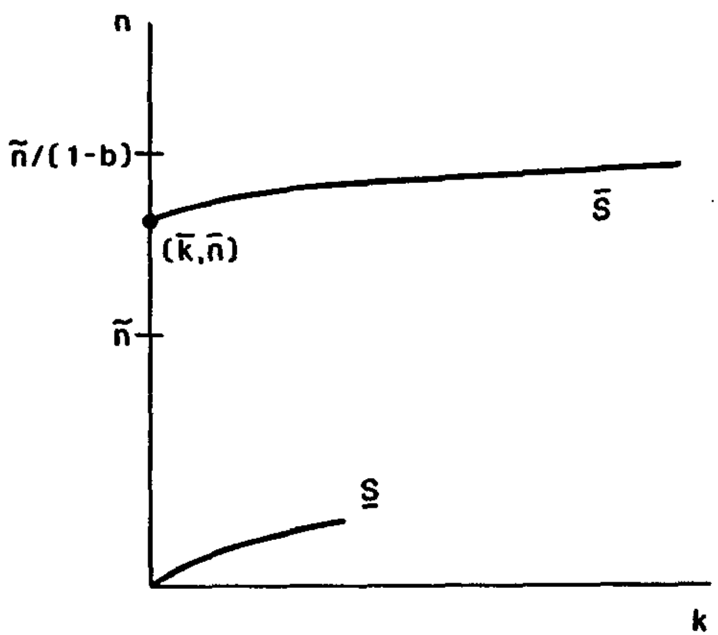


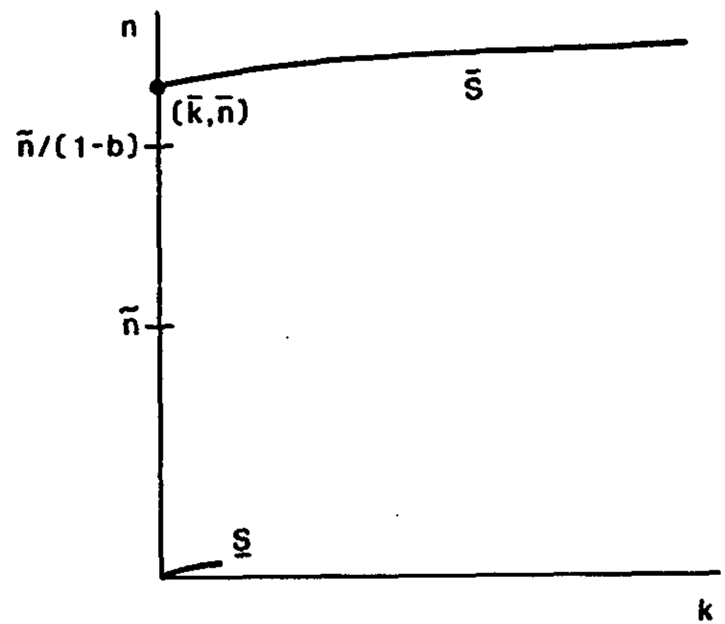
<u>Figure 2</u>: $K/(1-U) = \underline{K}$.



<u>Figure 3</u>: $K/(1-U)$ slightly above $\underline{K}$ and well below $\bar{K}$.



<u>Figure 4</u>: $K/(1-U)$ well above $\underline{K}$ and slightly below $\bar{K}$.



<u>Figure 5</u>: $K/(1-U) > \bar{K}$.

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$\underline{S}$ must contain the origin. (Only the lower portion of $\underline{S}$ is shown in Figures 2-5 since the rest is irrelevant.)

The curves $\underline{S}$ and $\bar{S}$ shift with the aggregate land stock K, the unemployment rate U, and the resulting equilibrium prices. Fortunately, this dependence is easily understood: $\underline{S}$ and $\bar{S}$ depend only on r and q, which in turn, depend only on the land/labor-power ratio $K/(1-U)$.

As Figure 6 shows, the equilibrium land/labor ratio can be as low as $\underline{K}$; it can be nearly as high as $\bar{K}$ even when $U > 0$; and it can equal or exceed $\bar{K}$ when $U = 0$. Also note that the statements $U \geq \underline{U}(K)$ and $K/(1-U) \geq \underline{K}$ are algebraically equivalent, and similarly, that $U < \bar{U}(K)$ and $K/(1-U) < \bar{K}$ are equivalent. The land/labor ratios labeled "3", "4", and "5" lead to the configurations of $\underline{S}$ and $\bar{S}$ depicted in Figures 3, 4 and 5.

The movement of $\bar{S}$ can be easily understood in terms of the point $(\bar{k},\bar{n}) \in \bar{S}$. When $K/(1-U) = \underline{K}$, $(\bar{k},\bar{n}) = (\underline{K},\tilde{n})$. As the ratio $K/(1-U)$ increases, the point $(\bar{k},\bar{n})$ slides along toward the corner $(0,\tilde{n})$. Once $(\bar{k},\bar{n})$ hits the corner, it moves up the vertical axis indefinitely.

The movement of $(\bar{k},\bar{n})$ can be easily interpreted: As the ratio $K/(1-U)$ increases from $\underline{K}$, the wage q also rises; and hence it takes less rental income $r\bar{k}$ in order to maintain the necessary nutrition level $\tilde{n}$.[26] When $(\bar{k},\bar{n})$ hits the corner $(0,\tilde{n})$, the wage q alone suffices: no rental income is required in order to maintain $\tilde{n}$. Then if $K/(1-U)$ continues to

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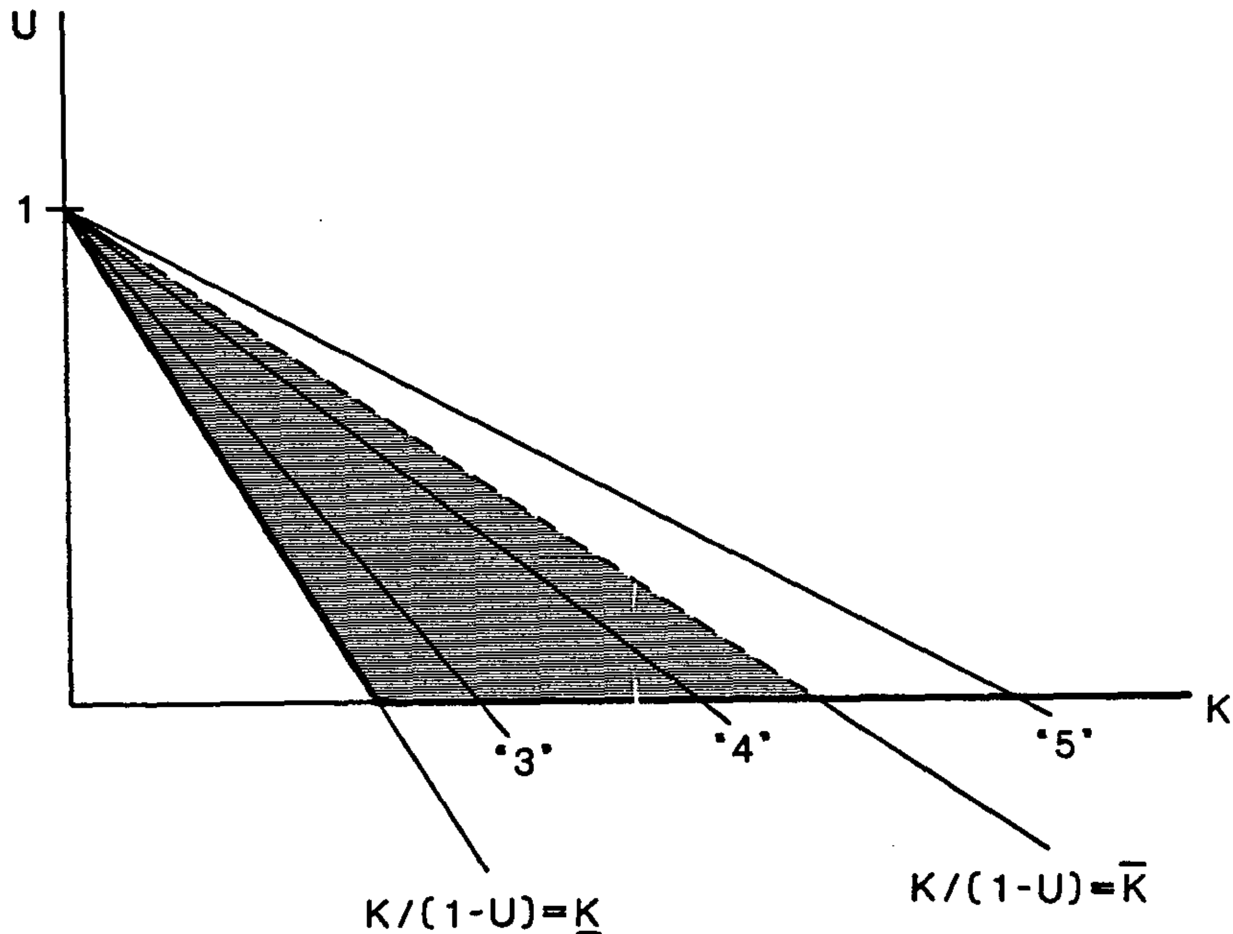

<u>Figure 6</u>: The straightforward relationship between the aggregate land stock K , the unemployment rate U , and the land/labor ratio K/(1-U) . The numbers "3", "4", and "5" label the land/labor ratios which are considered in Figures 3, 4, and 5.

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increase, a landless worker is able to maintain a nutrition level exceeding the minimum $\tilde{n}$.

4.2. Application

Consider an economy K and take an equilibrium unemployment rate U and the resulting prices as given. In such a stationary equilibrium, the (stationary) optimum of an unemployed agent can be represented by a land-nutrition pair in $\underline{S}$. The optimum of an employed agent can be represented by a point in $\bar{S}$.

Suppose that $\delta(n) < d$ for all $n < \tilde{n}$. In this case, no unemployed agent would maintain a positive stock of land, because his subjective discount factor $\delta(n)$ is less than d , which is the market discount factor determined by the discount factor of the employed agents. Thus, all the land is distributed among the employed agents. In terms of Figures 2-5, all unemployed agents are at $(0,0) \in \underline{S}$ and the employed agents are arbitrarily scattered along $\bar{S}$ with the one restriction that they divide up the aggregate land stock K among themselves. More formally, the set of all equilibrium distributions is characterized by[27]

1) If $a \in [0,U)$, then $(c(a),e(a),k(a),n(a)) = (0,0,0,0)$,
2) If $a \in [U,1]$, then $k(a) \geq \tilde{k}$ and $(c(a),e(a),k(a),n(a)) =$ $(\ rk(a)+q,\ 1,\ k(a),\ (\lambda(c(a))-x)/(1-b)\ )$,[28] and
3) $\int_{[U,1]} k\, d\mu = K$.

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Now suppose $\delta(n) = d$ for all n . In this case, unemployed agents would be willing to hold perpetually a positive amount of land. Thus, unemployed agents can be at points on $\underline{S}$ other than $(0,0)$. However, if an agent's land-nutrition pair is too far from the origin, he will choose to seek employment by accumulating the necessary stocks of land and nutrition. Lemma 8 (Appendix) demonstrates that if $K/(1-U) < \bar{K}$, there is some point on $\underline{S}$ other than the origin below which agents will choose to remain unemployed. Such agents do not choose to seek employment because the necessary accumulation of land and nutrition requires that they consume very little in the meantime. (This low consumption is very unattractive because of the concavity of the single-period utility function ν .) This point on $\underline{S}$ is called $(\underline{k},\underline{n})$, and accordingly, the set of all equilibrium distributions includes[29]

1) If $a \in [0,U)$, then $k(a) < \underline{k}$ and $(c(a),e(a),k(a),n(a)) = (\ rk(a),\ 0,\ k(a),\ \lambda(c(a))/(1-b)\ )$,[30] and
2) If $a \in [U,1]$, then $k(a) \geq \bar{k}$ and $(c(a),e(a),k(a),n(a)) = (\ rk(a)+q,\ 1,\ k(a),\ \{\lambda(c(a))-x\}/(1-b)\ )$, and
3) $\int_{[0,1]} k\, d\mu = K$.[31]

4.3. The Characterization Theorem, Revisited

There are two critical land/labor ratios: $\underline{K}$ and $\bar{K}$. The ratio $\underline{K}$ is critical because if a worker's labor is complemented with less than $\underline{K}$ units of land, his total output is less than $F(\underline{K},1)$, which is the minimum amount of consumption required to sustain his nutrition at $\tilde{n}$. Thus, no stationary equilibrium has a land/labor ratio below $\underline{K}$. Recall that as the land/labor ratio increases from $\underline{K}$, $\bar{k}$ declines from $\underline{K}$. This movement is depicted in Figures 2 and 3.

The ratio $\bar{K}$ is critical because if the land/labor ratio equals or exceeds $\bar{K}$, the marginal product of labor is sufficient in and of itself to adequately nourish a worker with a zero nutritional stock at the start of the period. In such a case, everyone will choose to be employed in every period. Thus, there is no stationary equilibrium with unemployment that has a land/labor ratio at or above $\bar{K}$.[32] Note that the statements $K/(1-U) < \bar{K}$ and $\tilde{n} < \tilde{n}/(1-b)$ are algebraically equivalent. The second inequality appears on the vertical axis of Figures 4 and 5.

We now relate our understanding of equilibrium distributions to Section 3's Characterization Theorem for equilibrium unemployment rates.

If $K < \underline{K}$, the minimum unemployment rate $\underline{U}(K)$ is obtained by putting $1-\underline{U}(K)$ workers precisely at $(\bar{k},\bar{n}) = (\underline{K},\tilde{n})$ and putting the rest at $(0,0)$ (Figure 2). The land/labor ratio is then $\underline{K}$ and every worker is just adequately nourished. The higher the K , the more workers can be provided with $\underline{K}$ units of land, and hence, the lower the minimum unemployment rate. If K is fixed and U increases from $\underline{U}(K)$ toward $\bar{U}(K)$, then the land/labor ratio increases, the wage q increases, and $(\bar{k},\bar{n})$ slides to the left and then up the vertical axis (Figures 3 and 4). Employed and unemployed people can be scattered on $\bar{S}$ and $\underline{S}$ as discussed in Subsection 4.2. The unemployment rate U can be made nearly as high as $\bar{U}(K)$. In other words, the land/labor ratio can be made nearly as high as $\bar{K}$. This bound is shown in Figure 4 by the fact that $(\bar{k},\bar{n})$ lies below $(0,\tilde{n}/(1-b))$.

Suppose $K = \underline{K}$. Here full employment is possible: See Figure 2 and set all agents precisely at $(\bar{k},\bar{n}) = (\underline{K},\tilde{n})$. As U is increased from 0 toward $\bar{U}(K)$, see first Figure 3 and then Figure 4, and scatter the agents on both $\bar{S}$ and $\underline{S}$ as discussed in Subsection 4.2. Again, the bound $U < \bar{U}(K)$ is shown in Figure 4 by $\bar{n} < \tilde{n}/(1-b)$.

Suppose $K \in (\underline{K},\bar{K})$. If $U = 0$, see Figure 3 or 4 and scatter agents on $\bar{S}$ alone. If $U \in (0,\bar{U}(K))$, see Figure 3 or 4 and scatter agents on both $\bar{S}$ and $\underline{S}$. Note that as K increases, the full-employment land/labor ratio rises, the full-employment wage rises, and the full-employment $(\bar{k},\bar{n})$ rounds the corner and climbs toward $(0,\tilde{n}/(1-b))$. Any positive unemployment rate entails a higher land/labor ratio, a higher wage, and a higher $\bar{S}$ curve.

When $K \geq \bar{K}$, there is no way to get the land/labor ratio below $\bar{K}$, and thus, unemployment becomes impossible. Consequently, each K entails a unique land/labor ratio (K itself) and a unique $\bar{S}$ curve whose $(\bar{k},\bar{n})$ lies on the vertical axis at or above $(0,\tilde{n}/(1-b))$.

5. Stability

5.1. Individual Stability

We divide our discussion of stability into two parts. First, we consider "individual stability", that is, we take stationary equilibrium prices as given and study an agent (of measure zero) whose initial land-nutrition pair entails a nonstationary optimum. We ask whether or not the optimum converges over time to a stationary optimum. Second, we make some remarks on "system stability", that is, we consider initial distributions of land and nutrition which are not stationary equilibrium distributions, and we ask how those distributions evolve over time in a nonstationary dynamic general equilibrium. In this case, prices can vary over time. Individual stability is addressed formally in this subsection, while the much more difficult issue of system stability is addressed verbally in Subsection 5.2.

Take an equilibrium land/labor ratio $K/(1-U)$ with associated equilibrium prices (p,q,r). A set Q consisting of stationary optimal land-nutrition pairs is a <u>global attractor</u> if at each initial $(k_0,n_0) \in \mathbb{R}^2_+$, every optimum $\langle(c_t,e_t,k_t,n_t)\rangle_{t=1}^{\infty}$ satisfies $\lim_{t\to\infty}(k_t,n_t) \in Q$. Such a set Q is a <u>local attractor</u> if there is an open set B containing Q such that if $(k_0,n_0) \in B$, every optimum $\langle(c_t,e_t,k_t,n_t)\rangle_{t=1}^{\infty}$ satisfies $\lim_{t\to\infty}(k_t,n_t) \in Q$. An attractor Q is <u>monotonic</u> if the optima converging to Q also satisfy either $(\forall t)\ k_{t+1} \geq k_t$ or $(\forall t)$ $k_{t+1} \leq k_t$.

<u>Stability Theorem 1</u>: Suppose that $K/(1-U) < \tilde{K}$ and that $\delta(n) < d$ for all $n < \tilde{n}$. Then $\{(0,0)\}$ and $\bar{S}\sim\{(\bar{k},\bar{n})\}$ are monotonic local attractors. However, $\bar{S}$ is <u>not</u> necessarily a monotonic local attractor. (See Figure 7, and Lemmas 5, 7, and 9 in the Appendix.)

<u>Stability Theorem 2</u>: Suppose that $K/(1-U) < \tilde{K}$, that $\delta(n) = d$ for all n , and that $\lim_{c\to 0}\nu'(c) = +\infty$. Then there is a land-nutrition pair $(\underline{k},\underline{n}) >> (0,0)$ such that $\underline{S} \cap \{(k,n) | (k,n) << (\underline{k},\underline{n})\}$ and $\bar{S}\sim\{(\bar{k},\bar{n})\}$ are monotonic local attractors. However, $\bar{S}$ is <u>not</u> necessarily a monotonic local attractor. (See Figure 8, and Lemmas 5, 8, and 9 in the Appendix.)

Essentially, this pair of stability theorems says that all the stationary optima discussed in previous sections are locally stable, with one exception: $(\bar{k},\bar{n})$. It is not at all surprising that $\bar{S}\sim\{(\bar{k},\bar{n})\}$ is stable, for near this set, the nutritional requirement of employment is not binding. Thus, the strict concavity of ν implies that agents consume the same amount in every period: $rk_0 + q$.

It is important that the low stationary optima on $\underline{S}$ are locally stable. If these optima were locally <u>un</u>stable, our entire argument could be crippled by arguing that any slight increment to the assets of the poor would permit them to climb

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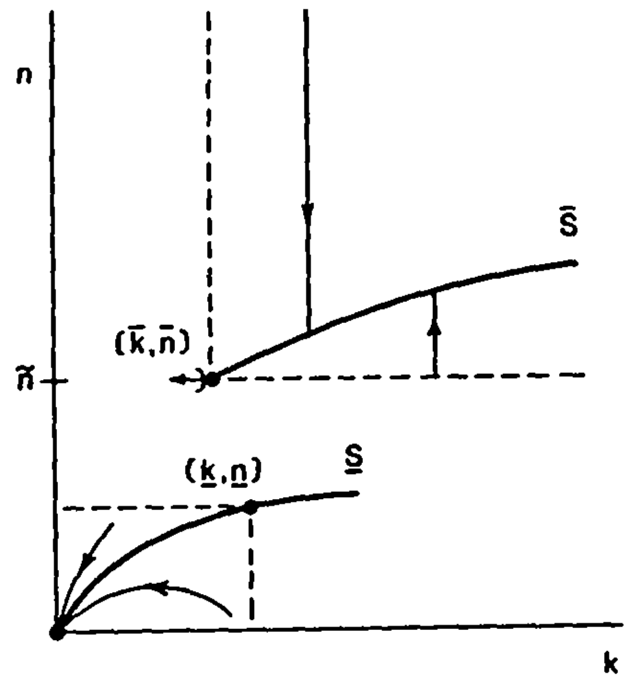

__Figure 7__: $K/(1-U) < \bar{K}$ and $\delta(n) < d$ for $n < \tilde{n}$. (Stability Theorem 1.)

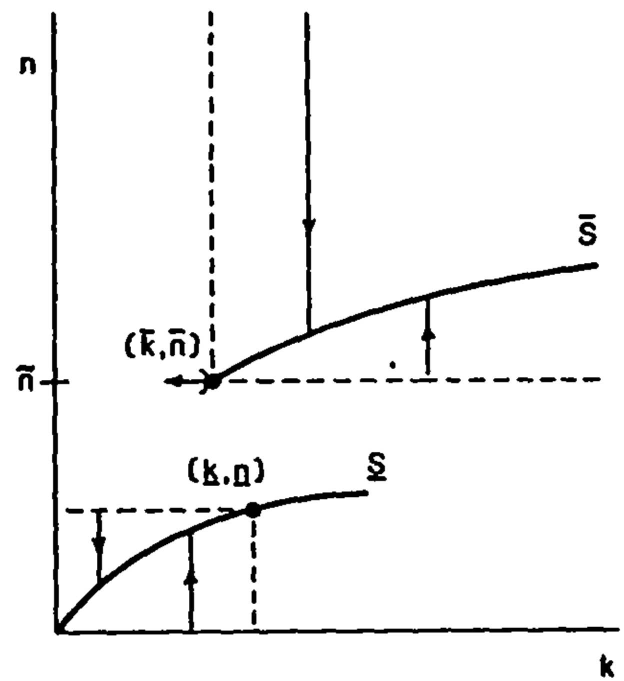

__Figure 8__: $K/(1-U) < \bar{K}$, $\delta(n) = d$ for all n , and $\lim_{c\to\infty}\nu'(c) = +\infty$. (Stability Theorem 2.)

out of poverty. Rather, our analysis shows that such small increments do not significantly alter the long-run behaviour of the poor.

The key to this result is the observation that there is some neighborhood of the origin from which agents would not ever seek employment by accumulating the necessary land and nutrition. Because land generates rental income and nutrition depreciates, an agent seeking to gain employment would first accumulate land rather than nutrition. By buying more land with the rental income, the agent's land stock could accumulate at a rate of d. This option is unattractive for one of two reasons: either (Stability Theorem 1) malnourished agents are less likely to live to reap the benefits of the accumulation ($\delta(n) < d$), and hence run down their land stock to finance current consumption, or (Stability Theorem 2) the strict concavity of v makes low consumption during the accumulation process very onerous (as discussed in Subsection 4.2).

Although it was surprising to us that $(\bar{k},\tilde{n})$ is unstable, the result is intuitive. If $n_0 = \tilde{n}$ and k_0 is just a little less that $\bar{k} > 0$, the agent could work for many periods while maintaining his land close to $\bar{k}$. In each period, he would sell a tiny bit of land to make up for the slight deficiency in rental income. In this case, $\langle k_t \rangle_{t=1}^{\infty}$ is clearly moving away from $\bar{k}$. The alternative to this scenario is to forego employment in the first period and to accumulate. This second option entails a large loss in consumption during the first period, and it is thus less attractive than the first option for agents whose k_0 is very close to $\bar{k}$.

It is technically difficult to solve the agent's maximization problem when (k_0, n_0) lies between $(\underline{k}, \underline{n})$ and $(\bar{k}, \bar{n})$.[33] These difficulties are illustrated by the instability of $(\bar{k}, \bar{n})$.

One further observation can be made in a casual manner: If $K < \bar{K}$, increases in U shrink the region of attraction surrounding the low stationary optima on $\underline{S}$.[34] This observation suggests that as unemployment increases, "rags-to-riches" stories become more prevalent, i.e. slight increments in the assets of a poor agent are more likely to have long-run effects. Thus the frequency of rags-to-riches stories varies directly, rather than inversely, with the degree of inequality, malnutrition, and unemployment.

Finally, if $K \geq \bar{K}$ (implying that $K/(1-U) \geq \bar{K}$), the nutritional requirement for employment is never binding and every agent consumes $rk_0 + q$ in every time period. Consequently, the following theorem is straightforward.

Stability Theorem 3: Suppose that $K/(1-U) \geq \bar{K}$. Then $\bar{S}$ is a monotonic global attractor. (See Figure 9, and Lemma 10 in the Appendix.)

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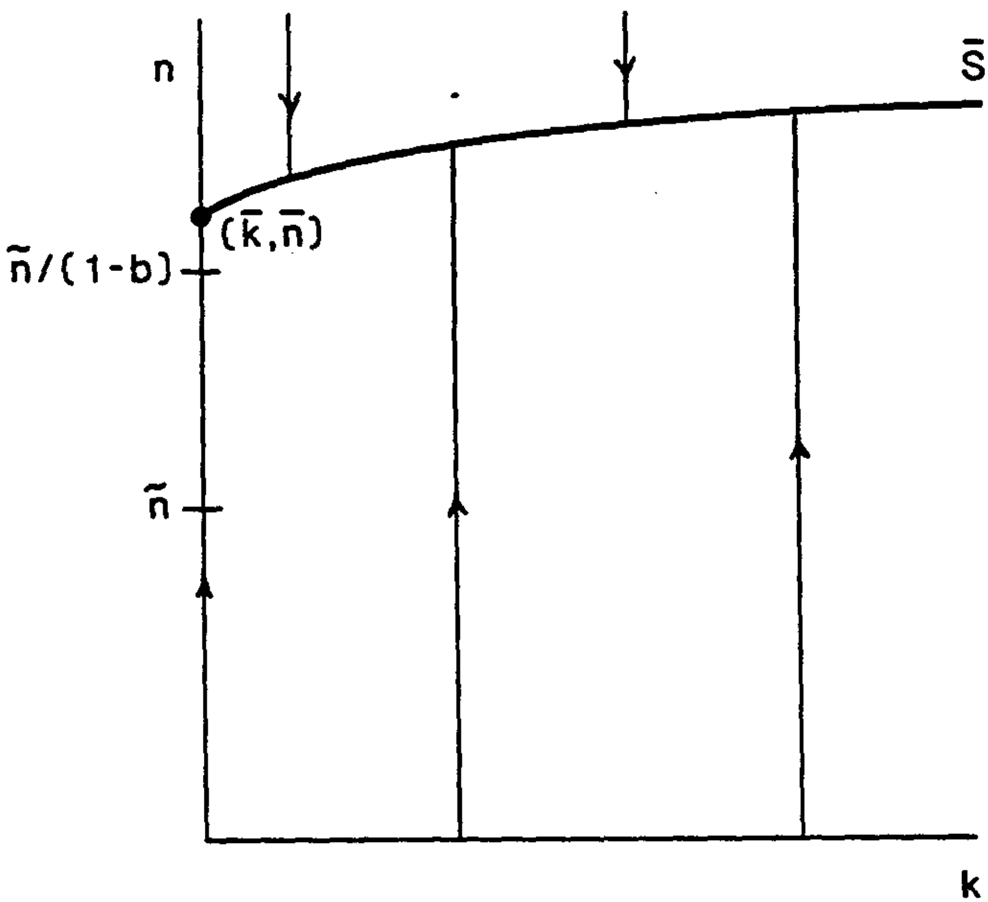

Figure 9: $K/(1-U) \geq \bar{K}$. (Stability Theorem 3.)

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5.2. System Stability

It is important to study system stability in order that we might understand the long-run effects of land reform.

Let us begin by considering a stationary equilibrium under the assumptions that $\delta(n) = d$ for all n and that $\lim_{c\to\infty} \nu(c) = +\infty$. Suppose that $K \in [\underline{K}, \bar{K})$, that $U > 0$, that all the unemployed are landless, and that all the employed have $K/(1-U)$ units of land each. Clearly, an equitable redistribution in which everyone has $K \geq \underline{K}$ units of land would eradicate unemployment and malnutrition.[35] However, actual land reforms are typically much less dramatic. Let β be the fraction of the complete land reform that is actually accomplished so that the land owned by each originally-unemployed agent is βK and by each originally-employed agent is $[(1-U\beta)/(1-U)]K$.

Stability Theorem 2 (Figure 8) indicates that sufficiently small land reforms (β near zero) will not have any effect on unemployment. Rather the stationary equilibrium prices will be unaffected, the new land distribution will not change over time, and the rental income will have been permanently redistributed to the benefit of the unemployed. On the other hand, a sufficiently large land reform (β near one) will employ all agents. A new stationary equilibrium will result with a lower land/labor ratio, the new land distribution will not change over time, and the originally-unemployed will now permanently receive both wages and a substantial rental income.

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We expect that somewhere in the unit interval there is a threshold land reform β^* such that if $\beta > \beta^*$, everyone will eventually be employed, and if $\beta < \beta^*$, the originally-unemployed will revert back to perpetual unemployment. The story, however, is complicated even under the simplifying assumption of certain survival ($\delta(n) = d$): If the originally-unemployed gain enough land to make them want to accumulate land for future employment, a dynamic general equilibrium with nonstationary prices will result.

Let us consider a similar land reform β under the alternative assumption that $\delta(n) < d$ for all $n < \tilde{n}$. Once again, small land reforms (β near zero) would not have any effect on unemployment (Stability Theorem 1). But, in contrast to the previous case, the new land distribution will not result in a stationary equilibrium. Rather, unemployed agents would seek to decumulate land because of their low survival probabilities. Since the unemployed want to sell land, the employed must be enticed to buy it. Yet the employed wish constant consumption streams because of the concavity of ν. As a result, the price of land would fall immediately following the land reform, and then it would climb back to its original level. Bigger land reforms under uncertain survival are very similar to those discussed previously under certain survival, except for the fact that variable discount factors will further complicate the nonstationary dynamic equilibria which must be studied in order to calculate the threshold land reform β^*.[36]

APPENDICES

Appendix 1. Existence of Optima

Here we prove the existence of a solution to the individual's optimization problem given stationary equilibrium prices. Our stability analysis (Section 5 and Appendix 2) relies upon this result.

Following Streufert (1987), let $Z = \mathbb{R}_+ \times \{0,1\} \times \mathbb{R}_+^2$ be the action space, let $z_0 = (0,0,k_0,n_0)$ be the exogenously-given initial action, and let $z_t = (c_t,e_t,k_t,n_t)$ denote the action in period t. Define the stationary Malinvaud production correspondence $G : Z \rightrightarrows Z$ by

$$G(z_{t-1}) = \{\ z \mid e = 1 \ \text{ and } \ (c,k,n) \in G^1(k_{t-1},n_{t-1})\ \}$$
$$\cup\ \{\ z \mid e = 0 \ \text{ and } \ (c,k,n) \in G^0(k_{t-1},n_{t-1})\ \}\ ,$$

where

$$G^1(k_{t-1},n_{t-1}) =$$
$$\{\ (c,k,n) \in \mathbb{R}_+^3 \mid n \geq \tilde{n} \ \text{ and } \ (k,n) \in f^1(c,k_{t-1},n_{t-1})\ \}\ ,$$
$$G^0(k_{t-1},n_{t-1}) = \{\ (c,k,n) \in \mathbb{R}_+^3 \mid (k,n) \in f^0(c,k_{t-1},n_{t-1})\ \}\ ,$$

$f^1 : \mathbb{R}_+^3 \to \mathbb{R}_+^2$ is defined by equations (1)-(2) , and $f^0 : \mathbb{R}_+^3 \to \mathbb{R}_+^2$ is defined by equation (3)-(4) . Both f^1 and f^0 are continuous.

<u>Lemma 1</u>: G is compact-valued.

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Proof: Take any z_{t-1} . The set $G^1(k_{t-1},n_{t-1})$ is the graph of a continuous function, namely $f^1(\cdot,k_{t-1},n_{t-1}) : \mathbb{R}_+ \to \mathbb{R}_+^2$, which is restricted by weak inequalities. Hence, $G^1(z_{t-1})$ is closed. Similarly, $G^0(z_{t-1})$ is closed. Furthermore, $G^1(z_{t-1}) \subseteq [\ 0,\ (p+r)k_{t-1} + q\ ] \times [\ 0,\ k_{t-1} + (rk_{t-1} + q)/p\ ] \times [\ \tilde{n},\ bn_{t-1} + \lambda((p+r)k_{t-1} + q) - x\ ]$. Similarly, $G^0(k_{t-1},n_{t-1}) \subseteq [\ 0,\ (p+r)k_{t-1}\ ] \times [\ 0,\ k_{t-1} + rk_{t-1}/p\ ] \times [\ 0,\ bn_{t-1} + \lambda((p+r)k_{t-1})\ ]$. Since both $G^1(z_{t-1})$ and $G^0(z_{t-1})$ are closed and bounded subsets of $\mathbb{R}^3$, $G(z_{t-1})$ is compact. Q.E.D.

Lemma 2: G is upper hemicontinuous.

Proof. Consider $<(z^m_{-1},z^m)>_{t=1}^{\infty} \to (z^0_{-1},z^0)$ such that $(\forall m)$ $z^m \in G(z^m_{-1})$. Since $e^0 \in \{0,1\}$, there is a subsequence $<(z^\lambda_{-1},z^\lambda)>_{t=1}^{\infty} \to (z^0_{-1},z^0)$ such that $(\forall\lambda)$ $e^\lambda = e^0$.

Suppose $e^0 = 1$. Then $<(k^\lambda_{-1},n^\lambda_{-1},c^\lambda,k^\lambda,n^\lambda)>_{t=1}^{\infty}$ is contained in the graph of G^1 . The graph of G^1 is closed because it is the graph of a continuous function, namely $f : \mathbb{R}_+^3 \to \mathbb{R}_+^2$, restricted by weak inequalities. Thus, $(c^0,k^0,n^0) \in G^1(k^0_{-1},n^0_{-1})$, and consequently, $z^0 \in G(z^0_{-1})$. A similar argument can be made if $e^0 = 0$. Q.E.D.

Following Streufert (1987), define $U : Z^\infty \to \bar{\mathbb{R}}$ by equation (5), and define $W : Z\times\bar{\mathbb{R}} \to \bar{\mathbb{R}}$ by

$$W(z_t,u_{t+1}) = \nu(c_t) + \delta(n_t)\cdot u_{t+1} .$$

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The aggregator W is continuous since ν and δ are continuous by assumption, and it is strictly increasing in u_{t+1} since $\delta(n_t) > 0$ by assumption. Clearly, W recursively expresses U.

<u>Lemma 3</u>: If $p = [d/(1-d)]r$, U is tail insensitive over G at any z_0.

<u>Proof</u>. First we derive an upper bound for consumption in period t. Suppose that the entire initial land stock k_0, as well as the entire wage q from periods $s = 1,\ldots t-1$ is saved to finance consumption in period t. Then, because savings (in land) grow at the rate $1 + r/p$, and because $(1 + r/p) = d^{-1}$ since $p = [d/(1-d)]r$ by assumption,

$$
\begin{aligned}
(9)\qquad \sup \pi_c G^t(z_0) &\le (1 + r/p)^t pk_0 + \Sigma_{s=1}^{t} (1 + r/p)^{t-s} q \\
&= d^{-t} pk_0 + q \cdot \Sigma_{m=0}^{t-1} (d^{-1})^m \\
&\le d^{-t} a_1 ,
\end{aligned}
$$

where $a_1 = pk_0 + q/\ln d^{-1}$.

Next we derive an upper bound for utility in period $t + 1$. Since the exponent of ν is asymptotically bounded below unity, there is $c^0 \ge 0$ and $e^0 \in (0,1)$ such that $\nu(c) \le \nu(c^0) + c^{e^0}$. Thus, by (9),

$$
\begin{aligned}
(10)\qquad & \Sigma_{s=1}^{\infty} d^{s-1} \nu(\sup \pi_c G^{t+s}(z_0)) \\
&\le \Sigma_{s=1}^{\infty} d^{s-1} (\nu(c^0) + (d^{-(t+s)} a_1)^{e^0}) \\
&= a_2 + d^{-e^0 t} a_3 ,
\end{aligned}
$$

where $a_2 = \nu(c^0)(1-d)^{-1}$ and $a_3 = (d^{-1}a_1)^{e^0}(1-d^{(1-e^0)})^{-1}$.

Finally, take any ${}_1\tilde{z} \in \times_{t=1}^{\infty} G^t(z_0)$. By (10),

$$\begin{aligned}
&\lim_{t\to\infty} \sup U(\ {}_1\tilde{z}_t,\ \times_{s=t+1}^{\infty} G^s(z_0)\ ) \\
&\quad \leq \lim_{t\to\infty} U(\ {}_1\tilde{z}_t,\ {}_{t+1}\tilde{0}\ ) + d^t \cdot \Sigma_{s=1}^{\infty} d^{s-1}\nu(\sup \pi_c G^{t+s}(z_0)) \\
&\quad \leq \lim_{t\to\infty} U(\ {}_1\tilde{z}_t,\ {}_{t+1}\tilde{0}\ ) + d^t a_2 + d^{(1-e^0)t} a_3 \\
&\quad = \lim_{t\to\infty} U(\ {}_1\tilde{z}_t,\ {}_{t+1}\tilde{0}\ ) \\
&\quad \leq \lim_{t\to\infty} \inf U(\ {}_1\tilde{z}_t,\ \times_{t=1}^{\infty} G^s(z_0)\ )\ .
\end{aligned}$$

Q.E.D.

__Lemma 4__: If $p = [d/(1-d)]r$, an optimum exists.

__Proof__. Lemmas 1-3 and Streufert (1987, Theorem A). Q.E.D.

Appendix 2. Stability of Optima

Here we study the stability of solutions to the individual's optimization problem given stationary equilibrium prices. Specifically, we take as parametric a land/labor ratio $K/(1-U) \geq \underline{K}$, and assume $q = F_E(K,1-U)$, $r = F_K(K,1-U)$, and $p = [d/(1-d)]r$. The lemmas of this Appendix yield the stability theorems of Section 5.

__Lemma 5__: Define $(\bar{k},\bar{n})$ as in Section 4. If $(n_0,k_0) \geq (\bar{k},\bar{n})$, there is a unique optimum $\langle(c_t,e_t,k_t,n_t)\rangle_{t=1}^{\infty}$, and for all t , $c_t = rk_0 + q$, $e_t = 1$, and $k_t = k_0$.

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Proof: Consider the stream $\langle(c_t, e_t, k_t, n_t)\rangle_{t=1}^{\infty}$ defined at each t by $(c_t, e_t, k_t) = (rk_0 + q, 1, k_0)$ and $n_t = bn_{t-1} + \lambda(rk_0 + q) - x$. Since $\lambda(rk_0 + q) - x$ is constant, $\langle n_t\rangle_{t=1}^{\infty}$ converges monotonically to $\{\lambda(rk_0 + q) - x\}/(1-b) > \{\lambda(r\bar{k} + q) - x\}/(1-b) \geq \bar{n}$. Then since $n_0 \geq \bar{n} \geq \tilde{n}$, each $n_t \geq \tilde{n}$. The argument employed in paragraphs 3 and 5 of Part 1 in the proof of the Characterization Theorem (Subsection 3.1) can now be straightforwardly applied to demonstrate that $\langle(c_t, e_t, k_t, n_t)\rangle_{t=1}^{\infty}$ is the unique optimum. Q.E.D.

Definitions: Recall that the derivative λ' is bounded above by the constant h (Subsection 2.2). Define $(k^*, n^*) \gg (0,0)$ to be a point on $\underline{S}$ such that if another (k,n) satisfies $phk + n \leq phk^* + n^*$, then (k,n) must also satisfy both

$$n < \tilde{n}/2 \text{ , and} \tag{11}$$

$$bn + \lambda[(p+r)k + q) - x < \tilde{n} \text{ .} \tag{12}$$

See Figure 10. Such a (k^*, n^*) exists because the subset of $\mathbb{R}_+^2$ which satisfies (11)-(12) is open and contains $(0,0)$ since $K/(1-U) < \bar{K}$ (Subsection 4.3).

Define $C = \{(n,k) |\ k \geq k^*\}$, $B = \sim C \cap \{(n,k) |\ phk + n \leq phk^* + n^*\}$, and $A = \sim C \cap \sim B$. See Figure 10.

Lemma 6: Suppose $K/(1-U) < \bar{K}$. If $\langle(c_t, e_t, k_t, n_t)\rangle_{t=1}^{\infty}$ is feasible from $(k_0, n_0) \in B$ and there is some s for which

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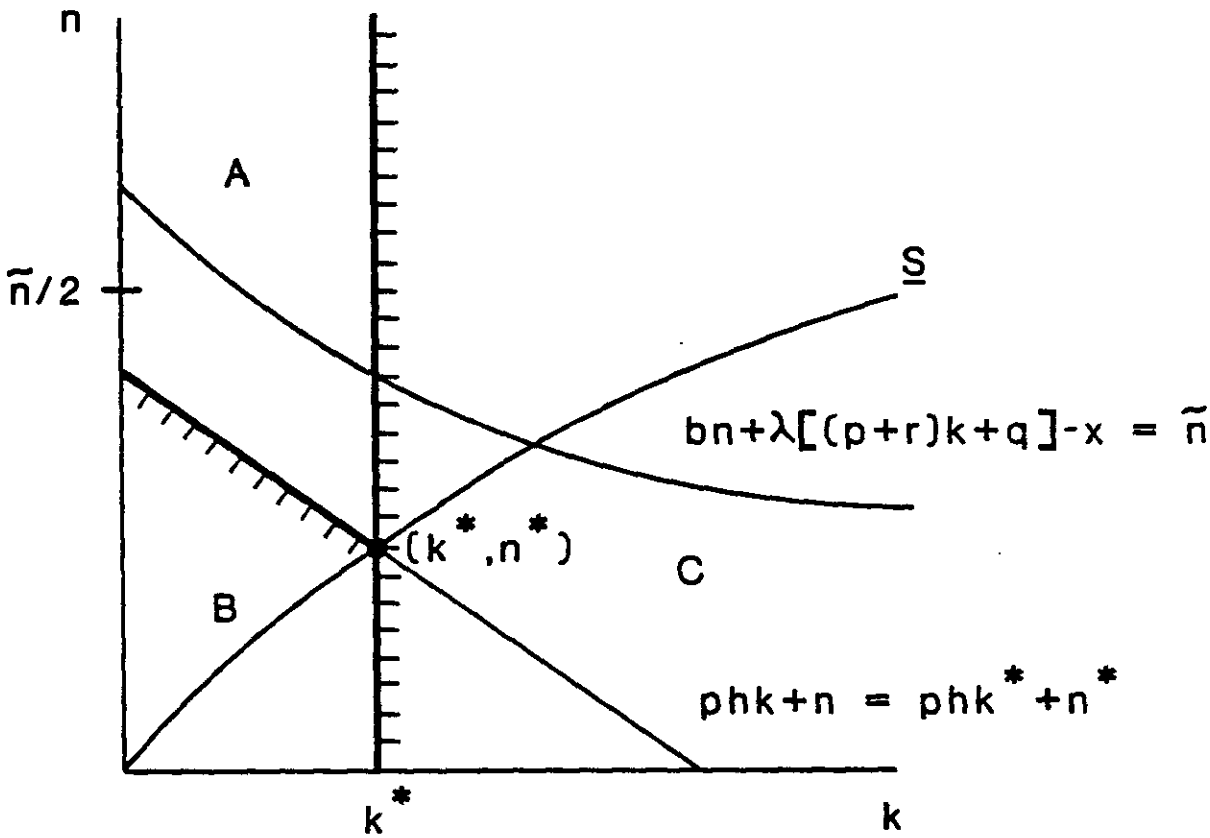

Figure 10: The point (k^*,n^*) and the sets A , B , and C . (Lemma 6.)

$n_s > \tilde{n}/2$, then there is a period T for which $k_T \geq k^*$ and $(k_t, n_t) \in B$ for $t < T$.

Proof: Consider any (k_{t-1}, n_{t-1}) on the border between A and B . Since $(k_{t-1}, n_{t-1}) \in B$, $e_t = 0$ by (12), and since it lies above $\underline{S}$, $\lambda(rk_{t-1}) - (1-b)n_{t-1} < 0$. Thus, equations (3)-(4) imply

$$n_t = bn_{t-1} + \lambda[rk_{t-1} + p(k_{t-1} - k_t)] ,$$
$$n_t \leq bn_{t-1} + \lambda(rk_{t-1}) + ph(k_{t-1} - k_t) ,$$
$$n_t + phk_t \leq n_{t-1} + phk_{t-1} + [\lambda(rk_{t-1}) - (1-b)n_{t-1}] , \text{ and}$$
$$n_t + phk_t \leq n_{t-1} + phk_{t-1} .$$

Therefore, if (k_{t-1}, n_{t-1}) lies on the border between A and B , (k_t, n_t) must not lie in A . Since any point in B is dominated by a point on the border between A and B , we have that $(k_{t-1}, n_{t-1}) \in B$ implies $(k_t, n_t) \in {\sim}A$.

Now take some feasible $\langle(c_t, e_t, k_t, n_t)\rangle_{t=1}^{\infty}$ from $(k_0, n_0) \in B$ and suppose that $n_s > \tilde{n}/2$. Then $(k_s, n_s) \in A \cup C$. Since $(k_{t-1}, n_{t-1}) \in B$ requires $(k_t, n_t) \in {\sim}A$ by the previous paragraph, there must be some $T \leq s$ such that $(k_T, n_T) \in C$ and $(k_t, n_t) \in B$ for $t < T$. Q.E.D.

Lemma 7: Suppose that $K/(1-U) < \bar{K}$, and that $\delta(n) < d$ for all $n < \tilde{n}$. Then there is a land-nutrition pair $(\underline{k}, \underline{n}) \gg (0,0)$ such that if $(k_0, n_0) \ll (\underline{k}, \underline{n})$, then every optimum $\langle(c_t, e_t, k_t, n_t)\rangle_{t=1}^{\infty}$ is such that each $e_t = 0$,

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$\langle k_t \rangle_{t=1}^{\infty}$ is monotonically decreasing, and $\lim_{t\to\infty} c_t = \lim_{t\to\infty} k_t = \lim_{t\to\infty} n_t = 0$.

<u>Proof</u>: Define $0 < \underline{k} < k^*$ so that

$$\nu'(\underline{k}/d) > v \cdot (k^*)^{-a} (\underline{k}/d)^{a-1} ,$$

where $v = \nu(rk^*/d + q)/(1-d)$ and $a = \ln \delta(\tilde{n}/2)/\ln d > 1$ (note $\ln \delta(\tilde{n}/2) < \ln d < 0$). Such a $\underline{k}$ exists because $\nu'(k/d)$ is nondecreasing as k appoaches 0 since ν is concave, and because $\lim_{k\to 0} v(k^*)^{-a}(k/d)^{a-1} = 0$ since $a > 1$. Define $\underline{n}$ to be the second coordinate of the point on $\underline{S}$ whose first coordinate is $\underline{k}$. Since $0 < \underline{k} < k^*$, $0 < \underline{n} < n^*$.

Now take any $(k_0, n_0) \ll (\underline{k}, \underline{n})$ and suppose that $\langle (c_t, e_t, k_t, n_t) \rangle_{t=1}^{\infty}$ is a feasible stream in which nutrition exceeds $\tilde{n}/2$ in some period. By Lemma 6, there is a period T such that $k_T \geq k^*$ and $(k_t, n_t) \in B$ for $t < T$.

Modify this stream by taking the land k_T and consuming in the first period its present discounted value $d^{T-1}k_T$. This yields a utility gain in the first period of

$$\nu(c_1 + d^{T-1}k_T) - \nu(c_1) .$$

The loss entailed in period $T+1$ and thereafter can be bounded. First, assume that the original $\langle (c_t, e_t, k_t, n_t) \rangle_{t=1}^{\infty}$ involved employment in period $T+1$ and every succeeding period, and that by stripping away k_T , we sacrifice all this employment. Second, $(k_{T-1}, n_{T-1}) \in B$ implies both $e_T = 0$ and $k_{T-1} < k^*$,

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and consequently, $k_T < (1 + r/p)k_{T-1} < (1 + r/p)k^* = k^*/d$. Thus, the current utility loss evaluated at period $T+1$ is at most

$$v = \nu(rk^*/d + q)/(1-d) .$$

Since $(k_t, n_t) \in B$ for $t < T$ implies $n_t < \tilde{n}/2$ for $t < T$, the utility loss evaluated at period 1 is at most

$$\begin{aligned} &\delta(\tilde{n}/2)^{T-1} dv \\ &\quad \le \delta(\tilde{n}/2)^{\ln (d^{T-1}k_T/k^*)/\ln d} v \\ &\quad = (d^{t-1}k_T/k^*)^a v , \end{aligned}$$

where $a = \ln \delta(\tilde{n}/2)/\ln d$ as defined earlier, and the inequality follows from the fact that $k_T \ge k^*$ and thus $T-1 \ge \ln (d^{T-1}k_T/k^*)/\ln d$.

By $c_1 < k_0/d < \underline{k}/d$ and the concavity of ν , the definition of $\underline{k}$, the two facts that $a > 1$ and $d^T k_T < k_0 < \underline{k}$, and finally algebraic manipulation,

$$\begin{aligned} &\nu(c_1 + d^{T-1}k_T) - \nu(c_1) \\ &\quad > \nu'(\underline{k}/d)d^{T-1}k_T \\ &\quad > [\underline{k}/k^*]^a [d^T k_T/\underline{k}][v/d^a] \\ &\quad > [\underline{k}/k^*]^a [d^T k_T/\underline{k}]^a [v/d^a] \\ &\quad = (d^{T-1}k_T/k^*)^a v . \end{aligned}$$

Thus the utility gained by increasing first-period consumption by $d^{t-1}k_T$ outweighs the utility lost by reducing land in period T

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from k_T to 0 . Since this modification is feasible, the original $\langle(c_t,e_t,k_t,n_t)\rangle_{t=1}^{\infty}$ cannot be optimal. Hence, every optimum has $n_t < \tilde{n}/2$ in every period.

Since every optimum exhibits perpetual malnutrition whenever $(k_0,n_0) << (\underline{k},\underline{n})$, every optimum exhibits perpetual unemployment. Since an optimum exists (Lemma 4), the maximization problem (1)-(5) then reduces to finding the optimal way to distribute consumption over time given the land endowment k_0 . Furthermore, $n_t < \tilde{n}/2$ for all t implies $\delta(n_t) < \delta(\tilde{n}/2) < d$ for all t . Therefore, the agent will decumulate land holdings: $\langle k_t\rangle_{t=1}^{\infty}$ is monotonically decreasing, and $\lim_{t\to\infty}k_t = \lim_{t\to\infty}c_t = \lim_{t\to\infty}n_t = 0$. Q.E.D.

<u>Lemma 8</u>: Suppose that $K/(1-U) < \bar{K}$, that $\delta(n) = d$ for all n , and that $\lim_{c\to 0}\nu'(c) = +\infty$. Then there is a land-nutrition pair $(\underline{k},\underline{n}) >> (0,0)$ such that if $(n_0,k_0) << (\underline{k},\underline{n})$, there is a unique optimum $\langle(c_t,e_t,k_t,n_t)\rangle_{t=1}^{\infty}$, and for all t , $c_t = rk_0$, $e_t = 0$, and $k_t = k_0$.

<u>Proof</u>: Define $0 < \underline{k} < k^*$ so that

$$\nu'(\underline{k}/d) > v/k^* ,$$

where $v = \nu(rk^*/d + q)/(1-d)$. Such a $\underline{k}$ exists because $\lim_{c\to 0}\nu'(c) = +\infty$. Define $\underline{n}$ to be the second coordinate of the point on $\underline{S}$ whose first coordinate is $\underline{k}$. Since $0 < \underline{k} < k^*$, $0 < \underline{n} < n^*$.

As in the proof of Lemma 7, we employ Lemma 6 to show that every optimum from some $(k_0, n_0) << (\underline{k}, \underline{n})$ exhibits perpetual malnutrition. Only two modifications are required. First, the utility loss evaluated at period 1 is at most

$$d^{T-1}dv \le d^{\ln (d^{T-1}k_T/k^*)/\ln d} v$$
$$= (d^{T-1}d_T/k^*)v \ .$$

Second, the gain from modifying the original stream is shown to exceed the loss by $c_1 \le k_0/d \le \underline{k}/d$, the concavity of ν, and the definition of $\underline{k}$:

$$\nu(c^1 + d^{T-1}k_T) - \nu(c_1) > \nu'(\underline{k})d^{t-1}k_T$$
$$> (d^{T-1}k_T/k^*)v \ .$$

Finally, since perpetual malnutrition implies perpetual unemployment and since an optimum exists (Lemma 4), the maximization problem (1)-(5) reduces to finding the optimal way to distribute consumption over time given the land endowment k_0. Since $p = [d/(1-d)]r$ and ν is strictly concave, there is a unique optimum $<(c_t, e_t, k_t, n_t)>_{t=1}^{\infty}$ such that for all t, $c_t = rk_0$, $e_t = 0$, and $k_t = k_0$. Q.E.D.

<u>Lemma 9</u>: Define $(\bar{k}, \bar{n})$ as in Section 4. If $\bar{k} > 0$, there is a $\varepsilon > 0$ such that if $<(c_t, e_t, k_t, n_t)>_{t=1}^{\infty}$ is optimal from $(k_0, \bar{n})$ and $k_0 \in (\bar{k}-\varepsilon, \bar{k})$, then $k_1 < k_0$.

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<u>Proof</u>: Define T such that

$$\Sigma_{t=1}^{T} d^{t-1}\nu(r\bar{k} + q) > \nu(r\bar{k} + dq)/(1-d) .$$

Such a T exists because $\Sigma_{t=1}^{\infty} d^{t-1}\nu(r\bar{k} + q) = \nu(r\bar{k} + q)/(1-d)$. Then define $0 < \varepsilon < d^T\bar{k}$.

Now take any $k_0 \in (\bar{k}-\varepsilon,\bar{k})$ and assume $n_0 = \tilde{n}$. Employment in period 1 requires that $n_1 \geq \tilde{n}$, hence that $c_1 \geq r\bar{k} + q$, and hence that $k_1 \leq k_0 - (\bar{k} - k_0)r/p < k_0$. Hence, if $k_1 \geq k_0$, then $e_1 = 0$, and consequently, the agent's lifetime income cannot exceed $[1/(1-d)]r\bar{k} + [d/(1-d)]q$. Thus, if $k_1 \geq k_0$, the agent's utility cannot exceed $\nu(r\bar{k} + dq)/(1-d)$.

We show it is suboptimal to choose $k_1 \geq k_0$ by constructing a feasible stream which exceeds this upper bound. Let $(c_t,e_t,n_t) = (r\bar{k} + q,\ 1,\ \tilde{n})$ for the first T periods. By the definition of $\bar{k}$, a consumption of $r\bar{k} + q$ is sufficient to maintain nutrition at $\tilde{n}$, and hence employment is feasible. This consumption in excess of factor incomes is maintained by whittling down the initial land holding during the first T periods. To be precise, $k_t = \bar{k} - d^{-t}(\bar{k} - k_0)$ for $t \leq T$, and by the definitions of k_0 and ε , $k_T = \bar{k} - d^{-T}(\bar{k} - k_0) > \bar{k} - d^T\varepsilon > 0$. This course of action yields a utility of at least $\Sigma_{t=1}^{T} d^{t-1}\nu(r\bar{k} + q)$, and by the definition of T , this exceeds the upper bound derived when $k_1 \geq k_0$. Q.E.D.

<u>Lemma 10</u>: Suppose $K/(1-U) \geq \bar{K}$. Then for any (k_0,n_0) there is a unique optimum $\langle(c_t,e_t,k_t,n_t)\rangle_{t=1}^{\infty}$, and for all t ,

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$c_t = rk_0 + q$, $e_t = 1$, and $k_t = k_0$.

Proof: Since $K/(1-U) \geq \bar{K}$, the nutritional requirement for employment is never binding-regardless of (k_0, n_0) (Subsection 4.3). Thus the agent will be employed and adequately nourished in every period. Constant consumption is implied by the strict concavity of ν . Q.E.D.

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NOTES

1. Leibenstein's observations have been extended by a number of writers. See, e.g., Mirrlees (1975), Stiglitz (1976), Bliss and Stern (1978a,b), and Dasgupta and Ray (1986, 1987a). The theories start from the postulate that there is a strong connection between an individual's consumption and his ability to perform productive work. This connection yields a number of interesting results regarding labor markets. The theory has also been subjected to testing-with mixed results depending on the exact form under review. See Bliss and Stern (1978b) for a summary of some of these tests.

2. In fact, the term "efficiency wage" has been carried over to embrace an entire group of theories where the payment of wages affects ability or effort, not necessarily for nutritional reasons. See Yellen (1984) and the recent collection of readings edited by Akerlof and Yellen (1986).

3. A person's nutritional history may affect his current ability in several ways. First, he may acquire stores of energy (such as body fat) which can be run down during a period of low consumption for the purposes of activity. Second, it has been claimed that his very history of intakes might create adaptive responses in his needs. Finally, nutritional history affects current activity by altering survival probabilities. The first and third aspects are explicitly considered in our model. For a survey of the clinical literature in this context, see Dasgupta and Ray (1987b).

4. We adopt in this paper the simplifying assumption that an individual is able to supply one unit of labor power if he is "adequately" nourished (i.e., if his nutrition stock exceeds the critical minimum), and none otherwise. This is a step function which can be generalized (at the cost of considerable technical complexity) to the smoothly increasing functions used in the static theory. However, we do allow fresh nutrition acquisition

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to be a smoothly increasing function of consumption (see below in the main text).

5. Our dynamic framework requires a refinement of this second notion of unemployment. In our model, any positive initial land holding enables the agent to work in some (perhaps very distant) time period if he consumes little and accumulates land in the meantime. As the initial land holding approaches zero, the waiting time increases without bound. Consequently, this option is unattractive for sufficiently small initial land holdings, due to the concavity of the objective function (Section 4.2) or the decreased survival probability (Section 5.1).

6. If the option of being employed is outside the feasible set, one might hesitate to call such unemployment involuntary. Imagine, for example, an individual who wishes to be a professional pianist and therefore rejects other lucrative job offers, but cannot find a job in the industry of his dreams because he has not talent as a musician. Is his unemployment in the music industry "involuntary"? Perhaps it should not be classified as such! In the present context, however, agents are constrained by their lack of adequate <u>nutrition</u>. The case for classifying such an individual as involuntarily unemployed is far more compelling.

7. Indeed, the characterization of stationary equilibria that we present below can be extended with no difficulty. Rather more demanding is the extension of the stability analysis in Section 5.

8. As noted by Bewley (1982, p. 234), stationary dynamic general equilibria are not unique and do depend upon initial conditions. In other words, initially wealthy individuals generally remain wealthy. In essence, a concave intertemporal production function uniquely determines the <u>aggregate</u> capital stock (just as it does in optimal growth theory, Gale, 1967), but the <u>distribution</u> of this aggregate quantity among individuals provides another degree of freedom. (Also see Coles (1983) and Yano (1984).)

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9. Our formal analysis deals with stationary equilibria where prices are indeed time-stationary. We could have, nevertheless, set up the agent's problem using a time-varying price sequence, but avoid this for notational ease.

10. The loss $(1-b)n_{t-1}$ approximates the energy required merely to maintain the agent's body given minimum activity levels. This requirement increases with the agent's weight (Bliss and Stern, 1978b, p. 369).

11. "Over-consumption" may even lead to a <u>decline</u> in nutritional well-being, but we are obviously not concerned with this extreme in the paper!

12. This imposes an upper bound on the efficiency with which the digestive system converts food into nutrition.

13. There is, of course, the energy requirements for basal matabolism and the maintenance of the body frame that must be expended. We capture this is the depreciation factor b .

14. We are referring to "partial equilibrium" models such as Leibenstein (1957), Mirrlees (1975), and Bliss and Stern (1978a); and "general equilibrium" models such as Dasgupta and Ray (1986,1987a). For examples of two-period models with a lagged nutrition-efficiency relationship, see, e.g., Gupta (1987) or Guha (1987).

15. All the references in footnote 14 deal with a nutrition-efficiency relationship that yields a smoothly increasing endowment of efficiency units as nutrition increases. For an example (in the static context) using the "step-function" case that we consider here, see Dasgupta and Ray (1986).

16. In the terminology of Brock and Gale (1969), we assume that the exponent of ν is asymptotically bounded below unity.

17. There is (as is to be expected) a vast clinical literature linking undernutrition to survival probabilities. See the references in Dasgupta and Ray (1987b).

18. The model assumes a constant population size although the survival probability may fall below unity. This requires

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some story regarding reproduction: Each agent who dies after period t is replaced in period $t+1$ by a perfect clone, interpreted as the child of the deceased. In particular, this child inherits the land holdings and nutritional stock of his parent. This story accords crudely with the fact that in relatively malnourished family lines, individuals have shorter lifetimes and generations elapse more quickly. The utility function of the individual living in period r is given by (5), where "t=1" is replaced by "t=r" and "s=1" is replaced by "s=r". Thus every individual in the family line maximizes expected utility, given that the utility of death is zero (Heal, 1973).

19. It is also required that c , e , and k are measurable with respect to Lebesque measure μ . This technicality is handled appropriately in the proof of the Characterization Theorem (Section 3).

20. The abstract theories of recursive utility and dynamic programming in Streufert (1986a, 1986b, 1987) can be straightforwardly applied to this unusual optimization problem having unbounded feasible streams, a variable discount factor, two state variables, and a discrete employment variable. Given the stated assumptions and that the price of land equals the present discounted value of the rental stream (i.e. $p = [d/(1-d)]/r$; this holds in any stationary equilibrium), we know, for any initial (k_0,n_0) , that an optimum exists (Lemma 4), and that the objective function (5) is finite over the feasible set (second paragraph in proof of Lemma 3).

21. Of course, by the constant-returns assumption, maximum profits are zero.

22. The fact that $\bar{K}$ is well-defined follows from the Inada condition on F , and, of course, the ability of λ to attain the value $\tilde{n} + x$.

23. $\bar{K}$ can be thought of as a threshold beyond which the "trickle-down" effects of growth can at least succeed in removing the sort of poverty trap created by malnutrition. Of course,

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issues of inequality will continue to be important even in this region.

24. There is another possible concept of the viable threshold, namely, the smallest economy that can provide everyone with an adequate level of nutrition ($\tilde{n}$), <u>regardless</u> of whether all are employed or not. One can show that this viable threshold is lower than the viable threshold defined in the text. Moreover, it may be possible to support such allocations as stationary equilibria (see footnote in Subsection 4.2). These equilibria would involve unemployment but no malnutrition.

25. The proof follows the first paragraph in the proof of the Characterization Theorem, and it is omitted here.

26. As the ratio increases, the rental income from a given amount of land also falls. This decrease in r is, however, always dominated by the increase in q .

27. Of course, we can derive many more equilibrium distributions by changing the names of the employed and the unemployed.

28. This implies that $n(a) \geq \tilde{n}$ and that $(k(a),n(a)) \in \bar{S}$.

29. This is not a characterization of all equilibrium distributions because $(\underline{k},\underline{n})$ does not exactly demarcate those points on $\underline{S}$ from which employment would not be sought. Rather, it gives a subset of those points.

30. This implies that $(k(a),n(a)) \in \underline{S}$.

31. There may be equilibrium prices in which certain points on $\underline{S}$ above $n = \tilde{n}$ are optima. Such points would represent unemployed agents who are nonetheless adequately nourished.

32. A non-Walrasian equilibrium concept involving unemployment rationing would allow us to obtain an unemployment rate of $\bar{U}(K)$ with a land/labor ratio of $\bar{K}$. The "involuntary unemployment" of Dasgupta and Ray is a very similar concept. In this case, landless workers with zero nutrition would be just capable of providing labor power but would be rationed out of the labor market. However, such workers are <u>not</u> capable of offering their

labor power for any wage less than the equilibrium wage. They are thus incapable of upsetting this extended equilibrium concept.

33. We do however know that an optimum always exists (Lemma 4), and that the abstract dynamic programming theory of Streufert (1987, Theorem A) is applicable.

34. This casual observation accords with the formal fact that if we try to set U above the upper limit $\bar{U}(K)$, everyone can work in every period and unemployment becomes impossible. It also accords with the fact that the $(\underline{k}, \underline{n})$ defined in Lemmas 7 and 8 shrinks toward $(0,0)$ as U increases toward $\bar{U}(K)$.

35. We neglect in this discussion how the initial distribution of nutrition is altered.

36. The dynamic stability of general equilibrium paths has been studied under convex structures (see Bewley (1982), Coles (1983), and Yano (1984)). However, the pervasive nonconvexities of our model preclude an application of the techniques developed in those papers. The formulation of an adequate nonstationary equilibrium theory in models such as ours remains a challenging task.

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