Long Run Policy Analysis and Long Run Growth

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45

1. Introduction

Countries differ widely in their average rates of economic growth. For example, in the sample studied by Kormendi and Meguire (1986), the fastest growing countries grew roughly seven times faster than the country with the lowest rate of expansion.[1] Economists and policy makers frequently perceive these cross-country differences to be linked to various aspects of economic policy, such as taxes, trade, property rights, etc.

The belief that policy matters for growth has found some empirical confirmation but has surprisingly weak theoretical foundations. The familiar neoclassical growth model is not a suitable environment for studying the effects of different policies on the long run rate of economic expansion. The question cannot really be posed, since neoclassical economies grow in the steady state at the rate of growth of exogenous technological progress.[2]

The main objectives of this paper are to (i) propose an alternative class of models to study the long run effects of policy; and (ii) begin to

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[1] The Kormendi-Meguire sample period was 1948-1977, subject to restrictions on data availability. The fastest growing countries were Brazil (9%), Israel (8.4%) and Japan (8%). The slowest growing country was Uruguay (1.3%).

[2] Throughout the paper, steady state growth is defined as a situation in which all variables grow at constant (but possibly different) rates. The terms steady state and balanced growth are used interchangeably. The expression long run growth refers, for the models we consider, to the behavior of the economy in the steady state.

explore the implications of this class of models.

In any model that seeks to identify the sources of economic growth and the impact of policy on these sources, productivity increases associated with economic expansion must be viewed as the outcome of actions taken by agents in the economy. Naturally, this leads us to consider models in which sustained growth in per capita output arises in the presence of time-stationary technologies. Following King and Rebelo (1986) we refer to these economies as "endogenous growth models", to emphasize that long run growth is not driven by exogenous factors. This class of environments is the outgrowth of recent work by Paul Romer (1986a) and Robert Lucas (1985).[3] These important contributions reopened the twin questions: (i) what are the engines of economic growth; and (ii) how should these forces be incorporated in simple descriptions of the mechanics of economic development.

Ideally, we would study the policy implications of an endogenous growth model which had been subjected to substantial theoretical scrutiny and careful empirical testing. Unfortunately, such a model is not yet available. We are left with the alternative of characterizing the policy implications of the entire class of endogenous growth models. To reduce the size of this task we focus on economies that possess a steady state path. There are several advantages to restricting the scope of the investigation to this

[3]Predecessors of these models are, among others, Arrow (1962) and Uzawa (1965).

47

subset of models. First, there is hope that the models studied have empirical relevance, since steady state economies typically conform with the stylized facts of economic development.[4] Second, the analysis is greatly simplified. Finally, it will be clear that any differences between the models we examine and the neoclassical model are not related to the question of whether or not economies possess a steady state growth path. A major disadvantage of focusing on steady state models is that stringent restrictions must be imposed on the functional forms employed to describe preferences and technology.

A direct presentation of a general model with multiple capital and consumption goods as well as endogenous labor supply is feasible but provides little information on the economic mechanisms at work in this class of environments. For this reason we follow an alternative path. The first step involves studying a simple endogenous growth model in which there is a single capital good and a single consumption good. The second step is to investigate how the results of this model change as we introduce more consumption and capital goods and make the supply of labor endogenous. The analysis of these richer environments leads to the conclusion that the implications of the basic model are surprisingly robust. The multiple capital good economies studied here also illustrate that steady state endogenous growth models do not necessarily rely on linearity in the

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[4] The stylized facts usually associated with the neoclassical model include the constancy of the real interest rate and of several ratios, such as the labor share, the share of consumption in output, the investment-output ratio, etc. See Kaldor (1961) for a complete list. It is controversial whether these facts are reasonably accurate descriptions of reality--see Romer (1986a,1987).

production function of the capital sector or on the exclusion of fixed factors from the production of all types of capital.[5]

To clarify the exposition, the discussion will be focused on two familiar policy issues--the long run effects of taxation and of anticipated inflation. However, the analysis presented is applicable to a wide range of policy questions. The effects of taxation are discussed because of their simplicity and general importance. Anticipated inflation is studied since endogenous growth models may provide new insights into the mechanics underlying the long run relationship between the real interest rate and the rate of inflation. In both cases only permanent changes in policy will be addressed. The focus will be on determining which permanent policy changes have a permanent effect on the growth rate of the economy.

The paper is organized as follows. Section 2 examines the nature of the growth phenomenon in the neoclassical model and reviews its main implications for the effects of taxation and anticipated inflation. Section 3 presents a simple endogenous growth model and contrasts it with the neoclassical paradigm. Section 4 discusses extensions of this model that incorporate multiple capital goods and endogenous labor supply. This section illustrates the robustness of the results of the basic model of section 3. A final section summarizes the main conclusions.

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[5]The expression "fixed factor" designates factors of production that are available in the same exogenous quantity in every point in time.

2. Policy Implications of the Neoclassical Model

While the neoclassical model and its policy implications are very familiar, it is nevertheless useful to summarize its key features.

Exogenous long run growth

In the neoclassical model, there is a single produced good. The technology of production is constant returns to scale, combining capital (K) and labor (N) augmented by exogenous technological progress (X):[6] $Y = F(K,NX)$.[7] The index of technological progress grows at a constant rate g_x.[8] The function F(K,NX) is twice differentiable, increasing in both arguments, concave and verifies the Inada conditions.[9] Furthermore, both factors are assumed to be essential in production. Capital is simply stored output so its path is given by $\dot{K}_t = I_t - \delta K_t$, where I_t is investment--the part of output devoted to capital accumulation--and δ is the depreciation rate.[10] Finally, the amount of output available for consumption (C_t) in each period is given by $Y_t - I_t \geq 0$.

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[6] The only type of technological progress consistent with steady state growth is labor-augmenting. See Swan (1963) and Phelps (1966).

[7] In order to simplify the exposition, we abstract from population growth throughout the paper, thus all variables are expressed in per capita terms.

[8] Throughout the paper, the notation g_i is used to denote the growth rate of variable i, provided this growth rate is constant.

[9] $Df(0) = \infty$ and $Df(\infty) = 0$, where $f(K/NX) = F[K/(NX),1]$ and Df() denotes the first derivative of f().

[10] The dot notation for the derivative with respect to time is used throughout the paper.

It is straightforward to show that in the single technologically feasible steady state per capita output, consumption and investment all grow at rate g_x. It is natural to ask when this balanced growth path is the outcome of a (sequential) competitive equilibrium under perfect foresight.[11] In the case of exogenous labor supply, this can be determined as follows.[12] Profit maximization by firms implies that, in every period, the real interest rate equals the marginal productivity of capital (net of depreciation), which is given by $D_1F(K/NX,1) - \delta$ and is constant in the steady state.[13] Households, faced with a constant interest rate, must then choose to expand their consumption at a constant rate. If their preferences are time-separable, this will only be optimal if their momentary utility is isoelastic, i.e. if it has the form, $u(C) = (C^{1-\sigma} - 1)/(1-\sigma)$. Assuming that this is the case, we can compute the interest rate that is compatible with consumption growing at rate $g_c = g_x$ by using the efficiency conditions for the household problem. These imply that $g_c = (r - \rho)/\sigma$ so that the steady state interest rate is $r = \sigma g_x + \rho$. The condition $D_1F(K/NX,1) - \delta = r$ pins down the steady state capital-labor ratio, where labor is in terms of efficiency units. It is

[11]For expositional convenience the operation of the various models is described in terms of a regime of sequential loan markets and spot labor markets, although this is not the only market structure that would support the optimal allocation.

[12]Provided that the appropriate restrictions on the utility function are used to guarantee that households choose a constant labor supply along the steady state path (see King, Plosser and Rebelo (1987)), this discussion can be easily adapted to incorporate endogenous labor-leisure choice.

[13]The notation $D^i f(x)$ for the ith total derivative of the function $f(x)$ and $D_i h(x_1, x_2, \ldots, x_n)$ for the ith partial derivative of $h(.)$ will be employed throughout.

well-known that this steady state is stable in the sense that, given any initial capital-labor ratio, the economy always converges to the steady state.[14]

Long run effects of taxation [15]

The introduction of a proportional income tax at rate τ in a neoclassical economy leads to a decrease in the steady state ratio of capital to labor in efficiency units. The steady state real interest rate continues to be equal to $\rho + \sigma g_X$, since consumption has to grow at rate g_X, but the marginal productivity of capital from the standpoint of the private agents, is $(1-\tau)\ D_1F(K/NX,1) - \delta$, (assuming that economic depreciation is not tax deductible). This implies a lower K/NX ratio, with correspondingly lower steady state levels of Y/X, C/X, and I/X.[16] In other words, the steady state paths of Y, K, C and I are shifted downward. If we graph the logarithms of

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[14]The proof in Cass (1965) can easily be adapted to encompass exogenous technological progress.

[15]Standard references are Sato (1967), Krzyzaniak (1967) and Feldstein (1974) who examined the consequences of capital income taxation in versions of the neoclassical model in which the savings rate is not optimally determined. Stiglitz (1978) studied similar problems in an optimizing version of the model, focusing on steady state effects. Recently, Becker (1985) and Judd (1985) have provided elegant analyses of issues concerning capital taxation and income redistribution along the adjustment path.

[16]The effects of a given tax are, of course, dependent on the allocation of its revenue. Standard assumptions, used to isolate the effects of taxation, are that this revenue is transfered to the private sector in a lump sum fashion, or that it is used to finance the provision of public goods and services that do not affect the marginal utility of private consumption or the production possibilities of the private sector. The effects described in the main text hold under either of these two cases.

these variables against time, this shift is parallel, since their steady state growth rate continues to be g_x.

A proportional income tax can be viewed as composed of two separate taxes--one on consumption and the other on investment. The individual effect of each of these two components is significantly different. The steady state K/NX ratio is lower the higher the tax on investment but it is invariant to the consumption tax rate.[17] This is because only the investment tax distorts the economy's production possibilities across time.

Money and Growth

The recent literature on the effects of anticipated inflation, which makes use of an optimizing framework, shares the conclusion that the steady state real interest rate is independent of the steady state inflation rate.[18] A representative sample of this literature is Stockman's (1981) study of the effects of anticipated inflation in a discrete time version of the

[17] A tax on consumption affects the consumption path but not the capital accumulation decisions. When labor supply is endogenous, a tax on consumption has an indirect effect on the accumulation decisions, since it will in general affect the number of hours devoted to work in the market, inducing a change in the marginal productivity of capital.

[18] An exception is the result obtained by Epstein and Hynes (1983) in a model with recursive preferences. However, in their model there is zero steady state growth. The negative effects of anticipated inflation on the real interest rate predicted by Mundell (1963) and Tobin (1965) stem from their use of a non-optimizing framework.

neoclassical model in which a cash-in-advance (CIA) constraint is introduced.[19] His results are similar to the ones we just described for taxation. If there is a CIA constraint on consumption or investment, anticipated inflation acts like a tax on consumption or investment. In the absence of technological progress higher rates of monetary expansion, which lead to higher inflation rates, are therefore associated with lower steady state capital stocks whenever there exists a CIA constraint on investment. In the presence of exogenous technological change, these level effects translate into parallel shifts in the paths of the logarithms of the variables.

3. The Basic Endogenous Growth Model

In order to rationalize the phenomenon of unceasing growth without resorting to exogenous time-shifts in the production technology we need two obvious requirements. First we have to specify the (stationary) technology so that unbounded growth is feasible. Second we must restrict preferences so that, given the production technology, unceasing growth is optimal. The class of models that has these two features is extremely large. For the reasons explained in section one, we narrow this class by imposing the additional restriction that the economy possesses a balanced growth path.

In a one capital good model, these requirements imply that the production

[19] In endogenous growth models, if real balances are introduced in the production or utility functions the results obtained depend crucially on the functional forms employed, and there is no sound basis for choosing among alternative functional forms. For this reason, the imposition of a CIA constraint seems to be a preferable device for generating a demand for real balances. See Baxter (1986) for a critical appraisal of the use of the CIA constraint.

function of the capital sector has to be linear in capital. This stringent restriction on technology is substantially weakened once we allow for the presence of more capital goods (see section 4).

The simplest environment with the features mentioned above is a one-sector model with a production technology linear in capital: $Y_t = B\,K_t$. In contrast to the neoclassical model, in which there is a single sustainable growth rate (g_x), this economy can sustain any growth rate of capital between $B-\delta$ and $-\delta$, with δ being the depreciation rate.

The model used in this section is a simple refinement of this basic structure. The economy has two sectors, one that produces capital with a linear technology ($Q_t = BK_t(1-\phi_t)$) and one that produces consumption goods with a technology given by $C_t = F(\phi_t K_t)$, where ϕ_t is the fraction of K_t devoted to the production of the consumption good. For it to be feasible for both C_t and K_t to grow at constant rates, we have to specialize the production function $F(.)$ to the form $C_t = A(\phi_t K_t)^{\alpha}$, $\alpha \leq 1$. Capital is assumed to depreciate at rate δ in both sectors, so the evolution of K_t is given by: $\dot{K}_t = Q_t - \delta K_t$.[20] Preferences are assumed to be time separable.

The competitive equilibrium under perfect foresight can be determined by exploiting the fact that, in this economy, it coincides with the optimal path, which is the solution to the following concave program:

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[20]Different rates of depreciation in the two industries can easily be considered without altering the basic results.

55

$$\max \quad \int_0^\infty u\,(C_t)\,\exp\,(-\,\rho t)\,dt$$

$$\begin{aligned}
\text{s.t.}\quad & C_t = A\,(\phi_t K_t)^\alpha \\
& Q_t = B\,K_t(1-\phi_t) \\
& \dot{K}_t = Q_t - \delta K_t \\
& 0 \le \phi_t \le 1, \qquad K_0 = \underline{K} > 0
\end{aligned}$$

The current-valued Hamiltonian for this problem can be written as:

$$H(C_t, \phi_t, K_t, \theta_t, \lambda_t) = u(C_t) + \theta_t[BK_t(1-\phi_t)-\delta K_t] + \lambda_t[A(\phi_t k_t)^\alpha - C_t]$$

Assuming an interior solution for ϕ_t,[21] the first order conditions are:[22]

(3.1) $\quad Du[A(\phi_t K_t)^\alpha] = \lambda_t$

(3.2) $\quad \lambda_t \alpha\, A\,(\phi_t K_t)^{\alpha-1} = \theta_t B$

(3.3) $\quad \dot{\theta}_t = \rho\theta_t - \lambda_t \alpha\, A\, \phi_t^{\alpha} K_t^{\alpha-1} - B(1-\phi_t)\theta_t + \delta\theta_t$

(3.4) $\quad \dot{K}_t = K_t B\,(1-\phi_t) - K_t\delta$

(3.5) $\quad \lim_{t\to\infty} \theta_t\, K_t\, \exp\,(-\rho t) = 0$

The shadow prices of consumption (λ_t) and capital (θ_t) implied by these efficiency conditions coincide, as usual, with the prices that clear the markets for both goods in a competitive equilibrium with perfect foresight. Consequently the equilibrium relative price of

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[21]See the Appendix for the conditions that guarantee an interior solution, as well as finiteness of life-time utility.

[22]A proof that the Euler equations and the transversality condition are sufficient and necessary for an optimum in all the "planning problems" used to compute competitive equilibria in this paper can be found in Romer (1983) and in Araujo and Scheinkman (1984).

56

capital in terms of consumption goods, p_t, will be given by θ_t/λ_t.

The efficiency conditions (3.1) - (3.5) can be used to determine the economy's path for a given initial capital stock. This route is pursued in the Appendix. Here we will use a more intuitive approach which is more informative about the economic mechanisms that underlying the solution. We start by examining the steady state.

The first step consists of determining the growth rates of capital and consumption that the economy can sustain at a constant level. We already concluded that capital can grow at any rate between $B-\delta$ and $-\delta$. Given that capital grows at some constant rate, g_k, the corresponding rate of expansion for consumption is $g_c = \alpha g_k$.

We now have to determine the conditions under which the economy chooses to grow at a constant rate and, if such conditions exist, what that rate(s) is. The same line of reasoning employed for the neoclassical model applies here. As before, we have to assume that momentary utility is isoelastic in consumption, so that households, faced with a constant interest rate, choose to expand their consumption at a constant rate. However, the existence of two commodities in the economy complicates matters. We can consider two interest rates, one in terms of capital goods (r_t^k) and the other in terms of consumption goods (r_t^c). These will be identical if the relative price of capital, p_t, is constant over time. This price can be determined by computing θ_t/λ_t, using equation (3.2). Alternatively (given that the optimal path is equivalent to the competitive equilibrium), it may be obtained from the fact that profit maximization by firms implies $p_t B = \alpha A(\phi_t K_t)^{\alpha-1}$.

That is, firms have to be indifferent at the margin between using (or renting to other firms) an extra unit of capital to produce consumption goods or capital goods. Unless $\phi_t K_t$ is constant, p_t will vary over time and the two interest rates will not be identical.

Since the (net) marginal productivity of capital in the sector that produces capital goods is constant and equal to $B - \delta$, equilibrium requires that $r^k_t = B - \delta$. A standard arbitrage argument implies that $r^c_t = r^k_t + \dot{p}_t/p_t$. The rate of change in the relative price of capital, $\dot{p}_t/p_t$, is equal to $(\alpha-1)\ (\dot{K}_t/K_t + \dot{\phi}_t/\phi_t)$. In the steady state $\dot{\phi}_t = 0$ and we have $r^c = B - \delta + (\alpha - 1)g_k$. The steady state growth rate of consumption is $g_c = \alpha g_k$.[23] The efficiency conditions for the consumer facing an interest rate r^c imply that $g_c = (r^c - \rho)/\sigma$. This, in turn, implies that $\alpha g_k = [B - \delta + (\alpha-1)g_k - \rho]/\sigma$. We then conclude that the steady state growth rate of capital is:

$$(3.6) \qquad g_k = \frac{B - \delta - \rho}{1 - \alpha(1-\sigma)}$$

One noticeable property of the steady state solution is that it is compatible with any level of the capital stock. This, naturally, leads us to conjecture that the economy grows always at the steady state growth rate g_k, regardless of its initial capital level. In the Appendix it is shown that this is, in fact, the only solution that

[23] Although production (measured in physical units) in the two industries grows at diferent rates, the share of consumption in net income, i.e. $C_t/(C_t + pQ_t - \delta K_t)$ is constant.

58

satisfies (3.1) - (3.5).

Determinants of the rate of growth

The growth rate of this economy is a function of the parameters of preferences and technology. It will be higher the greater the willingness of households to substitute across time (higher $1/\sigma$) and the lower their pure rate of time preference (ρ). It is worth emphasizing that the constant A (the level parameter in the production function for consumption goods) does not enter in the expression for the growth rate. The parameter A determines only the level of the consumption path.

In this economy, temporary disturbances typically generate permanent effects.[24] Suppose, for instance, that part of the economy's capital stock is destroyed. Immediately after the occurence of this shock, the economy will resume growing at the steady-state growth rate--there is no tendency for the economy to return to the previous capital path. As a consequence, there will be a permanent effect on the level of the capital stock. In economies with multiple capital goods, such as those described in Uzawa (1965), Lucas (1985) and in section 4, a temporary disturbance is followed by a period of transition toward the steady state, but the new steady state will be different from the original one in terms of levels.

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[24]The potential for temporary shocks to give rise to permanent effects is a general feature of endogenous growth models. This feature has important consequences for the traditional separation of growth and business cycles in both theoretical and empirical analyses. See King and Rebelo (1986) for a discussion of this issue.

Growth and the savings rate

In the original version of the neoclassical model, the savings rate (s) was fixed at an exogenous level. In that context, Solow (1956) concluded that the savings rate determines only the steady state levels of the different variables but not their growth rates. Although the speed of convergence toward the steady state depends on s, the steady state growth rate is exogenous and all s does is pin down the capital-labor ratio.

Our simple model can be used to illustrate that this result is an artifact of the exogenous nature of steady state growth in the neoclassical model. Net income per capita in our economy, choosing the consumption good as numeraire, is: $Y_t = p_tQ_t + C_t - \delta p_tK_t$. Since the savings rate is, by definition, the fraction of net income devoted to net investment,

$$s = \frac{p_tQ_t - \delta p_tK_t}{p_tQ_t + C_t - \delta p_tK_t}$$

Suppose we were to fix the savings rate at an exogenous level. Firms' profit maximization condition would still imply that $p_tB = \alpha A(\phi_tK_t)^{\alpha-1}$. Substituting p_t and rearranging, we can express the growth rate of capital as a function of the savings rate,

$$g_k = \frac{(B - \delta)s}{\alpha + (1-\alpha)s}$$

60

This expression makes clear the one-to-one relation that would exist between the rate of savings and the growth rate of the economy. It is easy to show that higher savings rates would lead to higher growth rates. Naturally, if we replace s in this equation with the optimal savings rate implicitly defined by (3.1)-(3.5), we obtain expression (3.6).

Long run effects of taxation

Given that in the neoclassical model a consumption tax has different implications from a tax on investment, it is useful to study these two taxes separately here, instead of simply considering an income tax. Government revenue, in terms of the consumption good, is given by: $T_t = \tau_c C_t + \tau_i p_t I_t$. We assume that this revenue is used to finance the provision of goods that do not affect the marginal utility of private consumption or the production possibilities of the private sector.

The competitive equilibrium is the solution to the following concave program:

$$\max \quad \int_0^\infty u\,(C_t)\,\exp\,(-\,\rho t)\,dt$$

$$\begin{aligned}
\text{s.t.} \quad & C_t\,(1+\tau_c) = A\,(\phi_t K_t)^{\alpha} \\
& Q_t = B\,K_t(1-\phi_t) \\
& I_t\,(1+\tau_i) = Q_t \\
& \dot{K}_t = I_t - \delta K_t \\
& 0 \le \phi_t \le 1, \qquad K_0 = \underline{K} > 0
\end{aligned}$$

The current-valued Hamiltonian for this problem is:

$$H(C_t, \phi_t, K_t, \theta_t, \lambda_t, \mu_t) = u(C_t) + \lambda_t[A(\phi_t K_t)^{\alpha} - C_t(1+\tau_c)] + $$
$$+ \mu_t[B(1-\phi_t)K_t - I_t(1+\tau_i)] + \theta_t(I_t - \delta K_t)$$

where $u(C_t)$ is isoelastic.

Assuming an interior solution for ϕ_t, the efficiency conditions are:

(3.7) $C_t^{-\sigma} = \lambda_t \, (1 + \tau_c)$

(3.8) $\lambda_t \, \alpha \, A \, (\phi_t K_t)^{\alpha-1} = \mu_t B$

(3.9) $\mu_t \, (1 + \tau_i) = \theta_t$

(3.10 $\dot{\theta}_t = \rho\theta_t - \lambda_t \alpha \, A\phi_t^{\alpha} K_t^{\alpha-1} - \mu_t B \, (1 - \phi_t) + \theta_t \delta$

(3.11) $\lim_{t\to\infty} \theta_t K_t \exp(-\rho t) = 0$

The equilibrium after-tax prices of the consumption good and the capital good are now $\lambda_t(1+\tau_c)$ and θ_t, respectively. It is straightforward to show that the new equilibrium growth rate is:

(3.12) $$g_k = \frac{B/(1+\tau_i) - \delta - \rho}{1 - \alpha \, (1-\sigma)}$$

As before, $g_c = \alpha \, g_k$.[25]

An increase in τ_i is equivalent to a decrease in B, and so it leads

[25] The equilibrium growth rate can be determined by using the efficiency conditions for households and firms, instead of solving (3.7)-(3.11). The difference with the respect to the economy without taxes is that firms' profit maximization condition now implies that:

$$(1 + r_k) = \frac{1}{1+\tau_i} [B + (1-\delta)] + \tau_i \, (1-\delta)\frac{1}{1+\tau_i} .$$

to a decrease in the economy's growth rate.[26] The equilibrium growth rate (3.12) does not depend on on τ_c. The influence of this policy variable is similar to that of the parameter A--it determines the level of the consumption path but exerts no influence on capital accumulation decisions since it introduces no intertemporal distortions in the economy (the consumption tax does not distort the decision of consuming now versus later).

Since a proportional tax on income amounts to taxing consumption and investment at the same rate, an increase in the income tax rate induces a decrease in the rate of growth of this economy.[27]

The empirical evidence on the long run effects of taxation is extremely scarce due to the difficulty of controlling for frequent changes in the tax structure. In a cross-country study, Kormendi and Meguire (1986) found no correlation between the share of government expenditures in GNP and the rate of growth. The model described in this section has no implications for the relation between these two variables. It predicts that it is the tax system, not the government expenditure share per se, that influences the growth rate. An empirical

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[26] Changing the fiscal treatment of depreciation will act as a change in δ and so it will induce as well a change in the growth rate of the economy. For instance, if the tax is on net investment, δ will also be divided by $(1+\tau_i)$ in (3.12).

[27] This discussion may shed light into the nature of the results in Boyd and Prescott (1985) where varying the income tax rate affects the economy's growth rate. This result is obtained simply because their model has a linear production technology and, as we have just shown, a change in the income tax schedule acts as a displacement to this technology, leading to a decrease in the rate of growth.

investigation structured on the basis of this model should be aimed at identifying empirical counterparts of τ_c and τ_i.

Money and Growth

A discrete time version of the model presented above with money introduced through the imposition of a cash-in-advance (CIA) constraint as in Stockman (1981) confirms the results one would expect given the effects of taxation just discussed. If there exists a CIA constraint on consumption a larger rate of monetary growth is associated with a larger rate of inflation which acts as a tax on consumption. Consequently, the rate of growth of the economy is not affected. A similar result holds for the neoclassical model.[28] However, if there is a CIA constraint on investment, the increase in the inflation rate induced by an increase in the rate of monetary expansion will act like a decrease in B, depressing the rate of capital accumulation (g_k), and decreasing the real interest rate (recall that $r = \rho + \sigma\alpha g_k$).

It is worthwhile emphasizing that the mechanism underlying this negative relationship between anticipated inflation and the real interest rate is radically different from the Mundell-Tobin effect where higher inflation leads to a higher steady state stock of capital and to a lower real interest rate. In this model, inflation has a negative effect on the real interest rate not because it leads to an increase in the capital stock but because it slows down economic growth.

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[28]Stockman (1981) shows that, in this case, there is long-run neutrality. Abel (1985) proves that neutrality holds as well during the transition toward the steady state.

Mishkin (1984,1985) has found evidence of a negative relationship between the expected rate of inflation and the real interest rate. A related empirical finding in the Finance literature reported by, among others, Fama and Schwert (1977) is the negative relationship between common stock returns and expected inflation rates.

4. Extensions of the Basic Model

The basic model studied in section 3 suggests that economies with endogenous balanced growth have the following two properties: (i) the steady state growth rate is invariant to linear transformations of the production function of the consumption sector; and (ii) the steady state growth rate varies with linear transformations of the production function of the capital sector. The policy implications discussed in section 3 were driven by these two properties. The effects of changing tax rates or of increasing the rate of monetary expansion can be identified with the effects of a linear transformation of the appropriate production function.[29] For this reason, it is important to determine whether properties (i) and (ii) hold for more general environments. In particular, it is of interest to establish whether the presence of a single capital and consumption good was important in determining the aforementioned properties.

There are two main conclusions in this section. First the results of section 3 continue to hold in the richer environments studied here.

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[29] Abel and Blanchard (1983) discuss in detail this equivalence for the effects of taxation in the neoclassical model.

Second in multiple capital good models there is a much wider range of production functions consistent with steady state growth. Namely we do not need linearity in the production function of the capital sector and can allow for capital goods produced with decreasing returns to scale technologies. It will also be clear that the feasibility of steady state growth does not depend on the assumption that all factors of production can be accumulated. Production factors fixed in quantity can play a role both in the consumption sector and in some of the capital sectors.

To facilitate the exposition, capital goods will be refered to as "primary" if their technology of production is constant returns to scale and their production process does not involve, directly or indirectly, factors of production fixed in quantity or produced with decreasing returns to scale technologies. They will be refered to as "secondary" capital goods otherwise. As we will see, this is an important distinction in terms of understanding the mechanics of economic growth in this economy.

Examining in detail each possible extension of the model of section 3 would be a very lengthy process. For this reason multiple consumption goods, multiple capital goods and endogenous labor supply are introduced one at a time in the basic model. The interaction among these various aspects is sufficiently unimportant that this is a valid strategy for investigating the properties of the general model in which all of these aspects coexist. The discussion will be limited to the main implications of each model. The mathematical derivations that lead to the results discussed are similar to those employed in section 3 and for this reason are omitted. For each economy we state the most general class of preferences and technologies that is consistent with steady state growth.[30] Table 1 summarizes the main features of the models studied in this section, including the steady state growth rate of consumption, capital and net income measured in consumption units and denoted by g_y.[31]

4.1 Multiple consumption goods

The novel features that arise in economies with multiple consumption goods can be illustrated simply by introducing a second consumption sector in the basic model. The structure of the economy is as follows:

Preferences : $\qquad U = \int_0^\infty u(C_{1t}, C_{2t}) \exp(-\rho t)\, dt$

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[30] We restrict attention to production functions that have non-increasing returns to scale to avoid problems of non-existence of competitive equilibrium.

[31] In an economy with n consumption goods and m capital goods net income per capita in terms of consumption good type one is defined as:

$Y = C_1 + \sum_{i=2}^{n} q_i C_i + \sum_{j=1}^{m} p_j(Q_j - \delta_j K_j)$. The variables p_i and q_j denote, respectively, the relative price of consumption type i and of capital type j in terms of type one consumption. Q_j denotes the production of capital type j.

67

where the function $u(C_{1t}, C_{2t})$ has to be such that the elasticities $\sigma_{ij} = \frac{D_{ij}u(C_1, C_2)}{D_i u(C_1, C_2)} C_j$ $(j=1,2;\ i=1,2)$ are constant.[32]

Technology:

$$C_{it} = A_i(\phi_t^i K_t)^{\alpha_i}, \qquad \alpha_i \leq 1 \quad i = 1,2$$

$$\dot{K}_t = B\,(1 - \sum_{i=1}^{2} \phi_t^i)\,K_t - \delta K_t, \qquad K_o = \underline{K} > 0$$

$$\phi_t^i \geq 0, \qquad i = 1,2; \qquad t = 1,2,\ldots$$

$$\sum_{i=1}^{2} \phi_t^i \leq 1 \qquad t = 1,2,\ldots$$

The steady state growth rate is given by:

$$g_k = \frac{B - \rho - \delta}{1-\alpha_1-\sigma_{11}\alpha_1-\alpha_2\sigma_{12}} \tag{4.1}$$

(see Table 1 for g_c and g_y). Inspection of the growth rate of capital reveals that properties (i) and (ii) hold. As a consequence the policy implications drawn in section 3 are valid for this model.

A bothersome feature of models with multiple consumption goods is that the presence of steady state growth implies that stringent restrictions between the parameters of preferences and technology have to be imposed. In this case, unless condition (4.2) holds, the economy does not grow at a constant rate.

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[32]This implies severe restrictions on the form of momentary utility. The function $u(C_{1t}, C_{2t})$ has to be either additively separable or multiplicatively separable.

64

$$(4.2) \qquad \alpha_1 + \sum_{i=1}^{2} \alpha_i \sigma_{1i} = \alpha_2 + \sum_{i=1}^{2} \alpha_i \sigma_{2i}$$

This condition garantees that it is not optimal to change the mix of consumption goods as the overall scale of consumption expands.

4.2 Multiple primary capital goods

In the basic model the technology of production in the capital sector had to be linear to make balanced growth feasible. This is not necessary in economies with multiple primary capital goods. The following model illustrates this point. In order for steady state growth to be feasible all we have to require is that the two primary capital goods be produced with constant returns to scale technologies.

Preferences : $$U = \int_o^\infty \frac{C_t^{1-\sigma} - 1}{1 - \sigma} \exp(-\rho t)\, dt$$

Technology:

$$C_t = G(\phi_t^{13}K_t^1, \phi_t^{23}K_t^2)$$

$$\dot{K}_t^i = F_i(\phi_t^{1i}K_t^1, \phi_t^{2i}K_t^2) - \delta_i K_t^i, \qquad i = 1,2$$

$$K_o^i = \underline{K}^i > 0 \qquad i = 1,2$$

$$\sum_{j=1}^{3} \phi_t^{ij} = 1 \qquad i = 1,2; \qquad t = 1,2\ldots$$

$$\phi_t^{ij} \geq 0, \quad i = 1,2,3; \quad j = 1,2; \qquad t = 1,2,\ldots$$

where the functions $F_i(.)$ are constant returns to scale, twice differentiable, increasing in both arguments, concave and satisfy the Inada conditions. Furthermore both factors are assumed to be essential in production. The function G() has these same properties except that the returns to scale, denoted by α, can be constant or decreasing.

This economy has transitional dynamics that were absent in section 3. It is straightforward to show that there is a single K^1/K^2 ratio consistent with balanced growth. If the initial values of the two capital stocks are not in this proportion, there will generally be an adjustment period during which the economy does not grow at constant rates. These dynamics are transitory; eventually the economy converges to the balanced growth path.[33]

In general we cannot solve for the rate of growth explicitly but we can establish that properties (i) and (ii) hold in the steady state. For sake of space limitations, this proof is not carried out here. However, the following example, in which we can solve for the growth rate, may be helpful in terms of visualizing the determinants of the steady state rate of expansion. Suppose that all the production functions are Cobb-Douglas: $F_i() = B_i(\phi_t^{1i}K_t^1)^{\gamma_i}(\phi_t^{2i}K_t^2)^{1-\gamma_i}$ and $G() = A(\phi_t^{13}K_t^1)^{\alpha_1}(\phi_t^{23}K_t^2)^{\alpha_2}$, $\alpha_1+\alpha_2 \leq 1$. Furthermore assume that both types of capital depreciate at the same rate $(\delta_1=\delta_2)$. In this case the balanced growth rate is given by:

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[33]See King and Rebelo (1986) for a study of the transitional dynamics in a discrete time version of this model. Transitional dynamics are generally present in models with multiple capital goods. In these economies one can seldom solve explicitly for the entire optimal growth path so one would like to appeal to an existence theorem. See Romer (1986b) for an existence proof that applies to these models.

$$(4.3)\quad g_{k^1} = g_{k^2} = \frac{B_1^{\left[\frac{\gamma_2}{1 - \gamma_1 + \gamma_2}\right]} B_2^{\left[\frac{1 - \gamma_1}{1 - \gamma_1 + \gamma_2}\right]} \quad m(\gamma_1, \gamma_2) - \rho - \delta}{1 - (\alpha_1 + \alpha_2)\ (1 - \sigma)}$$

where $m(\gamma_1, \gamma_2)$ is a positive function of γ_1 and γ_2. This expression makes clear that linear transformations of the production functions of the capital sectors (i.e. changes in B_1 or B_2) have an impact on the steady state growth rate, while linear transformations of the technology of the consumption sector (i.e. changes in A) do not.

4.3 Endogenous labor supply

There are two ways in which labor can play a role in the models discussed so far. First, labor can enter as a productive factor in the consumption sector. Consider the following extension of the basic model:

$$C_t = A\cdot(\phi_t K_t)^{\alpha}\ N_t^{1-\alpha}$$
$$\dot{K}_t = B\ K_t(1 - \phi_t) - \delta K_t$$
$$0 \leq \phi_t \leq 1, \qquad K_0 = \underline{K} > 0$$
$$N_t + L_t = 1, \qquad N_t,\ L_t \geq 0$$

where L_t denotes leisure and N_t time devoted to work. The class of

71

preferences consistent with steady state growth is:[34]

$$(4.4) \qquad u(C_t, L_t) = \begin{cases} \dfrac{C_t^{1-\sigma}}{1-\sigma} V_1(L_t) & \text{if } \sigma \neq 1 \\ \log (C_t) + V_2(L_t) & \text{if } \sigma = 1 \end{cases}$$

where $L_t = 1 - N_t > 0$ denotes leisure.

It is straightforward to show that the growth rate of this economy is given by (3.6). Consequently, properties (i) and (ii) hold for this economy.

An alternative way of giving labor a role in the models we examined is to interpret the capital stock K, in the basic model, as representing human capital. This reinterpretation implies that, as in the seminal work of Ben-Porath (1967), human capital must be viewed as having the same effects as labor augmenting technological progress. Production is a function of labor in efficiency units which is given by N_tH_t, where N_t denotes the amount of time worked and H_t the level of the worker's human capital. Since our economy is populated by identical agents, the level of human capital will be the same for all individuals. The production technology of this reincarnation of the basic model is given by:

[34]See King, Plosser and Rebelo (1987) for a proof and for the requirements on $V_1(.)$ and $V_2(.)$ that are necessary for concavity of life-time utility.

72

$$C_t = A\,(N_t^1 H_t)^{\alpha}$$

$$\dot{H}_t = B\,H_t N_t^2 - \delta H_t, \quad H_o = \underline{H} > 0$$

$$N_t^1 + N_t^2 + L_t = 1, \qquad L_t, N_t^i \geq 0, \quad i=1,2 \qquad t=1,2,\ldots$$

where N_t^1, N_t^2 and L_t denote, respectively, time devoted to the consumption industry, to human capital accumulation and to leisure activities.[35]

Not surprisingly, optimality of steady state growth requires restrictions on preferences. We can consider two classes of preferences depending on whether leisure enters in the utility function in terms of time units or in terms of efficiency units. In the first case, preferences have to be of the form (4.4) to be consistent with steady state growth. In the second case momentary utility is of the form $u = u(C, LH)$. This class of preferences, introduced by Heckman (1976), can be viewed as a particular formalization of Becker's (1965) concept of household production function.[36] Preferences in this class have been employed in the labor literature to rationalize the fact that hours devoted to work in the market have been fairly unresponsive to the observed secular increase in real wages. In order for the economy to possess a steady state path, $u = u(C, LH)$ has to be homogeneous in $C^{1/\alpha}$

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[35]Notice that human capital is embodied so specialization in its production cannot occur.

[36]Using Becker's household production concept would widen the class of preferences consistent with steady state growth.

and in L.[37] Momentary utility can then be written in the form:

$$(4.5) \qquad u(C_t, L_t H_t) = H_t^{\nu}\, u(C_t^{1/\alpha}/H_t, L_t)$$

where ν is the degree of homogeneity. Since in the steady state $g_c = \alpha g_h$, the ratio $C_t^{1/\alpha}/H_t$ is constant and the marginal utility of consumption and leisure expand at constant rates as is required for balanced growth to be optimal.

If momentary utility is given by (4.5), the steady state growth rate of this economy is:

$$(4.6) \qquad g_h = \frac{B - \rho - \delta}{1 - \nu}$$

If momentary utility is (4.4), it can be established that the rate of growth is a function of B, ρ, δ, α, σ and of the parameters of the $V_1(.)$ or $V_2(.)$ functions. The level parameter A does not affect the rate of growth. In consequence, properties (i) and (ii) hold for this economy.

We could introduce labor in the model with two capital goods using the class of momentary utility functions (4.4) or (4.5) to describe preferences and reinterpreting one of the capital goods as human capital. Not surprisingly, we would find that properties (i) and (ii) hold for the economy's steady state.

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[37] See King, Plosser and Rebelo (1987).

4.4 Secondary Capital Goods

In the models surveyed until now there are no capital goods produced with decreasing returns to scale technology. However it is perfectly feasible to introduce secondary capital goods provided there is at least one primary capital good in the economy. This is illustrated in the following model in which a secondary capital good (K_2) is added to the basic environment.

Preferences : $$U = \int_0^\infty \frac{C_t^{1-\sigma} - 1}{1-\sigma} \exp(-\rho t)\, dt$$

Technology: $$C_t = A\,(\phi_t^{13} K_t^1)^{\alpha_1} (\phi_t^{23} K_t^2)^{\alpha_2}, \quad \alpha_1+\alpha_2 \leq 1, \quad \alpha_1, \alpha_2 > 0$$

$$\dot{K}_t^1 = B_1\, \phi_t^{11}\, K_t^1 - \delta_1 K_t^1,$$

$$\dot{K}_t^2 = B_2 (\phi_t^{12} K_t^1)^{\gamma_1} (\phi_t^{22} K_t^2)^{\gamma_2} - \delta_2 K_t^2, \qquad \gamma_1+\gamma_2 < 1, \quad \gamma_1, \gamma_2 > 0$$

$$K_o^i = \underline{K}^i > 0 \qquad i = 1,2$$

$$\sum_{i=1}^{3} \phi_t^{1i} = 1, \quad \sum_{j=2}^{3} \phi_t^{2j} = 1 \qquad t = 1,2\ldots$$

$$\phi_t^{1i} \geq 0, \qquad i = 1,2,3; \qquad t = 1,2,\ldots$$

$$\phi_t^{2j} \geq 0, \qquad j = 2,3; \qquad t = 1,2,\ldots$$

where the function $F_i(.)$ is constant returns to scale, twice differentiable, increasing in both arguments, concave and satisfies the Inada conditions.

The steady state growth rate of the primary capital good is (see

Table 1 for g_{k_2}, g_c and g_y):

$$g_{k_1} = \frac{(1-\gamma_2)(B_1 - \rho - \delta)}{(1-\gamma_2)-(1-\sigma)[\alpha_1(1-\gamma_2) + \alpha_2\gamma_1]}$$

Not surprisingly the level parameter B_2 is absent from the growth rate expression. Since the rate of interest is pinned down by the technology of the primary capital industry, intertemporal opportunities are not affected by linear transformations of the production function of the secondary capital sector. Properties (i) and (ii) have to be revised: (i') the steady state rate of growth is invariant to linear transformations of the production function of the consumption sector or of the secondary capital goods sector; (ii') the steady state growth rate varies with linear transformations of the production function of the primary capital goods sector. This implies a slight change in the implications for policy: for instance, taxing investment in secondary capital goods does not affect the rate of growth. However, the implication that taxing income or total investment slows down the rate

76

of economic expansion continues to hold in this model.[38]

As in the model with two primary capital goods this economy will in general have transitional dynamics. If the initial values of the two capital stocks are not in a certain proportion, there will be a period during which the rate of growth will not be constant.

5. Conclusion

Exogenous technological progress is the only determinant of the rate of economic growth in neoclassical economies. For this reason neoclassical environments are not suitable for investigations of the long run effects of economic policy on long run growth. The stimulating

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[38] The model studied by Lucas (1985) is on the borderline between the model with one secondary capital good described here and the economy with two primary capital goods studied in subsection 4.2. The technology in Lucas's model can be written, in the notation of this paper, as follows:

$$\dot{H}_t = B_1(1 - \phi_t)H_t - \delta H_t$$

$$\dot{K}_t = B_2 K_t^{\gamma}(\phi_t H_t)^{1-\gamma} - C_t - \delta K_t$$

where H_t denotes human capital and K_t physical capital. We simplified the model by abstracting from externalities and setting the depreciation rate of physical capital equal to that of human capital.

It is clear that this is a particular case of the economy studied here where $\gamma_1 + \gamma_2 = 1$ and $\alpha_1 = \gamma_1$, $\alpha_2 = \gamma_2$. In terms of the economy with Cobb-Douglas technologies described in subsection 4.2, Lucas's model corresponds to having $\gamma_1 = \alpha_1$ and $1-\gamma_1 = \alpha_2$ (the technology is identical in the consumption sector and in the physical capital industry) and $\gamma_2 = 0$ (physical capital does not enter in the production of human capital). The steady state growth rate is given by $g_h = (B_1 - \rho - \delta)/\sigma$. Since B_2 does not enter in the expression for the rate of growth some of the policy implications are similar to those of the neoclassical model. For instance, taxes on income (which does not include the production of human capital) do not affect the steady state growth rate.

77

work of Romer (1986a) and Lucas (1985) has demonstrated that it is possible to build models that rationalize the phenomenon of unceasing growth without relying on exogenous sources of productivity increase. The class of models spanned by their work--"endogenous growth models"--show promise of providing us with a deeper understanding of the factors responsible for economic growth and of the influence of policy on these factors.

The simple model studied in section 3 highlights the main features of endogenous growth models that display steady state growth The determinants of the rate of growth in these economies are substantially different from those of the neoclassical model. Neoclassical economies imply restrictions on the classes of preferences that are consistent with steady state growth. However, provided these restrictions are satisfied, the rate of steady state growth is determined by a single aspect of technology--the rate of technological progress. Preferences play no role in determining how fast the economy should grow. In models with endogenous growth the rate of growth is generally a function of the savings rate. As a consequence, the same elements of preferences that were emphasized by Irving Fisher in his Theory of Interest as determinants of savings behavior--the pure rate of time preference and the elasticity of intertemporal substitution in consumption--are important factors in the determination of the rate of economic expansion.

The policy implications of the basic model of section 3 are strikingly different from those of the neoclassical environment. This basic model leads us to expect that hyperinflation, as well as taxes on income or investment, slow down economic growth.

Section 4 was devoted to examining the robustness of the results of the model of section 3. The basic model was extended by introducing more consumption and capital goods and by making the supply of labor endogenous. Three important conclusions come out of section 4. First, despite its stark simplicity, the basic model turned out to be a good representative of the class of steady state endogenous growth economies. Its main policy implications continue to hold in richer environments. Second, this class of models is compatible with the existence of production factors that cannot be accumulated. Fixed factors can play a role both in the production of consumption goods, as well as in the production of some capital goods.[39] Third, in models with multiple capital goods, it is not required that the production function of one of the capital sectors be linear.

Many questions remain to be answered, both at a theoretical and empirical level. At a theoretical level, a more detailed investigation of the properties of this class of models is desirable. In particular the characterization of the adjustment path that typically arises in multiple capital models and the study of models with asymptotic steady

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[39] Controversy on this issue has deep historical roots. See Knight (1944) and Hagen (1942).

state growth deserves further investigation.[40] Although the emphasis of this paper has been on positive analysis, it is clear that endogenous growth models have important normative implications in such areas as optimal taxation and trade policy. Exploring the empirical implications of endogenous growth models is a promising topic for future research. These models provide not only testable implications but also new guidance concerning variables and data transformations that are likely to be revealing. It remains to be seen what fraction of the cross-country disparity in rates of growth can be explained as a consequence of different policy regimes.

[40] Many of the restrictions that had to be imposed on preferences and technology can be relaxed in models in which steady state growth is asymptotic. In those models there are no initial values of the capital stock compatible with growth at constant rates. However, the rate of growth approaches asymptotically a constant.

80

Table 1
Endogenous Growth Models

	Number of Primary Capital Goods	Number of Secondary Capital Goods	Number of Consumption Goods	Endogenous Labor Supply?	Steady State Growth Rate	Transitional Dynamics?
Basic Model (Section 3)	1	0	1	No	$g_k = \frac{B-\rho-\delta}{1-\alpha(1-\sigma)}$ $g_c = \alpha g_k$ $g_y = g_c$	No
Two Consumption Goods (Section 4.1)	1	0	2	No	$g_k = \frac{B-\rho-\delta}{1-\alpha_1-\sigma_{11}\alpha_1-\sigma_{12}\alpha_2}$ $g_{c_1} = \alpha_1 g_k$ $g_y = \alpha_1 g_k$	No
Two Primary Capital Goods (Section 4.2)	2	0	1	No	Function of $\sigma, \rho, \delta, \alpha$ and of the parameters of $F_1(\bullet)$ and $F_2(\bullet)$	Yes
Endogenous Labor Supply (Section 4.3)	1	0	1	Yes	If preferences are (4.5) $g_h = \frac{B-\rho-\delta}{1-\upsilon}$ and $g_c = \alpha g_h$, $g_y = g_c$. If preferences are (4.4) g_h, g_c and g_y are function of $\rho, \sigma, \delta, \alpha, \beta$ and of the parameters of $V_1(\bullet)$ or $V_2(\bullet)$	No
Secondary Capital Goods (Section 4.4)	1	1	1	No	$g_{k_1} = \frac{(1-\gamma_2)(B_1-\rho-\delta_1)}{(1-\gamma_2)-(1-\sigma)[\alpha_1(1-\gamma_2) + \alpha_2\gamma_1]}$ $g_{k_2} = \frac{\gamma_1}{1-\gamma_2} g_{k_1}$ $g_c = \alpha_1 g_{k_1} + \alpha_2 g_{k_2}$ $g_y = g_c$	Yes

Appendix

Solution for the basic model

(3.1)-(3.3) imply:

$$\text{(A.1)} \qquad \dot{\theta}_t/\theta_t = \rho + \delta - B$$

Differentiating (3.1) and (3.2) with respect to time and using (3.4) and (A.1) we obtain the equation:

$$\text{(A.2)} \qquad \dot{\phi}_t = a\phi_t + B\phi_t^{\,2}$$

Where $a = [(B - \delta)\alpha(1 - \sigma) - \rho]/[1 - \alpha\ (1 - \sigma)]$. This is a Riccati equation which has the following solution:

$$\text{(A.3)} \qquad \phi_t = a/[(a/\phi_0 + B)\ \exp\ (-at) - B]$$

As $a < 0$ (see condition (A.8)) if ϕ_0 is different from $- a/ B$, ϕ_t diverges from $- a/ B$.

There is an infinite number of paths consistent with equations (3.1)-(3.4). To determine the optimal path we need to use the transversality condition (3.5) (this amounts to determining ϕ_0).

Using (3.4) and (A.1) we have:

$$\text{(A.4)} \qquad K_t = K_0\ \exp\ [(B - \delta)t - B\int_0^t \phi_s ds]$$

$$\text{(A.5)} \qquad \theta_t = \theta_0\ \exp\ [(\rho + \delta - B)\ t]$$

Using these results in (3.5) we obtain the condition,

$$\text{(A.6)} \qquad \lim_{t\to\infty} K_0\theta_0\ B\{[1 - \exp(at)]/a + 1/\phi_0\} = 0$$

This implies that $\phi_0 = -a/B$ so equation (A.3) reduces to: $\phi_t = \phi_0 = -a/B$.

We have been assuming an interior solution for ϕ_t, i.e., $0 < \phi_t < 1$. This imposes the following conditions on the parameters:

(A.7) $\rho > \alpha(1-\sigma)(B - \delta)$

(A.8) $B > \rho + \alpha(1-\sigma)\delta$

The interpretation of these conditions follows naturally if we compute the life-time utility associated with an arbitrary constant growth path for capital. Condition (A.7) is necessary for life-time utility to be finite, while (A.8) garantees that growth is desirable.

Money and Endogenous Growth in a Cash-in-Advance Model

Consider a discrete time version of the model in section 3 augmented by a CIA constraint of the form:

(A.10) $(m_{t-1} + e_t)/P_t \geq \xi_c C_t + \xi_i p_t I_t$

where e_t is a lump sum transfer that exhausts the revenue associated with the inflation tax, P_t is the general price level, and m_{t-1} represents the nominal balances carried by each agent from period t-1 to period t.

For the cases of interest this constraint will be binding, so we proceed to write it as an equality. The budget constraint for the representative consumer has to be modified to account for changes in the money stock:

$$A(\phi_t K_t)^{\alpha} + p_t B(1-\phi_t)K_t - C_t - p_t I_t + (m_{t-1}+e_t)/P_t - m_t/P_t = 0$$

Assume that preferences can be written as $\sum_{t=0}^{\infty} \beta^t u(C_t)$, where $u(C_t)$ is isoelastic and $\beta = 1/(1+\rho)$. Setting $\delta = 0$ to simplify, the efficiency conditions for the representative agent's problem (notice that e_t is viewed as exogenous by each agent) are:

83

$$C_t^{-\sigma} = \lambda_t + \chi_t \xi_c$$

$$(\lambda_t + \chi_t \xi_i)\, p_t = \theta_t$$

$$\alpha A \phi_t^{\alpha-1}\, K_t^{\alpha} = p_t B K_t$$

$$\lambda_t \alpha A \phi_t^{\alpha} K_t^{\alpha-1} + \lambda_t p_t\, B\, (1-\phi_t) = \theta_{t-1}/\beta - \theta_t$$

$$\frac{\lambda_t}{p_t \beta} = \frac{\chi_{t+1}}{p_{t+1}} + \frac{\lambda_{t+1}}{p_{t+1}}$$

where χ_t, λ_t and θ_t are, respectively, the Lagrange multipliers (expressed in present value terms, i.e., divided by β^t) associated with the CIA constraint, the budget constraint and the equation for capital accumulation.

In equilibrium $C_t = A(\phi_t K_t)^{\alpha}$ and $I_t = B(1-\phi_t)K_t$. Introducing these conditions in the first order equations and simply computing the growth rates for the different variables, it can be shown that an increase in the rate of monetary expansion leads to a contraction in the growth rate only if $\xi_i > 0$. When $\xi_i = 0$, given the assumption that all revenue from seignorage is transfered to the private sector, money is superneutral. As in the taxation example, if the inflation tax revenue is retained by the government the path for the logarithm of consumption, decreases in a parallel fashion but the growth rate of the economy remains the same.

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