Solving Nonlinear Rational Expectations Models With Asymmetric Adjustment Costs

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Empirical studies have begun to detect substantial conditional heterogeneity in the laws of motion of some economic time series. For example, heterogeneity in exchange rate dynamics has been documented by Baillie and Bollerslev (1988) and Hsieh (1988) using ARCH and GARCH models and by Gallant, Hsieh, and Tauchen (1988) using Gallant and Tauchen's (1988) seminonparametric (SNP) technique. Hussey (1988) applies the SNP technique to an aggregate employment series and finds important heterogeneity in the conditional density of that data also. Structural economic models for these time series, to be successful, need to explain the sort of conditional heterogeneity in the data that statistical models such as ARCH and SNP have identified. In the case of exchange rates, Gallant, Hsieh, and Tauchen show how a conditional mixture model with uneven flow of information to the market is consistent with the form of conditional heterogeneity they detect in the data. The objective of this paper is to evaluate whether a structural factor demand model with costly adjustment, which has been used to model employment dynamics (Sargent, 1978), can be specified in a way that will generate forms of conditional heterogeneity similar to that found in the employment data.

The factor demand model with costly adjustment is usually specified as a linear-quadratic model because that specification admits an explicit solution. However, that solution is a linear law of motion for the control variable, which implies that there is heterogeneity only in the mean of the variable's conditional distribution. Thus an L-Q model could not explain the conditional heterogeneity in employment detected by Hussey (1988).[1] Yet it is possible that heterogeneity in higher order moments could be explained by a model that maintains the costly adjustment structure but relaxes the quadratic restriction on the specification.

In the next section of this paper I describe a numerical method for obtaining approximate solutions to general specifications of factor demand models with costly adjustment. The method combines the work of Tauchen (1987) on obtaining discrete approximations to continuous forcing functions and of Coleman (1987) on solving rational expectations models with endogenous state variables, given a discrete approximation to the forcing functions. The third section of the paper presents the optimal decision rules that this method finds for both a linear-quadratic specification and an alternative specification of the factor demand model.

Of particular interest are specifications of the factor demand model in which the adjustment costs are non-quadratic, or not symmetric about a zero adjustment. Asymmetric adjustment costs have been suggested as an explanation for some features of employment dynamics such as in Weiss (1986). Also, Neftci's (1984) work on asymmetry in unemployment rates makes asymmetric adjustment costs for labor an appealing concept. Neftci finds that unemployment rates are more persistent during decreasing phases than during increasing. Gradual, persistent declines in the unemployment rate could be associated with significant costs to rapidly increasing employment during business cycle upturns, while sharper, less persist increases in the unemployment rate might be the result of relatively smaller costs to decreasing employment during declining phases of the business cycle.

The last sections of the paper are devoted to the question of whether the law of motion for the control variable implied by the asymmetric adjustment cost model is similar to the that of actual employment data. To make this comparison, Gallant and Tauchen's SNP technique is used to characterize the conditional distributions of the two variables. This technique, which is described in section four, is basically a flexible form model for time series;

it can accommodate both non-Gaussian distributions and conditional heterogeneity in the mean, variance, and higher order moments of distributions. For the asymmetric adjustment cost model, the decision rule obtained from solving the model is used to simulate a long realization of the control variable. SNP models are then fitted to the simulated data and to actual employment data, and the estimated conditional distributions are compared in section five.

II. Numerical Method for Solving Factor Demand Models With Costly Adjustment

The factor demand model with costly adjustment can be described as follows. A firm selects the level of its single factor of production n_t to maximize its expected discounted cash flow

$$E_t\left[\sum_{j=0}^{\infty} \beta^j [f(n_{t+j},\epsilon_{t+j}) - w_{t+j}n_{t+j} - c(n_{t+j} - n_{t+j-1})]\right] \tag{2.1}$$

subject to n_{t-1} given, where n_t is the level of the factor at time t, w_t is the real rental rate for the factor, $f(n_t,\epsilon_t)$ is the production function, ϵ_t is a random shock to technology, and $c(\cdot)$ is the adjustment cost function. The adjustment cost function is usually specified such that $c(0) = 0$ and $c(x) \geq 0$ for $x \neq 0$. The Euler equation for the firm's optimization problem is

$$f_1(n_t,\epsilon_t) - w_t - c'(n_t - n_{t-1}) + \beta E_t[c'(n_{t+1} - n_t)] = 0. \tag{2.2}$$

For the exogenous variables $z_t = (\epsilon_t,w_t)'$ the firm faces a given law of motion, which in this paper is specified as a linear Gaussian autoregression

$$z_t = A_0 + A_1 z_{t-1} + A_2 z_{t-2} + \ldots + A_L z_{t-L} + u_t, \qquad u_t \sim N(0,\Sigma)$$
$$E_{t-1}[u_t] = 0. \tag{2.3}$$

More complex laws of motion for the exogenous variables can be accommodated by the solution method of this paper, but the point of interest is the type of conditional heterogeneity that an asymmetric adjustment cost structure will imply given relatively simple forcing functions.

With $y_t = (z'_t, z'_{t-1}, \ldots, z'_{t-L+1})$, the solution to the firm's optimization problem is the decision rule

$$n_t = \Psi(y_t, n_{t-1})$$

that satisfies the Euler equation. This rule relates the current value of the factor to current and lagged values of the exogenous variables and to the value of the factor in the previous period.

Coleman's (1987) algorithm for solving rational expectations models with endogenous state variables can be applied to this problem. The algorithm solves the problem by iterating over possible forms for the decision rule to find one that satisfies the Euler equation. Given some postulated form of the decision rule $\Psi_0(y_t, n_{t-1})$, the left side of the Euler equation can be written as

$$f_1(n_t, \epsilon_t) - w_t - c'(n_t - n_{t-1}) + \beta E_t[c'(\Psi_0(y_{t+1}, n_t) - n_t)].$$

The algorithm determines an updated "guess" for the decision rule, $\Psi_1(y_t, n_{t-1})$, by substituting this new rule into the Euler equation for n_t and finding the form of $\Psi_1(y_t,n_{t-1})$ that satisfies

$$f_1(\Psi_1(y_t,n_{t-1}),\epsilon_t) - w_t - c'(\Psi_1(y_t,n_{t-1}) - n_{t-1})$$
$$+ \beta E_t[c'(\Psi_0(y_{t+1},\Psi_1(y_t,n_{t-1})) - \Psi_1(y_t,n_{t-1}))] = 0. \tag{2.4}$$

After each iteration, $\Psi_1(y_t,n_{t-1})$ is renamed to be the old guess at the decision rule, $\Psi_0(y_t,n_{t-1})$, and a new $\Psi_1(y_t,n_{t-1})$ is determined with the next iteration. This process continues until the decision rule updates converge.

To implement the algorithm, a discrete state-space of the exogenous and endogenous variables is constructed. If one uses N discrete values for z, there are $I = N^L$ discrete values for y, the vector of L concatenated z's. With I discrete values for the exogenous variables,

$$y_i, \qquad i = 1,2,\ldots,I,$$

and J discrete values for the endogenous variable,

$$n_j, \qquad j = 1,2,\ldots,J,$$

there are I•J states in the system. State "ij" is defined as the variables taking on the values (y_i,n_j). At each iteration, solving for an updated form of the decision rule amounts to solving for an updated value of the decision rule for each state. Thus for the discrete system, the decision rule $\Psi(\cdot,\cdot)$ can be thought of as an I x J matrix of values of the continuous decision rule evaluated at the I•J discrete values of (y_i,n_j).

For any set of discrete state values for the exogenous variables, $\{y_i\}$, equation (2.3) implies a matrix of transition probabilities between those states. (How the discrete values are selected and the transition probabilities computed is discussed later in this section.) The probability of going from state i of the exogenous variables in one period to state k in the following period can be expressed as $g(y_k|y_i)$. These transition probabilities can be used to evaluate the expectations operator in a discrete version of the Euler equation.

Equation (2.4) for the discrete system is

$$
\begin{aligned}
& f_1(\Psi_1(y_i,n_j),\epsilon_i) - w_i - c'(\Psi_1(y_i,n_j) - n_j) \\
& \quad + \beta \sum_{k=1}^{I} [c'(\Psi_0(y_k,\Psi_1(y_i,n_j)) - \Psi_1(y_i,n_j))\cdot g(y_k|y_i)] = 0, \\
& \qquad i = 1,2,\ldots,I \quad \text{and} \quad j = 1,2,\ldots,J,
\end{aligned}
\tag{2.5}
$$

for which $\Psi(y_i,n_j)$ must be determined for each state ij. (In the above equation, ϵ_i is the first element and w_i is the second element of the vector y_i.) To reduce the complexity of equation (2.5), one can let $x_{ij} = \Psi_1(y_i,n_j)$, which is the updated value of the decision rule in state ij, so the equation becomes

$$
\begin{aligned}
& f_1(x_{ij},\epsilon_i) - w_i - c'(x_{ij} - n_j) \\
& \quad + \beta \sum_{k=1}^{I} [c'(\Psi_0(y_k,x_{ij}) - x_{ij})\cdot g(y_k|y_i)] = 0, \\
& \qquad i = 1,2,\ldots,I \quad \text{and} \quad j = 1,2,\ldots,J.
\end{aligned}
\tag{2.6}
$$

Each iteration consists of finding the value of x_{ij} that satisfies equation (2.6) for each state ij. These x_{ij} then become the estimated values of the decision rule that are used as $\Psi_0(y_i,n_j)$ for the next iteration.[2] The algorithm is considered to have converged when

$$\max_{(y_i,n_j)} \mid \Psi_1(y_i,n_j) - \Psi_0(y_i,n_j) \mid \leq \xi$$

where ξ is some small number.

In order to get reasonable solutions with this technique, one would like to have a rigorous way of selecting the discrete values of the exogenous variables and specifying the transition probabilities between those values. This goal can be accomplished by using Tauchen's (1987) quadrature-based method, which provides a discrete state-space model that approximates a given continuous law of motion. The translation from a continuous law of motion to a discrete one is particularly useful since economists are usually better able to determine continuous characterizations for the laws of motion of forcing variables. For example, in the problems considered in the next section, it would be difficult to know how to chose directly discrete values and transition probabilities for the exogenous variables. However, it is possible to get a reasonable estimate of those variables' continuous law of motion, which Tauchen's method can then use to construct a discrete model.

Tauchen's method relies on quadrature rules, which are sets of values and weights that form discrete probability distributions. For N discrete values of z_t, N points and weights are selected to form a discrete probability distribution whose first 2N-1 moments are the same as those of the continuous distribution of z_t characterized by equation (2.3). The values and weights of this distribution are in turn used to construct a discrete probability distribution for y_t, and using equation (2.3), transition probabilities between the exogenous states $\{y_i\}$ can be calculated. Tauchen shows that when the discrete state values and the probability transition matrix are determined from a quadrature rule, the properties of the continuous law of motion can be closely approximated with a very coarse state-space.

Using Tauchen's method in conjunction with Coleman's thus has several advantages. First, it provides a rigorous way to specify the discrete values and the transition probabilities between the discrete states of the exogenous variables. And second, it limits the computational effort needed to solve the costly adjustment problem by allowing one to work with a coarse state-space for the exogenous variables.

III. Optimal Decision Rules

The solution method for factor demand models with costly adjustment was implemented for two specifications of the model, one linear-quadratic and the other with asymmetric adjustment costs. The algorithm performed well, converging smoothly in a reasonable amount of time and producing both sensible and interesting decision rules.

The solution method could be viewed as attempting to find a "fixed point" for $\Psi(\cdot,\cdot)$. Neither the existence nor the uniqueness of a fixed point for these problems has been proven theoretically, but several aspects of the behavior of the algorithm indicate that the resulting solutions are unique. First, though it is necessary to postulate an initial form for the decision rule $\Psi_0(y_i,n_j)$ to start the algorithm, the solution does not seem to be sensitive to the initial guess. Also, the distance measure between decision rule updates, max $|\Psi_1(y_i,n_j) - \Psi_0(y_i,n_j)|$, decreases monotonically with successive iterations, indicating that adjustments are being made smoothly toward the correct rule with no arbitrary choices between alternative updates.

At each iteration, the method requires finding roots for I•J equations, that is finding the x_{ij} that satisfy equation (2.6) for each state. The Van Wijngaarden-Dekker-Brent method described in Press, Flannery, Teukolsky, and Vetterling (1986) is used to find the roots. This method combines bisection and inverse quadratic interpolation to converge on a root within a bracketed interval. The more common Newton-Raphson method for finding roots is inappropriate for this problem because it requires that the first derivative be specified, and the function in this problem is piecewise linear and thus not differentiable.

The time required for the algorithm to converge depends primarily on the number of endogenous and exogenous states and the tightness of the convergence criterion. As an example, an asymmetric adjustment cost problem with 15 exogenous state values and 71 endogenous state values took 40 iterations and $1\frac{3}{4}$ hours to converge to six significant digits when run on a Compaq 386/16.

1. <u>The Linear-Quadratic Specification</u>

The linear-quadratic problem is solved primarily to test the algorithm's ability to converge to the correct decision rule since an explicit solution to the linear-quadratic problem is available for comparison. The specification of the functions of the problem are as follows:

$$f_{LQ}(n_t, \epsilon_t) = (a + \epsilon_t)n_t - \frac{1}{2}bn_t^2 + c$$

$$c_{LQ}(n_t - n_{t-1}) = \frac{1}{2}d(n_t - n_{t-1})^2$$

$$\begin{bmatrix} \epsilon_t \\ w_t \end{bmatrix} = \begin{bmatrix} \rho & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \epsilon_{t-1} \\ w_{t-1} \end{bmatrix} + \begin{bmatrix} v_t \\ 0 \end{bmatrix} \qquad \Sigma = \begin{bmatrix} \sigma_v^2 & 0 \\ 0 & 0 \end{bmatrix}$$

where a, b, c, d, ρ, and σ_v^2 are constants.[3] As a simplification, the autoregressive specification of the forcing functions implies that the real rental rate of the endogenous factor is constant, $w_t = w$. In the notation of the previous section, this specification implies $y_t = z_t = (\epsilon_t, w)'$, so using I discrete values for y_t amounts to using I discrete values for ϵ_t.

The explicit solution to this specification of the optimization problem is a decision rule for n_t in which n_t is a linear function of n_{t-1} and ϵ_t. Since ϵ_t follows an AR(1) process, the reduced form law of motion for n_t can be estimated as a Gaussian VAR with two lags,

$$n_t = \alpha_0 + \alpha_1 n_{t-1} + \alpha_2 n_{t-2} + e_t, \qquad e_t \sim N(0, \sigma_e^2), \tag{3.1}$$

where the α's and σ_e^2 are functions of the structural parameters a, b, d, ρ, σ_v^2, w, and β.

The above equation was estimated for quarterly production worker employment in the manufacturing durables sector from 1947I through 1986IV. The data were first exponentially detrended by regressing their natural log on a trend and seasonal dummies and keeping the residuals. Since the factor demand model is formulated in terms of levels of the data, the detrended data were rescaled to the original units by taking their exponential and then multiplying by the mean of the raw data. Some summary statistics on the raw data and the detrended data are reported in Table 1. The resulting estimated VAR is

$$n_t = \underset{(169.76)}{648.11} + \underset{(.066)}{1.4928}\, n_{t-1} - \underset{(.066)}{.5772}\, n_{t-2} + \hat{e}_t, \quad s_e = 167.6. \qquad (3.2)$$

Values of the underlying structural parameters are estimated from the reduced form VAR coefficients so they can be used to calibrate the linear-quadratic problem for the numerical algorithm. Of the seven structural parameters only four are identified, so assumptions are made about the values of three of them and the values of the others are determined relative to these assumptions. The data are quarterly, so β is assumed to equal .99. a and w merely scale the problem and are set equal to 1.2 and 1 respectively. The solution obtained by the numerical algorithm will be insensitive to the values chosen for β, a, and w as long as the other parameter values are scaled relative to the values of β, a, and w. However, a mapping from the linear-quadratic to the asymmetric adjustment cost problem is specified below, and the relative values of a and w do affect the implied decision rule for the asymmetric adjustment cost problem because a and w do not enter that objective function of that problem symmetrically as they do in the linear-quadratic problem. Experimentation with different values for a revealed that a value of 1.2 put the decision rule for the asymmetric adjustment cost problem in approximately the same range as the linear-quadratic problem.

An additional problem arises in determining the values of the structural parameters because there is no real value of ρ that satisfies the implied relationship between the reduced form VAR coefficients in equation (3.2) and the structural parameters of the linear-quadratic specification. However, the imaginary part of the implied value of ρ is small so it is ignored. The values of the other structural parameters are determined then relative to β, a, w, and the modified estimate of ρ such that the unconditional mean of the VAR is kept equal to the mean of the data.

The implied values of the structural parameters are as follows:

$$\beta = .99$$

$$w = 1$$

$$\left.\begin{array}{l} a = 1.2 \\ b = .000026 \end{array}\right\} \quad f_{LQ}(n_t, \epsilon_t) = (1.2 + \epsilon_t)n_t - .000013n_t^2 + c$$

$$d = .00030 \quad \} \quad c_{LQ}(n_t - n_{t-1}) = .00015(n_t - n_{t-1})^2$$

$$\left.\begin{array}{l} \rho = .75 \\ \sigma_v^2 = .00088 \end{array}\right\} \quad \epsilon_t = .75\epsilon_{t-1} + v_t, \qquad v_t \sim N(0, .00088)$$

The numerical algorithm was used to solve the linear-quadratic problem with the above specification of the production, cost, and forcing functions. Figure 1 is a graphic representation of the resulting decision rule $n_t = \Psi(y_t, n_{t-1})$, which for the above specification of the forcing functions could be written $n_t = \Psi(\epsilon_t, n_{t-1})$. n_{t-1} appears on the horizontal axis and each line on the graph is for a different discrete value of ϵ_t. Thus given values for n_{t-1} and ϵ_t, the figure indicates how much one should increase or decrease n from its value in the previous period. The problem was solved with 29 discrete values of the endogenous variable n_t equally spaced over the interval 5000 to 12000 and seven discrete values of the exogenous variable ϵ_t. The values of ϵ_t and their stationary distribution, which are derived from a seven-point quadrature rule, along with the probability transition matrix implied by the autoregression for ϵ_t are reported in Table 2.

The algorithm converged easily to a very close approximation of the correct decision rule. The decision rule for the linear-quadratic problem should be of the form

$$n_t = \beta_0 + \beta_1 n_{t-1} + \beta_2 \epsilon_t$$

where the β's are explicit functions of the underlying structural parameters. Thus the numerical algorithm correctly found that $(n_t - n_{t-1})$ should be a linear function of n_{t-1} and ϵ_t. To gain some indication of the accuracy of the approximation, the values of the approximated decision rule were compared to the values of the correct decision rule for the seven values of ϵ_t at the boundaries 5000 and 12000 where the differences would be the most extreme. The maximum difference is about 25, which can easily be accepted given the coarseness of the state-space and the fact that the parameter values were only specified to two significant digits.

2. The Asymmetric Adjustment Cost Specification

Since the numerical algorithm performed well in solving the linear-quadratic problem, it was then used to solve an asymmetric adjustment cost specification of the factor demand model, for which no explicit solution exists. Because an explicit solution is no longer attainable, the quadratic restriction can be relaxed on the specification of the production function as well as the adjustment cost function. However, a linear specification for the forcing function is maintained since the interest is in the type of conditional heterogeneity that an asymmetric adjustment cost structure will imply given a relatively simple forcing function. The production function, forcing function, and adjustment cost function are specified as

$$f_{AS}(n_t, g_t) = n_t^{\gamma} g_t \qquad g_t = \exp(u_t),$$

$$(u_t - \mu_u) = \nu(u_{t-1} - \mu_u) + \eta_t \qquad \eta_t \sim N(0, \sigma_\eta^2)$$
$$\mu_u = E[u_t]$$

$$c_{AS}(x_t) = \delta\left[\frac{1}{\tau}[\exp(\tau x_t) - 1] - x_t\right] \qquad x_t = n_t - n_{t-1}.$$

The production function is a one factor Cobb-Douglas function, and the adjustment cost function has the properties that $c(0) = 0$, $c(x) > 0$ for $x \neq 0$, and $c(x) > c(-x)$ for $x > 0$. Thus adjustment costs rise more rapidly for increases in the control variable than for decreases.

To determine reasonable values for the parameters of the above functions, a mapping between the estimated parameters of the linear-quadratic specification, $\{a, b, \rho, \sigma_v^2, d\}$, and the parameters of the nonlinear specification, $\{\gamma, \nu, \mu_u, \sigma_\eta^2, \delta, \tau\}$, is defined around the mean of the employment series. The same values of w and β are used for both specifications. The parameters γ, ν, μ_u, and σ_η^2 are determined by satisfying the following criteria:

1) $$\frac{\partial f_{LQ}(\bar{n}, \bar{\epsilon})}{\partial n} = \frac{\partial f_{AS}(\bar{n}, \bar{g})}{\partial n}$$

2) $$\frac{\partial^2 f_{LQ}(\bar{n}, \bar{\epsilon})}{\partial n^2} = \frac{\partial^2 f_{AS}(\bar{n}, \bar{g})}{\partial n^2}$$

$$3)\ \mathrm{Var}\left[\frac{\partial f_{LQ}(\bar{n},\epsilon_t)}{\partial n}\right] = \mathrm{Var}\left[\frac{\partial f_{AS}(\bar{n},g_t)}{\partial n}\right]$$

$$4)\ \mathrm{Cov}\left[\frac{\partial f_{LQ}(\bar{n},\epsilon_t)}{\partial n}, \frac{\partial f_{LQ}(\bar{n},\epsilon_{t-1})}{\partial n}\right] = \mathrm{Cov}\left[\frac{\partial f_{AS}(\bar{n},g_t)}{\partial n}, \frac{\partial f_{AS}(\bar{n},g_{t-1})}{\partial n}\right]$$

where $\bar{n} = E[n_t]$, $\bar{\epsilon} = E[\epsilon_t]$, and $\bar{g} = E[g_t]$. The parameters of the adjustment cost function, δ and τ, are determined by satisfying

$$5)\ c'_{LQ}(0) = c'_{AS}(0)$$

$$6)\ c'_{LQ}(2\sigma_x) = 3\cdot c'_{AS}(2\sigma_x)$$

where σ_x is the standard deviation of $(n_t - n_{t-1})$ in the detrended data.

With the above mapping and the estimated values of the L-Q parameters, the implied values of the structural parameters are

$$\beta = .99$$

$$w = 1$$

$$\gamma = .80 \quad \} \quad f_{AS}(n_t, g_t) = n_t^{.8}\cdot \exp(u_t)$$

$$\left.\begin{array}{l} \nu = .75 \\ \mu_u = 2.02 \\ \sigma_\eta^2 = .00088 \end{array}\right\} \quad u_t = .51 + .75u_{t-1} + \eta_t, \qquad \eta_t \sim N(0, .00088)$$

$$\left.\begin{array}{l} \delta = .063 \\ \tau = .0046 \end{array}\right\} \quad c_{AS}(x_t) = .063\cdot\left[\frac{1}{.0046}[\exp(.0046x_t) - 1] - x_t\right].$$

Graphs of the asymmetric and quadratic adjustment cost functions appear in Figure 2, and graphs of their derivatives appear in Figure 3. Both functions have the value zero and the same slope at $x_t = 0$. The asymmetric function shows more rapidly rising costs than the quadratic function for increases in the control variable and less rapidly rising costs for decreases.

The factor demand model with the above nonlinear production function and asymmetric adjustment costs was solved using 71 discrete values for the endogenous variable equally spaced over the interval 5000 to 12000 and 15 discrete values of the exogenous variable. The discrete values of the exogenous variable and their stationary probability distribution are report in Table 3. The estimated decision rule is graphed in Figure 4, with the eight lines corresponding to every other of the 15 discrete values of u_t.

Figure 4 shows that the decision rule for this specification reflects significant departures from the linear decision rule associated with the linear-quadratic specification. The curved shape of each line in the figure indicates that for any given value of the exogenous variable, the optimal value for n_t depends nonlinearly on n_{t-1}. Also, the increasing spacing of the lines indicates that for any given value of n_{t-1}, the optimal value for n_t depends nonlinearly on u_t. One would thus expect the law of motion for n_t to display complex forms of conditional heterogeneity. For example, the wider spacing of the lines at higher values of n_{t-1} is probably indicative of conditional dependence of the variance of n_t on the past.

By examining Figure 4, one can see that a time series generated by this decision rule would display many small increases and some small and large decreases from one period to the next. This result is sensible for a problem in which costs of adjusting the variable upward increase more rapidly than costs of adjusting downward. It is also consistent with the story told in the introduction about Neftci's finding of asymmetry in unemployment rates. Unemployment rate time series are essentially mirror images of employment time series, so gradual, persistent decreases in the unemployment rate reflect gradual, persistent increases in employment. Likewise, more rapid, less persistent increases in the unemployment rate reflect more rapid, less persistent decreases in employment. From the decision rule, one would expect to see just such gradual, persistent increases and more rapid, less persistent decreases in n_t.

IV. Seminonparametric Models

The decision rule for the asymmetric adjustment cost problem implies a law of motion for n_t. This section presents *Gallant-Tauchen seminonparametric* (SNP) method, which will be used to characterize this law of motion. [See Gallant and Tauchen (1988) and *Gallant, Hsieh,* and Tauchen (1988).] The SNP technique allows one to estimate the conditional distribution of a variable without imposing strong restrictions on the form of the distribution; it can accommodate conditional heterogeneity in the mean, in the variance, and in higher order moments of the distribution. This flexibility is important for modeling the behavior of n_t since the decision rule derived in the last section suggests that the conditional distribution of n_t may display complex forms of heterogeneity.

The SNP technique estimates the distribution of an M-dimensional vector y_t conditional on its past. The only restrictions are that the data $\{y_t\}_{-L+1}^{n}$ are a realization from a stationary time series $\{y_t\}_{t=-\infty}^{\infty}$ and that the *conditional* distribution of y_t *given the entire past depends on a finite* number L of lagged values of y_t. With

$$x_{t-1} = (y'_{t-L}, y'_{t-L+1}, \ldots, y'_{t-1})',$$

the conditional distribution of y_t can then be expressed as $h(y_t | x_{t-1})$.

The SNP model approximates the conditional distribution as a truncated Hermite expansion, which is a polynomial expansion around the Gaussian density function. The terms of the polynomial accommodate departures from Gaussianity in the conditional distribution of y_t.

Gallant and Tauchen find that, rather than expanding around a Gaussian distribution for y_t, their method performs better if they approximate the conditional distribution of a parametric transformation of y_t that preserves Gaussianity. Thus they would model the conditional distribution of z_t given x_{t-1}, $f(z_t | x_{t-1})$, where*

$$z_t = R^{-1}(y_t - b_0 - Bx_{t-1}), \qquad RR' = \Omega,$$

and Ω is the variance-covariance matrix of $(y_t - b_0 - Bx_{t-1})$. The conditional distribution of y_t is easily retrievable from $f(\cdot|\cdot)$ as

$$h(y_t | x_{t-1}) = f[R^{-1}(y_t - b_0 - Bx_{t-1}) | x_{t-1}]/\det(R).$$

The SNP approximation of the conditional distribution is, without the time subscripts,

$$f_K(z | x) = \frac{\left[P_K(z, x) \right]^2 \varphi(z)}{\int \left[P_K(u, x) \right]^2 \varphi(u) \, du}$$

* This z_t is not the same as the z_t used for the exogenous variables in Section 2.

where $P_K(z, x)$ is a polynomial of degree K and φ is the multivariate standard Gaussian density function. The polynomial is squared to maintain positivity, and the integral in the denominator insures that the distribution integrates to one.

Gallant and Tauchen represent the polynomial as

$$P_K(z, x) = \sum_{|\alpha|=0}^{K_z} \left[\sum_{|\beta|=0}^{K_x} a_{\alpha\beta} \, x^\beta \right] z^\alpha$$

where the a's are the coefficients of the polynomial,

$$\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_M)' \qquad \beta = (\beta_1, \beta_2, \ldots, \beta_{ML})'$$

are multi-indices (vectors with integer elements), and

$$|\alpha| = \sum_{i=1}^{M} \alpha_i \qquad |\beta| = \sum_{i=1}^{ML} \beta_i$$

$$z^\alpha = \prod_{i=1}^{M} (z_i)^{\alpha_i} \qquad x^\beta = \prod_{i=1}^{ML} (x_i)^{\beta_i} .$$

In order for $f(z| x)$ to be estimated consistently, both K_z and K_x must grow with sample size.[4]

The parameters of the model are the elements of b_0, B, and R and the coefficients of the polynomial, $a_{\alpha\beta}$, with the constant term of the polynomial always normalized to equal one. For particular values of K_z and K_x, these parameters can be estimated with standard maximum likelihood techniques. If

$K_z = K_x = 0$, the polynomial is just a unit constant, so $f_K(z|\, x)$ is the standard Gaussian density and $h_K(y|\, x)$ is a Gaussian VAR. If $K_z > 0$ and $K_x = 0$, P_K is a polynomial in current values of z, which allows shape departures from Gaussianity in $f_K(z|\, x)$. If $K_z > 0$ and $K_x > 0$, the coefficients of the polynomial also depend on the history of the series, so there can be nonlinear conditional dependence in the estimated density. By making K_z and K_x sufficiently large, the SNP model can approximate any smooth conditional density arbitrarily accurately. On a given data set, SNP models with different values for the tuning parameters L, K_z, and K_x are estimated, and standard model selection criteria are used to select between the different SNP(L,K_z,K_x) specifications.

Because of the initial transformation made to y_t for the leading term of the polynomial expansion, the SNP model nests the Gaussian VAR. It can also nest the ARCH model (Engle, 1982) if Ω is made to depend on the squared elements of x_{t-1}. Rather than parameterize this specifically, Gallant and Tauchen instead make R depend linearly on the absolute value of x_{t-1} to simplify the estimation. Asymptotically this modification to the SNP specification is not necessary for SNP to accommodate ARCH type behavior, but with finite samples, the degrees of the polynomial expansion for which the SNP model can be estimated will be limited. Thus it is easier to explain conditional heterogeneity in both the mean and the variance with the leading term of the polynomial expansion and to let the other polynomial terms accommodate additional conditional heterogeneity beyond the first and second moments.

The modified SNP model has the same form as the above SNP model except z is now specified as

$$z = R_x^{-1}(y - b_0 - Bx), \text{ and}$$

$$\mathrm{vec}(\text{upper\_triangular } R_x) = p_0 + P \cdot \mathrm{abs}(x)$$

where abs(x) is a vector of the absolute values of the L elements of x. The parameters of the SNPRX model are b_0, B, p_0, P, and the $a_{\alpha\beta}$.

V. Conditional Distribution Estimates

In this section, the SNP model is used to compare the law of motion of the detrended employment data and the law of motion implied by the asymmetric adjustment cost model. To apply the SNP model to the asymmetric adjustment cost problem, the decision rule for the problem is used to simulate a realization of 1000 observations on n_t and the SNP model is estimated on this data.[5]

Some summary statistics on the simulated data are reported in Table 1. The mean, standard deviation, and standard deviation of first differences are all somewhat larger for the simulated series than for the detrended employment data. Such differences are not surprising since the mapping between the linear-quadratic problem and the asymmetric cost problem is not exact. Also, the goal of this section of the paper is to compare the conditional distributions of the two series qualitatively, not to match moments exactly, and these small but essentially proportional differences should not hinder such comparison. A less magnified simulated series with very similar conditional distribution properties could probably be generated by the asymmetric adjustment cost model if the production function was multiplied be a constant less than one, which would be equivalent to using a smaller value for the mean of u_t.

Engle's (1982) test for the absence of ARCH was performed on both the detrended data and the simulated data to determine whether to estimate SNP or SNPRX models. The test requires estimating an autoregression of the data, squaring the residuals, then estimating another autoregression of the squared residuals. Engle shows that a TR^2 test on the second autoregression is the Lagrange multiplier test of the null hypothesis of no ARCH in the data. The null hypothesis is not rejected for the detrended employment data, but it is rejected for the simulated data. However, Hsieh (1983) shows that this test probably has low power for finite samples indicating that not rejecting the null hypothesis is not strong evidence against ARCH. Because of the test results, both SNP and SNPRX models are estimated on the detrended data, but only the SNPRX model is estimated on the simulated data.

Results from estimating the SNPRX model on the detrended employment data are reported in Table 4. Each line of the table corresponds to a different specification of the SNPRX(L,K_z,K_x) model, where a specification is characterized by the values of L, K_z, and K_x. The fourth column of the table indicates the number of parameters estimated for each specification and the fifth column contains the corresponding maximized log likelihood value.

Three model selection procedures are used to compare the different specifications: upward testing with chi square statistics, the Schwarz criterion, and the Akaike information criterion (AIC). P-values for the likelihood ratio chi square tests are reported in columns six through eight. A p-value in the K_x column, for example, compares that line's specification, SNPRX(L,K_z,K_x), to its successor, SNPRX(L,K_z,K_x+1). There is one exception:

the K_z p-values for the SNPRX(L,0,0) specifications correspond to a test between that specification and the SNPRX(L,2,0) specification.

The p-values clearly indicate that two lags are sufficient for modeling the data. However, they also indicate that one should use specifications with higher and higher values of K_z and K_x. The Schwarz criterion, which is known to be conservative, rarely selects specifications with $K_x > 0$. However, for this data the Schwarz criterion selects an SNPRX(2,2,1) specification. Like the p-values, the Akaike criterion calls for very large dimensional models. In this case, it selects an SNPRX(2,4,1) specification. The Akaike criterion might have selected even larger specifications such as an SNPRX(2,5,1) or an SNPRX(2,4,2), but these specifications were not estimated because their dimensionality would have been excessive for a data set of 156 observations.

Figures 5 and 6 show plots of the one step ahead density of employment, $h(y| x)$ or $h(n_t| n_{t-2}, n_{t-1})$, implied by the SNPRX(2,2,1) and SNPRX(2,4,1) estimates. Each of the three plots in the figures is conditional on different values of n_{t-2} and n_{t-1}. The center plot is the conditional distribution of n_t given that n_{t-2} and n_{t-1} were equal to the unconditional mean of the series. For the left plot, the distribution of n_t is conditional on n_{t-2} equal to the unconditional mean and n_{t-1} equal to the unconditional mean minus one standard deviation of the first differences of the series. The conditional path for the right plot is the mirror image of that for the left: n_{t-2} is equal to the unconditional mean and n_{t-1} is equal to the unconditional mean plus one standard deviation of the first differences of the series.

Clearly heterogeneity in higher order moments is important in the conditional distribution of employment. A linear Gaussian law of motion conditional on the same paths would have generated three Gaussian densities all with the same variance but with different means. But the plots in Figures

5 and 6 indicate substantially different dynamic behavior. The primary insight revealed by the plots is that the conditional variance of employment is much smaller when the series has been increasing than when it has been decreasing. This result is consistent with the evidence cited earlier of greater persistence and more gradual changes in unemployment rates during declining periods than during rising periods.

The SNP model was also estimated on the detrended employment data because the test for absence of ARCH described above did not result in a rejection. However, the SNP results are not reported in detail here, primarily because the SNP model is nested in the SNPRX model for which results are reported above. Plots of estimated conditional densities from the SNP model, though, are similar to those from the SNPRX model. They also detect tighter variance following an increasing history of the series than following a decreasing history.

The SNPRX model was also estimated on data simulated from the decision rule for the asymmetric adjustment cost model to see to if that model implies a conditional distribution similar to that of the actual employment data. A full table of different SNPRX specifications was not estimated for this data because the criteria used for selecting between different specifications are based on sample size, which is obviously not a consideration when working with simulated data. There is not necessarily a finite specification of the SNPRX model that completely characterizes the law of motion for the simulated data, so more polynomial terms will always improve the approximation of the true conditional density. Thus with larger and larger samples, one would expect the selection criteria to choose higher and higher dimensional specifications of the SNPRX approximation to the actual conditional density. Rather than estimating extremely large specifications, the law of motion of the simulated data is approximated with the SNPRX(2,2,1) and SNPRX(2,4,1) specifications, which are the specifications chosen by the Schwarz and Akaike criteria for the true employment data. This allows one to compare the projections of the two laws of motion onto the same dimensional SNPRX models.

Plots of the estimated densities of the simulated data conditional on different lagged values appear in Figures 7 and 8. The paths on which the three densities in each figure are conditioned are the same as those for Figures 5 and 6 except they are based on the unconditional mean and standard deviation of first differences of the simulated series rather than the true employment series. (The reader is cautioned about comparing density plots between the figures because the horizontal and vertical scaling is not the same for each figure. The relevant factor to consider is the relative shapes of the densities in each figure.)

Figures 5 and 6 for the employment data compare very favorably with Figures 7 and 8 for data simulated from the asymmetric adjustment cost model. Though the plots for the actual and simulated data are not precisely the same, the asymmetric adjustment cost model clearly captures the important conditional variance properties in the actual employment data described above. All of the plots show that the conditional variance is greater when the series has been decreasing than when it has been increasing.

VI. Conclusion

Recent empirical studies have found strong evidence of nonlinearity and conditional heterogeneity in the dynamic behavior of employment and unemployment time series. Existing structural economic models for employment, such as the linear-quadratic factor demand model with costly adjustment, cannot explain this complex behavior. However, it is possible that a modification of the costly adjustment model could better capture the dynamic properties of employment data. Neftci's work on asymmetry in unemployment rates motivates considering a modification in which adjustment costs for labor are asymmetric, where costs of increasing the size of the labor force rise more rapidly than costs of decreasing.

The dynamic factor demand model with costly adjustment has been studied with a linear-quadratic specification because that specification admits and explicit solution. In this paper a numerical method is developed to approximate the solution to a more general specification of the problem. Coleman's work on solving rational expectations models with endogenous state variables and Tauchen's work on obtaining discrete approximations to continuous laws of motion are combined in this method. Applying the method to the asymmetric adjustment cost problem reveals a complex nonlinear decision rule for the choice variable.

Gallant and Tauchen's seminonparametric method is used to compare the law of motion for a time series of employment in the durables manufacturing sector and the law of motion implied by the asymmetric adjustment cost model. This method is appropriate for characterizing these laws of motion because it can accommodate complex forms of conditional heterogeneity which one would expect to be found in these data. The employment data are found to display important condition variance properties, with the conditional variance smaller when the series has been increasing and larger when the series has been decreasing. Data simulated from the solution to the asymmetric adjustment cost model displays similar conditional variance properties indicating that asymmetric adjustment costs may be important for explaining aggregate employment dynamics.

Table 1

Production Worker Employment in Durables Manufacturing

and Simulated Data from the Asymmetric Adjustment Cost Model

	Raw Data	Detrended Data	Simulated Data
Mean	7670	7695	8915
Standard Deviation	721	617	788
Standard Deviation Of First Differences	213	205	275
Minimum	5818	6248	6884
Maximum	9219	8833	11225

Table 2

Discrete Probability Model for ϵ_t

Stationary Distribution

	ϵ_i	$f(\epsilon_i)$
1	-.11126	.018735
2	-.070209	.098513
3	-.034245	.23138
4	.0	.30275
5	.034245	.23138
6	.070209	.098513
7	.11126	.018735

Probability Transition Matrix

.40616	.46488	.11990	.00887619	.00018130	7.6718E-07	2.7903E-10
.088318	.42493	.38566	.094598	.00640222	9.5331E-05	1.4575E-07
.00969246	.16410	.44847	.31425	.060755	.00272417	1.4655E-05
.00054827	.030757	.24012	.45714	.24012	.030757	.00054827
1.4655E-05	.00272417	.060755	.31425	.44847	.16410	.00969246
1.4575E-07	9.5331E-05	.00640222	.094598	.38566	.42493	.088318
2.7903E-10	7.6718E-07	.00018130	.00887619	.11990	.46488	.40616

Table 3

Discrete Probability Model for u_t

Stationary Distribution

	u_i	$f(u_i)$
1	1.85121	.000050483
2	1.88604	.00077481
3	1.91552	.00529675
4	1.94243	.021766
5	1.96784	.060702
6	1.99234	.12225
7	2.01629	.18395
8	2.04000	.21041
9	2.06371	.18395
10	2.08766	.12225
11	2.11216	.060702
12	2.13757	.021766
13	2.16448	.00529675
14	2.19396	.00077481
15	2.22879	.000050483

Table 4

SNPRX(L,K_z,K_x) for Production Workers in Durables Manufacturing

				Log	p-values				
L	K_z	K_x	p_θ	Likelihood	L	K_z	K_x	Schwarz	AIC
1	0	0	4	-0.30678	0.0001	0.0026		-0.3715	-0.3324
1	2	0	6	-0.26854	0.0001	0.0007	0.2121	-0.3657	-0.3070
1	3	0	7	-0.23099	0.0001	0.9230	0.1096	-0.3443	-0.2759
1	4	0	8	-0.23096	0.0001		0.0715	-0.3604	-0.2822
1	2	1	9	-0.25411	0.0001	0.0007	0.0631	-0.3998	-0.3118
1	3	1	11	-0.20679	0.0001	0.2731		-0.3848	-0.2773
1	4	1	13	-0.19847	0.0001			-0.4089	-0.2818
1	2	2	12	-0.23073	0.0001			-0.4250	-0.3077
2	0	0	6	-0.09628	0.7964	0.0420		-0.1934	-0.1347
2	2	0	8	-0.07595	0.7743	0.0122	0.0001	-0.2054	-0.1272
2	3	0	9	-0.05578	0.0595	0.0142	0.0001	-0.2014	-0.1135
2	4	0	10	-0.03649	0.0872		0.0001	-0.1983	-0.1006
2	2	1	14	0.04315	0.9031	0.0026	0.0111	*-0.1834	-0.0466
2	3	1	17	0.08880	0.4040	0.0162		-0.1864	-0.0202
2	4	1	20	0.12184				-0.2019	-0.0064*
2	2	2	23	0.11172				-0.2605	-0.0357
3	0	0	8	-0.09482	0.2283	0.0408		-0.2243	-0.1461
3	2	0	10	-0.07431	0.8882	0.0008	0.0001	-0.2362	-0.1384
3	3	0	11	-0.03769	0.7332	0.0219	0.0001	-0.2157	-0.1082
3	4	0	12	-0.02085				-0.2151	-0.0978
3	2	1	19	0.04823		0.0009		-0.2593	-0.0736
3	3	1	23	0.10859				-0.2637	-0.0388
4	0	0	10	-0.08535		0.1587		-0.2472	-0.1495
4	2	0	12	-0.07355		0.0006		-0.2678	-0.1505
4	3	0	13	-0.03570				-0.2461	-0.1190

p_θ = number of estimated parameters

* maximum selection criterion value

DECISION RULE FOR LINEAR-QUADRATIC PROBLEM

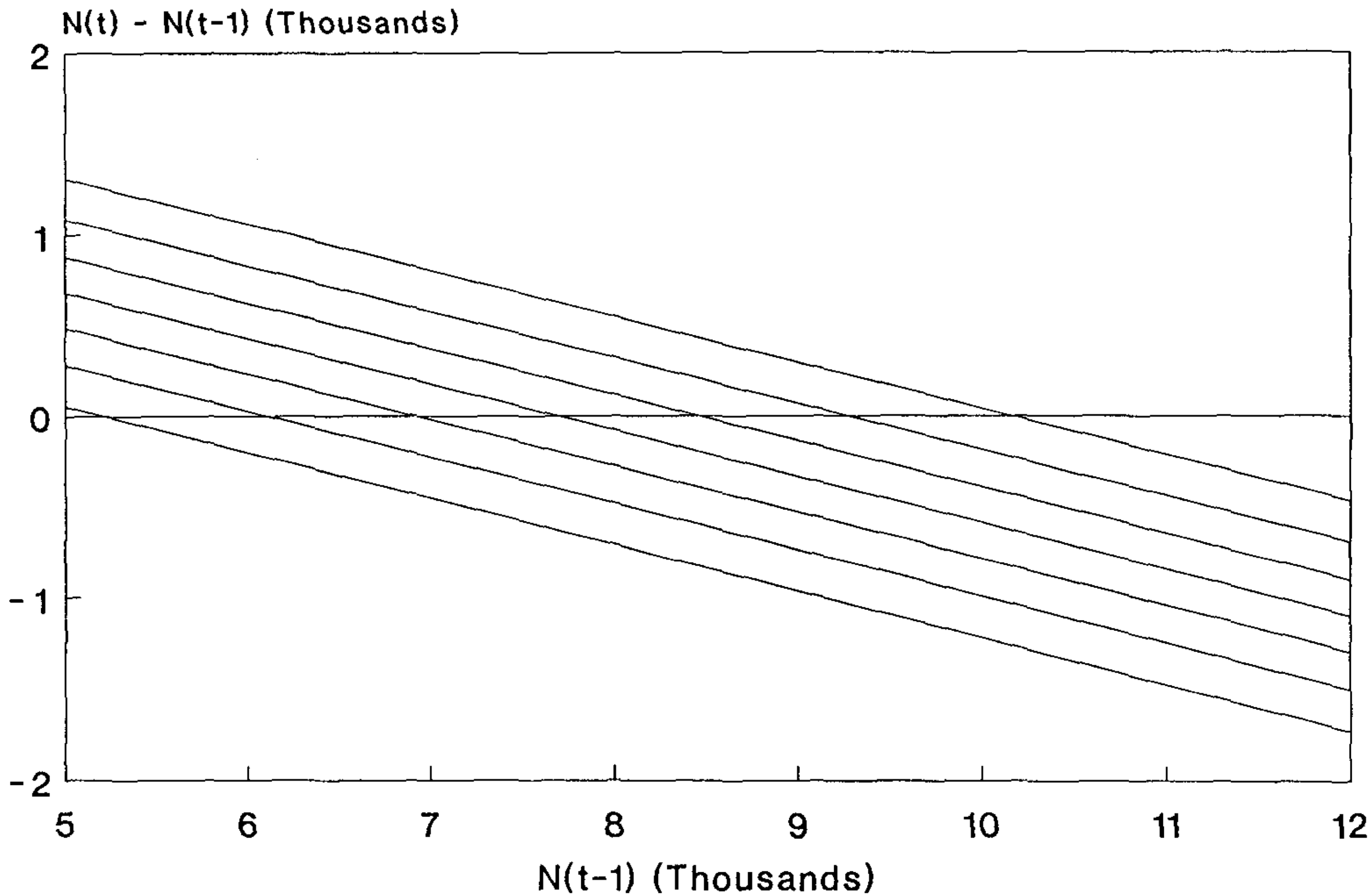

FIGURE 1

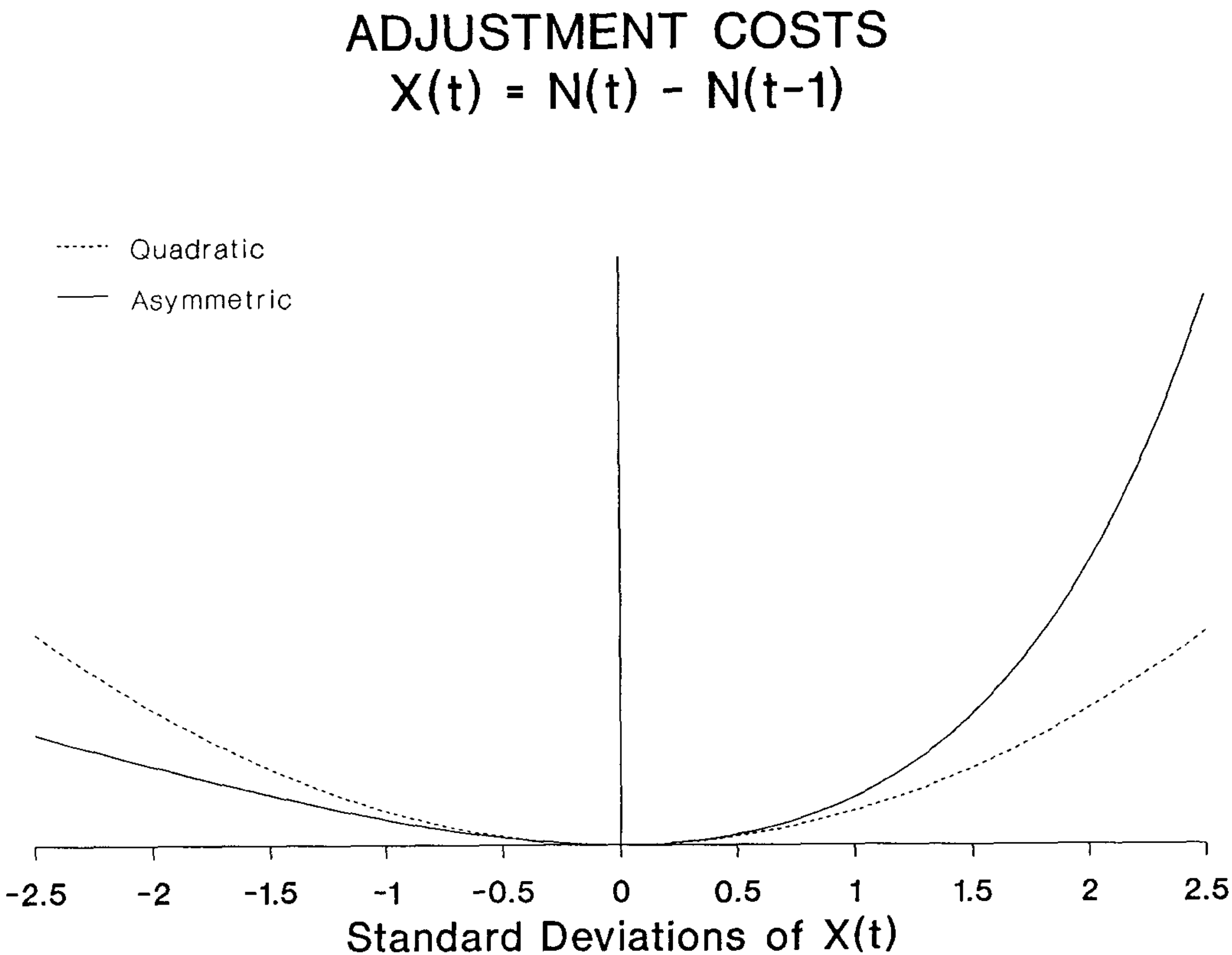
ADJUSTMENT COSTS
X(t) = N(t) - N(t-1)
Quadratic
Asymmetric
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
Standard Deviations of X(t)

FIGURE 2

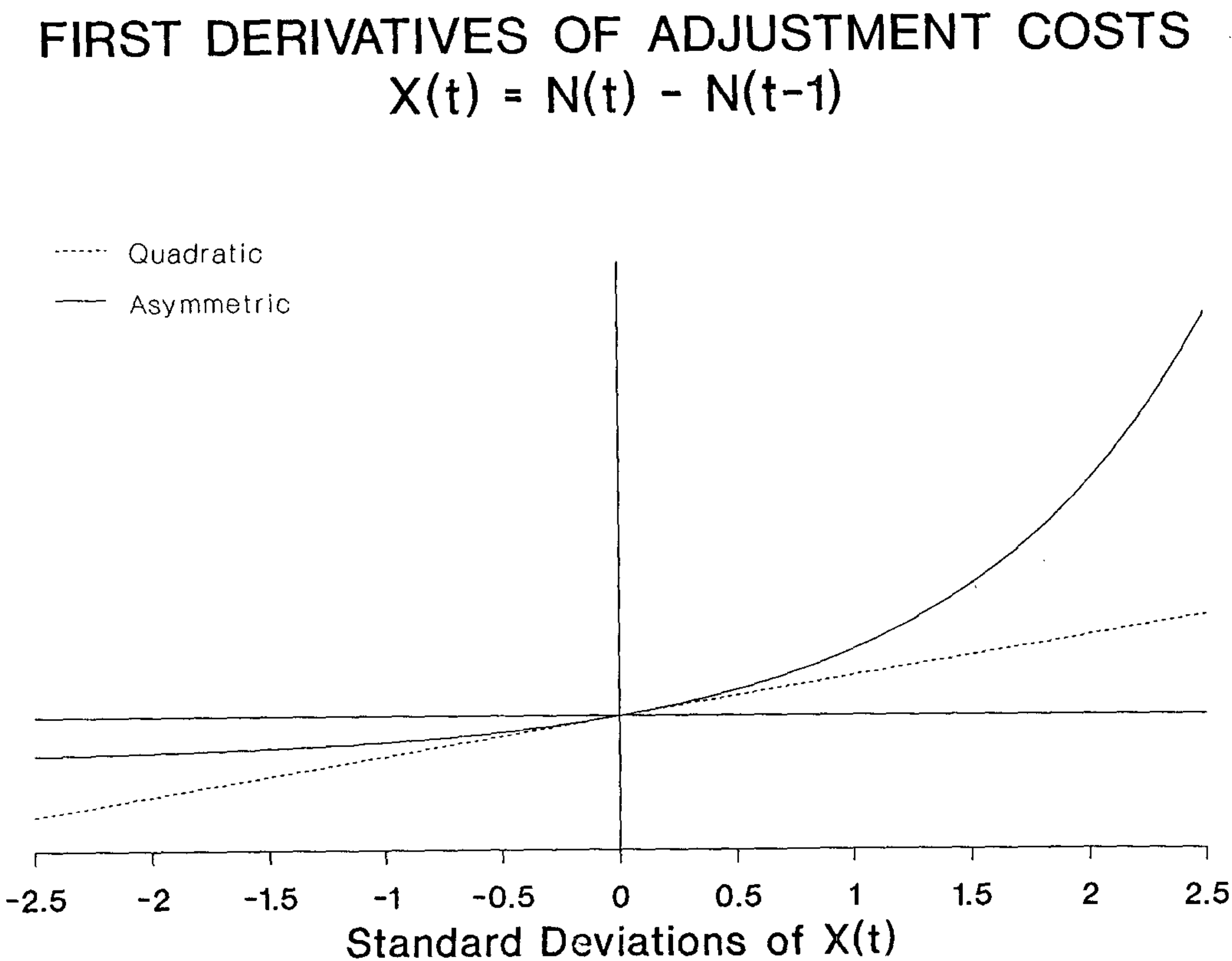
FIRST DERIVATIVES OF ADJUSTMENT COSTS
X(t) = N(t) - N(t-1)
Quadratic
Asymmetric
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
Standard Deviations of X(t)

FIGURE 3

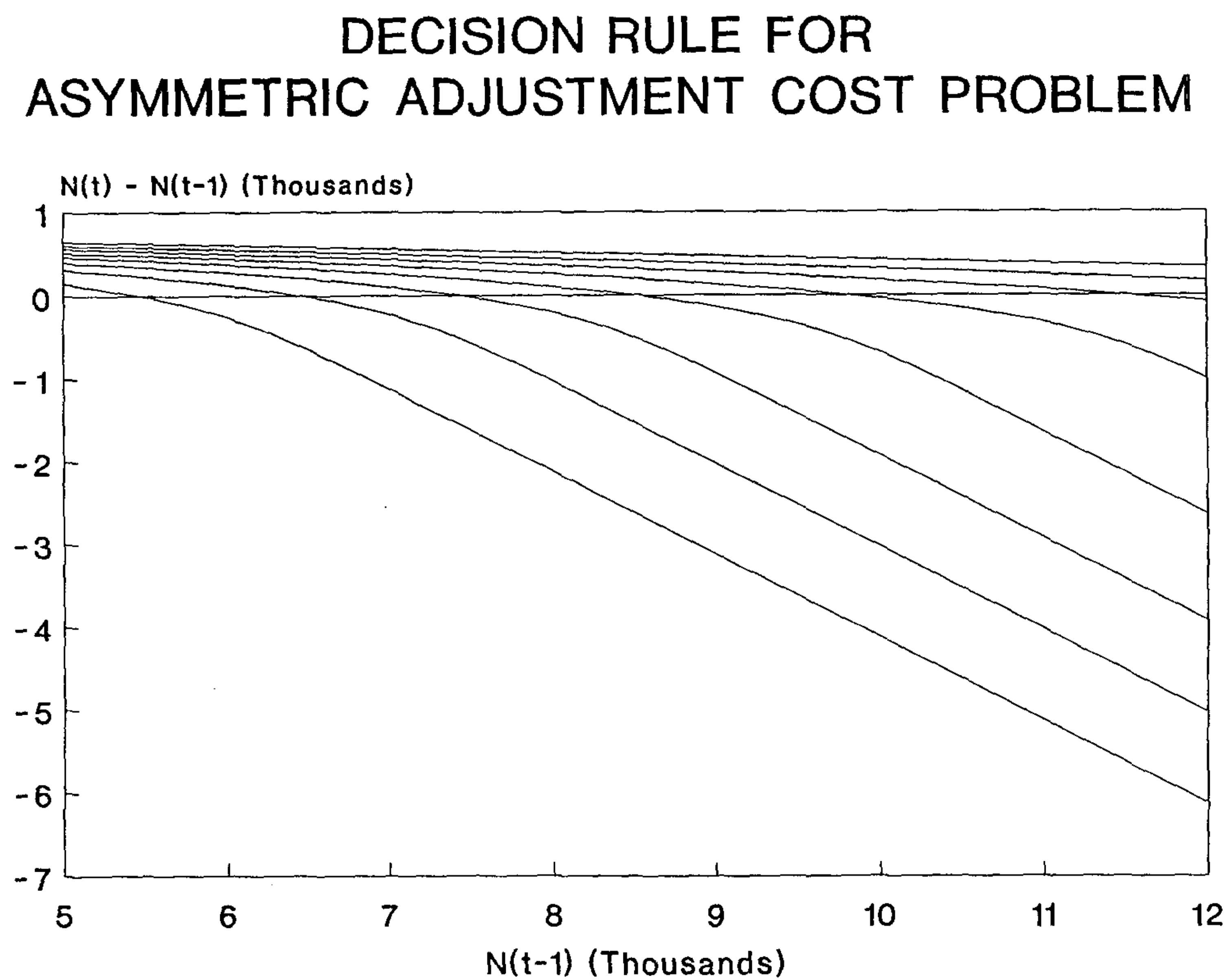
DECISION RULE FOR
ASYMMETRIC ADJUSTMENT COST PROBLEM
N(t) - N(t-1) (Thousands)
1
0
-1
-2
-3
-4
-5
-6
-7
5
6
7
8
9
10
11
12
N(t-1) (Thousands)

FIGURE 4

SNPRX(2,2,1): DURABLES EMPLOYMENT

N(t-2) = MEAN

N(t-1) = (MEAN - 205, MEAN, MEAN + 205)

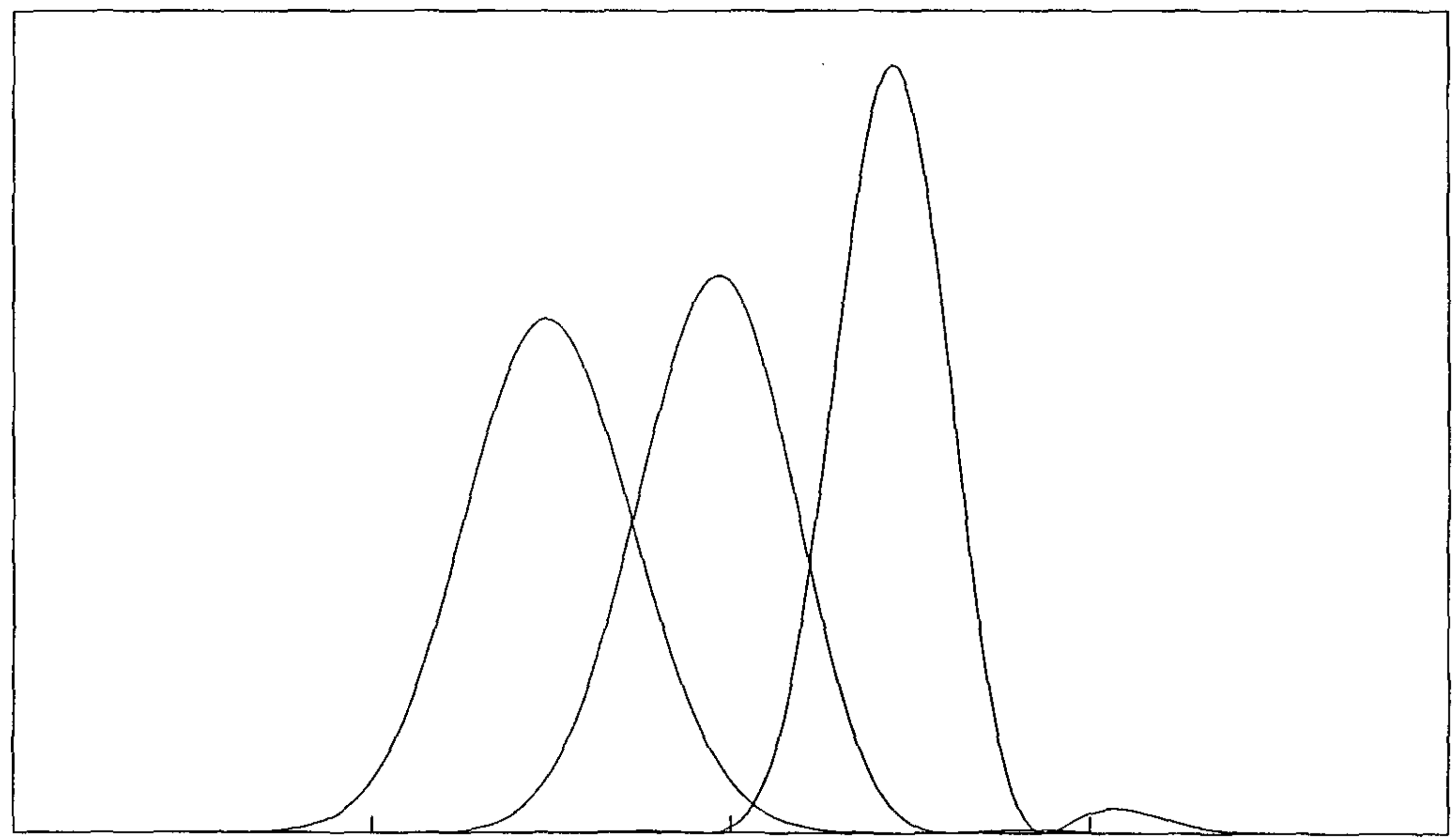

FIGURE 5

SNPRX(2,4,1): DURABLES EMPLOYMENT

N(t-2) = MEAN

N(t-1) = (MEAN - 205, MEAN, MEAN + 205)

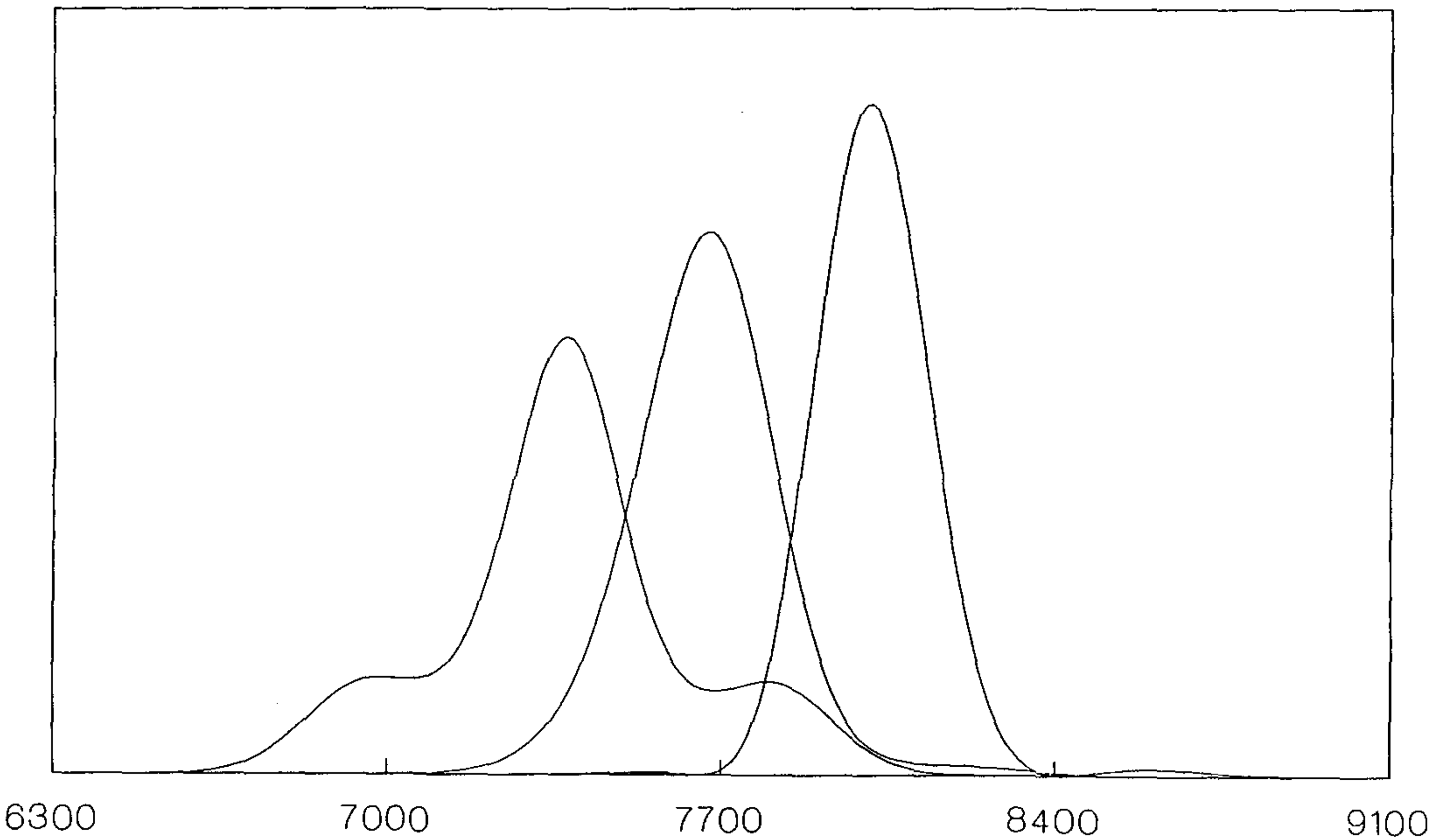

FIGURE 6

SNPRX(2,2,1,): SIMULATED DATA

N(t-2) = MEAN

N(t-1) = (MEAN - 275, MEAN, MEAN + 275)

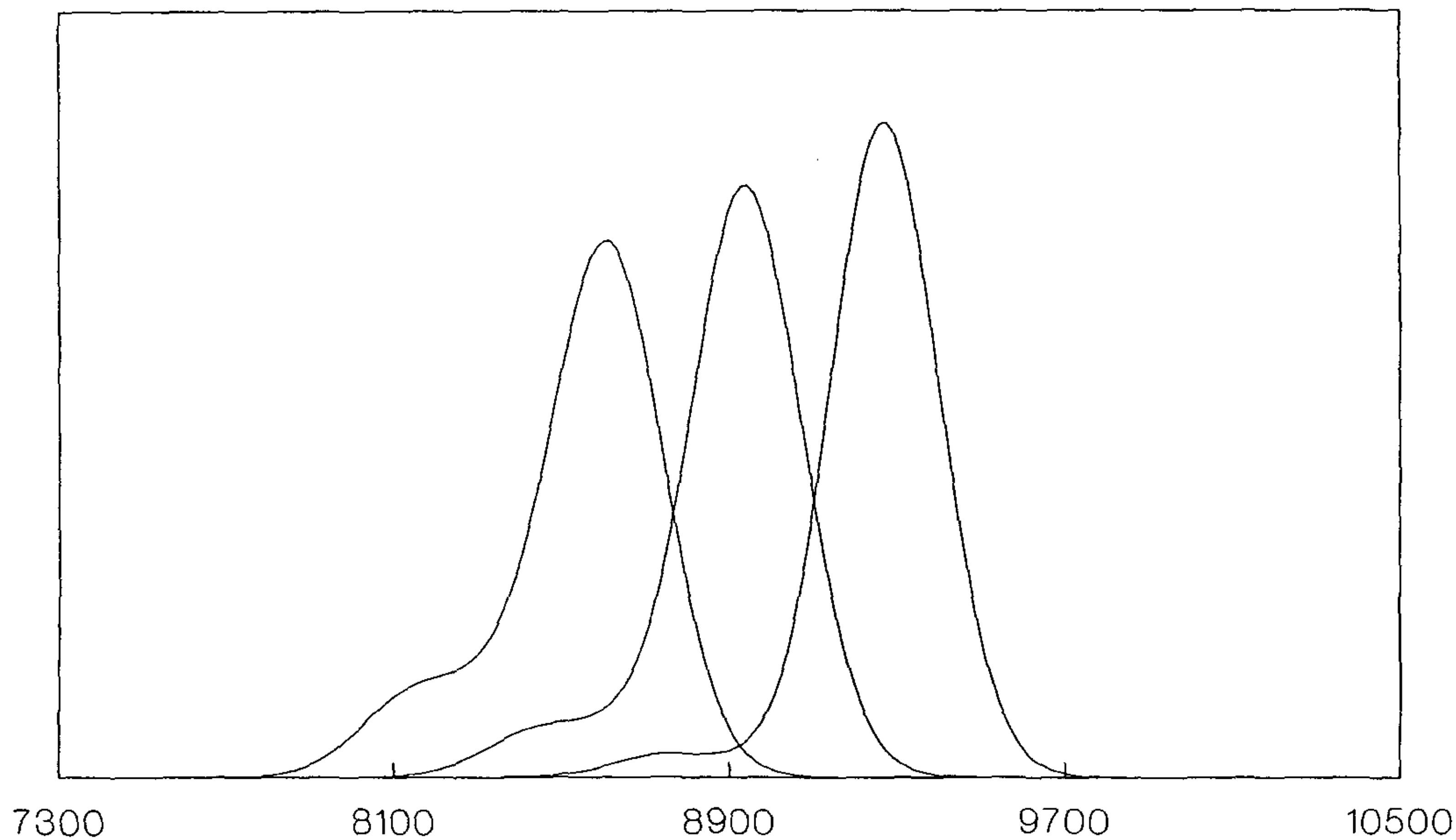

FIGURE 7

SNPRX(2,4,1): SIMULATED DATA

N(t-2) = MEAN

N(t-1) = (MEAN - 275, MEAN, MEAN + 275)

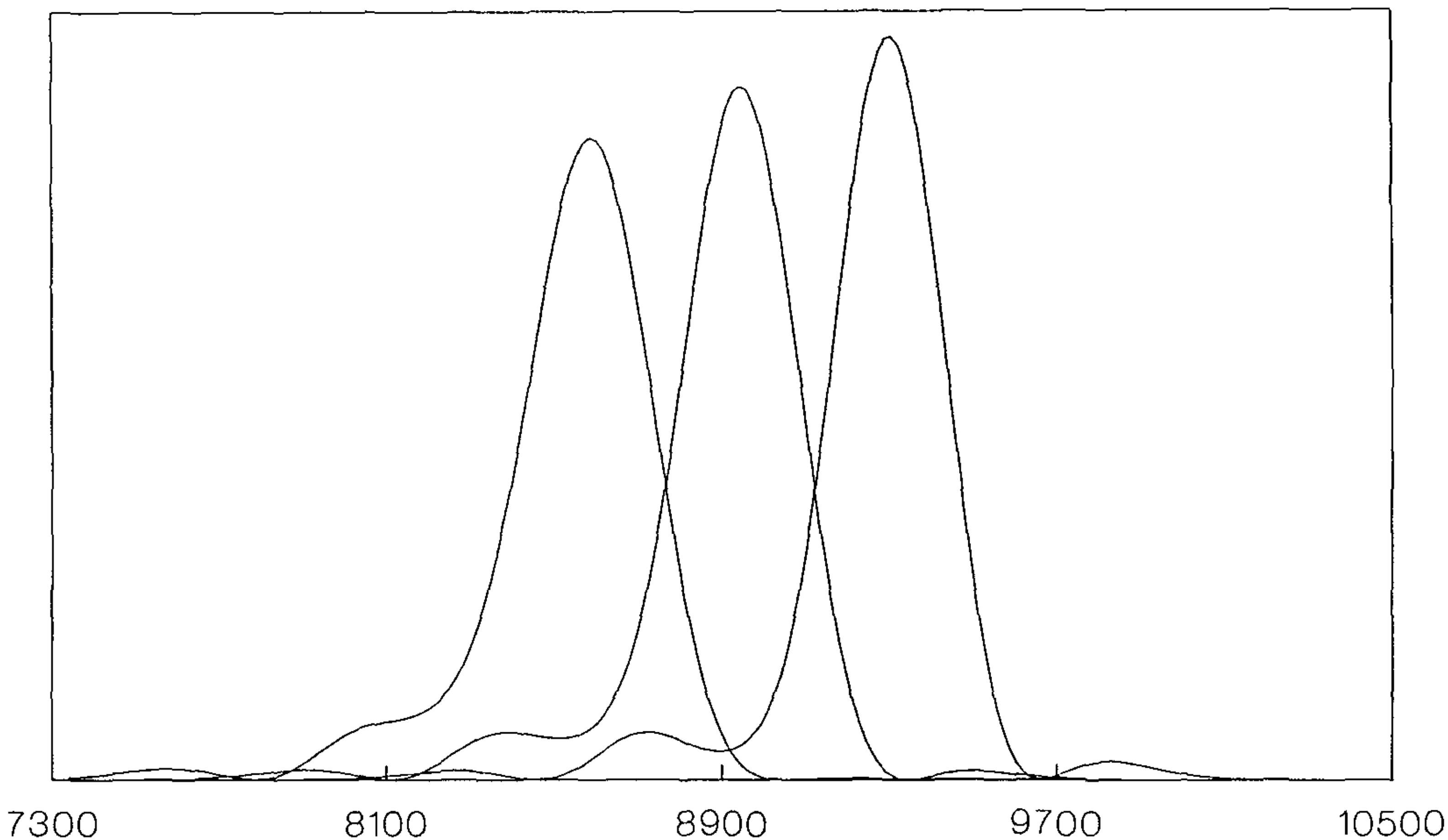

FIGURE 8

NOTES

1. Nonlinearity in aggregate employment data has also been found by Brock and Sayers (1986). In unemployment rate series, which are essentially mirror images of employment series, nonlinearity has been well documented as in Neftci (1984), Stock (1987), and Brock and Sayers (1986).

2. Solving the Euler equation for x_{ij} requires evaluating Ψ_0 at various values for x_{ij}, but the value of Ψ_0 is only known at discrete points. To overcome this limitation, the value of Ψ_0 between discrete points is estimated by interpolating linearly between the values at the two points.

3. The value of c is essentially irrelevant to the law of motion for n_t because it does not appear in the Euler equation, but a positive value of c is implied by the mapping defined in the next subsection between the quadratic and the nonlinear problem.

4. For a precise statement of the regularity conditions required for consistency, see Gallant and Nychka (1987).

5. Starting the simulation requires seed values for ϵ_t and n_{t-1}. For ϵ_t, a random draw is taken from its discrete stationary distribution. The distribution for n_t is unknown, so the seed value for n_{t-1} is chosen as the center of the grid over which the decision rule is determined, 8500. To overcome the arbitrary selection of n_{t-1}, the simulated observations are not collected until 500 realizations have been generated.

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