The Optimal Inflation Path in
a Sidrauski-type Model with Uncertainty

Wouter den Haan   June 1988

G.S.I.A.

Abstract

The objective of this paper is to investigate whether, in a Sidrauski type model with uncertainty, welfare maximization calls for following the famous "Chicago Rule". This question will be answered in the affirmative in this paper, i.e. social welfare optimization calls for a zero nominal interest rate on one-period bonds. The zero nominal interest rate, however, does not imply in an uncertain world that there is no systematic difference between the expected rate of deflation and the rate of time preference in an economy without growth. The magnitude of this difference turns out to be small, however.

Numerical welfare comparisons are made between the optimal policy and policies in which the growth rate of money is fixed. The optimal policy requires that the monetary authorities react every period to the available information and they choose a growth level of the money stock that will set the interest rate equal to zero. If we compare the time paths of the real variables under the optimal policy with the time paths if the money supply decreases at a rate equal to the rate of time preference, then we see hardly any differences. The price dynamics can be very different, however.

The paper also investigates the issue of superneutrality and finds that the quantitative deviations from superneutrality are substantial if a model with a shopping time technology is used.

The neo-classical models in this paper are solved numerically using a technique developed in Marcet (1988).

1. Introduction

The objective of this paper is to investigate whether, in a Sidrauski-type model with uncertainty, welfare maximization calls for following the famous "Chicago Rule". In McCallum (1987) it is shown that in a perfect-foresight version of Sidrauski (1967), social optimality requires following the "Chicago Rule". In a perfect-foresight model this means to deflate at a rate that is equal to the real rate of return on a one-period bond. A zero nominal interest rate is therefore a necessary condition for a Pareto-efficient competitive equilibrium. The deflation rate also equals the real return on capital. In a stationary state this means that the optimal rate of deflation is equal to the rate of time preference[1].

The literature suggests that there are reasons for a systematic difference between the optimal inflation rate in the perfect-foresight and in the uncertainty case. The basic reason is that the introduction of uncertainty adds an important characteristic to money. That is: Holdings of real money balances could increase or decrease the variability of the consumption stream. Consequently the rate of return on real money balances - i.e. the deflation rate - at which people will hold the satiation level of real money balances will be different from the perfect-foresight case[2].

Nevertheless the question of whether the "Chicago Rule" remains optimal in a Sidrauski-type model when there is no perfect-foresight in the model has not yet been answered by the literature[3]. This question will be answered in the affirmative in this paper, i.e. social welfare optimization calls for a zero nominal interest rate on one-period bonds. The zero nominal interest rate, however, does not imply in an uncertain world that the expected deflation is equal to the expected real return on capital investments. Nor does it

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[1] It should be noted that this paper only deals with the "shoe-leather" cost of inflation. This cost occurs since inflation causes people to economize on a service-yielding asset that is costless to produce. See Fischer (1984) for a discussion on the "non-shoe-leather" cost of inflation.

[2] See Hahn (1971).

[3] Lucas & Stokey (1983) and Krugman, Persson and Svensson (1985) find the expression for the inflation rate that will induce agents to hold the optimal amount of real money balances in an intertemporal model with a cash-in-advance constraint.

mean that there is no systematic difference between the expected rate of deflation and the rate of time preference in an economy without growth. Although there are some factors that could create a systematic difference between the perfect-foresight and the uncertainty case, the magnitude of this difference, however, turns out to be very small. This conclusion is reached by numerically solving the model using techniques developed in Marcet (1988). Welfare maximization therefore calls for a deflation rate that is on average close to the rate of time preference in the uncertainty case as well as in the perfect-foresight case.

Numerical welfare comparisons are made between between the following two regimes:
1st - The monetary authorities react every period to the available information and they choose a growth level of the money stock that will set the interest rate equal to zero. With the numerical procedure it is possible to solve for the optimal policy rule.

2nd - The money stock changes every period with a fixed percentage. Of particular interest is of course a reduction of the money supply at a rate equal to the rate of time preference. The welfare difference between this policy rule and the optimal monetary policy turns out to be negligible. The same is true for the time paths of the real variables, but the price dynamics can be very different.

By comparing the regimes with different growth rates of the money supply process, we can check whether the model satisfies the superneutrality property. For the numerical part of the paper a utility function of consumption, real money balances and labor supply is used, that is equivalent to using a shopping time technology. In McCallum (1987) it is shown for the perfect-foresight case that the superneutrality property holds for a wide class of utility functions. The utility function derived from the shopping time technology does not, however, belong to this class. The quantitative deviations from superneutrality turn out to be substantial.

This paper is organized as follows: Section 2 contains a description of the model, the necessary and sufficient conditions for maximizing behavior of a representative agent and the definition of a rational expectations equilibrium. In Section 3 the optimal rate of

inflation is calculated and the differences with a perfect-foresight model are discussed. The numerical procedure and the results are described in section 4.

2. The Model

The economy of this paper consists of a large number of similar households that live forever. At the beginning of every period t they decide how much to consume that period, c_t, how much labor to supply h_t, and how much to save. They can save by accumulating nominal money balances M_{t+1}, by investing in real capital k_{t+1} and by buying bonds B_{t+1}. All three variables denote quantities held at the beginning of period $t+1$.

An important characteristic of the model is that an increase in the amount of real money balances m_t ($= M_t / p_t$) decreases the time spent in conducting transactions. p_t is the money price of the consumption good. If an agent possesses an amount of real money balances equal to m_t and wants to buy c_t, the shopping time v_t is given by the following function $v : R_+^2 \rightarrow [0,1]$

$$v_t = v(m_t, c_t)$$

The first partial derivatives are assumed to satisfy $v_m(m_t, c_t) \leq 0$ and $v_c(m_t, c_t) \geq 0$. It is assumed that for every level of c_t, there exists a level of real money balances for which the partial derivative with respect to money balances is equal to zero.

Capital is output that is not consumed and output y_t is produced by combining capital k_t and labor h_t.

$$y_t = f(k_t, h_t) x_t + \delta k_t$$

The production function $f: R_+ \times [0,1] \to R_+$ is assumed to be well-behaved, so a unique positive value of k_{t+1} and h_t will be chosen each period. x_t is a stochastic variable that follows a Markov process with a stationary transition density.

The third investment possibility is a one-period bond with a certain nominal return. The bonds that agents buy in period t, B_{t+1}, are sold at the nominal price q_t and yield one unit of money in period t+1. The nominal rate of interest thus equals $(1-q_t)/q_t$. The number of bonds B_t divided by the price of the commodity in that period is denoted by b_t.

Each household gets in period t a pecuniary lump-sum transfer (net of taxes) of the amount $p_t t_t$ from the government.

At every period t the households face the following budget constraints:

$$
(2.1) \quad c_{t+j} + k_{t+j+1} + (1+\pi_{t+j}) \, m_{t+j+1} + (1+\pi_{t+j}) \, q_{t+j} \, b_{t+j+1} =
$$
$$
f(k_{t+j}, h_{t+j}) \, x_{t+j} + \delta k_{t+j} + m_{t+j} + b_{t+j} + t_{t+j} \qquad \text{for all } j \in \{0,1,2,...\}
$$

where $\pi_{t+j} = (p_{t+j+1} - p_{t+j})/p_{t+j}$ is the inflation rate between t+j and t+j+1.

The representative household maximizes the expected discounted sum of a von Neumann-Morgenstern utility function $W: R_+ \times [0,1] \times R_+ \to R$:

$$
E_t \sum_{j=0}^{\infty} \beta^{t+j} \, W(c_{t+j}, l_{t+j}, \xi_{t+j})
$$

E_t is the expectations operator conditioned on the information available to the agents at period t. $l_t = 1 - h_t - v_t$ is the amount of leisure in period t and ξ_t is a stochastic preference shock that follows a Markov process with a stationary transition density. If the definitions of l_t and v_t are substituted into this utility function we get a utility function of consumption, real money balances, labor supply and the preference shock.

$$U(c_t, m_t, h_t, \xi_t) \quad = \quad W(\, c_t, \, 1\text{-}h_t\text{-}v(c_t, m_t)\,, \, \xi_t\,)$$

For the theoretical discussion about optimal inflation it does not matter whether we work with $U(.)$ or $W(.)$. For the issue of superneutrality we have to be more careful. In McCallum (1987) it is pointed out that if the ratio $U_h(.)/U_c(.)$ does not depend on m_t the stationary state values of real variables are not affected by the the growth rate of the money supply process[4]. If $U(.)$ would have a CES structure for instance then this assumption would be satisfied. If, however, $U(.)$ has the structure of $W(.)$, than it is very unlikely that the ratio $U_h(.)/U_c(.)$ does not depend on m_t.

The law of motion for the money supply process is given by:

$$(2.2) \quad \underline{M_{t+1}} = \quad (1 + \mu_t)\,\underline{M_t}$$

μ_t is assumed to be a fixed function of the state variables in the economy, including η_t, and is thus also stationary[5]. η_t is an independent white noise affecting the money supply process. One of the experiments will be to increase the variance of η_t.

The expected discounted sum of the utility function is maximized at every period t with respect to M_{t+j+1}, B_{t+j+1}, k_{t+j+1}, h_{t+j} and c_{t+j}, $j \in \{0,1,2,...\}$ subject to the budget constraints (2.1). The function $U(.)$ is taken to be well-behaved, so an interior solution will be obtained. In order to solve this problem the agents need predictions for the price of the commodity, the price of the one-period bond and the transfer from the government. It is assumed that households think that these variables are governed by

$p_t \quad = \quad p(s_t, \sigma_t)$

$q_t \quad = \quad q(s_t, \sigma_t)$

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[4] Except for real money balances of course.

[5] To be correct we have to assume that all functions we use are measurable.

$$t_t = t(\underline{s}_t, \sigma_t)$$

where $\underline{s}_t$ stands for $[\underline{k}_t, \underline{M}_t, \underline{B}_t]$. A bar denotes the per capita value of the variable in the economy. The number of households in the economy is assumed to be so large, that an individual household can not influence the average levels in the economy. σ_t stands for $[x_t, \xi_t, \eta_t]$. Note that x_t, ξ_t and η_t are not underlined because these shocks are assumed to be the same for every household. It is assumed that $\underline{s}_t$ and σ_t are elements of the information set at period t.

The consumption, time allocation and investment decision at period t are influenced by the initial real wealth, i.e. by m_t, b_t and k_t. Recall the assumption that the stochastic processes x_t, ξ_t and η_t are Markov processes and that t_t and the prices p_t and q_t are functions of $\underline{s}_t$ and σ_t. Because of these two assumptions we can say that all relevant information on the current and future state of the household's world is captured by the vector $[s_t, \underline{s}_t, \sigma_t]$. Every period our representative household solves an optimization problem of the same form but every period the vector $[s_t, \underline{s}_t, \sigma_t]$ has a different value. The actual choice of the representative household can thus be described by fixed decision rules:

$$c_t = c(s_t, \underline{s}_t, \sigma_t)$$

$$M_{t+1} = M(s_t, \underline{s}_t, \sigma_t)$$

$$B_{t+1} = B(s_t, \underline{s}_t, \sigma_t)$$

$$k_{t+1} = k(s_t, \underline{s}_t, \sigma_t)$$

$$h_t = h(s_t, \underline{s}_t, \sigma_t)$$

Necessary and sufficient conditions for a maximum are that the functions $c(s_t, \underline{s}_t, \sigma_t)$, $h(s_t, \underline{s}_t, \sigma_t)$, $M(s_t, \underline{s}_t, \sigma_t)$, $B(s_t, \underline{s}_t, \sigma_t)$ and $k(s_t, \underline{s}_t, \sigma_t)$ besides the budget constraint (2.1) satisfy[6]:

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[6] A proof for a simplified version of this model is given in Levhari and Srinivasan (1969).

(2.3) $-U_c(c_t,m_t,h_t,\xi_t) + \beta E_t [(f_k(k_{t+1},h_{t+1}) x_{t+1} + \delta) U_c(c_{t+1},m_{t+1},h_{t+1},\xi_{t+1})] = 0$

(2.4) $E_t [B_{t+1} \{ -U_c(c_t,m_t,h_t,\xi_t) q_t + \beta [(p_t/p_{t+1})U_c(c_{t+1},m_{t+1},h_{t+1},\xi_{t+1})] \}] = 0$

(2.4*) $-U_c(c_t,m_t,h_t,\xi_t) q_t + \beta E_t [(p_t/p_{t+1}) U_c(c_{t+1},m_{t+1},h_{t+1},\xi_{t+1})] \leq 0$

(2.5) $-U_c(c_t,m_t,h_t,\xi_t) + \beta E_t [(p_t/p_{t+1}) (U_c(c_{t+1},m_{t+1},h_{t+1},\xi_{t+1}) +$
$U_m(c_{t+1},m_{t+1},h_{t+1},\xi_{t+1}))] = 0$

(2.6) $U_c(c_t,m_t,h_t,\xi_t) f_h(k_t,h_t) + U_h(c_t,m_t,h_t,\xi_t) = 0$

(2.3') $\lim_{i\to\infty} E_t \beta^{i-1} U_c(c_{t+i},m_{t+i},h_{t+i},\xi_{t+i}) k_{t+i+1} = 0$

(2.4') $\lim_{i\to\infty} E_t \beta^{i-1} q_{t+i} b_{t+i+1} U_c(c_{t+i},m_{t+i},h_{t+i},\xi_{t+i}) (1+\pi_{t+i}) = 0$

(2.5') $\lim_{i\to\infty} E_t \beta^{i-1} m_{t+i+1} U_c(c_{t+i},m_{t+i},h_{t+i},\xi_{t+i}) (1+\pi_{t+i}) = 0$

Equation (2.4*) has to hold with equality if B_t is positive. The equations (2.1), (2.3), (2.4) & (2.4*), (2.5) and (2.6) solve for the functions $c(s_t,\underline{s}_t,\sigma_t)$, $h(s_t,\underline{s}_t,\sigma_t)$, $M(s_t,\underline{s}_t,\sigma_t)$, $B(s_t,\underline{s}_t,\sigma_t)$ and $k(s_t,\underline{s}_t,\sigma_t)$. With the following two definitions we can also calculate $m(s_t,\underline{s}_t,\sigma_t)$ and $b(s_t,\underline{s}_t,\sigma_t)$.

(2.7) $m_t = \dfrac{M_t}{p_t}$

$b_t = \dfrac{B_t}{p_t}$

Note that for bonds we have the two-part first-order conditions, because in this model agents are not allowed to sell bonds. If they would be allowed to have debts, it would be optimal and possible to finance consumption today by borrowing and to finance the redemption of the debt next period by borrowing again, etc.[7]. That is, agents can consume without sacrificing anything. Note that the transversality condition (2.4') does not prevent this behavior. It only prevents (the absolute value of) B_t from growing too fast.

Competitive Equilibrium

There are three equilibrium conditions in the model that can be used to calculate the price of the commodity and the price of the bond. The first condition is that the demand of the representative household is equal to the average money supply, i.e.,

$$(2.8) \quad M(s_t, \underline{s}_t, \sigma_t) = \underline{M}_{t+1} = (1 + \mu_t) \, \underline{M}_t$$

In the notation of the equilibrium conditions the assumption is used, that all the households are similar.

The supply of bonds is given by the budget constraint of the government (2.9).

$$(2.9) \quad \underline{B}_{t+1} = \frac{\underline{B}_t + p_t \underline{t} + \underline{M}_t - \underline{M}_{t+1}}{q_t}$$

Thus the second equilibrium condition is given by

$$(2.10) \quad B(s_t, \underline{s}_t, \sigma_t) = \underline{B}_{t+1} = \frac{\underline{B}_t + p_t \underline{t} + \underline{M}_t - \underline{M}_{t+1}}{q_t}$$

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[7] Another lower bound than 0 could also have been chosen.

Instead of (2.8) or (2.10) we can also take the overall constraint (equilibrium on the commodity market) as an equilibrium condition

$$(2.11) \quad f(k_t, h_t) + \delta k_t = k_{t+1} + c_t$$

Note that because every household is the same we have $s_t = \underline{s}_t$. Let $z_t = [s_t, \underline{s}_t, \sigma_t]$. We can now define a rational expectations equilibrium.

<u>Definition</u>: A rational expectations equilibrium is a price function $p(s_t,\sigma_t)$, a bond price function $q(s_t,\sigma_t)$, a transfer function $t(s_t,\sigma_t)$, a demand function for next period's money balances $M(s_t,\underline{s}_t,\sigma_t)$, a demand function for bonds $B(s_t,\underline{s}_t,\sigma_t)$, an investment function $k(s_t,\underline{s}_t,\sigma_t)$, a labor supply function $h(s_t,\underline{s}_t,\sigma_t)$ and a consumption function $c(s_t,\underline{s}_t,\sigma_t)$, such that:

$$(2.12) \quad -Lab_t + \beta E_t [(x_{t+1} f_k(k(z_t),h(z_{t+1})) + \delta)Lab_{t+1}] = 0$$

$$(2.13) \quad E_t [B(z_t) \{ -Lab_t q(z_t) + \beta E_t [\frac{p(z_t)}{p(z_{t+1})} Lab_{t+1}] \}] = 0$$

$$(2.13^*) \quad -Lab_t q(z_t) + \beta E_t [\frac{p(z_t)}{p(z_{t+1})} Lab_{t+1}] \leq 0$$

$$(2.14) \quad -Lab_t + \beta E_t [\frac{p(z_t)}{p(z_{t+1})} \{ Lab_{t+1} + U_m(c(z_{t+1}),m(z_{t+1}),h(z_{t+1}),\xi_{t+1})] = 0$$

$$(2.15) \quad U_c(c(z_t),m(z_t),h(z_t),\xi_t) f_h(k(z_t),h(z_t)) + U_h(c(z_t),m(z_t),h(z_t),\xi_t) = 0$$

$$(2.16) \quad M(z_t) = (1+\mu_t) M_t$$

$$(2.17) \qquad B(z_t) = \frac{B_t + p(z_t) t(z_t) + M_t - M(z_t)}{q(z_t)}$$

$$(2.18) \qquad f(k(z_t), h(z_t)) x_t + \delta k_{t-1} = k(z_t) + c(z_t)$$

Where

$$Lab_t = U(c(z_t), m(z_t), h(z_t), \xi_t)$$

Unfortunately, there is no general theory that may be applied to show the existence and uniqueness of a rational expectations equilibrium. So I will assume that a unique rational expectations equilibrium exists[8].

It is clear that if the price function is homogeneous of degree one in the nominal supply of money that the policy functions of the real variables are homogeneous of degree zero in money. On the other hand if the policy functions of the real variables are not affected by the level of the nominal money stock, the equilibrium price function is indeed homogeneous of degree one in money. I restrict myself to equilibria for which this consistency property holds[9]. This will be a very convenient in the numerical part of the paper.

I will exclude the possibility that nominal interest rates can be negative. A sufficient condition would be that government supply of bonds is positive and $v_m(c,m) \geq 0$ for all values of c and m.

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[8] Danthine and Donaldson (1986) show the existence and uniqueness in a simplified version of this model if $U_{cm}(.) = 0$. It must be noted that if U(.) has the structure of W(.), this assumption is not likely to be true.

[9] Also see Danthine and Donaldson (1986).

3.1 The Optimal Inflation Path

The similarity of the agents makes it easier to calculate the social optimum. Let us assume that the social planner is not able to influence the stochastic properties of x_t or ξ_t. Without doing any calculations it is clear that there is a continuum of optimal paths for governments bonds if the households are taxed in a lump-sum fashion. The reason is that given the optimal time path for M_t, a change in the time path of B_t changes the time path of t_t in such a way that the budget set of the households stays exactly the same, that is, the model satisfies the Ricardian equivalence property. The time path of B_t is therefore taken as given.

The problem of the social planner is then to maximize the agent's utility function subject to the overall resource constraint (2.11) with respect to c_{t+j}, h_{t+j}, k_{t+j+1} and m_{t+j}, $j \in \{0,1,2,...\}$.

There are no Pareto improvements possible, if the solution of the rational expectations equilibrium satisfies the conditions for maximizing the social planner's problem.

The necessary and sufficient conditions for the social planner's problem are:

$$(2.3) \quad -U_c(c_t,m_t,h_t,\xi_t) + \beta \, E_t [\, (f_1(k_{t+1},h_{t+1}) \, x_{t+1}+\delta) \, U_c(c_{t+1},m_{t+1},h_{t+1},\xi_{t+1})] \quad = 0$$

$$(3.1) \quad U_m(c_t,m_t,h_t,\xi_t) \quad = 0$$

$$(2.6) \quad U_c(c_t,m_t,h_t,\xi_t) \, f_h(k_t,h_t) + U_h(c_t,m_t,h_t,\xi_t) \quad = 0$$

$$(2.3') \quad \lim_{i \to \infty} E_t \, \beta^{i-1} \, U_c(c_{t+i},m_{t+i},h_{t+i},\xi_{t+i}) \, k_{t+i+1} \quad = 0$$

Note that $U_m(.) = - W_l(.) \, v_m(.)$, where $W_l(.)$ is the partial derivative of the original utility function with respect to leisure. Thus (3.1) is satisfied if (3.2) holds

(3.2) $v_m(.) = 0$

Equation (3.2) implies that he social planner chooses that level of real money balances at which consumers are exactly satiated with money for facilitating transactions. Note that since the law of motion for prices is given by $p(s_t, \sigma_t)$ the law of motion for real money balances in a competitive equilibrium can be represented as $m(s_t, \sigma_t)$. At the beginning of the period s_t, x_t, η_t and ξ_t are known. In a competitive economy the monetary authorities would want to choose μ_t (by choosing M_{t+1}) in such a way that $v_2(c_t, m_t)$ is equal to 0. The question is whether the function $m(s_t, \sigma_t)$ is such that μ_t can indeed be chosen to make the amount of real money balances held by economic agents equal to the satiation level. Since the functional forms are unknown it is impossible to find the restrictions on $m(s_t, \sigma_t)$. It is not hard, however, to find conditions for the time path of inflation which are necessary for a Pareto-efficient competitive equilibrium.

By comparison with the conditions for the competitive equilibrium it can be seen that if $v_m(c_t, m_t)$ and thus $U_m(.)$ is to be set equal to zero every period, the following has to be true.

(3.3) $-U_{ct} + \beta E_t \left[\frac{P_t}{P_{t+1}} U_{ct+1} \right] = 0$

We can see directly from equation (2.4*) that this can only be true if $q_t \geq 1$. But since we assumed that nominal interest rates cannot be negative (i.e. $q_t \leq 1$), we conclude that for this model the *rule for the optimum quantity of money is that it will be attained by a rate of*

price deflation that makes the nominal rate of interest equal to zero [10,11]. In this sense we find that welfare maximization calls for following the "Chicago Rule".

By rewriting (3.3) we can derive the implications for the expected inflation. But let us first recall the optimal inflation in a perfect-foresight version of the Sidrauski model. McCallum (1987) finds the following for the optimal change in prices $(p_t/p_{t+1})_{pf}$:

$$(3.4) \quad \left(\frac{p_t}{p_{t+1}}\right)_{pf} = \frac{U_{c,t}}{\beta\, U_{c,t+1}}$$

In a stationary state $(p_t/p_{t+1})_{pf}$ reduces to β^{-1}. If $U_c(c_t,m_t) > U_c(c_{t+1},m_{t+1})$ we have that $(p_t/p_{t+1})_{pf} > \beta^{-1}$. If $(p_t/p_{t+1})_{pf}$ is bigger than 1 then the optimal rate of inflation is negative. Because $\beta < 1$ we have that in a stationary state and in a growing economy the optimal inflation is negative.

By rewriting (3.3) we get for the optimal expected change in prices for the uncertainty case

$$(3.5) \quad E_t\left(\frac{p_t}{p_{t+1}}\right)_{un} = \frac{\dfrac{1}{\beta} - COV_t\left(\dfrac{p_t}{p_{t+1}}, \dfrac{U_{c,t+1}}{U_{c,t}}\right)}{\dfrac{E_t U_{c,t+1}}{U_{c,t}}}$$

By rewriting (2.3) and substituting for the expected change in the marginal utility of consumption in (3.5), we get

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[10] This sentence is copied from Friedman (1969).

[11] Without the assumption that $q_t \leq 1$, we would conclude that non-positive nominal interest rates are a necessary condition for a Pareto-efficient competitive equilibrium.

$$(3.6) \quad E_t \left(\frac{P_t}{P_{t+1}}\right)_{un} \approx \frac{\frac{1}{\beta} \cdot COV_t\left(\frac{P_t}{P_{t+1}}, \frac{U_{c,t+1}}{U_{c,t}}\right)}{\frac{1}{\beta} \cdot COV_t\left(f_{k,t+1} x_{t+1}, \frac{U_{c,t+1}}{U_{c,t}}\right)} E_t\left[f_{k,t+1} x_{t+1} + \delta\right]$$

This expression tells us that the expected change in prices is only equal to the expected gross rate of return on capital if the two covariance terms in (3.6) are equal to each other which is unlikely to be true. Note that equal covariance terms corresponds with equal risk premia on one-period bonds and capital investments.

To get some more grip on the optimal rate of inflation, it is useful to note that the optimal expected inflation can only be positive if the expected real interest rate on one-period bonds is negative. To see why this is true, rewrite equation (2.4*) in the same way as equation (3.3) is rewritten. The result is that in a competitive equilibrium it must always be true that the gross real return on one-period bonds $E_t(p_t/p_{t+1}q_t)$ is smaller than the right-hand side of equation (3.5). If the optimal expected inflation is positive we have that $E_t(p_t/p_{t+1})<1$ and thus $E_t(p_t/p_{t+1}q_t)<1$, which corresponds with a negative expected real interest rate.

3.2 Comparing the Perfect-Foresight with the Uncertainty Case

There are two reasons why the optimal rate of inflation in a perfect-foresight economy might be different from the optimal rate of inflation in a world with uncertainty.

1st The first reason is that investing in real money balances changes the variance of consumption and leisure over time. This effect is indicated by the covariance term between the unexpected price change and the unexpected change in the marginal utility of consumption. A negative covariance means that people have an unexpectedly low return on their money holdings if they "need" the money because their marginal utility

of consumption is higher than expected and that they have an unexpectedly high return on their money holdings if they do not "need" the money because the marginal utility of consumption is unexpectedly low. Investing in real money balances is less attractive when the covariance is negative than when it is zero (like it is in the perfect-foresight economy). A negative covariance thus corresponds with a higher rate of <u>deflation</u> at which people will hold the level of real balances for which the marginal utility of money is equal to zero. A priori, there is no reason to expect a negative or positive covariance. The optimal inflation in the model with uncertainty might thus be higher or lower than in the perfect-foresight model.

2nd In a world without perfect-foresight there is uncertainty about next period's consumption and next period's real value of money balances. This raises the expected margnal utility of next period's consumption if $U_1(c_{t+1}, m_{t+1}, h_{t+1}, \xi_{t+1})$ is a convex function[12]. Using equation (3.5) we know that this decreases the optimal deflation. This uncertainty makes risk-averse agents more willing to substitute funds from this period to the next period. If agents are more willing to substitute funds from one period to the other period, then they will hold the satiation level of money balances at a lower rate of deflation.

4.1 Solution Method

In this section I give an intuitive description of the numerical procedure to solve the model. A more formal analysis can be found in Marcet (1988). The procedure will be described for the model of section 2 without bonds and without stochastic money supply. For this version of the model we have 5 equations, (2.2), (2.3), (2.5), (2.6) and (2.11) to solve for c_t, h_t, k_{t+1}, M_{t+1} and p_t given k_t, x_t, ξ_t and M_t. First multiply both sides of

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[12] If the utility function has the structure of the "shopping time technology utility function", then it is not clear that this function is convex.

equation (2.5) with M_{t+1}. This is legitimate since M_{t+1} is an element of the information set at period t[13].

$$(2.5^*) \qquad M_{t+1}/p_t = m_t(1+\mu) = \beta E_t [(M_{t+1}/p_{t+1}) (U_{c,t+1} + U_{m,t+1}) / U_{c,t}] \qquad = 0$$

The conditional expectations in equations (2.3) and (2.5*) are functions of the state variables. If we would know the functional form of the conditional expectations, we would have 5 equations in 5 unknowns. Let us suppose that the conditional expectations are a power function of the state variables. A reason for doing this is that the conditional expectation in the simple growth model in Brock & Mirman (1972) has this form. We then get

$$(4.1) \qquad U_c(c_t, m_t, h_t, \xi_t) \qquad = \beta \; a1 \; k_t^{a2} \; x_t^{a3} \; x_t^{a4}$$

$$(4.2) \qquad m_t(1+\mu) \qquad = \beta \; b1 \; k_t^{b2} \; x_t^{b3} \; x_t^{b4}$$

$$(4.3) \qquad U_h(c_t, m_t, h_t, \xi_t) \qquad = U_c(c_t, m_t, h_t, \xi_t) \; f_h(k_t, h_t) \; x_t$$

$$(4.4) \qquad f(k_t, h_t)x_t + \delta k_t \qquad = k_{t+1} + c_t$$

$$(4.5) \qquad M_{t+1} \qquad = (1+u) M_t$$

Note that money does not enter as a state variable in the specification of the conditional expectations. The reason is that according to the discussion at the end of section 2, real variables do not depend on M_t. Since the lefthand side of equation (4.1) and (4.2)

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[13] In this case I also could have multiplied the equation with M_t. For the optimal policy I have to multiply with M_{t+1} and to make the comparisons between the different policies as easy as possible. I chose to multiply with M_{t+1} in this case as well.

only contains real variables it would therefore be inconsistent to include nominal balances on the righthand side. Since real money balances in this case are only a function of the variables k_t, x_t and ξ_t, we also do not have to include m_t on the righthand side. This would also be true if μ_t would be a stochastic variable, since μ_t is assumed not to depend on the level of the nominal money stock.

In principle we could now solve the model because we have 5 equations in 5 unknowns. The remaining problem is of course to find the parameters of the power functions. To find these parameters the following iteration scheme is used. Each iteration consists of two steps.

Step 1. In the first step we solve the model with the parameters from the preceeding iteration and create time series for the 5 endogeneous variables. Besides parameter values, we need a capital stock and a money stock for the first period and we need to generate the random shocks x_t and ξ_t. The random shocks are generated only once and are kept the same in every iteration.

Step 2. In this step we estimate the parameters of the power functions with the data generated in step 1. For estimating the power function of (4.1), for instance, this would involve estimating the following equation with non-linear least squares

$$(4.6) \qquad g_t = a1\, k_t^{a2}\, x_t^{a3}\, x_t^{a4} + \varepsilon_t$$

where $\quad g_t = (f_k(k_{t+1},h_{t+1})\, x_{t+1} + \delta)\, U_c(c_{t+1},m_{t+1},h_{t+1},\xi_{t+1})$
and ε_t is an error term.

We conclude that the iteration scheme is converged if the parameters that are used for the simulation of the series are "close" to the estimated parameters in the second step of the

iteration[14]. See Marcet (1988) for a method for finding good initial values of the parameters. If we interpret every iteration as a mapping from the space of power functions to itself, then we can say that the solution of the model is a fixed point of this mapping.

The question now is whether the resulting series are close to the series of a true rational expectations equilibrium. In (4.1) and (4.2) the conditional expectations are replaced by projections on the space of first-order power functions. The equilibrium (the fixed point) found in this way is an approximate rational expectations equilibrium in the following sense. If agents use as a forecast rule the power function with the parameters of the fixed point, then for the times series generated with this forecast rule, this rule was indeed the best forecast rule in the space of first-order power functions. The conditional expectations in a rational expectations equilibrium can not in general be restricted to be of this form. We can check, however, whether the simulated series change if we take a second-order power function instead of a first order. It turns out that the changes are very small.

Note that the five equations (4.1) ... (4.5) are still a complicated system of the unknowns. Equation (4.1), however, can be transformed to an equation with only consumption on the lefthand side. This would not change the righthand side of equation (4.2). It would change the term g_t in equation (4.6) of course. Then only equation (4.3) is a non-linear equation that needs to be solved numerically for the labor supply.

Optimal Program

In simulating the model under the optimal policy, equation (4.5) is replaced by equation (4.7).

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[14] Another convergence criterium I used was that the maximum change in the simulated series from one iteration to the next should be smaller than 0.05%.

(4.7) $v_m(c_t, m_t) = 0.$

Again we have a system of five equations and five unknowns. Given parameter values for the power functions, a transformation of equation (4.1) solves for c_t. Equation (4.7) can then be used to solve for m_t and since M_t is known we can solve for the price level. M_{t+1} is calculated from the new version of equation (4.2) and h_t and k_{t+1} from equation (4.3) and (4.4) respectively.

4.2 Results

In this section I give the results of the simulation procedures. It must be stressed that I do not try to mimic "stilized facts" of the real world. The simulations are done to get some more insight in the properties of the model. (When I talk about simulating a model I mean the whole procedure untill convergence.)

The model was simulated under two types of monetary policy. The first type is the optimal monetary policy in which the monetary authorities - given the values for the capital stock and the realisations of the stochastic shocks - choose that growth level of the nominal money stock for which the nominal interest rate equals zero. The second type consists of fixed growth rates of the money supply.

Functional forms and parameter values

The following constant relative risk aversion utility function was used

$$U(c_t, l_t, \xi_t) \;=\; \xi_t \frac{(c_t^\gamma \, l_t^{1-\gamma})^\tau - 1}{\tau}$$

The technology is represented by

$$y_t = x_t \, k_t^{\alpha} \, h_t^{1-\alpha}$$

Park (1985) works with the following shopping time technology

$$v_t = v_1 \, c_t \left(\frac{M_t}{P_t \, c_t} \right)^{\frac{-v_2}{1-v_2}}$$

with $v_1 > 0$, $0 < v_2 < 1$

This correspond with a Cobb-Douglas technology in which shopping time and real money balances are combined to produce services needed to buy consumption. That is

$$c_t = v_1^{\,v_2-1} \, v_t^{\,1-v_2} \, m_t^{\,v_2}$$

If $v_2 = 1$ this technology corresponds with a Cash-In-Advance economy and when $v_2 = 0$ money does not provide any liquidity services. Note that for this technology $v_m(c_t, m_t)$ is never equal to zero. The following function is therefore chosen instead

$$v_t = v_1 \, c_t \left(\frac{M_t}{P_t \, c_t} \right)^{\frac{-v_2}{1-v_2}} + v_3 \frac{M_t}{P_t}$$

The auditional term can be interpreted as storage costs ($v_3 > 0$). I need this term to make the model well defined if the inflation is close to -2 %. I have run the model for higher rates of inflation with $v_3 = 0$ and obtained similar results.

Parameters

The basic parameter set consists of the following[15]

$$\tau = -0.50 \qquad \alpha = 0.33 \qquad \beta = 0.98 \qquad v_1 = 1.00$$

$$\gamma = 0.28 \qquad \delta = 0.70 \qquad\qquad\qquad v_2 = 0.45, \; 0.65, \; 0.85$$

$$v_3 = 0.01$$

The natural logarithms of the stochastic processes x_t and ξ_t follow a first-order autoregressive process

$$\log(x_t) = \rho_x \log(x_{t-1}) + \varepsilon_{x,t} \qquad\qquad \rho_x = 0.9$$

$$\log(\xi_t) = \rho_\xi \log(\xi_{t-1}) + \varepsilon_{\xi,t} \qquad\qquad \rho_\xi = 0.9$$

$\varepsilon_{x,t}$ and $\varepsilon_{\xi,t}$ are normally distributed processes with zero mean and with the following standard deviations

$$\sigma_x = \sigma_\xi = 0.025$$

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[15] The value for γ is from Park (1985). In this paper a model with a shopping time technology is estimated. His estimate for v_2 is close to 0.65. The representative agent in my model spends - depending on the growth rate of the money supply - around 1% percent of his total time endowment less on shoping than the representative agent in Park's model. The value for α is also used in Kydland and Prescott (1982). The value for δ that is used is lower than in Kydland and Prescott (1982), since it takes much more effort to run the simulation procedure for high values. It must be noted, however, that "capital" is the only non-labor input in the production process and there is no reason to think that "capital" only stands for machinery and buildings. If we consider this, then there is no reason to call δ unrealisticly low.

It must be noted that the standard deviations are rather high, but in a paper that wants to compare a stochastic model with a perfect-foresight model this choice is natural. Besides the parameter values reported I simulated the model for different values of τ, v_1, σ_x and σ_ξ. The results were very similar to the results presented and they are not reported.

Optimal monetary policy versus a reduction of the money supply at a rate equal to the rate of time preference.

For both regimes the model was simulated 5 times for 1200 observations. For every simulation some first and second moments were calculated[16,17]. The averages of these moments and the corresponding standard errors are given in table 1. The first important observation from the tabel is that - even with the enormous standard deviation of the model - the average deflation rate is indeed close to the rate of time prefence. The same conclusion can be drawn for different values of τ and for different combinations of supply and demand shocks, that is for different values of σ_x and σ_ξ.

In table 1 we also see that the averages for the real variables are almost exactly the same. Much more is true. For the separate simulations the moments and even the time paths of the real variables (real money balances excluded) are very similar. An example is given in figure 1 in which we see that the change in consumption follows an almost identical path under both policy variants. The price dynamics are very different, however, although in both cases there is an average deflation of 2%. This difference becomes most clear when the demand shocks are more important than the supply shocks. For this reason I simulated the economy also when $\sigma_\xi = 0.025$ and $\sigma_x = 0.001$. The results are given in figure 1.

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[16] I also tried to run the model 20 times for 300 obervations. For some parameter values this creates a difficulty in estimating the parameters, since the state variables are correlated. The obvious solution to improve the efficiency in the estimation part of the simulation is of course to increase the number of observations. When simulating with a small sample was possible the results were very similar.

[17] The second moments are calculated using a slow moving trend as in Kydland and Prescott (1982). To calculate the second moments in this way, I had to limit the number of observations, since I had to invert a matrix. The second moments are therefore calculated using a subsample (116 observations) of each of the simulated series.

Suppose that there is a positive preference shock. In an economy with a fixed growth rate of money this means that there is an unexpected increase in consumption and prices. If consumption goes up, however, the optimal amount of real balances also goes up and under the optimal monetary policy prices would have to go down. The difference in price dynamics is clearly visible in figure 1. Prices move procyclical under the fixed growth rate and acyclical under the optimal monetary policy. Note that prices already move acyclical when there are only supply shocks.

Although the sum of the discounted utility levels for the optimal monetary policy is higher in every simulation for the optimal program the differences with the fixed growth rule are neglible. The increase in expected utility corresponds with a permanent increase of consumption and leisure of 0.006%. Compared with a zero growth rate of the money supply the welfare gain of the optimal program is equivalent with a permanent increase in consumption and leisure of 1.30 %. It is not surprising that the difference in the standard deviation of prices (prices are much more stable under the optimal monetary policy) does not create a welfare loss, since under both regimes the economy operates close to the satiation level of real money balances, i.e. $v_m(c,m) \approx 0$. (See, however, also the section "More variable money supply"). Unexpected changes in prices and real money balances therefore have a small effect on shopping time and consequently on other real variables.

Different growth rates of money
Of course it is of interest to investigate the changes that occur in moving to growth rates of the money supply that are not equal to -2%. I therefore simulated the economy for money growth rates between -2% and 10% and investigated whether the model satisfies the property of superneutrality.

Danthine, Donaldson & Smith (1988) discuss the issue of superneutrality in a Sidrauski-type model with a fixed labor supply. In the perfect-foresight version of their model the stationary values of real variable are not affected by the growth rate of money.

Danthine, Donaldson & S...ith find that the deviations of the superneutrality property are quantitatively unimportant when the model is enriched with uncertainty. It is pointed out in McCallum (1987) that after relaxing the "fixed labor supply" assumption the perfect-foresight model still possesses the superneutrality property for a large class of utility functions. In section 2 it was pointed out that the utility function, that is equivalent to using the shopping time technology of this paper does not belong to this class of functions.

In figure 2 we see that the quantitative deviations from superneutrality are substantial. Economic agents clearly reduce their consumption and increase their amount of leisure. It is worthwhile to focus the readers attention on the sharp decline of the capital stock. It is often mentioned in the literature that an increase in inflation would induce people to substitute real money balances for capital. This is the well-known Tobin effect [18]. In this model an increase in inflation makes consumption more expensive in terms of shopping time, leisure becomes relatively cheaper and the amount of hours worked reduces. The latter reduces the marginal productivity of capital. Capital therefore has to decrease in order to attain an equilibrium return on capital investments[19].

The substitution effects are bigger for lower values of v_2. This is intuitive since in a Cash-In-Advance economy ($v_2 = 1.0$) there are less possibilities to substitute. The welfare losses are given in table 2.

More variable money supply

An interesting experiment is investigating an increase in monetary uncertainty. Let the money supply process be given by

$$M_{t+1} = (1+\mu) \eta_t M_t$$

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[18] See Tobin (1965).

[19] In the appendix I show that in comparing steady states real money balances, consumption, capital and labor hours have to be lower for higher inflation rates. The effect on leisure is ambiguous.

The logarithm of η_t is normally distributed with mean zero and a standard deviation of 0.05. For this experiment $\mu = 0.025$. The results are given in table 3. The differences are surprisingly small. I think that the main reason for the similarity is that an unexpected change in the money supply never has a wealth effect. If we look at the overall constraint (2.11) we see that the consumption and investment opportunities only change if the amount of labor supplied changes. An unexpected increase in the money supply is always offset by a reduction in the supply of government bonds or by a reduction in the lump-sum tax, leaving the wealth of the agent unaffected.

5   Concluding Remarks

The purpose of this paper is to see whether in a Sidrauski-type model with stochastic shocks the optimal rate of deflation would be equal to the rate of time preference if there is on average no growth. It was shown that in a model with stochastic shocks even on average the optimal rate of deflation might be different from the optimal rate of deflation in the perfect-foresight version of the model. Quantitatively, however, the differences turn out not to be substantial. Moreover, just like in the perfect-foresight case and in other (stochastic) models it is optimal to deflate at a rate which results in a zero nominal interest rate on one-period bonds[20].

It seems to be an important assumption that the monetary authorities have the possibility to tax agents in a lump-sum fashion. Phelps (1973) has argued that in this type of model - with real money balances as an argument of the utility function - the "Chicago Rule" is no longer the social optimum, if a lump-sum mechanism is not available. But the utility function is an indirect utility function: money has utility because it facilitates transactions. Kimbrough (1986) has argued that when money is explicitly modelled as

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[20]   See: Krugman, Persson and Svensson (1985) and Lucas and Stokey (1983).

being useful in facilitating transactions, the inflation tax is analogous to a tax on an intermediate good. These taxes are not part of a socially optimal tax package[21,22,23]. Kimbrough accourdingly finds that the "Chicago Rule" is again the social optimum.

I consider this paper in the first place as a theoretical exersize to get more insights in the properties of an important model in monetary economics. I think that before the model can be used for policy recommendations it is important to add the cost aspects of raising taxes and issuing money to the model. In this paper it is assumed that everybody pays their taxes and that raising taxes and printing money does not involve any costs. If raising taxes is more expensive than printing money, then deviations from the "Chicago Rule" are very well possible. This cost difference should be compared with the welfare losses caused by deviating from the "Chicago Rule". The calculations in this paper suggest that these can be substantial.

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[21] See: Diamond and Mirrlees (1971).

[22] Kimbrough's model does not have capital or stochastic shocks, but the same result holds if capital is added to the model. If the model is enriched with uncertainty, we have to assume that there are complete markets, to get that the nominal interest rate has to equal zero at the social optimum. Proofs are available from the author.

[23] Lucas and Stockey (1983) find for the same reason that money should not be taxed in a cash-in-advance model.

FIGURE 1A: OPTIMAL MONETARY POLICY
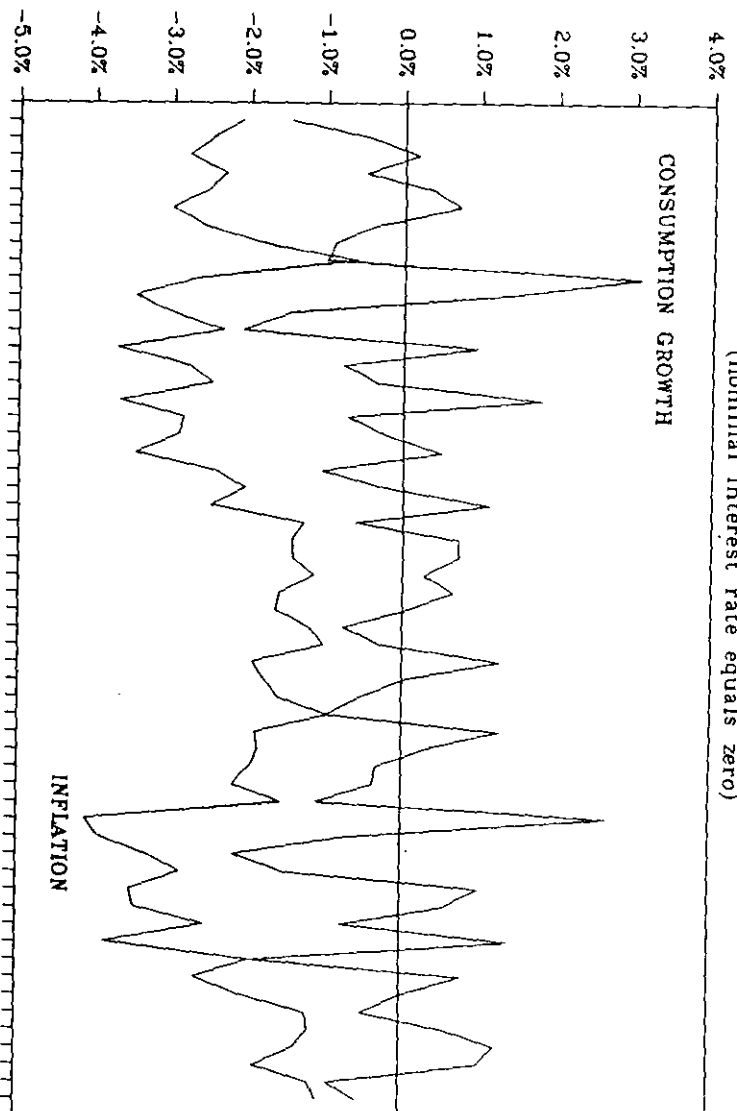(nominal interest rate equals zero)
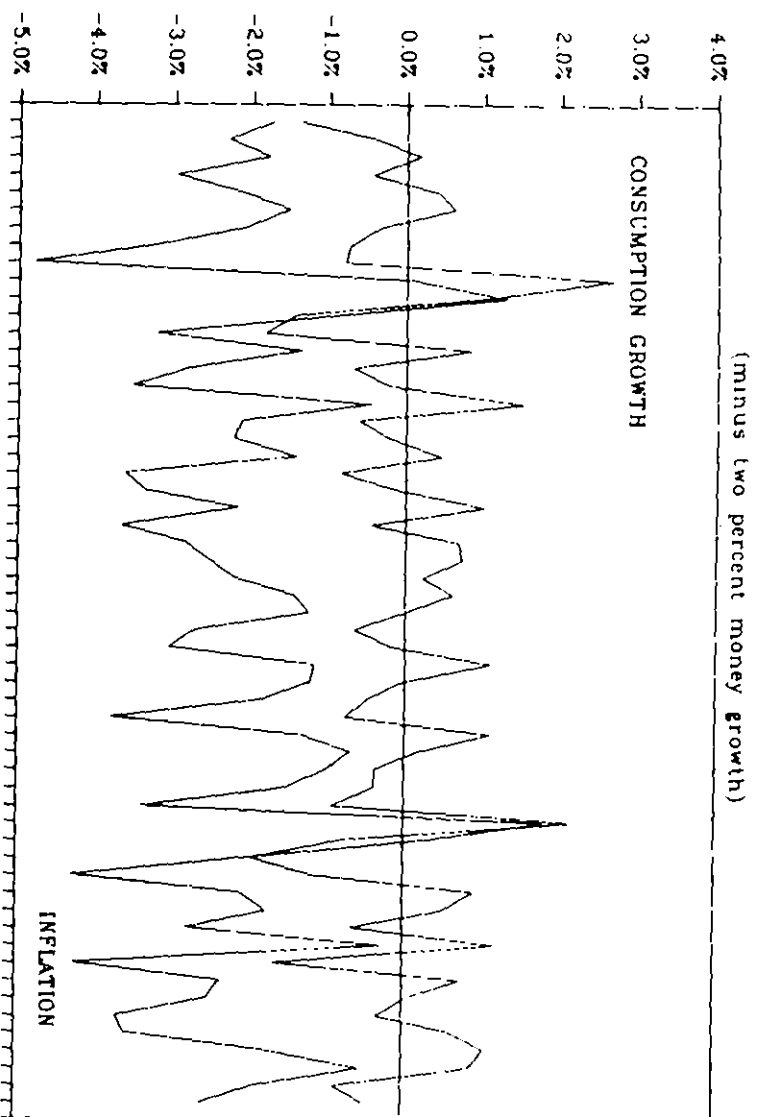
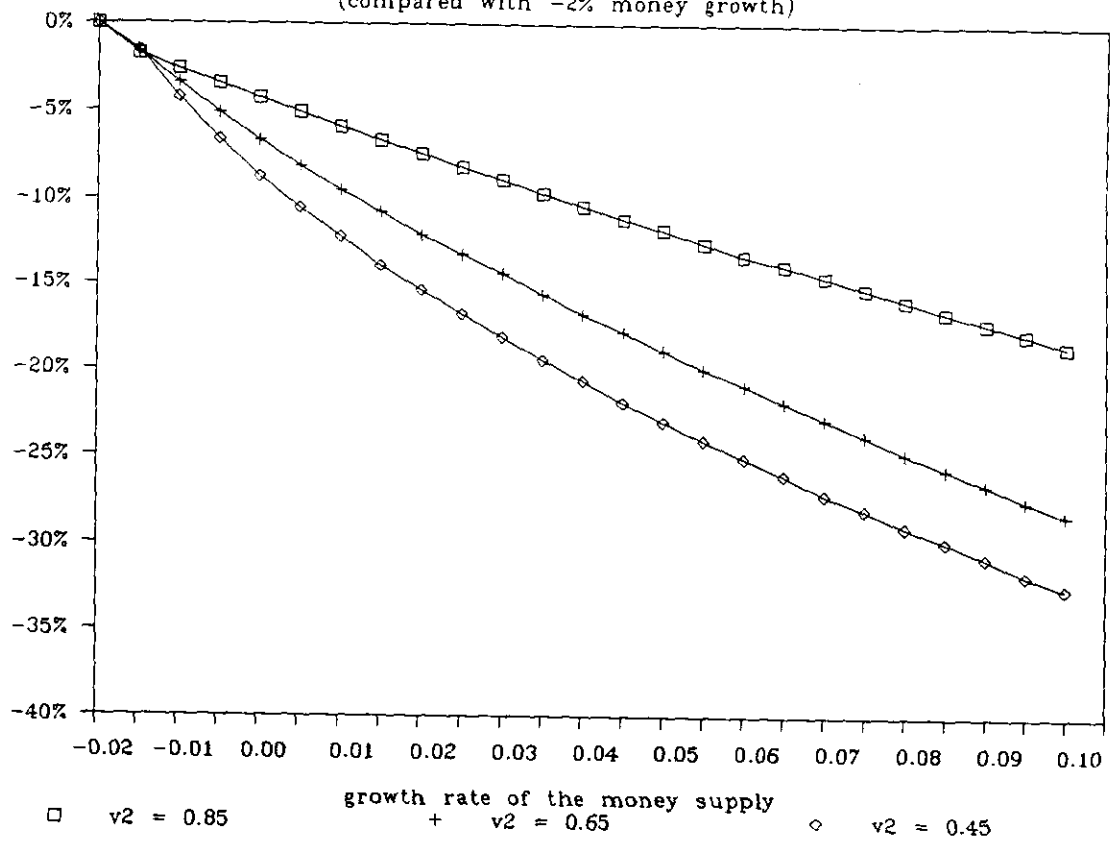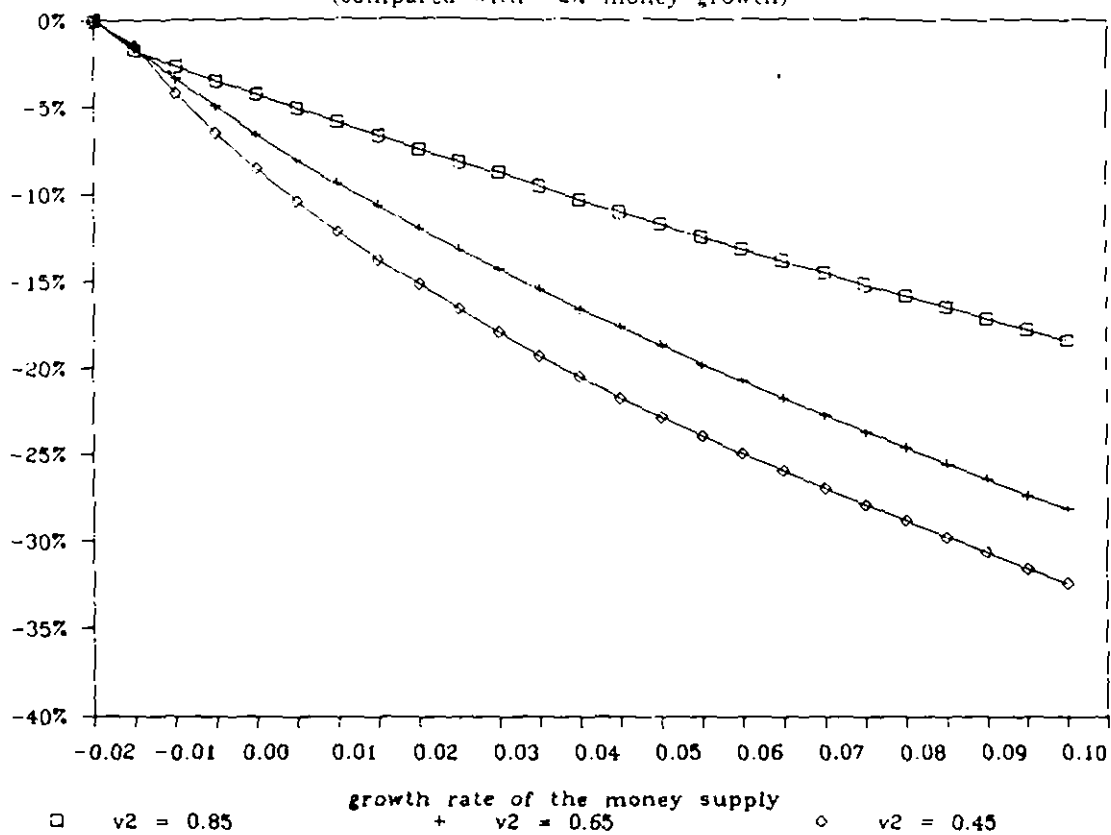FIGURE IB: APPROXIMATE OPTIMAL POLICY
(minus two percent money growth)

FIGURE 2A: CHANGES IN CAPITAL
(compared with −2% money growth)

growth rate of the money supply

□   v2 = 0.85   +   v2 = 0.65   ◇   v2 = 0.45

FIGURE 2B: CHANGES IN CONSUMPTION
(compared with -2% money growth)

growth rate of the money supply

□ v2 = 0.85   +   v2 = 0.65   ◇   v2 = 0.45

FIGURE 2C: CHANGES IN LABOR SUPPLY
(compared with -2% money growth)

growth rate of the money stock

□ v2 = 0.85   +   v2 = 0.65   ◇   v2 = 0.45
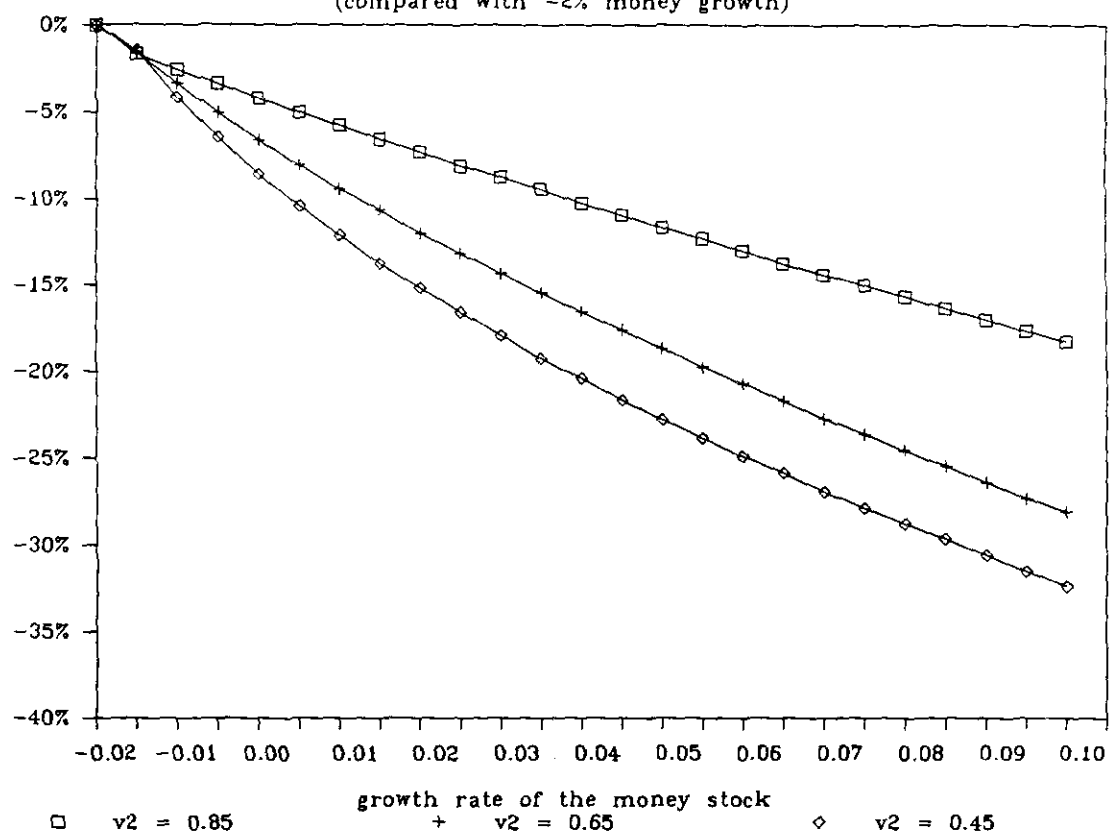
FIGURE 2D: CHANGES IN LEISURE

(compared with -2% money growth)

growth rate of the money supply

◻ v2 = 0.85   +   v2 = 0.65   ◊   v2 = 0.45
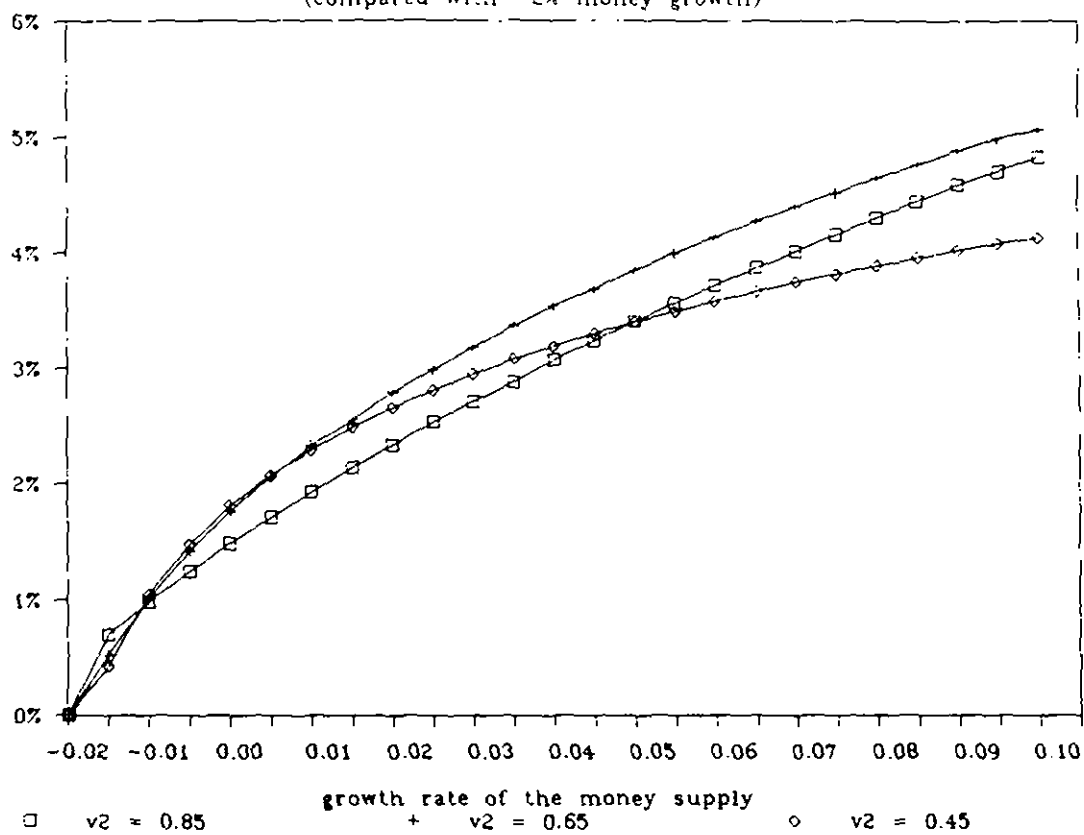
FIGURE 2E: CHANGES IN REAL MONEY

(compared with -2% money growth)

growth rate of the money supply

□   v2 = 0.85   +   v2 = 0.65   ◇   v2 = 0.45
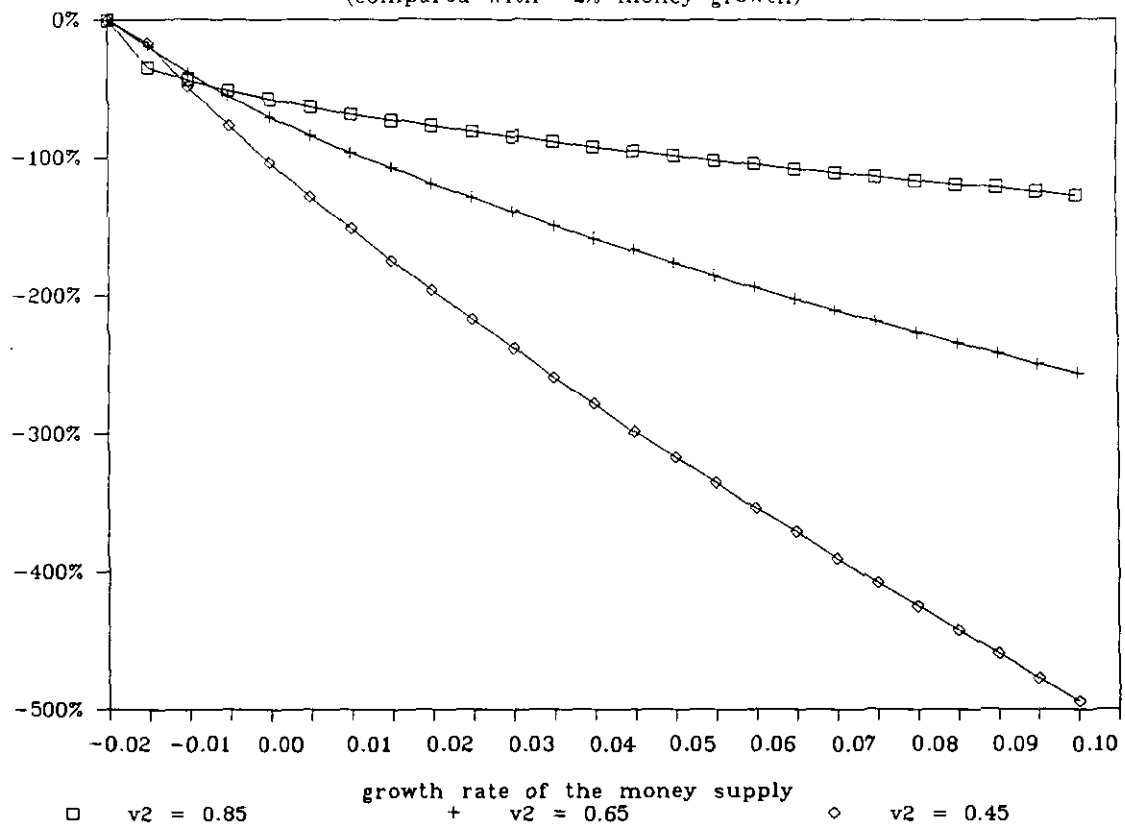
Appendix A: Regression Results

In table 4 I give the results of the regressions of the last iteration of the simulation procedure. I also give the results of the regression for the bond equation. Since we do not need the bond price to solve the model we can estimate the conditional expectation of the first-order condition connected with bonds (2.4), after the simulation procedure has converged.

Below I rewrite equation (2.3) and I repeat equation (2.5) for the convenience of the reader

$$(A.1) \qquad c_t^{\gamma\tau-1} \qquad = \beta E_t [c_t^{\gamma\tau-1} (f_{k,t+1} x_{t+1} + \delta) U_{c,t+1}/U_{c,t}]$$

$$(A.2) \qquad M_{t+1}/P_t \qquad = \beta E_t [(M_{t+1}/P_{t+1}) (U_{c,t+1} + U_{m,t+1}) / U_{c,t}]$$

For the bond equation we get

$$(A.3) \qquad c_t^{\gamma\tau-1} q_t \qquad = \beta E_t [c_t^{\gamma\tau-1} (p_t/p_{t+1}) U_{c,t+1}/U_{c,t}]$$

The condtional expectations in these three equations are approximated with the first-order power functions. Thus first the time series for all the endogenous variables are calculated, using equations (4.1),..., (4.5)[1], and some initial parameter values for the power functions. Then the terms inside the conditional expectations above are calculated for every period. Regressing these time series on the power function gives us new parameters for the power function.

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[1] For equation (4.1) the transformed version, i.e. (A.1), should be used.

To do stationary state comparisons we need the following four equations.

(B.1) $1 = \beta[\,\alpha(\tfrac{h}{k})^{1-\alpha} + \delta\,]$

(B.2) $\dfrac{p'}{p} = \beta[\,1 + v_1\dfrac{v_2}{1-v_2}(\tfrac{m}{c})^{\frac{-1}{1-v_2}}(1-\alpha)(\tfrac{k}{h})^{\alpha}\,]$

(B.3) $\dfrac{y}{c} = \dfrac{1-\gamma}{L}\,v_1\dfrac{v_2}{1-v_2}(\tfrac{m}{c})^{\frac{-v_2}{1-v_2}} + \dfrac{1-\gamma}{L}\dfrac{1}{(1-\alpha)(\tfrac{k}{h})^{\alpha}}$

(B.4) $c = k^{\alpha}h^{1-\alpha} + \delta k$

(B.5) $L = 1 - h - v_1(\tfrac{m}{c})^{\frac{-v_2}{1-v_2}}\,c$

with, $0 < \alpha < 1,\ 0 < \delta < 1$

$0 < \gamma < 1$

$v_1 > 0,\ 0 < v_2 < 1$

The first four equations are the stationary state versions of equations (2.3), (2.5), (2.6) and (2.11) for the functions specified in section 4. (p'/p) indicates the stationary state

rise in the price level. Equation (B.5) defines the shopping time and specifies the time constraint.

The first claim is that it is impossible that the labor supply h increases if (p'/p) increases. Supposes to the contrary that h does increase. Then we know from equation (B.1) that capital also has to increase and from equation (B.4) that consumption also increases. Moreover, the ratio (h/k) does not change. Therefore if (p'/p) increases, the ratio (m/c) has to decrease to satisfy equation (B.2). This with an increase in c means that shopping time will increase, which implies that leisure has to decrease. The above implies that the lefthand side of equation (B.3) will go down, while the righthand side will go up. Clearly a contradiction.

The second claim is that real money balances have to go down. From the first claim we know that consumption can not increase. Since the ratio (m/c) has to go down, real money balances have to go down.

The third claim is that shopping time cannot decrease. If we multiply both sides of equation (B.3) with c and L and use equation (B.5) to eliminate L we get for the shopping time v

$$(B.6) \quad v = \frac{1 - h - \dfrac{1-\gamma}{\gamma} \dfrac{1}{f_h} c}{\dfrac{1-\gamma}{\gamma} \dfrac{v_2}{1-v_2} + 1}$$

where f_h stands for the marginal product of labor. Since h and c can not go up v can not go down.

Now we are able to show the fourth claim that consumption, capital, labor supply and shopping time have to change. We know that the ratio (m/c) has to decrease. This implies that shopping time has to increase unless a decrease in consumption exactly offsets this change. But this is impossible. The reason is that consumption and labor supply move in the same direction and from (B.6) we know that the shopping time changes if the labor supply and consumption change in the same direction.

But if the shopping time does strictly increase, we are also done since equation (B.6) implies that h and c cannot stay the same if v changes.

The fifth claim is that leisure will go up as a response to a higher inflation if

$$[(1-v_2) \frac{1-\beta\delta+\alpha\beta\delta}{1 - \alpha - \beta\delta + \alpha\beta\delta} - v_2] \frac{1}{\gamma + v_2 - 2\gamma v_2} < 0$$

If we combine equation (B.3) and (B.5) to eliminate the shopping time, then we get

$$L = \frac{(1-\gamma)}{\gamma + v_2 - 2\gamma v_2} [v_2 - v_2 h + \frac{(1-v_2)}{f_h} c]$$

From equation (B.4) we know that

$$c \approx [(\frac{k}{h})^\alpha + \delta (\frac{k}{h})] h$$

and from equation (B.1) we get that

$$(\frac{k}{h})^{1-\alpha} = \frac{\alpha\beta}{1-\beta\delta}$$

If we combine the last three equations we get that

$$L = \frac{(1-\gamma)\,v_2}{\gamma + v_2 - 2\gamma v_2} - \frac{(1-\gamma)}{\gamma + v_2 - 2\gamma v_2} \left[(1-v_2)\frac{1-\beta\delta+\alpha\beta\delta}{1-\alpha-\beta\delta+\alpha\beta\delta} - v_2 \right] h$$

Note that I did not include storage costs in the shopping time technology, i.e. $v_3 = 0$. All claims remain valid except the third and the fifth. If $v_3 > 0$, then it is possible that the shopping time decreases as inflation goes up, since the reduction of real money balances reduces "storage time". Of course the condition mentioned in the fifth claim would change if $v_3 > 0$. Note that the condition mentioned in the fifth claim is not valid for our parameter set if $v_2 = 0.45, 0.65$. Nevertheless we find that leisure goes up if inflation increases in the simulations, thus the reduction in storage time causes leisure to go up as a response to a higher rate of inflation. If v_3 does not equal zero, we get for leisure

$$L = \frac{(1-\gamma)\,v_2}{\gamma + v_2 - 2\gamma v_2}\,[\,1 - v_3\,m] - \frac{(1-\gamma)}{\gamma + v_2 - 2\gamma v_2} \left[(1-v_2)\frac{1-\beta\delta+\alpha\beta\delta}{1-\alpha-\beta\delta+\alpha\beta\delta} - v_2 \right] h$$

TABLE 1: THE OPTIMAL POLICY AND THE -2% RULE
AVERAGES OF 1st and 2nd MOMENTS
(standard deviations in parenthesis)

	Optimal program	-2% rule
means		
Consumption	0.180457 (0.001502)	0.180217 (0.001520)
Capital	0.269753 (0.002278)	0.269409 (0.002298)
Leisure	0.725745 (0.000015)	0.726088 (0.000102)
Shopping Time	0.017281 (0.000144)	0.017295 (0.000144)
Real Money Balances	1.123235 (0.009351)	1.106842 (0.011994)
Inflation	-0.019879 (0.000150)	-0.020177 (0.000130)
Sum Discounted Utility	-142.805306 (0.580878)	-142.809333 (0.580771)
Standard Deviations		
Output	4.131190 % (0.149062)	4.187260 % (0.173604)
Consumption	3.451827 % (0.091138)	3.491590 % (0.124803)
Leisure	0.281401 % (0.012541)	0.325592 % (0.023463)
Real Money Balances[1]	3.451827 % (0.091138)	4.757446 % (0.640376)
Inflation	2.030363 % (0.106245)	2.767305 % (0.137543)
Price Level	3.898927 % (0.287973)	4.830944 % (0.706006)

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[1] Note that the standard deviation for consumption equals the standard deviation for real money balances. The reason is that there is a fixed relation between consumption and real money balances in the optimum program and that the standard deviations are scaled.

TABLE 2: WELFARE LOSSES OF POSITIVE MONEY GROWTH
Permanent decrease in consumption and leisure equivalent to welfare change
(compared with a zero growth rate of money)

money growth rate	$v_2 = 0.45$	$v_2 = 0.65$	$v_2 = 0.85$
0.025 (= 2.5 %)	1.24 %	0.76 %	0.24 %
0.050	2.22 %	1.45 %	0.50 %
0.075	2.95 %	1.92 %	0.57 %
0.100	3.69 %	2.52 %	0.85 %

TABLE 3: INCREASING THE VARIANCE OF THE MONEY SUPPLY
AVERAGES OF 1st and 2nd MOMENTS
(standard deviations in parenthesis)

	$\sigma_\eta = 0.0$	$\sigma_\eta = 0.05$
means		
Consumption	0.159067 (0.001306)	0.159164 (0.001594)
Capital	0.237791 (0.001953)	0.237974 (0.002397)
Leisure	0.748473 (0.000038)	0.748374 (0.000625)
Shopping Time	0.024736 (0.000112)	0.024932 (0.000346)
Real Money Balances	0.485019 (0.005364)	0 .487316 (0.011336)
Inflation	0.024736 (0.000127)	0.025552 (0.002110)
Sum Discounted Utility	-143.731035 (0.603096)	-143.723325 (0.578605)
Standard Deviations		
Output	4.132109 % (0.205720)	4.120616 % (0.182771)
Consumption	3.415992 % (0.160012)	3.519375 % (0.131557)
Leisure	0.269880 % (0.023636)	0.279920 % (0.020988)
Real Money Balances	4.763364 % (0.504757)	6.611483 % (0.304696)
Inflation	2.711944 % (0.078779)	5.507807 % (0.323215)
Price Level	4.875882 % (0.576581)	7.580988 % (1.033256)

TABLE 4: REGRESSION RESULTS[1]

OPTIMAL POLICY
($V_2 = 0.65$)

	constant	k_t	x_t	ξ_t	R^2
Equation (A.1)	4.006084	-0.447427	-0.919103	-0.130855	
	(0.006263)	(0.000226)	(0.000521)	(0.000252)	0.9993
Equation (A.2)	1.965936	0.427498	0.766715	0.015597	
	(0.001502)	(0.000226)	(0.000506)	(0.000228)	0.9994
Equation(A.3)	3.949279	-0.458058	-0.918846	-0.139980	
	(0.005825)	(0.000216)	(0.000499)	(0.000241)	0.9994

MINUS 2% RULE
($V_2 = 0.65$)

	constant	k_t	x_t	ξ_t	R^2
Equation (A.1)	3.9793330	-0.452922	--0.920631	-0.078325	
	(0.005405)	(0.000197)	(0.000465)	(0.000228)	0.9994
Equation (A.2)	2.083859	0.480828	0.894943	-0.682185	
	(0.001502)	(0.000226)	(0.000506)	(0.000228)	1.0000
Equation(A.3)	3.976162	-0.453559	-0.916933	-0.093905	
	(0.000099)	(0.000004)	(0.000008)	(0.000004)	1.0000

Note that in the regression results, we also see the difference in the price dynamics. A positive preference shock lowers the price level in the optimal program (a negative effect on (M_{t+1}/p_t)) and a positive effect on the price level in the -2% rule.

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1 Standard errors are given in parentheses.

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