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TOWARD A THEORY OF INTERNATIONAL CURRENCY

by

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Abstract

Our goal is to provide a theoretical framework in which both positive and normative aspects of international currency can be addressed in a systematic way. To this end, we use the framework of random matching games and develop a two country model of the world economy, in which two national fiat currencies compete and may be circulated as media of exchange.

There are multiple equilibria, which differ in the areas of circulation of the two currencies. In one equilibrium, the two national currencies are circulated only locally. In another, one of the national currencies is circulated as an international currency. There is also an equilibrium in which both currencies are accepted internationally. We also find an equilibrium in which the two currencies are directly exchanged. The existence conditions of these equilibria are characterized, using the relative country size and the degree of economic integration as the key parameters.

In order to generate sharper predictions in the presence of multiple equilibria, we discuss an evolutionary approach to equilibrium selection, which is used to explain the evolution of the international currency as the two economies become more integrated.

Some welfare implications are also discussed. For example, a country can improve its national welfare by letting its own currency circulated internationally, provided the domestic circulation is controlled for. When the total supply is fixed, however, a resulting currency shortage may reduce the national welfare.

Keywords: Best Response Dynamics, Evolution of International Currency, Money as a Medium of Exchange, Multiple Currencies, (Nonuniform) Random Matching Games

1. Introduction

International economic activity, just like domestic activity, requires the use of money, whose primary function is "to lubricate the wheels of commerce." Although there have always been hundreds of local and national currencies, only a few, and often only one, of them served as a generally accepted means of payment in international transactions. The classical example is the gold solidus of the Byzantine Empire in the medieval Mediterranean World. In his article, "The Dollar of the Middle Ages," Lopez (1951, p.209) quoted a sixth century Greek monk, who proudly stated that the gold coin of the Byzantine Empire "is accepted everywhere from end to end of the earth. It is admired by all men and in all kingdoms, because no kingdom has a currency that can be compared to it." The dominance of the Byzantine coin continued until the seventh century, when it was partially replaced by the dinar of the Arabs. In the thirteenth century Mediterranean, the florino of Florence became eminent, which was then taken over, two centuries later, by the ducato of Venice [Cipolla (1956, Ch. 2)]. More recently, the pound sterling played the dominant role in international commerce until 1914. And, of course, the U.S. dollar was the vehicle currency of the Bretton-Woods system after World War II [Yeager (1976)].

Unlike national currencies, whose domestic circulation may be enforced (though not always successfully) by the national governments through a variety of legal restrictions, the rise and fall of national currencies as a medium of international commerce are largely due to the process of "the Invisible Hand." It would be thus sensible for an economist to ask; which characteristics of a national economy make its national currency a natural candidate for the vehicle currency?; how could local and national currencies, with their limited acceptance, survive and coexist with the universally accepted means of payment in the absence of legal restrictions?; how would an international currency emerge as national economies become more integrated?; what would be the benefits and costs of having its own currency circulated as the international medium of exchange? The significance of these questions attracted a number of economists over the years, such as Swoboda (1969), Cohen (1971), McKinnon (1979), Kindleberger (1981) and Krugman (1980, 1984), yet formal modelling has been illusive.

The goal of our project is to provide a theoretical framework in which both positive and normative aspects of international currency can be addressed in a systematic way. To this end, we build on the recent literature of decentralized exchange processes, in particular, on the random matching model of Kiyotaki and Wright (1989).[1] This literature is based on the two observations. First, agents often specialize in production; they cannot always produce what they need to consume, which gives agents an incentive to exchange. Second, there is no centralized market in which all transactions can be settled simultaneously and multilaterally. Actual exchange processes often need to be bilateral and quid pro quo. Jevons's (1875) "double coincidences of wants" problem naturally arises in such an environment. And a certain object may emerge as a medium of exchange, as long as agents believe that it will. The random matching game is the natural framework within which to formalize this idea.

We modify the Kiyotaki and Wright model in two important ways. First, our main concern here is a choice among fiat monies, rather than the issue of commodity money versus fiat money or the issue of barter versus monetary

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[1]Jones (1976) is the seminal work on decentralized monetary exchanges. Recent contributions include Iwai (1988), Kiyotaki and Wright (1990,1991), and Oh (1989).

exchanges. We thus assume that all commodities are storable only by their producers, which implies no agent accepts any commodity unless he wants to consume it. In this sense, our model is similar to cash-in-advance models; we require (to be more exact, make the assumption that leads to) the use of fiat money in all transactions. Unlike cash-in-advance models, however, we do not specify <u>which</u> fiat money needs to be used. Second, the Kiyotaki and Wright model, as well as most other random matching models, assumes that the random process in which agents are matched in pairs is uniform; the matching of any pair of agents is equally likely as that of any other pairs.[2] Here, we divide agents into two groups and assume that a pair of agents that belong to the same group is more likely to be matched than a pair of agents that belong to different groups. Such a nonuniform matching process gives rise to a natural definition of the two regional economies.

The use of money as a medium of exchange critically hinges on the strategic externality and economies of scale; you are more willing to accept a currency, to the extent that you feel confident that people you will meet in the future would do the same. This suggests that an agent's incentive to accept a nation's currency would depend upon, among other things, how likely you would meet a member of that country. We thus treat the relative size of the two economies and the degree of economic integration as the key parameters, as well as the discount rate and the degree of specialization.

The paper is organized as follows. In section 2, we describe our two-country, two-currency model of the world economy and the equilibrium concept in detail, and then discuss the general properties of the model. We

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[2]A few exceptions are, not surprisingly, Matsui and Matsuyama (in process) and Matsuyama (1991b).

characterize the existence conditions of the equilibria in section 3. In section 4, we address the issues concerning multiplicity of equilibria. In order to generate sharper predictions, we discuss an evolutionary approach to equilibrium selection, and show how this idea can be applied to explain the emergence of an international currency as the two economies become more integrated. Some welfare implications are discussed in section 5. In section 6, we construct an example of the mixed strategy equilibrium in which an exchange of two currencies takes place. In section 7, we summarize the main findings, discuss the drawbacks of the model, and suggest the directions for future research. The proof and details of calculation are provided in Appendix.

2. The Model

2.1 The Physical Environment

Time is discrete and extends from zero to infinity. The world economy is populated by a continuum of infinitely lived agents with unit mass. The agents are divided in two regions, Home and Foreign. Let $n \in (0,1)$ be the size of Home population, so that $\theta = (1-n)/n$ represents the relative size of the Foreign country. There are k ($k \geq 3$) types of indivisible commodities, and within each economy, there are equal proportions of k types of agents, who specialize in consumption, production and storage. A type i agent derives utility only from consumption of commodity i. After he consumes commodity i, he is able to produce one and only one unit of commodity $i+1$ (mod k) costlessly, and he also knows how to store his production good costlessly up to one unit; he can neither produce nor store other types of goods. With k being greater than or equal to three, the patterns of specialization assumed here imply that there is no "double coincidence of wants" in this economy.

Only one k-th of the population derives the utility from each good. We interpret k as the degree of specialization.

Let $u > 0$ be the instantaneous utility from consuming his own consumption good, and $\delta > 0$ his discount rate (both independent of the type and the nationality). The expected discounted utility of an agent as of time t is given by

$$V_t = E[\sum_{s=0}^{\infty} (1+\delta)^{-s} I_{t+s} u | \Omega_t] ,$$

where I_{t+s} is a random indicator function that equals one if the agent consumes his consumption good at period t+s and zero otherwise; Ω_t is the information available at period t.[3] With a positive discount rate and zero production and storage cost, an agent, if lucky enough to acquire his consumption good, will consume it immediately and produce a unit of his production good, which he carries over to the next period.

In addition to the commodities described above, there are two distinguishable <u>fiat monies</u>, objects with zero intrinsic worth, which we call the Home currency and the Foreign currency. It is assumed that each currency is indivisible, and can be stored costlessly up to one unit by every agent if he does not carry his production good or the other currency. This implies that, at any date, the inventory of each agent contains either one unit of the Home currency, one unit of the Foreign currency, or one unit of his

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[3]For the time being, one may assume that the information set, Ω_t, includes the entire structure of the economy, the entire history up to period t, as well as which equilibrium is being played (i.e., the strategy profile is common knowledge among players). Such an informational assumption may seem heroic in a game with a continuum of players. However, as long as steady state equilibria are concerned, the informational requirement can be made less stringent. We will come back to this issue later when discussing the evolution of an international currency.

production good, but no more than one object at the same time.[4] We simply assume that agents never dispose of their inventories, but this is actually not restrictive, because they do not gain from doing so.

We use the following notations for inventory and money holdings. Let m_h (m_f) be the fraction of Home agents holding the Home (Foreign) currency. The fraction of Home agents holding production goods is then $1 - m_h - m_f$, so that the inventory distribution among Home agents can be summarized by a row vector

$$X = (1 - m_h - m_f, m_h, m_f) .$$

Likewise, the inventory distribution among Foreign agents is $X^* = (1 - m_h^* - m_f^*, m_h^*, m_f^*)$, where m_h^* (m_f^*) denotes the fraction of Foreign agents holding the Home (Foreign) currency. Next, let m and $m^* \in (0,1)$ denote the supply of the Home currency per Home agent and that of the Foreign currency per Foreign agent, respectively. Then,

$$nm = nm_h + (1-n)m_h^* , \qquad (1-n)m^* = nm_f + (1-n)m_f^* .$$

We treat both m and m^* as exogenous parameters.

There is no centralized market in which all agents could meet together and exchange commodities multilaterally. Instead, agents are matched randomly in pairs. And when agents are matched, they must decide whether or not to trade bilaterally without any outside authority to impose any

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[4]The indivisibility of commodities and monies, as well as the restriction on the inventory holding, simplify the following analysis substantially because the trade between agents entails one-for-one swap of inventories under these assumptions and thus the agent's problem can be reduced into a three-state Markov decision problem. The major drawback of these assumptions is that they make the model ill adapted for the issues of exchange rate stability. These assumptions are also responsible for some of the results that may or may not be viewed as attractive features of the model, as will be pointed out below.

arrangement. Trade entails a one-for-one swap of inventories, and takes place if and only if they both agree to trade. The matching technology is given in Table 1. For example, each period, a Home agent runs into another Home agent with probability n; he runs into a Foreign agent with probability $\beta(1-n)$; he does not meet anybody with probability $(1-\beta)(1-n)$. The crucial assumption here is that a pair of agents who live in different countries meet less frequently than a pair of agents who live in the same country: $\beta \in (0,1)$ represents the relative frequency. It is also assumed that an increase in β does not reduce the frequency in which a pair of agents from the same country meets. We interpret β as the degree of economic integration. It should be pointed out that the relative chance in which a Home agent meets a Foreign agent, instead of his fellow citizen, is equal to $\beta\theta$: it depends not only on β, but also on the relative country size.[5]

2.2 Strategy and Equilibrium

An agent chooses a trade strategy to maximize his expected discounted utility, taking as given the strategies of other agents and the distribution of inventories. Such a trade strategy can be most generally described as a random function of his type, nationality and inventory, those of his opponent, as well as the date, and everything that has happened to him up to that point. However, we restrict our attention to pure strategies which only depend on his nationality and the objects he and his opponent have in inventory. Thus, the Home agent's trade strategy can be described simply as

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[5]We chose the period length so that all agents would be matched with probability one if the economic integration were complete ($\beta = 1$). Note also that this matching process implies increasing returns in trading. For any $\beta < 1$, an agent living in the larger economy has better chance of meeting potential trading partners.

$$\tau_{ab} = \begin{cases} 1 & \text{if he agrees to trade object a for object b ,} \\ 0 & \text{otherwise ,} \end{cases}$$

where a, b = g, h, or f, and a ≠ b. Similarly, the Foreign agent's trade strategy is given by $\tau^*_{ab} = 0$ or 1. For example, $\tau_{gf} = 0$ means that a Home agent does not agree to trade his production good for the Foreign currency, and $\tau^*_{hg} = 1$ means that a Foreign agent agrees to trade the Home currency for his consumption good.

In other words, we impose the following restrictions on the strategy space. First, we consider only time independent strategies, given that the physical environment here is stationary and the planning horizon is infinite: this effectively limits our attention to steady state equilibria in what follows. [In section 4.2, we will discuss an alternative justification for focusing on steady states.] Second, we assume that this is an anonymous sequential game [Jovanovic and Rosenthal (1988)]; that is, a strategy does not depend on the type and the nationality of the agent with which he is currently matched. Third, given the symmetry imposed in the environment, agents of all types with the same nationality follow the same strategies: that is, we focus on the symmetric equilibria. Furthermore, we assume at least until section 6 that an agent agrees to trade if and only if the trade results in a strict increase in his expected discounted utility. This also rules out randomized strategies.

Trade strategies, τ and τ^*, inventory distributions, X and X^*, as well as the matching technology, jointly generate the Markov process that each agent's inventory follows. It can be summarized by two transition matrices,

$$\Pi = \begin{bmatrix} 1-P_{gh}-P_{gf} & P_{gh} & P_{gf} \\ P_{hg} & 1-P_{hg}-P_{hf} & P_{hf} \\ P_{fg} & P_{fh} & 1-P_{fg}-P_{fh} \end{bmatrix}, \quad \Pi^* = \begin{bmatrix} 1-P^*_{gh}-P^*_{gf} & P^*_{gh} & P^*_{gf} \\ P^*_{hg} & 1-P^*_{hg}-P^*_{hf} & P^*_{hf} \\ P^*_{fg} & P^*_{fh} & 1-P^*_{fg}-P^*_{fh} \end{bmatrix},$$

where P_{ab} (P^*_{ab}) is the transition probability with which a Home (Foreign) agent switches his inventory from object a to object b. For example, the conditional probability with which a Home agent will acquire the Foreign currency, provided that he has his production good in his inventory, is given by $P_{gf} = \tau_{gf}[nm_f\tau_{fg} + \beta(1-n)m^*_f\tau^*_{fg}]/k$. This is because the opportunity for a Home agent to trade his production good for the Foreign currency arrives in two ways; Either when he is matched with another Home agent (with probability n), who carries the Foreign currency (with probability m_f), and whose consumption good is his production good (with probability 1/k), or when he is matched with a Foreign agent (with probability $\beta(1-n)$), who carries the Foreign currency (with probability m^*_f), and whose consumption good is his production good (with probability 1/k). In either case, the trade takes place if and only if mutually agreeable; $\tau_{gf}\tau_{fg} = 1$ or $\tau_{gf}\tau^*_{fg} = 1$. Note also that the steady state requires that X, X^*, Π and Π^* are constant and satisfy $X\Pi = X$ and $X^*\Pi^* = X^*$.

In sum, we consider a steady-state, symmetric, pure strategy Nash equilibrium of this economy, which is a set of strategies, τ and τ^*, together with the steady state inventory distributions, X and X^*, and steady state transition matrices, Π and Π^*, that satisfy; (a) maximization: given the strategies of other agents, and steady state distributions, X and X^*, each agent chooses a trading strategy to maximize his expected utility, and (b) rational expectations: the steady state transition matrices and inventory distributions are consistent with the strategies chosen by the agents.

One can immediately and trivially show that barter trade cannot take place in this economy under the assumed patterns of specialization in consumption and in production, which imply no "double coincidence of wants," and the impossibility of storage of a good except by its producer. Our goal is to determine the extent to which two fiat currencies are accepted in different equilibria and to characterize the existence conditions and welfare properties of these equilibria.

2.3 Some General Results

Before proceeding further, we describe some general properties of the model that will prove useful. In a steady state equilibrium, each agent faces a stationary environment (which includes not only the physical environment, but also the strategies chosen by other agents), which allows us to formulate each agent's decision problem in a dynamic programming framework. Let V_g, V_h and V_f be the value functions of a Home agent in a particular equilibrium: that is, the equilibrium values of his expected discounted utility conditional on that he has in inventory his production good, the Home currency, and the Foreign currency, respectively. Then, Bellman's equations are

(2-1) $$V_g = [(1-P_{gh}-P_{gf})V_g + P_{gh}V_h + P_{gf}V_f]/(1+\delta) ,$$

(2-2) $$V_h = [P_{hg}(u+V_g) + (1-P_{hg}-P_{hf})V_h + P_{hf}V_f]/(1+\delta) ,$$

(2-3) $$V_f = [P_{fg}(u+V_g) + P_{fh}V_h + (1-P_{fg}-P_{fh})V_f]/(1+\delta) ,$$

Note that, as shown in both (2-2) and (2-3), the value of acquiring the consumption good is equal to $u + V_g$, the utility directly derived from consumption plus the value of holding the production good, because consumption makes the agent capable of producing. The value functions and equilibrium strategies must satisfy the following incentive compatibility constraints:

(2-4) $\tau_{gb} = 1$ iff $V_g < V_b$ (b = h, or f) ,

(2-5) $\tau_{ag} = 1$ iff $V_a < u + V_g$ (a = h, or f) ,

(2-6) $\tau_{ab} = 1$ iff $V_a < V_b$ (a, b = h, or f) .

For example, if a Home agent can increase his utility from trading his production good for the Foreign currency ($V_g < V_f$), he agrees to trade (τ_{gf} = 1). On the other hand, he does not agree to trade, $\tau_{gf} = 0$, if $V_g \geq V_f$. This inequality actually states that a Home agent cannot improve his utility from a one-shot deviation; that is, "accept the Foreign currency once, and then follow the equilibrium strategy in what follows." [Note that V_f is the <u>equilibrium</u> expected utility and that, in any steady state equilibrium with $\tau_{gf} = 0$, the Home agent holding the Foreign currency is an <u>out-of-equilibrium</u> event.] However, a principle of dynamic programming, the "unimprovability" criterion, guarantees that an agent cannot improve his utility from any deviation if he cannot improve it from a one-shot deviation; see, for example, Kreps (1990). Thus, $V_g \geq V_f$ is the necessary and sufficient condition for a Home agent not to trade his production good for the Foreign currency. One can similarly define the value functions of a Foreign agent, which also satisfy the relations analogous to (2-1) through (2-6).

We are now ready to state some general properties of the model.

<u>Proposition:</u> In any steady state equilibrium,

a) $0 \leq V_g, V_h, V_f < u + V_g$.

b) $\text{Max}\,\{V_h, V_f\} > V_g > 0$, or $V_g = V_h = V_f = 0$.

c) $V_h \gtrless V_f$ iff $P_{hg} \gtrless P_{fg}$,

d) $V_h \gtrless V_g$ iff $P_{hg}(\delta+P_{gf}+P_{fg}+P_{fh}) + P_{hf}P_{fg} \gtrless P_{gf}P_{fg}$,

e) $\qquad V_f \gtreqless V_g \quad \text{iff} \quad P_{fg}(\delta+P_{gh}+P_{hg}+P_{hf}) + P_{fh}P_{hg} \gtreqless P_{gh}P_{hg}$,

f) $\qquad \delta[(1-m_h-m_f)V_g+m_hV_h+m_fV_f] = [m_hP_{hg} + m_fP_{fg}]u$.

The same relations hold for a Foreign agent, with relevant variables starred.

The first inequality in Proposition a) should be obvious from the assumption of zero production and storage costs. The second inequality states that he is always willing to trade his inventory for his own consumption good. Proposition b) means that the return for production is strictly positive if and only if there is at least one currency he wants to obtain (because he could use it as a medium of exchange for acquiring his consumption good). Proposition c) states that holding the Home currency is more valuable than holding the Foreign currency, if and only if the Home currency gives him the better chance of acquiring his consumption good. Proposition d) can be interpreted as that a Home agent accepts the Home currency instead of waiting to meet Foreign currency holders, unless the possibility of indirect exchange through the Foreign currency, $P_{gf}P_{fg}$, is very large. Proposition e) can be interpreted similarly. Both d) and e) will prove useful below when describing the existence conditions of a particular equilibrium. Proposition f) is a direct consequence of the risk neutrality of the agents; the steady state utility level of a Home agent is proportional to the fraction of Home agents that consume their consumption goods in each period. This substantially simplifies the welfare evaluations.

Proposition gives us a simple three-step algorithm for finding equilibria. Step 1: propose a ranking of the Home agent's value functions, V_g, V_h, V_f, subject to the constraints given in Proposition a) and b). From (2-4) through (2-6), this determines the equilibrium strategies that the Home agent would follow. Do the same for the Foreign agent. Step 2: calculate the steady state inventory distribution and transition probabilities implied by these equilibrium strategies. Step 3: check to see if the ranking of value functions proposed in Step 1 and the transition probabilities calculated in Step 2 in fact satisfy the restrictions given in Proposition c) through e) and their Foreign counterparts.

It turns out that ten different types of equilibria could exist in this model. Instead of going through all possibilities, we restrict our attention to the equilibria in which the Home currency is accepted in the Home country and the Foreign currency is accepted in the Foreign country. Even with this restriction, there are four different types of equilibria.[6] Characterizing the existence conditions of these equilibria is the subject of the next section.

3. Existence

3.1 Equilibrium with Two Local Currencies: Equilibrium A

We first consider the following equilibrium, in which:

a) *A Home agent trades his production good for the Home currency, the Home currency for his consumption good, but does not accept the Foreign currency* $(u + V_g > V_h > V_g \geq V_f)$.

b) *A Foreign agent trades his production good for the Foreign currency, the Foreign currency for his consumption good, but does not accept the Home currency* $(u + V_g^* > V_f^* > V_g^* \geq V_h^*)$.

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[6]There are six equilibria that do not satisfy this restriction. First, there are three equilibria in which at least one currency is not accepted by anybody. It is straightforward to show that these equilibria always exist. We call the equilibrium in which the Home (Foreign) currency is the unique universally accepted medium of exchange Equilibrium HH (FF). Second, one could generate one equilibrium from each of Equilibria A, F, and H, which will be discussed in detail below, by simply relabeling the currencies.

Since there will be no trade between the two countries in this equilibrium, we call it Equilibrium A: A for autarky.

In the steady state, only Home agents hold the Home currency and only Foreign agents hold the Foreign currency and thus the inventory distributions are simply given by $X = (1-m, m, 0)$ and $X^* = (1-m^*, 0, m^*)$. The transition probabilities in this equilibrium for a Home agent are

$$(3\text{-}1) \qquad \begin{cases} P_{gh} = nm/k , & P_{hg} = n(1-m)/k , \\ P_{fg} = \beta(1-n)(1-m^*)/k , & P_{gf} = P_{hf} = P_{fh} = 0 . \end{cases}$$

For example, $P_{gh} = nm/k$ because, in this equilibrium, a Home agent trades his production good for the Home currency only when he is matched with another Home agent (with probability n), who holds the Home currency (with probability m), and whose consumption good is his production good (with probability 1/k). The other expressions in (3-1) can be interpreted similarly. Although a Home agent would want to trade the Foreign currency for the Home currency in this equilibrium, he would be unable to do so ($P_{fh} = 0$), since only Home agents hold the Home currency and none of them is willing to accept the Foreign currency. Likewise, the transition probabilities for a Foreign agent are

$$(3\text{-}2) \qquad \begin{cases} P^*_{gf} = (1-n)m^*/k , & P^*_{fg} = (1-n)(1-m^*)/k , \\ P^*_{hg} = \beta n(1-m)/k , & P^*_{gh} = P^*_{fh} = P^*_{hf} = 0 . \end{cases}$$

From Proposition a) and b), a Home agent follows the prescribed equilibrium strategy, if the following inequality holds:

$$(3\text{-}3) \qquad V_g \geq V_f .$$

which ensures that a Home agent does not accept the Foreign currency. Similarly, a Foreign agent follows the prescribed equilibrium strategy, if

(3-4) $$V^*_g \geq V^*_h ,$$

which ensures that a Foreign agent does not accept the Home currency. Thus, Equilibrium A exists if and only if the two incentive constraints, (3-3) and (3-4), are satisfied, given the transition probabilities. From Proposition e) and (3-1), (3-3) can be rewritten to

(3-5) $$\beta \leq A(n) \equiv m(1-m)n^2/(1-m^*)(1-n)(k\delta+n) .$$

Similarly, (3-4) becomes

(3-6) $$\beta \leq A^*(n) \equiv m^*(1-m^*)(1-n)^2/(1-m)n(k\delta+1-n) ,$$

which can also be obtained from (3-5) by exchanging the roles of m and m^* and those of n and 1 − n.

Given m, m^*, δ and k, (3-5) and (3-6) give the existence conditions on (n,β) space, a unit square, as depicted in Figure 1. The graph of $\beta = A(n)$ is upward sloping, while that of $\beta = A^*(n)$ is downward sloping; they intersect once inside the box, since $A(n)A^*(n) = mm^*n(1-n)/(k\delta+n)(k\delta+1-n) < 1$. Equilibrium A exists in the shaded region. It shows that, for any n, the two currency areas could co-exist side-by-side without interacting each other, if the degree of economic integration is sufficiently small. But, it also shows that, for any β, a sufficiently small or large n is not consistent with this equilibrium. If the Foreign country is large enough, the Home agents would have an incentive to accept the Foreign currency. Likewise, when the Home country is sufficiently large, the Foreign agents would be willing to accept the Home currency. For a given n, a sufficiently large β would also eliminate this equilibrium; as the two economies are more integrated with each other and the chance of running into foreigners increases, the incentive to accept foreign currencies would be higher.

These equilibrium conditions depend on the other parameters as follows. The graph of $\beta = A(n)$ shifts upward as one increases m^* or $m(1-m)$. Home commodity holders find the Foreign currency less attractive as the fraction of the commodity holders in the Foreign country declines, or as m^* increases. They also have stronger incentive to wait for the Home currency, instead of accepting the Foreign currency, if the chance of running into the Home currency holders (proportional to m) and the chance of running into the commodity holders willing to accept the Home currency (proportional to 1-m) are high. Likewise, $\beta = A^*(n)$ shifts upward as one increases m or $m^*(1-m^*)$. On the other hand, an increase in k or δ shifts down both graphs, reducing the equilibrium region: as the degree of specialization increases, or as the agents become more impatient, incentives for accepting foreign currencies would be higher.

3.2 Equilibria with One Local Currency and One International Currency: Equilibria F and H

We now turn to the possibility of endogenous emergence of an international currency. First, let us consider what we call Equilibrium F, in which the Home currency is circulated locally at Home, and the Foreign currency becomes an international medium of exchange; that is,

a) *A Home agent trades his production good both for the Home and Foreign currencies, and trades both currencies for his consumption good* ($u + V_g > V_h$, $V_f > V_g$).

b) *A Foreign agent trades his production good for the Foreign currency, the Foreign currency for his consumption good, but does not accept the Home currency* $(u + V^*_g > V^*_f > V^*_g \geq V^*_h)$.

When agents follow these strategies, $m_h = m$, $m^*_h = 0$, $m^*_f < m^*$, and $m_f > 0$ and the inventory distributions in this equilibrium are $X = (1 - m - m_f, m, m_f)$ and $X^* = (1 - m^*_f, 0, m^*_f)$. The steady state also requires that the ratios of commodity holders to the Foreign currency holders in the two countries should be equalized, thus $m^*_f(1-m-m_f) = (1-m^*_f)m_f$, or $m_f = (1-m)m^*_f$. Therefore,

$$X = ((1-m)(1-m^*_f), m, (1-m)m^*_f) , \qquad X^* = (1 - m^*_f, 0, m^*_f) , \tag{3-7}$$

so that the total supply and domestic circulation of the Foreign currency satisfy

$$(1-n)m^* = n(1-m)m^*_f + (1-n)m^*_f = (1-nm)m^*_f . \tag{3-8}$$

On the other hand, the transition probabilities are given by

$$\left\{ \begin{array}{ll} P_{gh} = nm/k , & P_{gf} = [n(1-m)+\beta(1-n)]m^*_f/k , \\ P_{hg} = n(1-m)(1-m^*_f)/k , & P_{fg} = [n(1-m)+\beta(1-n)](1-m^*_f)/k , \\ P_{fh} = P_{hf} = 0 , & \end{array} \right. \tag{3-9}$$

and

$$\left\{ \begin{array}{l} P^*_{gf} = [\beta n(1-m)+(1-n)]m^*_f/k , \quad P^*_{hg} = \beta n(1-m)(1-m^*_f)/k , \\ P^*_{fg} = [\beta n(1-m)+(1-n)](1-m^*_f)/k , \\ P^*_{fh} = P^*_{hf} = P^*_{gh} = 0 , \end{array} \right. \tag{3-10}$$

where m^*_f satisfies (3-8). Note that (3-9) shows that $P_{hg} < P_{fg}$, which implies $V_h < V_f$ from Proposition c): the Foreign currency is more valuable than the Home currency even for the Home agent, because the Foreign currency is, as the international medium of exchange, more widely accepted. (Thus, a Home agent is willing to trade the Home currency for the Foreign currency, but unable to do so because other agents, including Foreign agents, also value the Foreign currency more.)

That $V_h < V_f$, as well as Proposition a) and b), implies that it is sufficient to satisfy the following condition in order to make a Home agent follow the equilibrium strategy:

(3-11) $$V_g < V_h ,$$

which states that a Home agent has an incentive to accept the Home currency. For a Foreign agent, it suffices to check

(3-12) $$V_g^* \geq V_h^* ,$$

which ensures that a Foreign agent does not accept the Home currency. Thus, Equilibrium F exists if and only if the two incentive constraints, (3-11) and (3-12), hold given (3-9) and (3-10). From Proposition d), (3-8) and (3-9), one can rewrite (3-11) to,

(3-13) $$f(n,\beta) \equiv n(1-m)(1-nm)[k\delta+n(1-m)+\beta(1-n)] - m^*(1-n)[n(1-m)+\beta(1-n)]^2 > 0 .$$

Similarly, (3-12) becomes, using the Foreign equivalent of Proposition d), (3-8) and (3-10),

(3-14) $$f^*(n,\beta) \equiv \beta n(1-m)(1-nm)[k\delta+\beta n(1-m)+(1-n)] - m^*(1-n)[\beta n(1-m)+1-n]^2 \leq 0 .$$

Figure 2 depicts the equilibrium region defined by (3-13) and (3-14) on the (n,β) space. The locus of $f = 0$ has a positive slope, and the $f^* = 0$ locus has a negative slope. They intersect once at $\beta = 1$, and Equilibrium F exists in the shaded region. This demonstrates the possibility that a local currency may survive and co-exist with the universally accepted means of payment in the absence of legal restrictions. It also shows, however, that the existence requires that, for any β, the Home country cannot be too small; Facing a large Foreign country, the Home commodity holders would find it advantageous to wait for the Foreign currency, instead of accepting the Home currency. Nor can the Home country be too large in order to prevent Foreign commodity holders from accepting the Home currency. The range of the relative country size which satisfies both of these constraints becomes narrower as the degree of economic integration increases, and in fact would disappear if the distinction between the two economies are to become irrelevant ($\beta = 1$).

An increase in m shifts both $f = 0$ and $f^* = 0$ loci to the right, since a high m reduces the fraction of the Home commodity holders more than the fraction of the Foreign commodity holders, which make it less attractive to accept the Home currency. An increase in m^* also shifts both loci to the right. A high m^* increases the fraction of Foreign currency holders both among Home and Foreign agents, so that it becomes more advantageous to wait for them, rather than accepting the Home currency. An increase in k or δ, on the other hand, shifts both loci to the left: a high degree of specialization or more impatience make the Home currency more attractive for both Home and Foreign agents.

One may also consider the following equilibrium, Equilibrium H, in which the Foreign currency is circulated locally, and the Home currency becomes an international currency, that is,

a) *A Home agent trades his production good for the Home currency, the Home currency for his consumption good, but does not accept the Foreign currency* $(u + V_g > V_h > V_g \geq V_f)$,

b) *A Foreign agent trades his production good both for the Home and Foreign currencies, and trades both currencies for his consumption good* $(u + V_g^* > V_f^*,\ V_h^* > V_g^*)$.

The equilibrium conditions for Equilibrium H can be obtained by replacing m for m* and n for 1 - n in (3-13) and (3-14), as follows:

$$(3\text{-}15)\qquad h^*(n,\beta) = (1-n)(1-m^*)(1-(1-n)m^*)[k\delta+(1-n)(1-m^*)+\beta n] - mn[(1-n)(1-m^*)+\beta n]^2 > 0 ,$$

$$(3\text{-}16)\qquad h(n,\beta) = \beta(1-n)(1-m^*)(1-(1-n)m^*)[k\delta+\beta(1-n)(1-m^*)+n] - mn[\beta(1-n)(1-m^*)+n]^2 \leq 0 .$$

Equation (3-15) states the incentive constraint that Foreign agents accept the Foreign currency, while (3-16) ensures that Home agents do not accept the Foreign currency. The nature of the region defined by these constraints can be analyzed as in the case of Equilibrium F, and thus will not be repeated here.

3.3 Equilibrium with the Unified Currency: Equilibrium U.

Finally, let us briefly discuss what we call Equilibrium U, in which the two currencies are unified and become perfect substitutes; that is,

a) *A Home agent trades his production good both for the Home and Foreign currencies, and trades both currencies for his consumption good* $(u + V_g > V_h,\ V_f > V_g)$,

b) *A Foreign agent trades his production good both for the Home and Foreign* *currencies, and trades both currencies for his consumption good* ($u + V_g^* > V_f^*,\ V_h^* > V_g^*$) .

In the steady state, a complete mixing of inventories is achieved: $X = X^*$, and thus $m_h = m_h^* = nm$, and $m_f = m_f^* = (1-n)m^*$. The transition probabilities are

$$(3\text{-}17)\quad \begin{cases} P_{gh} = nm[n+\beta(1-n)]/k\ , \qquad P_{gf} = [n+\beta(1-n)](1-n)m^*/k\ , \\ P_{hg} = P_{fg} = [n+\beta(1-n)][n(1-m)+(1-n)(1-m^*)]/k\ , \\ P_{fh} = P_{hf} = 0\ , \end{cases}$$

$$(3\text{-}18)\quad \begin{cases} P^*_{gh} = nm[\beta n+(1-n)]/k\ , \qquad P^*_{gf} = [\beta n+(1-n)](1-n)m^*/k\ , \\ P^*_{hg} = P^*_{fg} = [\beta n+(1-n)][n(1-m)+(1-n)(1-m^*)]/k\ , \\ P^*_{fh} = P^*_{hf} = 0\ , \end{cases}$$

From Proposition c), $P_{hg} = P_{fg}$ and $P^*_{hg} = P^*_{fg}$ imply that all agents are indifferent between the two currencies, and, from Proposition b), they prefer both currencies to their production goods. Thus, given that every other agents follow the equilibrium strategies, each agent has an incentive to follow his. Equilibrium U exists for any $(n,\beta) \in (0,1)^2$.

4. Multiple Equilibria and Evolution

We set out comparing equilibria, particularly Equilibrium A, F, and H, which have been discussed separately. In Section 4.1, we compare the existence regions of these equilibria. The multiplicity of equilibria poses some conceptual difficulty when using our model in predicting the emergence of an international currency. In an attempt to overcome this difficulty, we propose an "evolutionary" story of equilibrium selection in Section 4.2. It helps to determine which steady state the economy would converge after an exogenous shock to the fundamentals dislodges the economy from the original steady state. We will apply this idea to explain how an international currency would emerge as the degree of economic integration rises.

4.1 Coexistence of Equilibria

Throughout this section we will restrict ourselves to discuss the limit case, $\delta \to 0$, and $m = m^*$. Then, the existence conditions for Equilibrium A, H, and F become, from (3-5), (3-6), and, (3-13) through (3-16),

Equilibrium A: $\beta < \text{Min}\ \{\ mn/(1-n),\ m(1-n)/n\ \}$,

Equilibrium F: $\beta < \text{Min}\ \{\ (1-m)^2 n/m(1-n)^2,\ m(1-n)^2/(1-m)^2 n\ \}$,

Equilibrium H: $\beta < \text{Min}\ \{\ mn^2/(1-m)^2(1-n),\ (1-m)^2(1-n)/mn^2\ \}$,

Figures 3a and 3b depict the cases of $0 < m < 1 - 1/\sqrt{2}$ and $1 - 1/\sqrt{2} < m < 1/2$, respectively. In these cases, the existence of Equilibrium H would require a larger Home country compared with that of Equilibrium F. In this sense, this model, if currency supplies are small enough, supports the common sense idea: the national currency of a large country is more likely to become the international medium of exchange. Unfortunately, this result would not hold if $m > 1/2$. For sufficiently high currency supplies, the existence of Equilibrium H and nonexistence of Equilibrium F require that the Home country is smaller than the Foreign country.

These results for the case with high money supplies may be puzzling given the discussion in the previous section. When discussing the condition for Equilibrium F, it was shown that a larger Home country would give both Home and Foreign agents a stronger incentive to accept the Home currency, <u>given the steady state inventory distribution implied by Equilibrium F</u>. It should be noted, however, that a shifting from one equilibrium to another changes the steady state distribution. The inventory distributions in Equilibrium H are

$$X = ((1-m)/(1-m+nm),\ nm/(1-m+nm),\ 0)\ ,$$

$$X^* = ((1-m)^2/(1-m+nm),\ nm(1-m)/(1-m+nm),\ m)\ ,$$

while those in Equilibrium F are

$$X = ((1-m)^2/(1-nm),\ m,\ (1-n)m(1-m)/(1-nm))\ ,$$

$$X^* = ((1-m)/(1-nm),\ 0,\ (1-n)m/(1-nm))\ ,$$

Under the assumption on inventory holding restrictions, a switch from Equilibrium H to F would reduce the fraction of commodity holders among Home agents, while increasing it among Foreign agents. When m is high, this effect on the steady state inventory distribution becomes dominant, which is responsible for the perverse result stated above. We are not happy about this particular implication of our assumption on inventory restrictions, which was adopted to make the agent's trade strategy decision tractable. Dropping this assumption, although highly desirable, is beyond our present investigation. Instead, we will focus on the case of low money supplies, $m = m^* < 1/2$, for the remainder of this section.

Even with these restrictions, the model has multiple steady state equilibria for any $(n, \beta) \in (0, 1)$. We regard the multiplicity as a virtue of our model, since the use of money necessarily involves factors such as confidence, faith and social custom. In fact, we believe that it is a property that any good model of money ought to have. Nevertheless, it poses a serious problem concerning the predictive content of the model. For example, suppose that (n,β) belongs to the region where Equilibrium H exists but neither A nor F exist. One cannot conclude from this observation that only the Home currency becomes an international currency, because there are other equilibria as well, in which the Foreign currency is accepted in both countries. [One of them is Equilibrium U. The other is the equilibrium in which the Home currency is not accepted in either economy and the Foreign currency is accepted in both economies.] The multiplicity also presents a conceptual problem in predicting the impacts of an exogenous shock to the fundamentals of the economy.[7] In order to generate sharp predictions on the impact of such a shock after the economy is dislodged from the original equilibrium, one needs to tell some story of equilibrium selection. We will attempt to do precisely this in the next subsection.

4.2 Economic Integration and Emergence of an International Currency: Evolutionary Approach

Let us begin our discussion of equilibrium selection by first pointing out that there are two alternative ways of interpreting steady state equilibrium in our model.

When deriving the equilibrium conditions, we have assumed that the agents have sufficient knowledge and ability to analyze the game in a rational manner. In particular, it was assumed that the agents know the entire structure of the game and also agree on which equilibrium is being played. In other words, the strategy profile is assumed to be common knowledge among the agents, so that they know how to coordinate or to focus on a specific equilibrium. According to this interpretation, which is more in the spirit of

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[7] In fact, a switch from one equilibrium to another might occur even in the absence of any intrinsic change, as long as there exist some correlated devices, which make it possible for agents to coordinate their actions. This property could be used to construct endogenous, stationary fluctuations in the present model. See Kiyotaki and Wright (1990b, Sec. VI) for an example of stationary sunspot equilibria in a model of money as a medium of exchange.

the introspective, or to use Binmore's (1990) term, "eductive" approach in game theory, the game is played once, and an equilibrium is achieved through (timeless and experienceless) contemplation. Any possible dynamics in the model is considered to take place along a nonsteady state equilibrium path. Interpreted this way, the steady state assumption is a rather ad-hoc restriction imposed on the set of all equilibria. The eductive reasoning, while standard and logically consistent, has two drawbacks for our purpose. First, the assumption that the strategy profile is common knowledge among players seems too stringent in a game with many players like ours. Second, it is powerless in explaining which steady state equilibrium is chosen and how it might emerge.[8]

An alternative interpretation, however, can be given to a steady state equilibrium in our model. According to this interpretation, which is more in the spirit of the evolutionary game theory,[9] the agents are not required either to have extensive knowledge on the structure of the model at the outset or to undertake complicated optimizing exercises. Instead, the agents encounter similar situations repeatedly. They follow simple rules of thumb and use trials and errors in revising their rules on the basis of information they acquire through local experiences and observations. It is assumed that there are substantial inertia in this process because of limited information: Agents' observations may be imperfect, their knowledge of how payoffs depend

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[8]This is true even if some kind of inertia are imposed in changing strategies: see Matsuyama (1991a) for the limitations of equilibrium dynamics in selecting a steady state.

[9]Recent studies in adaptive and evolutionary approach include Binmore (1990), Fudenberg and Maskin (1990), Gilboa and Matsui (1990), Kandori, Mailath and Rob (1991) and Matsui (1990). For somewhat related work, see Marimon, McGratten, and Sargent (1990).

on strategy choices may be inaccurate, and changing behavioral patterns may be costly. Given the presence of inertia, only a small fraction of agents changes their rules each period. And those who change will adopt the strategy that is the best response to the current strategy distribution among the population: they know that only a small fraction of the population changes its behavior at any given point in time and, hence, rules that proved to be effective today are likely to remain effective for some time to come. The economy evolves along the best response dynamics. A steady state equilibrium is considered as a stationary point in this evolutionary dynamic process.[10] This adaptive, or to use Binmore's term, "evolutive" interpretation of a steady state equilibrium seems particularly appropriate in our model for two reasons. First, it only requires that the agents follow simple rules or behavioral patterns, from which no agent would be interested in deviating unilaterally, and thus provides a description of monetary exchange as a social custom. Second, it helps to explain how a particular equilibrium may emerge in a dynamic context.

To illustrate the second point, consider the case of $1 - 1/\sqrt{2} < m < 1/2$, the situation depicted in Figure 3b and assume that $n < 1/2$. Imagine that, at the beginning, there is no interaction between the two economies ($\beta = 0$) and Equilibrium A prevails. Then the process of economic integration begun, and β started increasing gradually. A small fraction of Home agents may notice the change in the environment and accept the Foreign currency on an experimental basis. But, as long as β remains small and less than $A(n)$, no Home agent has

[10]To quote Lucas (1986, p.S403), "Technically, I think of economics as studying decision rules that are steady states of some adaptive process, decision rules that are found to work over a range of situations and hence are no longer revised appreciably as more experience accumulates."

an incentive to switch his rule and accept the Foreign currency. Nor does a Foreign agent have an incentive to accept the Home currency. As β continues to rise, it eventually crosses the locus of $\beta = A(n)$. At this point, $V_f > V_g$ holds, and some Home agents start revising their rules and accepting the Foreign currency. Other agents in the Home economy may notice that the Home agents who accept the Foreign currency are doing well, and start imitating. As the fraction of the Home agents accepting the Foreign currency increases, the incentive for the rest of the Home agents to do the same becomes even stronger. This process would continue until all Home agents accept the Foreign currency. On the other hand, no Foreign agent has an incentive to change his rules throughout this process. As long as (n,β) belongs to the region where Equilibrium F exists and $\beta < A^*(n)$ is satisfied, accepting the Foreign but not the Home currency remains the Foreign agent's best response, provided that other Foreign agents follow the same rule, no matter what fraction of the Home agents accepts the Foreign currency. The economy would thus converge to Equilibrium F. The Foreign currency emerges as the international currency.

After Equilibrium F is reached, how a further increase in β affects the evolution of the economy depends on whether $n < m$ or $m < n < 1/2$. If $n < m$, then an increase in β eventually leads to $f(n,\beta) < 0$, or $V_g > V_h$. Some Home agents start rejecting the Home currency, and this process would continue until no agent accepts the Home currency. The Foreign agents have no incentive to change their rules. The economy would thus converge to the equilibrium where only the Foreign currency is accepted in each economy; it becomes the unique medium of exchange, which is circulated worldwide. The situation that resembles dollarization appears in this case. On the other hand, if $m < n < 1/2$, then an increase in β leads to $f^*(n,\beta) > 0$, or $V_h^* > V_g^*$. Some Foreign agents start accepting the Home currency, and this process continue until Equilibrium U emerges.

The case of $n > 1/2$ is similar. As β increases, Equilibrium H is reached first, and the Home currency emerges as the international currency. If $1/2 < n < 1 - m$, then a further increase in β leads to Equilibrium U. If $1-m < n$, then the Foreign currency is eventually abandoned. The Home currency becomes the only medium of exchange and circulated worldwide. The evolutionary outcomes described above are summarized in Figure 4b.

For the case of $0 < m < 1 - 1/\sqrt{2}$, the situation given in Figure 3a, the evolutionary process would be similar patterns, unless $\nu < n < 1 - \nu$, where ν is defined by $(1-\nu)^3 = \nu^2(1-m)^2$. When $\nu < n < 1/2$, then an increase in β would eventually eliminate Equilibrium A because Home agents start accepting the Foreign currency as soon as the economy crosses $\beta = A(n)$. But this process would not last forever without causing an additional change in behavioral patterns of Foreign agents. [This can be seen because the economy now belongs to the region in which Equilibrium F does not exist.] The circulation of the Foreign currency in the Home country, although the higher acceptance rate of the Foreign currency makes it even more attractive than the Home currency, would create the shortage of a medium of exchange in the Foreign country. When $m = m^*$ is small, this dilution effect makes the Home currency attractive as an alternative medium of exchange for the Foreign commodity holders ($V_h^* > V_g^*$). Thus, some Foreign agents will start accepting the Home currency. Once both Home and Foreign agents start changing their behavioral patterns, this process would accelerate and continue until Equilibrium U emerges. Similarly, when $1/2 < n < 1 - \nu$, an increase in β

eliminates Equilibrium A, by first inducing the Foreign agents to accept the Home currency, which in turn leads to the acceptance of the Foreign currency by the Home agents. Again, Equilibrium U emerges. The evolutionary outcomes for the case of $0 < m < 1 - 1/\sqrt{2}$, are depicted in Figure 4a.

In either case, the evolutionary dynamics discussed above helps us to tie down the equilibria that would emerge over the process of economic integration. First, the currency of a larger country emerges as the international currency. If the size distribution of the two economies is sufficiently uneven, a further integration would eliminate the local currency. Otherwise, both currencies would be eventually circulated in both countries.

5. Welfare Implications

We now turn to some welfare implications. We focus on steady state utility levels. In view of Proposition f), it suffices to evaluate $W = m_hP_{hg}+m_fP_{fg}$ for the Home welfare level and $W^* = m_h^*P_{hg}^*+m_f^*P_{fg}^*$ for the Foreign welfare level. Table 2 lists the values of W and W^* (multiplied by k) in Equilibria A, F, H, and U. Several points seem to deserve special emphasis.

First, W (W^*) is increasing in m (m^*) for (0,1/2) and decreasing for (1/2,1) in Equilibrium A. Similarly, both W and W^* are increasing in the world per capita currency supply, $nm + (1-n)m^*$, for (0,1/2) and decreasing for (1/2,1) in Equilibrium U. A rise in per capita money supply initially increases the rate of consumption by facilitating transactions among agents. But it eventually decreases the rate of consumption, since too much money means too few commodity holders under the assumption of inventory holding restrictions. The possibility of welfare reducing money supply increases may be plausible if consumption is reinterpreted as inputs to production. In an economy where the price level is artificially fixed at a too low level, few firms are able to produce because of difficulty in acquiring inputs. And, because of low production levels, few firms can in fact get hold of inputs in spite of huge cash balances. The economy is thus trapped into underproduction equilibrium.[11]

Second, suppose $m = m^*$, then $W < W^*$ if $n < 1/2$ and $W > W^*$ if $n > 1/2$ in Equilibrium A. Similarly, for any m and m^*, $W < W^*$ if $n < 1/2$ and $W > W^*$ if $n > 1/2$ in Equilibrium U. This is to say that, once the per capita currency supply is controlled for, agents living in a larger economy enjoy a higher level of the steady state utility. This result is a reflection of increasing returns inherent in the matching technology.

Third, if $m = m^*$, the welfare level is higher in Equilibrium U than in Equilibrium A in both economies. In this sense, our model predicts that too many currency areas may coexist in the absence of any intervention, and unification of currencies could be welfare enhancing.[12]

Fourth, one may examine net benefits of an international currency from the viewpoint of the country issuing it.[13] This can be done by comparing W in Equilibria A and that in Equilibrium H (or comparing W^*'s in Equilibria A

[11]Arguably, this situation captures a problem of a centrally planned economy with suppressed inflation, where "too much money is chasing too few goods." If money were divisible in our model, inflation would reduce real balances and thus a welfare reducing money supply increase would not occur in equilibrium.

[12]This result may be of some interest in view of recent debates on European monetary integration. We are not aware of any previous studies which demonstrated the possibility of too many currency areas in a formal model. The benefits of common or unified currency have usually been assumed, despite their essential role, in the literature of optimal currency area, which dates back at least to Mundell (1961). Much effort in this literature has been devoted to explain the costs or difficulty of monetary unification.

[13]This problem has also attracted much attention in policy debates. For example, Cohen's (1971) main concern was the costs and benefits of pound sterling as an international currency from the Britain's viewpoint.

and F). As is clear from Table 2, if the domestic circulation of the Home currency, m_h, is controlled for, a switch from A to H would improve the welfare of Home agents. One can show, however, that it could reduce the level of Home welfare if the total supply of the Home currency, m, is fixed; for example, this is the case if $n < (1-m)[1+n-(1-n)m^*]$ and β is sufficiently small. This possibility arises because circulation of the Home currency abroad may create currency shortage at Home.

On the other hand, if the Home country is short of currency supply, then a switch for Equilibrium A to F could increase its welfare level; a sufficient condition is given by $m < (1-m)(1-m_f^*)$, where m_f^* is the domestic circulation of the Foreign currency <u>at Equilibrium F</u>. Likewise, the Foreign agents would be better off in Equilibrium H than in A, if $m^* < (1-m^*)(1-m_h)$, where m_h is the domestic circulation of the Home currency <u>at Equilibrium H</u>.

6. Currency Exchanges: Mixed Strategy Equilibria

In all equilibria we have investigated so far, there is no trade entailing an exchange of the two currencies ($P_{fh} = P_{hf} = P_{fh}^* = P_{hf}^* = 0$). This is a direct consequence of our assumption that an agent agrees to trade if and only if the trade results in a strict increase in his expected utility. This can be proved as follows. In order to generate currency exchanges, the steady state inventory distributions of both countries needs to have positive stocks of both currencies. This means that both Home and Foreign agents accept both currencies in such an equilibrium. Under the assumption mentioned above, this requires that all commodity holders <u>always</u> accept both currencies. This makes the two currencies perfect substitutes, and therefore there will be no currency exchanges. In this section, we show that, once agents are allowed to trade even when they are indifferent, equilibria in which the two currencies are exchanged can be constructed.

In particular, we consider what we call Equilibrium M, in which

a) *A Home agent always trades his production good for the Home currency, the Foreign currency for both the Home currency and his consumption good, and the Home currency for his consumption good. He is indifferent between his production good and the Foreign currency, and trades the former for the latter with a positive probability* ($u + V_g > V_h > V_g - V_f$).

b) *A Foreign agent always trades his production good for the Foreign currency, the Home currency for both the Foreign currency and his consumption good, and the Foreign currency for his consumption good. He is indifferent between his production good and the Home currency, and trades the former for the latter with a positive probability* ($u + V^*_g > V^*_f > V^*_g - V^*_h$).

Let $\pi = \tau_{gf}$ ($\pi^* = \tau^*_{gh}$): the probability with which a Home (Foreign) agent accepts the Foreign (Home) currency. Then, the transition probabilities for a Home agent satisfy

$$(6\text{-}1)\qquad \begin{cases} P_{gh} = [nm_h + \beta(1-n)m^*_h]/k\ , & P_{gf} = [nm_f + \beta(1-n)m^*_f]\pi/k\ , \\ P_{hg} = [n(1-m_h-m_f)+\beta(1-n)(1-m^*_h-m^*_f)\pi^*]/k\ , & \\ P_{fg} = [n(1-m_h-m_f)\pi+\beta(1-n)(1-m^*_h-m^*_f)]/k\ , & \\ P_{fh} = \beta(1-n)m^*_h\ , & P_{hf} = 0. \end{cases}$$

The transition probabilities for a Foreign agent can be given similarly. Furthermore, the steady state requires

$$(6\text{-}2)\qquad \begin{cases} (1-m_h-m_f)m^*_f\pi = m_f[(1-m^*_h-m^*_f) + km^*_h]\ , \\ (1-m^*_h-m^*_f)m_h\pi^* = m^*_h[(1-m_h-m_f) + km_f]\ . \end{cases}$$

From Proposition a) and b), it suffices to check the two incentive constraints

$$(6\text{-}3) \qquad V_g = V_f , \qquad V_g^* = V_h^* ,$$

in order to make all agents willing to follow their prescribed equilibrium strategies. Equilibrium M exists if there exist π and $\pi^* \in (0,1)$ for which (6-3) holds, given (6-1) and (6-2). The volume of currency exchanges is equal to $nm_f P_{fh} = (1-n)m_h^* P_{hf}^* = \beta n(1-n)m_f m_h^*$.

We restrict our attention to the symmetric case, $n = 1/2$ and $m = m^* \in (0,1)$, and search for the symmetric equilibrium, $\pi = \pi^* \in (0,1)$ and $m_f = m_h^* \in (0, m)$. After some algebra, one can show that the symmetric Equilibrium M exists uniquely if and only if

$$(6\text{-}4) \qquad \beta < m/(1+2k\delta) .$$

As in the other equilibria, the degree of economic integration cannot be too large to support Equilibrium M, and the range would be smaller if the degree of specialization and the discount rate is high. One can also show that both $\pi = \pi^*$ and $m_f = m_h^*$ depend negatively on β, k and δ. As $\beta(1+2k\delta)$ approaches m, they go down to zero, and so does the volume of currency exchanges. This is because, as the two economies are more integrated or with a higher degree of specialization, a Foreign agent would find the Home currency more attractive, given π or m_f. In order to keep him indifferent between the Home currency and his production good, the probability with which he could exchange the Home currency for the Foreign currency needs to be reduced, which requires a lower fraction of Home agents carrying the Foreign currency; that is, a lower π and a lower m_f. Note also the similarity of (6-4) with the conditions for Equilibrium A, as can be seen from imposing $m = m^*$ and $n = 1/2$ in (3-5) or (3-6), which yields $\beta \leq m/(1+2k\delta)$.[14]

7. Concluding Remarks

We have formulated a two-country, two-currency model of the world economy as a random matching game of monetary exchanges. Because of nonuniformity of the matching process, the two national fiat currencies can compete and may be circulated as media of exchange. As one would expect in any model of money, there are multiple equilibria. In our model, equilibria differ in the areas of circulation of the two currencies. In order to generate sharper predictions on the evolution of an international currency in spite of multiple equilibria, we discuss an evolutionary approach to equilibrium selection, which is used to explain how the international medium of exchange emerges as the world economy becomes more integrated. By comparing different equilibria, the model also provided some implications on costs and benefits of an international currency.

The model is highly stylized and is not meant to be the final product. In particular, the structure is not rich enough to allow for any policy discussion. First of all, the national governments are not modelled explicitly. One possible way of formalize the national government in this model would be to introduce one large agent in each economy, whose measure is strictly positive and who can make a commitment of accepting only the national currency. The second shortcoming of the model is the indivisibility of fiat currencies as well as the strong restriction on inventory holding,

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[14]We conjecture, but have not demonstrated, that the existence region for Equilibrium A is equal to the closure of the existence region for Equilibrium M.

which makes it impossible to talk about a variety of important issues, such as inflation, exchange rate stability. Despite these limitations, however, we believe that our model has yielded many new insights on the fundamental issues in international monetary economics. It is hoped that our model will serve as a first step toward a more satisfactory theory of international currency.

Appendices

Proof of Proposition:

Equations (2-1) through (2-3) can be rewritten to, in the short hand notations,

(A-1) $$[(1+\delta)I - \Pi]V = uQ\ ,$$

where I is the 3x3 identity matrix, $V' = [V_g, V_h, V_f]$ and $Q' = (0, P_{hg}, P_{fg})$. Since Π is a stochastic matrix, its Frobenius root is one. Thus, $(1+\delta)I - \Pi$ has the nonnegative inverse matrix for a $\delta > 0$, so $V = u[(1+\delta)I - \Pi]^{-1}Q$. This proves the first inequality in Proposition a). One can also show, after some algebra,

(A-2) $$u + V_g - V_h = (\delta+P_{hf}+P_{fh}+P_{fg})(\delta+P_{gf}+P_{gh})\delta u/\Delta > 0\ ,$$

(A-3) $$u + V_g - V_f = (\delta+P_{hf}+P_{fh}+P_{hg})(\delta+P_{gf}+P_{gh})\delta u/\Delta > 0\ ,$$

(A-4) $$V_h - V_f = (P_{hg}-P_{fg})(\delta+P_{gf}+P_{gh})\delta u/\Delta\ ,$$

(A-5) $$V_h - V_g = [P_{hg}(\delta+P_{gf}+P_{fg})+P_{fh}P_{hg}+P_{hf}P_{fg}-P_{gf}P_{fg}]\delta u/\Delta\ ,$$

(A-6) $$V_f - V_g = [P_{fg}(\delta+P_{gh}+P_{hg})+P_{fh}P_{hg}+P_{hf}P_{fg}-P_{gh}P_{hg}]\delta u/\Delta\ ,$$

where $\Delta = \det[(1+\delta)I - \Pi] > 0$. The second inequality in Proposition a) is from (A-2) and (A-3). Proposition c), d) and e) are from (A-4), (A-5), and (A-6), respectively. To prove Proposition b), let us first suppose Max $\{V_h, V_f\} \le V_g$. Then, from (2-1), $(1+\delta)V_g \le V_g$, or $V_g = 0$, which in turn implies $V_h = V_f = 0$. Next, if $V_h > V_g$, then $P_{gh} = [nm_h + \beta(1-n)m_h^*]/k > 0$, so $P_{gh}V_h > 0$. Likewise, $P_{gf}V_f > 0$ if $V_f > V_g$. Thus, Max $\{V_h, V_f\} > V_g$ implies $P_{gh}V_h + P_{gf}V_f > 0$, which in turn implies $V_g > 0$ from (2-1). This proves Proposition b). Finally, multiplying X from the left on both sides of (A-1) and using $X\Pi = X$ yield $\delta XV = uXQ$, or $\delta[(1-m_h-m_f)V_g+m_hV_h+m_fV_f] = [m_hP_{hg} + m_fP_{fg}]u$. This proves Proposition f).

Appendix on Equilibrium F: All properties of the equilibrium region, (3-13) and (3-14), discussed in the text can be verified by noting that f and f^* satisfy:

i) $f(n,1) = f^*(n,1)$, $f(0,0) = f^*(1,0) = 0$,

ii) $f(n,\beta)/n^3 = (1-m)(1+\theta-m)[k\delta(1+\theta)+\beta\theta+1-m] - m^*\theta(1-m+\beta\theta)^2 = \Phi(\theta)$, where $\theta = (1-n)/n$. Since, for any $\beta \in (0,1]$, Φ is a third order equation in θ satisfying $\Phi(0) > 0$, $\Phi'(0) > 0$ and $\Phi''' < 0$, $\Phi = 0$ is a unique positive solution. Thus, for any $\beta \in (0,1]$, $f(n,\beta) = 0$ has a unique solution $n(\beta) \in (0,1)$, and $f(n,\beta) < 0$ for $n \in (0,n(\beta))$ and $f(n,\beta) > 0$ for $n \in (n(\beta),1)$. Similarly for f^*.

iii) f (f^*) is quadratic in β, has a negative (positive) coefficient on β^2, and a positive (negative) constant term, so that $\beta f_\beta|_{f=0} = [\beta f_\beta - f]|_{f=0} < 0$ ($\beta f^*_\beta|_{f^*=0} = [\beta f^*_\beta - f^*]|_{f^*=0} > 0$).

iv) f is a third order equations of 1-m that has positive coefficients on $(1-m)^2$ and $(1-m)^3$, and a negative constant term, so that $(1-m)f_{(1-m)}|_{f=0} = [(1-m)f_{(1-m)} - f]|_{f=0} > 0$. Similarly for f^*.

iv) $f_{k\delta}$, $f^*_{k\delta} > 0$, and f_{m^*}, $f^*_{m^*} < 0$.

Appendix on Equilibrium M: Using (6-1) and applying the symmetry properties, $n = 1/2$, $\pi = \pi^*$, $m_h = m^*_f$, $m_f = m^*_h$, $m = m^* = m_h + m_f = m^*_h + m^*_f$, (6-2) can be rewritten to,

(A-7) $$(1-m)(m-m_f)\pi = m_f(1-m+km_f) \ ,$$

and (6-3) becomes

(A-8) $$M(\pi,m_f) = (\pi+\beta)\{2k\delta+(1-m)(1+\beta\pi)\} + k\beta m_f[1+\beta\pi] - [m-m_f+\beta m_f](1-\beta)(1-\pi) = 0 \ .$$

The symmetric equilibrium exists if there are $\pi \in (0,1)$ and $m_f \in (0, m)$, which satisfy (A-7) and (A-8) for a given $m \in (0,1)$. As shown in Figure A, the locus of (A-7) passes (0,0); it is upward-sloping and goes to infinity asymptotically as $m_f \to m$. On the other hand, for a given $m_f \in (0, m)$, $M(\pi, m_f)$ is increasing in $\pi > 0$ and positive at $\pi = 1$. Thus, $M(\pi, m_f) = 0$ has a unique solution in $\pi \in (0,1)$ for a given $m_f \in (0, m)$, if and only if $M(0, m_f) < 0$, or

(A-9) $$m > \beta(2k\delta+1) + \{1+\beta(k+\beta-2)\}m_f .$$

Since $\beta(k+\beta-2) > 0$, the locus of $M = 0$ does not intersect that of (A-7) in the relevant range if $m \leq \beta(2k\delta+1)$. On the other hand, if $m > \beta(2k\delta+1)$, $M = 0$ intersects with $m_f = 0$ at $\pi \in (0,1)$; it intersects with $\pi = 0$ at $m_f = \{m-\beta(2k\delta+1)\}/\{1+\beta(k+\beta-2)\} \in (0,m)$, and it has a negative slope between the two intersections, as shown in Figure A. (The monotonicity comes from that M_{m_f} is independent of m_f.) This establishes that (6-4) is the necessary and sufficient condition for the existence of the unique symmetric equilibrium. Furthermore, an increase in β, k, or δ shifts down the $M = 0$ locus (since M_β, M_k, $M_\delta > 0$), therefore reduces both π and m_f along the locus of (A-7).

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TABLE 1: The Matching Technology

	Home agent	Foreign agent	Nobody
Home agent	n	$\beta(1-n)$	$(1-\beta)(1-n)$
Foreign agent	βn	$1-n$	$(1-\beta)n$

TABLE 2: Welfare Evaluations

Equilibrium A:

$$kW = nm(1-m) ,$$
$$kW^* = (1-n)m^*(1-m^*) .$$

Equilibrium F:

$$kW = (1-m)(1-m_f^*)[nm+m_f^*(n(1-m)+\beta(1-n))] ,$$
$$kW^* = m_f^*(1-m_f^*)[1-n+\beta n(1-m)] ,$$
$$\text{with } m_f^* = (1-n)m^*/(1-nm) .$$

Equilibrium H:

$$kW = m_h(1-m_h)[n+\beta(1-n)(1-m^*)] ,$$
$$kW^* = (1-m^*)(1-m_h)[(1-n)m^*+m_h((1-n)(1-m^*)+\beta n)] ,$$
$$\text{with } m_h = nm/(1-(1-n)m^*) .$$

Equilibrium U:

$$kW = [nm+(1-n)m^*][n(1-m)+(1-n)(1-m^*)][n+\beta(1-n)] ,$$
$$kW^* = [nm+(1-n)m^*][n(1-m)+(1-n)(1-m^*)][\beta n+(1-n)] .$$

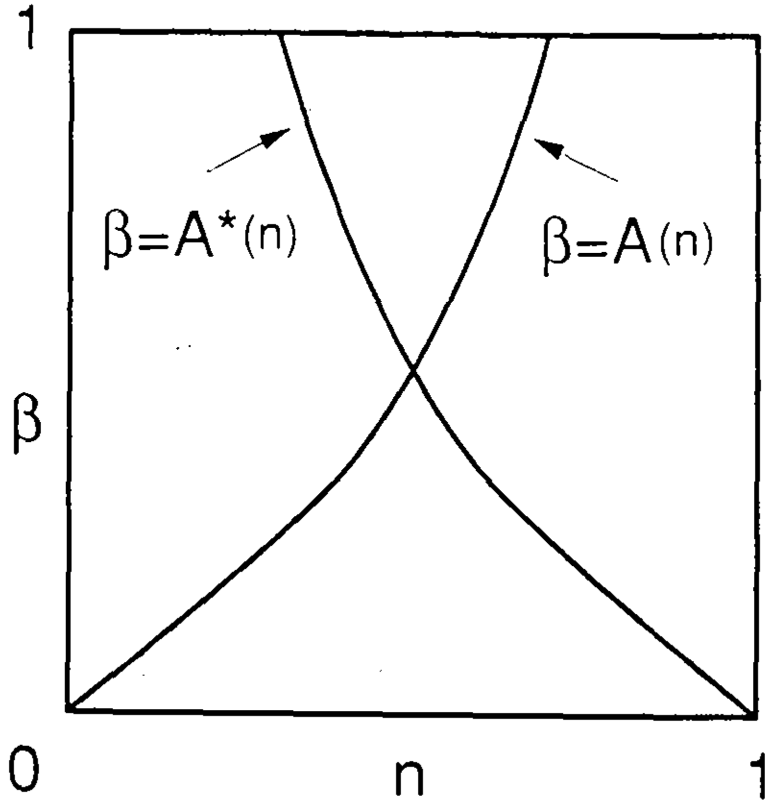

Figure 1: Equilibrium A

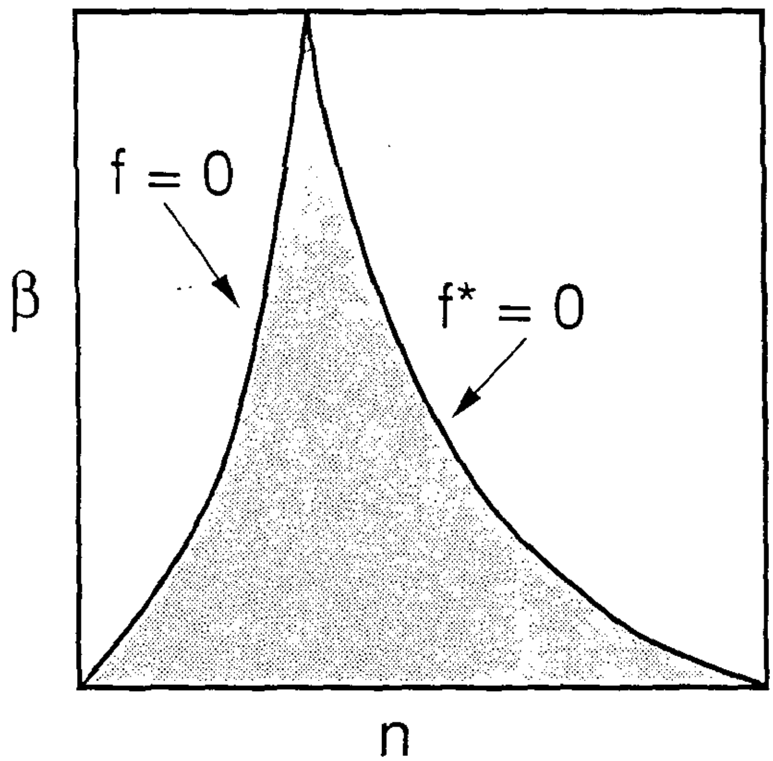

Figure 2: Equilibrium F

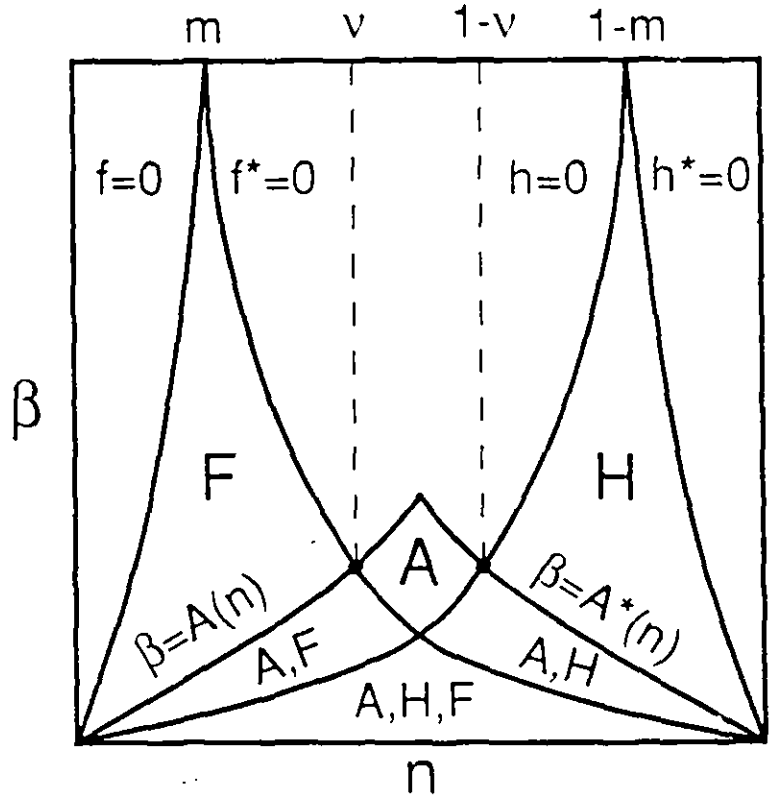

Figure 3a

$(0<m<1-\frac{1}{\sqrt{2}})$

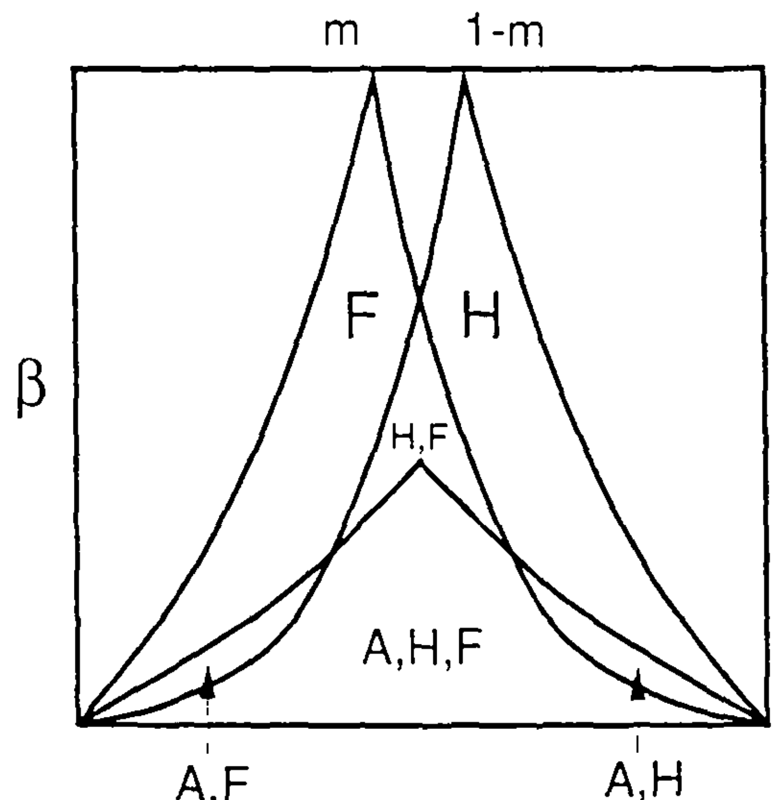

Figure 3b

$(1-\frac{1}{\sqrt{2}}<m<\frac{1}{2})$

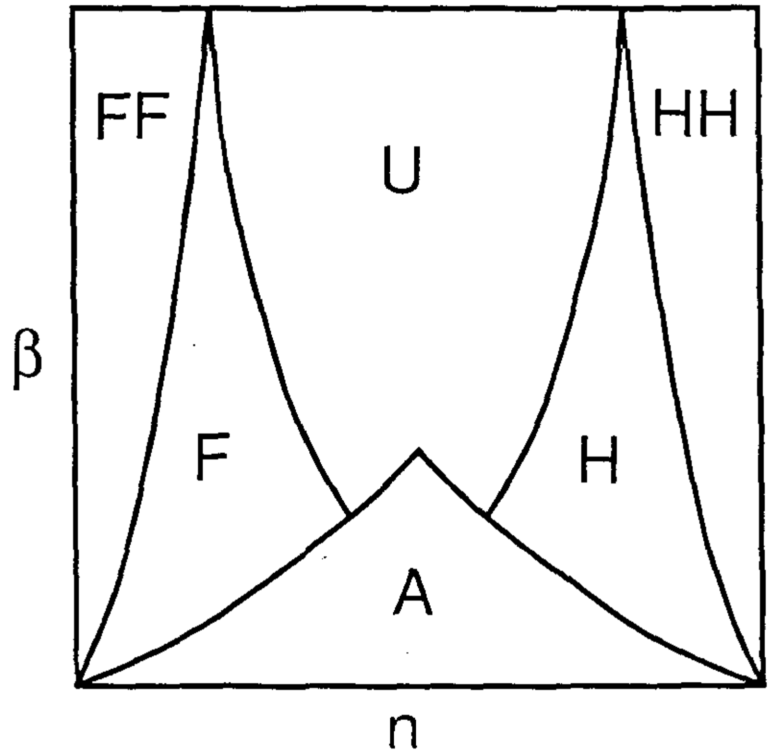

Figure 4a

$(0<m<1-\frac{1}{\sqrt{2}})$

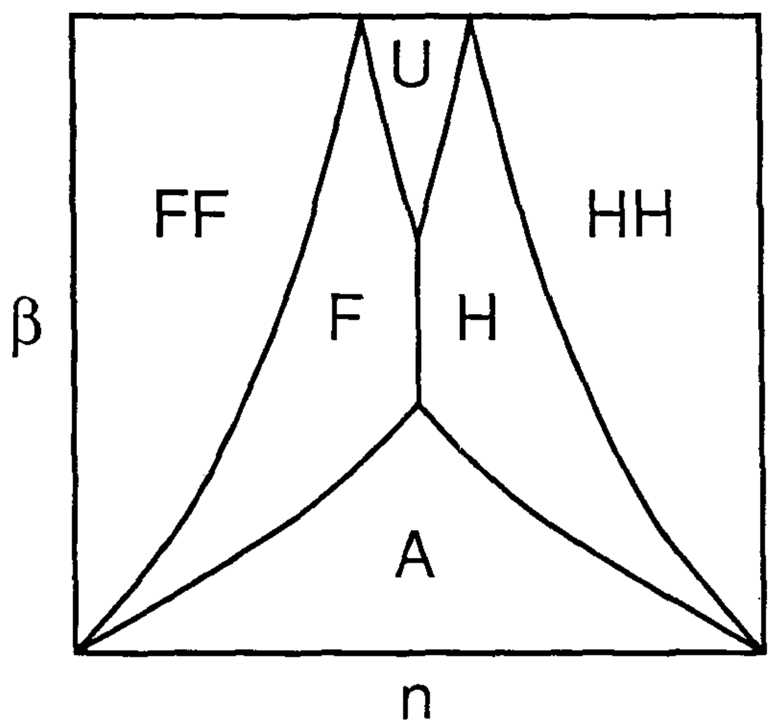

Figure 4b

$(1-\frac{1}{\sqrt{2}}<m<\frac{1}{2})$

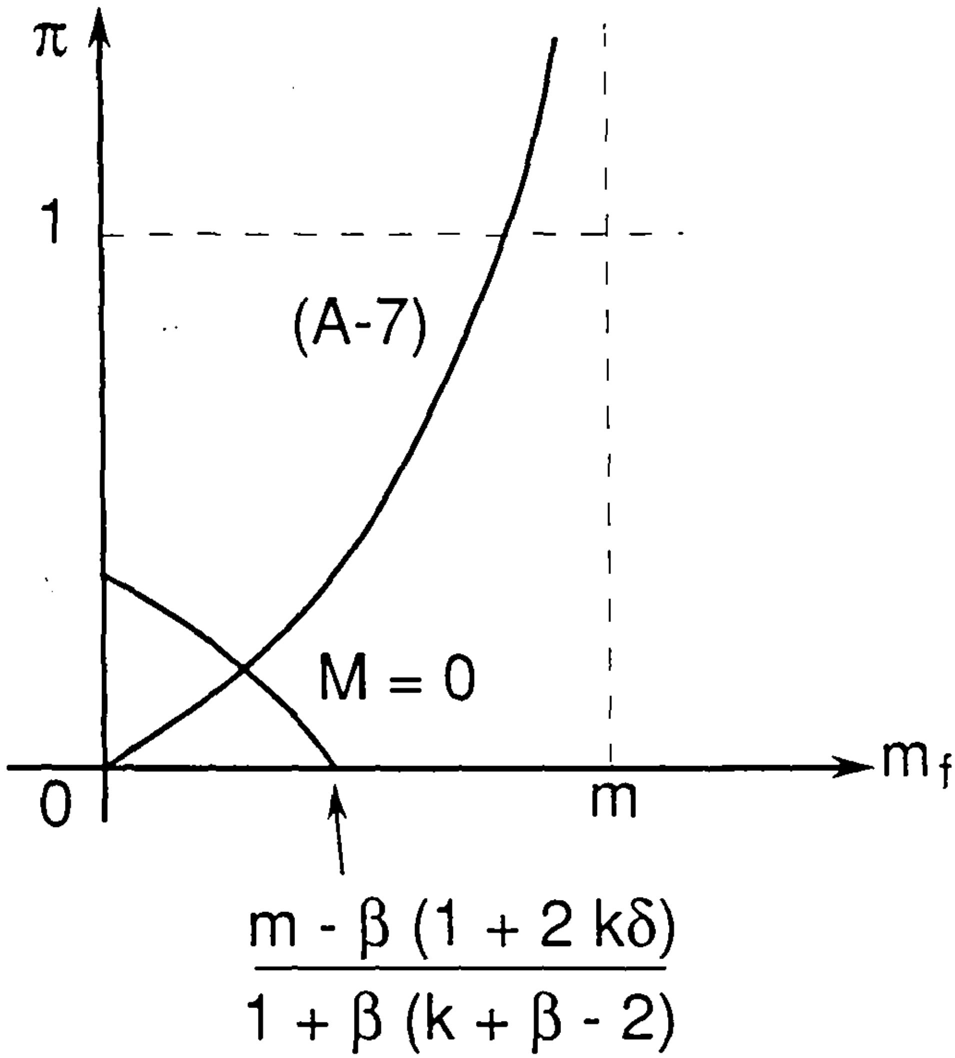

Figure A: Equilibrium M