ADVERSE SELECTION IN A NEOCLASSICAL GROWTH MODEL

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July 1991

* We have benefitted from comments on earlier versions of this paper by seminar participants at Duke, Iowa, the University of Western Ontario, UCLA, USC, Stanford, Berkeley, NYU, Johns Hopkins, Cornell and the Board of Governors of the Federal Reserve System. Roger Guesnerie's encouragement and advice make him a virtual coauthor of this paper, except for errors which remain our responsibility.

1. INTRODUCTION

A. The Issues

We explore in this paper interactions between financial and capital deepening in the one-sector neoclassical growth model of Diamond (1965) with heterogeneous producers, idiosyncratic technological uncertainty and two alternative credit-market structures: complete financial intermediation, and partial intermediation. Complete intermediation, made possible by public information about agent characteristics, exploits fully the law of large numbers. All investment is financed by bank loans; all consumption is deterministic; and individual risks do not affect capital accumulation, which is well described by a scalar dynamical system.

Partial intermediation appears when individual characteristics are privately known. We study the implications for growth of Rothschild-Stiglitz separating equilibria induced by credit rationing. These equilibria require consumers and producers to bear part of their individual risk. Capital accumulation then generally satisfies a planar dynamical system with two state variables: the capital stocks of rationed and unrationed producers. Comparing equilibria under the two structures will permit us to explore how private information complexifies the dynamical behavior of neoclassical growth models and gauge the effect of credit rationing on the economywide capital-labor ratio in the steady state. In addition, the credit rationing induced by private information has implications for the distribution of income, which we investigate.

Neoclassical growth models have become increasingly useful tools with which colleagues investigate traditional macroeconomic issues at business cycle and lifecycle frequencies. Wouldn't it be nice to know how robust the properties of those growth models are to the assumption that factors of production are hired in perfect markets? In particular, does the introduction of private information about the personal characteristics of borrowers influence the existence, number and asymptotic stability of steady states?

The questions posed in the previous paragraph are not merely pedagogical: they are directed at long-standing issues in economic theory and in growth accounting. At the most theoretical level we wish to know whether adverse selection in credit markets contributes to economic volatility: Does it strengthen an economy's tendency towards multiple steady states or its propensity to cycle? At the most applied level, we need to specify for empirical work what measures of credit market "thickness", if any, belong on the right-hand side of growth regressions. Is there a connection between economic growth and measures of financial market "depth"?

We are directed to the first of these questions by work on the excess volatility of consumption [Hayashi (1985), Zeldes (1989)], on borrowing constraints in dynamical general equilibrium [Bewley (1986), Woodford (1988)], on atemporal general equilibrium with incomplete markets [Geanakoplos and Mas-Colell (1989)], and on financial fragility [Bernanke and Gertler (1989)]. All four literatures study situations in which economic agents are no longer bound by a single budget constraint, as they would be if markets were complete or perfect. Individuals face instead a sequence of dated, state-contingent or idiosyncratic constraints whose consequence is some form of economic volatility. That volatility appears as unusual sensitivity of consumption to current household income, of corporate investment to retained earnings, or else as an unusually large number of locally non-unique general equilibria, including periodic equilibria. We propose to study adverse selection because it furnishes one convenient reason why some markets are imperfectly developed or missing altogether.

The second, more applied, question is one of long standing in economic development, dating back to research on financial deepening by Gurley and Shaw (1967) who investigated the tendency of the financial sector to expand relative to the economy as a whole along most observed growth paths.

By "financial deepening" we generally mean the evolution of credit markets toward the ideal mode of operation assumed in the theory of competitive general equilibrium: borrowers and lenders of the same risk type face the same interest rate, and everyone may borrow up to the present value of his lifetime income.

Judging how close actual credit markets come to being perfect is not a trivial matter; one may use various yardsticks to measure the efficiency of financial intermediaries, e.g., their aggregate size, the gap between the yield on low-risk bank loans and bank deposits, the incidence of credit rationing, etc.[1] For the time being we take the view that economic growth and financial depth are in part joint consequences of secular reductions in informational frictions that allow creditors to process more cheaply signals correlated with the personal attributes of borrowers. Institutions like credit bureaus, instalment plans and mortgages develop which facilitate the hiring of capital at some cost and, in turn, stimulate growth; see Greenwood and Jovanovic (1990) for related work along this line.

B. Intermediation and Growth

What are banks supposed to do in a growing economy? What service is performed by intermediaries that makes the credit market so important to economic development? The very term "intermediary" means a go-between, in this case one that links ultimate borrowers with ultimate lenders and helps smooth lifecycle consumption. If this link were broken, producers would be unable to go into debt and all investment would have to be financed by retained earnings and other internal sources of funding.

Another important function of banks, pointed out by Tobin (1963), is to spread the risk of individual investment projects over a large and diverse asset portfolio, much as insurance companies spread the risks of individual accidents over a large number of policy holders. Each loan finances an investment project that will go bankrupt with some probability and force the intermediary to take a loss of interest or even of the principal amount originally lent out. Intermediaries allow for these losses by adjusting their loan rates to reflect the risk of bankruptcy, exactly as insurance companies adjust the premia of their auto insurance policies for risks of fire, collision and other accidents.

If the risks of bankruptcy for individual projects are independent, an intermediary that finances many relatively small projects (i.e., one that does not tie up a significant fraction of its loans with any single borrower) is assured by the law of large numbers that the actual rate of return on its loan portfolio will be very close to the rate the bank expected to earn before it learned the outcome of the investment projects it is funding. That means well-diversified loan portfolios will earn normal rates of return.

Economies with highly developed credit markets enable financial intermediaries to hold loan portfolios that are free from idiosyncratic risks specific to individual borrowers. These banks are able, in turn, to offer their depositors private insurance in the form of contracts that guarantee the yield on deposits. Such guarantees are not possible in economies with imperfect credit markets; in these, borrowers must finance some fraction of their investment costs out of their own equity income, from retained earnings or current cash flow, and are thereby obliged to bear some or all of their individual investment risks.

To grasp what smoothly operating financial markets mean for economic development, we shall compare the operating properties of two benchmark economies: a fairly standard one-sector overlapping generations economy of heterogeneous producers with a perfect credit market; and an economy in which lenders ration credit to induce self-selection among privately informed borrowers. Heterogeneity is essential to this comparison: any lack of financial intermediation is unlikely to affect a representative-agent economy in which the typical individual saves in youth exactly as much as he will need to borrow in old age.

What general equilibrium implications should one expect from adverse selection? Azariadis and Smith (1991) explored this issue in a pure-exchange economy of overlapping generations, and report three main results. Given certain technical conditions (including normality and gross substitutability in consumption), adverse selection produces lower yields on inside money than does complete information; it biases outside money equilibria away from negatively valued national debt towards positively valued debt; and it does not necessarily injure the welfare of borrowers, even if they are rationed, for it is likely to lower loan rates compared to full information. The intuitive explanation for these results in that credit rationing discourages borrowers at every interest rate and converts "classical" economies, in the sense of Gale (1973), to "Samuelson" ones. If this bias becomes sufficiently large, adverse selection may monetize economies that would have no role for fiat money under full information.

In a production economy, of course, adverse selection will influence factor incomes as well as exchange prices. This observation has several implications. First, in the absence of national debt, steady state equilibrium interest rates must be lower under private than under full information. [Recall that Azariadis-Smith (1991) produces an analogous result for exchange economies.] Under the same conditions, private information may support either a smaller or a larger steady-state economywide capital stock per worker than would arise under full information. Interestingly, then, credit rationing need not reduce long-run capital accumulation. Furthermore, under full information our economy must have agents whose endowments are ex ante equivalent earning identical incomes. Under private information, a non-trivial distribution of income will emerge. We also consider the same issues in the presence of national debt. When national debt is non-zero, credit rationing must reduce capital formation in the steady state. However, credit rationing will continue to result in an unequal distribution of income.

These and other possibilities are explored in the remainder of this paper which is organized as follows: Section 2 introduces notation and reviews dynamical equilibria under public information in the one-sector growth model with heterogeneous producers. Section 3 brings in private information, discusses financial contracts and proves the existence of proper separating equilibria. General equilibrium takes up section 4 which discusses how credit rationing causes qualitative changes in the dynamical behavior of a growing economy, especially in the number of stable steady states and the speed of convergence to them. These changes are explored further in section 5 with the help of several illustrative examples. Section 6 sums up our findings.

2. PUBLIC INFORMATION

This section describes a simple generalization of Diamond's (1965) one-sector overlapping-generations model of capital accumulation. Our aim here is to set up a benchmark by adding heterogeneous producers to an otherwise well-known dynamical economy, and look at its full-information equilibrium. Readers who are familiar with this sort of exercise may wish to skim the basic equations and move on to sections 3 and 4.

We consider an infinite horizon economy, with discrete time indexed by $t = 0,1,\ldots$. The economy consists of an infinite sequence of three-period-lived overlapping cohorts. Each generation is identical in size and composition, containing a continuum of agents with unit mass.

There are two factors of production, labor and capital, used to produce a single, non-storable consumption good. Young agents are endowed with neither labor, capital, nor the consumption good. Middle-aged agents are endowed with one unit of labor which they supply inelastically to producers and nothing else.[2] Old agents have no endowment of labor, capital, or the consumption good. Finally, we assume that labor is a <u>non-traded</u> factor of production, so that each middle-aged agent is self-employed. The reasons for this assumption are discussed in section 3.

Middle-aged agents own a technology that converts labor and capital into the consumption good. Capital is essential to production, and it must be put in place one period in advance of production. Thus, a young individual must borrow whatever capital is to be employed in middle age. Capital depreciates fully after use.

In particular, we assume that each young agent borrows from an intermediary. A loan contract obtained at t-1 specifies a loan quantity, D_t, and a gross loan rate, R_t. Young agents then choose an investment level, $k_t \in [0, D_t]$, which is unobservable by the intermediary. Any uninvested portion of the loan, $D_t - k_t$, can be anonymously re-deposited at another intermediary. Deposits earn the sure gross deposit rate r_t between t-1 and t.

Young individuals come in two types indexed by $i \in \{H, L\}$. A young person of type i, who invests k_t units in capital at t-1, has a random output of Q_t^i at t. Q_t^i has the probability distribution

$$\begin{cases} f(k_t)/p_i & \text{with probability } p_i \\ 0 & \text{with probability } 1-p_i \end{cases} \tag{1}$$

where $f(k) = F(k,1)$ is an intensive production function[3] derived from the neoclassical production function $F: \mathbb{R}_+^2 \to \mathbb{R}_+$. We assume, as usual,[4] that f is twice continuously differentiable, increasing, concave and such that $f(0)=0$. Finally, we suppose that $1 > p_L > p_H$, so that type-H agents have a high probability of failure. The fraction of young agents who are of type H is $\lambda \in (0,1)$.

Loans are repaid only in the observable event of positive production. If, on the other hand, output is zero, producers are in a state of bankruptcy: their entire capital is foregone without any damage to them; the loss is borne by financial intermediaries which process reliable advance information as to the type of each borrower. Let y_t^i denote the income at t of any agent who agreed to the loan contract (D_t, R_t) at t-1, invested $k_t \in [0, D_t]$, and was a successful producer at t. Then

$$y_t^i = f(k_t)/p_i - R_t D_t + r_t(D_t - k_t) \tag{2}$$

Similarly, if $\hat{y}_t^i$ denotes the income of an otherwise identical agent who was not a successful producer at t, then

$$\hat{y}_t^i = r_t(D_t - k_t) \tag{3}$$

To simplify the exposition at little cost in generality, we suppose that agents care only about old age consumption, denoted c. Let $u: \mathbb{R}_+ \to \mathbb{R}$ be the common utility function of all individuals, and assume that u is thrice continuously differentiable, increasing and concave. Given this pattern of individual consumption, all middle-aged income is re-deposited with intermediaries at t, earning the gross deposit yield r_{t+1} between t and t+1. A young agent at t-1 who obtains the loan contract (D_t, R_t), and invests k_t achieves an expected utility level of $p_i u(r_{t+1} y_t^i) + (1-p_i) u(r_{t+1} \hat{y}_t^i)$. There will be no loss of generality in regarding all savings as intermediary deposits.

We will consider the behavior of this economy with and without valued national debt. Let z_t denote the outstanding quantity of national debt at t. This debt is default-free, and hence pays the sure gross return r_t between t-1 and t.

A. Credit and Investment

A key component of the model is the relationship between investment and loan contracts. We describe here the optimal choice of $k_t \in [0, D_t]$ for a type i agent who has obtained an arbitrary loan contract (D_t, R_t).

For a given loan contract, a type i agent chooses $k_t \in [0, D_t]$ to maximize $p_i u(r_{t+1} y_t^i) + (1-p_i) u(r_{t+1} \hat{y}_t^i)$, subject to (2) and (3). Assuming an interior solution, the optimal choice of k_t satisfies the first order condition

$$[f'(k_t) - p_i r_t] u'(r_{t+1} y_t^i) = (1-p_i) r_t u'(r_{t+1} \hat{y}_t^i) \tag{4}$$

Then equations (2)-(4) give k_t as a function of (D_t, R_t, r_t, r_{t+1}), and the parameter p_i which represents the agent type. In other words, there is a function ϕ such that

$$k_t = \phi(D_t, R_t, r_t, r_{t+1}; p_i) \tag{5}$$

It is straightforward to check that the partial derivatives of ϕ have the following properties: $\phi_1 > 0$ if $R_t \geq r_t$ (which will hold in equilibrium); $\phi_2 > 0$; $\phi_5 > 0$ if $y_t^i \geq \hat{y}_t^i$; and ϕ_3 is of ambiguous sign. In addition, $\phi_1 < 1$ if $f'(k) \geq p_i R_t$ (which again will hold in equilibrium).

Finally, the ratio ϕ_4/ϕ_1 plays an important role in subsequent analysis. Define $\rho(c) \equiv -cu''(c)/u'(c)$ to be the coefficient of relative risk aversion. Then it is easy to show that

$$\frac{\phi_4}{\phi_1} = \frac{[\rho(r_{t+1}\hat{y}_t^i) - \rho(r_{t+1}y_t^i)]/(r_t r_{t+1})}{[\rho(r_{t+1}\hat{y}_t^i)/\hat{y}_t^i] + (1-p_i/p_i)[\rho(r_{t+1}y_t^i)/y_t^i]} \tag{6}$$

Thus ϕ_4 has the same sign as $\rho(r_{t+1}\hat{y}_t^i) - \rho(r_{t+1}y_t^i)$. Finally observe that $f'(k) \geq r_t$ iff $y_t^i \geq \hat{y}_t^i$.

It will also be useful to have some notation for the maximized value of utility, given D_t, R_t, r_t, and r_{t+1}. Hence we define the indirect expected utility function

$$\begin{aligned} V(D_t, R_t, r_t, r_{t+1}; p_i) \equiv \max_{k_t \in [0, D_t]} \{p_i u\{r_{t+1}[f(k_t)/p_i - \\ R_t D_t + r_t(D_t - k_t)]\} + (1-p_i)u\{r_{t+1}\, r_t(D_t - k_t)\}\} \end{aligned} \tag{7}$$

V describes expected utility as a function of the loan contract and of the interest-rate sequence observed during an agent's lifetime.

B. Financial Intermediaries

The second integral component of our analysis is the behavior of intermediaries. On the deposit side we assume that intermediaries are competitive, acting as if they can obtain any desired quantity of deposits at the competitive gross deposit rate r_t. On the loan side we assume that intermediaries are Nash competitors who announce loan contract terms (D_t^i, R_t^i) to agents of type i, taking the announcements of other intermediaries as given. We assume that there is free entry into the activity of intermediation and no cost in converting deposits into loans. Borrowers simply accept their most preferred loan terms from among the set of offered contracts. We will show how these assumptions reproduce Walrasian outcomes under full information.

The assumption of free entry implies that, in equilibrium, intermediary profits must be zero, so that

$$R_t^i = r_t/p_i \quad \text{for} \quad i = H,L. \tag{8}$$

In addition, competition among intermediaries for borrowers implies that equilibrium loan contracts maximize the indirect expected utility function $V(D_t^i, R_{it}, r_t, r_{t+1}, p_i)$ subject to (8). In other words,

$$D_t^i = \underset{D\geq 0}{\operatorname{argmax}}\, V(D, r_t/p_i, r_t, r_{t+1}; p_i) \tag{9}$$

At any interior maximum, then

$$V_1(D_t^i, r_t/p_i, r_t, r_{t+1}; p_i) = 0 \tag{10}$$

From the definition of V, equation (10) is easily shown to be equivalent to

$$f'[\phi(D_t^i, r_t/p_i, r_t, r_{t+1}; p_i)] = p_i r_t^i = r_t. \tag{10'}$$

Thus all agents invest to the point where the expected marginal product of capital equals the implicit rental rate, that is, the gross deposit rate. Moreover, from equations (4) and (10′), we have

$$y_t^i = \hat{y}_t^i \quad \text{for} \quad i = H,L. \tag{11}$$

Thus all consumers obtain complete insurance against the idiosyncratic risks they face.

Using equations (2), (3) and (11), we see easily that

$$D_t^i = f(k_t)/r_t = [f(k_t)-r_t k_t]/r_t+k_t = [f(k_t)-k_t f'(k_t)]/r_t+k_t, \tag{12}$$

Equivalently, we define $w(k) \equiv f(k) - kf'(k)$ and obtain instead

$$D_t^i = k_t + w(k_t)/r_t. \tag{12'}$$

Furthermore, $y_t^i = \hat{y}_t^i = w(k_t)$ for each i. Thus, middle-aged agents receive income equal to the marginal product of labor. Each young agent borrows enough to guarantee an income of $w(k_t)$ in middle age, independently of how successful his investment project is, plus enough to finance a capital investment k_t such that $r_t=f'(k_t)$.

C. The Geometry of Loan Contracts

To facilitate future discussion we need a diagram showing how Nash equilibrium contracts are determined. In figure 1, then, the loci $R_t^i = r_t/p_i$ represent the zero profit contracts for type i agents. In addition, the preferences of type i agents with respect to loan contracts can be represented by indifference maps. An indifference curve of a type i agent in figure 1, for given values r_t and r_{t+1}, is a locus of the form $V(D,R,r_t,r_{t+1};p_i) = \beta$, where β is a parameter. Then the slope of a type i agent indifference curve through any contract (D,R) is given by

$$\left(\frac{dR}{dD}\right)_{dV_i=0} = -\frac{V_1(D,R,r_t,r_{t+1};p_i)}{V_2(D,R,r_t,r_{t+1};p_i)} = \frac{f'[\phi(D,R,r_t,r_{t+1};p_i)]}{p_i D} - \frac{R}{D}. \tag{13}$$

Thus an indifference curve has the shape shown in figure 1, with a peak at $f'[\phi(\cdot)] = p_i R$.

In equilibrium (D_t^i, R_t^i) maximizes $V(D,R,r_t,r_{t+1};p_i)$ subject to $p_i R_t^i \geq r_t$. In other words, equilibrium contracts occur at points of tangency between type-i agent indifference curves and the corresponding zero profit locus (points H and L in figure 1).

It will be useful for future reference to know something about the relative slopes of the type H and type-L agent indifference curves through any (D,R) pair. We now state

__Lemma 1.__ $f'[\phi(D,R,r_t,r_{t+1};p_i)]/p_i$ is a decreasing function of p_i.

This Lemma is proved in the appendix. From (13), and Lemma 1 it follows that, at any (D,R) pair, type-H agents' indifference curves have an algebraically larger slope than do those of type-L agents.

D. __General Equilibrium without National Debt__

Dynamical equilibria are situations in which net borrowing by young individuals equals net saving by middle-aged ones. Each young borrower receives a loan $k_t + w(k_t)/r_t$, and re-deposits

$w(k_t)/r_t$. Thus net borrowing in period t by each young consumer is k_t. Similarly, each middle-aged agent has an income $w(k_{t-1})$ in period t, all of which is saved. Thus, the equality of savings to net borrowing requires for each $t \geq 1$[5]

$$k_{t+1} = w(k_t). \tag{14}$$

Equation (14) is exactly the equilibrium condition that emerges from Diamond's OLG growth model when there is no national debt, and the savings rate is one. Under full information, our model reduces to Diamond's if agents care only about old age consumption. It bears emphasizing that the provision of complete insurance against individual risk prevents the equilibrium law of motion from depending on individual characteristics such as λ, p_H, or p_L at any date other than the initial one. Solutions to equation (14) are <u>independent of personal attributes</u> because the law of large numbers operates perfectly under public information; it neutralizes the higher moments of all idiosyncratic random variables and permits investment projects to be evaluated by their expected yield alone. This will change under private information. Moreover, all agents have identical incomes for any $t \geq 1$. This, too, will change under private information.

Given some fairly mild conditions[6], equation (14) has at least two steady-state equilibria, a trivial one with $k_t = 0\ \forall t$, and a positive one with $k^* = w(k^*)\ \forall t$. The positive steady state will be asymptotically stable, and the equilibrium sequence $\{k_t\}$ will be either monotonically increasing or decreasing. These properties are illustrated in figure 2. We assume henceforth that (14) has at least one positive stationary solution, as in figure 2.

E. General Equilibrium with National Debt

National debt is a bubble in OLG models. When government expenditures are zero, national debt expands at the interest rate, i.e.,

$$z_{t+1}/z_t = r_{t+1}, \quad t \geq 1 \tag{15}$$

In addition, net saving must equal net borrowing by young agents plus the value of outstanding national debt. The market-clearing condition is replaced with

$$k_{t+1} = w(k_t) - z_t, \quad t \geq 1 \tag{16}$$

Equations (15), (16), and $r_t = f'(k_t)$ are the equilibrium conditions for this economy. All these conditions are independent of λ, p_H, and p_L; the income distribution in this economy is a trivial one.

There are now generally three steady-state equilibria under our assumptions: the two inside-money equilibria discussed previously with $z_t=0$, plus a third one, our outside-money equilibrium, with $z_t \neq 0$ and $f'(k_t)=1$ for all t. The asymptotic stability properties of these equilibria depend on whether k^*, the positive stationary solution to (14), is above or below the golden rule.[7]

3. PRIVATE INFORMATION: LOAN CONTRACTS

We assume from now on that the type of each agent is private information. Given r_t and r_{t+1}, Nash equilibrium contract announcements by financial intermediaries must necessarily be incentive compatible. Thus, for i = H,L, equilibrium loan contracts (D_t^i, R_t^i) satisfy the self-selection constraints

$$V(D_t^H, R_t^H, r_t, r_{t+1}; p_H) \geq V(D_t^L, R_t^L, r_t, r_{t+1}; p_H) \tag{17}$$

$$V(D_t^L, R_t^L, r_t, r_{t+1}; p_L) \geq V(D_t^H, R_t^H, r_t, r_{t+1}; p_L) \tag{18}$$

for each t. Following Rothschild and Stiglitz (1976) and Wilson (1977), we assume that each contract offered must earn non-negative expected profits, so that

$$R_t^i \geq r_t/p_i \ \forall(i,t) \tag{19}$$

For a contract that pools agents in their population proportions, the loan rate must satisfy[8]

$$R_t \geq r_t/[\lambda p_H + (1-\lambda)p_L] \tag{20}$$

Observe that the equilibrium loan contracts of section 2 violate equation (17). This follows from the fact that, under full information, $D_t^H = D_t^L$, $R_t^H > R_t^L$, and that indirect expected utility is a decreasing function of the loan rate. We now turn our attention to characterizing Nash equilibrium loan contracts under private information.

In order to do so, we note first that Lemma 1 and equation (13) imply a standard "single crossing" property for the indifference curves of type H and L agents. In particular, at any point (D,R) in figure 1, the indifference curve of a type-H agent has an algebraically larger slope than the indifference curve of a type-L agent through the same point. Then arguments identical to those given by Rothschild and Stiglitz (1976) establish that any Nash equilibrium contract announcements induce self-selection. Free entry into the intermediary sector implies, in turn that equilibrium loan rates satisfy (19) with equality. Finally, competition among intermediaries for borrowers ensues that equilibrium contracts for type i agents must be maximal for them among the set of contracts that (a) earn non-negative profits for banks, and (b) and are incentive compatible in the presence of other announced contracts.

Now is a good time to explain why we assumed that labor is not a traded input. If it were traded and production occurred under constraint returns to scale, as one ordinarily assumes in growth theory, proper separating equilibria would not exist at all. The size of each firm would be indeterminate if profit per unit input is zero. On the other hand, if profit per unit input is positive, then the loan demand by constant-returns producers is likely to be infinite: a bank loan yields positive profits when the borrower is solvent and zero profits (no cost, no revenue) when the borrower is bankrupt. In the absence of bankruptcy costs and similar non-convexities, the use of credit rationing against low-risk borrowers will induce self-selection but equilibrium is not well defined; any pair of separating contracts can be pareto-dominated by another pair that charges the same loan rate and extends more credit to producers. That is why we need to regard all work as firm specific or entrepreneurial.

Based on these observations we conjecture, and then verify, that any Nash equilibrium in pure strategies specifies (D_t^H, R_t^H) in the same way as under full information. Thus $R_t^H = r_t/p_H$ and

$$D_t^H = \underset{D \geq 0}{\operatorname{argmax}}\, V(D, r_t/p_H, r_t, r_{t+1}; p_H)$$

for each t. Arguments identical to those in section 2 then establish that type-H agents choose a capital stock k_t so that $f'(k_t) = r_t$, and hence

$$D_t^H = w(k_t)/r_t + k_t \tag{21}$$

This contract provides type-H agents with complete insurance, so that the expected utility they obtain under it is given by $u[r_{t+1}\, w(k_t)]$.

The equilibrium contract for type-L agents must be maximal for them among the set of contracts that earn non-negative profits when taken by type L agents, and are incentive compatible in the presence of the contract (D_t^H, R_t^H). As figure 1 shows, this contract is defined by the point labelled L_1, where the type-H indifference curve through $(D_t^H, r_t/p_H)$ intersects the type-L agent's zero profit locus. Thus D_t^L is the (smallest) solution to

$$V(D_t^L, r_t/p_L, r_t, r_{t+1}; p_H) = u[r_{t+1}\ w(k_t)] \tag{22}$$

We denote the solution to equation (22) by

$$D_t^L = d[w(k_t), r_t, r_{t+1}].$$

Properties of the function d are investigated below. For future reference we note that $(D_t^L, r_t/p_L)$ lies on the upward sloping portion of a type-L agent indifference curve. Therefore

$$V_1(D_t^L, r_t/p_L, r_t, r_{t+1}; p_L) = r_{t+1}u'(r_{t+1}y_t^L)[f'(\phi(\cdot\,; p_L)) - p_L R_t^L] > 0, \tag{23}$$

which implies

$$f'[\phi(\cdot\,; p_L)] > r_t \tag{24}$$

The contracts just derived constitute a Nash equilibrium if they are incentive compatible, and if no intermediary can profitably offer any other contracts in their presence. Incentive compatibility follows from the construction of the contracts and the single crossing property for preferences; in particular, it is apparent from figure 1 that type L agents prefer $(D_t^L, r_t/p_L)$ to $(D_t^H, r_t/p_H)$. We now investigate the incentives to offer other loan contracts.

A. Existence of a Nash Equilibrium in Pure Strategies

Given the contract (D_t^H, r_t, p_H), no intermediary can find an alternative contract that attracts type-H agents alone and earns a non-negative expected profit. Similarly, given the contract $(D_t^L, r_t/p_L)$, no intermediary can offer a contract that attracts type-L agents only, and earns a non-negative profit. Thus, if any intermediary offers an alternative contract, that contract must attract all agents in their population proportions. For such a contract to earn zero profit or better, the contract must charge a loan rate that satisfies (20). The contract must offer a loan quantity D and an interest rate R that L-type agents prefer to $(D_t^L, r_t/p_L)$. Thus

$$V(D,R,r_t,r_{t+1};p_L) > V(D_t^L,r_t/p_L,r_t,r_{t+1};p_L) \tag{25}$$

Then a Nash equilibrium in pure strategies fails to exist iff there exists a pair (D,R) satisfying (20) and (25).

Such a (D,R) pair cannot exist iff

$$\max_{D\geq 0} V[D, r_t/(\lambda p_H + (1-\lambda)p_L), r_t, r_{t+1}; p_L] \\ \leq V(D_t^L, r_t/p_L, r_t, r_{t+1}; p_L) \tag{26}$$

From figure 1 (and in particular, from the single crossing property for preference maps) it is apparent that (26) holds for $\lambda=1$, and hence by continuity, for any λ sufficiently close to one. As in Rothschild-Stiglitz (1976), Nash equilibrium loan contracts in pure strategies exist if λ is large enough.[9]

We restrict our attention to situations that satisfy equation (26).

B. <u>Equilibrium Loan Quantities</u>

Equation (22) gives the equilibrium loan quantity $D_t^L = d[w(k_t), r_t, r_{t+1}]$. Observing that $r_t = f'(k_t)$ holds for all t, we may write

$$D_t^L = d[w(k_t), f'(k_t), f'(k_{t+1})] \equiv g(k_{t+1}, k_t). \tag{27}$$

The properties of the function g play an important role in subsequent analysis. We summarize these in

<u>Lemma 2:</u> The function $g: \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$ is decreasing in its second argument. It is increasing (decreasing) in its first argument if relative risk-aversion is decreasing (increasing).

Lemma 2 is proved in the appendix. Nash equilibrium loan contracts are completely described by equations (21), (27), and (19) taken as an equality.

It is worth noting that type-H and type-L agents who invest the same amount in capital will have identical expected returns on their investments. However, type-H investors will have a higher probability of loan default, and in this sense, constitute lower quality borrowers. It may appear somewhat odd to ration the higher quality type-L borrowers group, so we offer two additional comments. First our result is similar to automobile insurance contracts that require self-declared "safe" drivers to choose a policy with a large deductible and, thus, to bear some accident risk. Second, development economists commonly assert [see, e.g., McKinnon(1973), p. 8] that many "low quality" projects are fully funded while "higher quality" projects are not because of rationing. Our result provides some explanation why this outcome is observed.

4. DYNAMICAL EQUILIBRIA WITH CREDIT RATIONING

A. Inside-Money Dynamics

Denote by

$$k_t^L \equiv \phi(D_{Lt}, r_t/p_L, r_t, r_{t+1}; p_L) \tag{28}$$

the optimal capital investment at t-1 by a young type-L agent having the loan contract $(D_t^L, r_t/p_L)$. Then k_t^L is also net borrowing by that agent. Similarly, net borrowing by a young type-H person at t-1 is $k_t = (f')^{-1}(r_t)$. Net saving by middle-aged type-H individuals at t is then $w(k_t)$, while net saving by a successful (unsuccessful) middle-aged L-type producers is $y^L_t(\hat{y}^L_t)$, where

$$y^L_t \equiv f(k_{Lt})/p_L - (r_t D_{Lt})/p_L + \hat{y}^L_t \tag{29}$$

$$\hat{y}^L_t \equiv r_t(D_{Lt} - k_{Lt}) \tag{30}$$

Dynamical equilibria require net borrowing by young agents to equal net saving by middle-aged agents at each t, that is,

$$\lambda k_{t+1} + (1-\lambda)k^L_{t+1} = \lambda w(k_t) + (1-\lambda)[p_L y^L_t + (1-p_L)\hat{y}^L_t]. \tag{31}$$

From (29, (30) and $k_t = f'(r_t)$, one obtains

$$p_L y^L_t + (1-p_L)\hat{y}^L_t = f(k^L_t) - r_t k^L_t = f(k^L_t) - k^L_t f'(k_t) \tag{32}$$

Substituting (32) into (31) yields

$$\lambda k_{t+1} + (1-\lambda)k^L_{t+1} = \lambda w(k_t) + (1-\lambda)[f(k^L_t) - k^L_t f'(k_t)] \tag{33}$$

Furthermore, from equations (27) and (28), we obtain

$$k^L_t = \phi[g(k_{t+1}, k_t), f'(k_t)/p_L, f'(k_t), f'(k_{t+1}); p_L]. \tag{34}$$

Equations (33) and (34), which hold for each $t \geq 1$, are the equilibrium conditions for this economy.

The properties of dynamical equilibria depend heavily on the partial derivative term

$$\frac{\partial k_t^L}{\partial k_{t+1}} = \phi_1[g_1 + (\phi_4/\phi_1)f''(k_{t+1})].$$

Since $\phi_1 > 0$, $\partial k_t^L/\partial k_{t+1}$ has the same sign as $g_1 + (\phi_4/\phi_1)f''(k_{t+1})$. If this term is zero, k_t^L depends only on k_t in (34); then (33) and (34) constitute a <u>first-order dynamical system</u>. If $g_1+(\phi_4/\phi_1)f''$ does not vanish, then equation (34) can be implicitly solved for k_{t+1} to obtain

$$k_{t+1} = \eta(k_t, k_t^L) \tag{35}$$

The pair of equations (33) and (35) are a <u>second-order dynamical system</u>.

It is therefore of interest to understand the conditions under which $g_1+(\phi_4/\phi_1)f''$ vanishes. From equation (6) and Lemma 2, $g_1+(\phi_4/\phi_1)f'' = 0$ if $\rho(c)$ is a constant function. In this event, the economy displays <u>first-order</u> dynamics. However when $\rho(c)$ is not constant, $g_1+(\phi_4/\phi_1)f''$ is quite difficult to sign. Below we present examples in which that expression may be of either sign when evaluated at a non-trivial steady state and, hence, equations (33) and (35) constitute a <u>second-order</u> non-linear dynamical system.

If λ is sufficiently near one, the existence of a non-trivial steady state equilibrium under full information guarantees the existence of a non-trivial steady state equilibrium under private information. it is quite difficult to characterize analytically even the local dynamics of (33) and (35) in the neighborhood of a steady state. However, we have computed a number of numerical examples (see subsection E below). In each of them the non-trivial steady state equilibrium is a saddlepoint.

It appears that the presence of private information substantially complicates the dynamic behavior of this economy. We next turn our attention to stationary solutions of (33) and (35) under private information.

B. Stationary States

Steady state equilibria are pairs $(\tilde{k},\tilde{k}_L)$ that satisfy

$$\lambda\tilde{k}+(1-\lambda)\tilde{k}_L = \lambda w(\tilde{k})+(1-\lambda)[f(\tilde{k}_L)-\tilde{k}_L f'(\tilde{k})] \tag{36}$$

$$\tilde{k} = \eta(\tilde{k},\tilde{k}_L) \tag{37}$$

Throughout we focus on non-trivial solutions $(\tilde{k},\tilde{k}_L) > 0$ to these equations. Note immediately that

Proposition 1: $\tilde{k} > k^* > \tilde{k}_L$.

Proposition 1 is proved in the appendix. It asserts that, at a non-trivial steady state, type-H agents invest more under private information than they do under full information, while type-L agents invest less. An interesting question concerns the relationship between average per capita capital stocks under full versus private information. Below we present examples where $\lambda\bar{k}+(1-\lambda)\tilde{k}_L \gtrless k^*$. Thus credit rationing does not necessarily lower the steady state capital-labor ratio when national debt is zero. However, since $\bar{k}>k^*$ and $r=f'(\bar{k})$, credit rationing does reduce real interest rates in the absence of national debt. This result is in accordance with standard assertions in the literature on financial markets and development [see, e.g., McKinnon (1973) and Shaw (1973)], and parallels similar results for pure exchange economies [Azariadis and Smith (1991)].

C. Income Distribution

Under full information, all middle-age individuals have income $w(k_t)$ at time t. In a private information equilibrium, middle-age agents continue to receive real income $w(k_t)$ if they choose a capital stock of k_t. The expected income of a middle-aged type L agent at t is given by equation (32). Since $k_t^L < k_t$ holds, the income of these agents is $f(k_t^L)-k_t^L f'(k_t) < w(k_t)$, falling short of what H-type individuals earn from production.

In fact, at each date there will be a three-point income distribution under private information: all type-H agents will have income $w(k_t)$, successful type-L producers have income y_t^L, and unsuccessful type-L producers will earn $\hat{y}_t^L$. We demonstrate by example below that, at a steady state equilibrium, either $y^L > w(\bar{k})$

$> \hat{y}^L$ or $w(\tilde{k}) > y^L > \hat{y}^L$ may hold. Thus credit rationing gives rise to a non-trivial distribution of income. Who will be at the top of the income distribution is generally ambiguous.

We also make one additional point. It is often asserted [see, for instance, Kuznets (1966)] that income inequality is greatest in wealthier economies. In section 5 we present an example where, depending on initial conditions, an economy will either have k_t converging to zero or else to $\bar{k} > 0$. Economies with $k_t \to 0$ will achieve asymptotic income equality while economies with $k_t \to \tilde{k}$ will experience permanent income inequality. These equilibria seem to agree with observed relations between income levels and income inequality.

D. Welfare Considerations

One generally expects adverse welfare consequences from credit rationing and similar incentive constraints. But who bears these consequences? In partial equilibrium, all welfare losses fall on those experiencing binding constraints[9]. Agents whose presence creates an adverse selection problem, (e.g., our type-H borrowers) would neither gain nor lose as a result of the adverse selection problem.

This is not the case, in general equilibrium: type-H agents may either gain or lose as a result of adverse selection. To see this, note that the utility of a type-H agent in a steady state equilibrium is $u[f'(k)w(k)]$. These agents are better (worse) off under private than under full information if, and only if

$$f'(\tilde{k})w(\tilde{k}) > (<) \; f(k^*)w(k^*) \tag{38}$$

assuming that the steady state is positive.

The function $I(k) \equiv f'(k)w(k)$ is easily seen to be increasing in k if

$$f(k)-2kf'(k) \leq 0 \tag{39}$$

If (39) holds as a strict inequality for all $k \geq k^*$ then $u[f'(\tilde{k})w(\tilde{k})] > u[f'(k^*)w(k^*)]$, and type-H borrowers benefit in the steady state from the presence of adverse selection. If (39) fails for all $k \geq k^*$, then H-types are injured by adverse selection. Since the inequality sign in (39) can easily go either way, type-H agents may either benefit or lose from private information.[10] If they lose, then type-H agents are injured by the option of misrepresenting their type.

Unless type-H producers benefit from private information, type-L agents are hurt by it. This follows from the observation that, at a steady state equilibrium, expected consumption of type-L agents under private information is $f'(\tilde{k})[f(\tilde{k}_L)-\tilde{k}_L f(\tilde{k})]<f'(\tilde{k})w(\tilde{k})\leq I(k^*)$, whereas under full information, their sure consumption is $I(k^*)$. In this event, type-L agents derive lower expected consumption and experience greater risk with private information than they do with public information.

E. Some Examples

We illustrate the main points of this section in a series of numerical examples which demonstrate that: (i) under private information, equations (33) and (35) describe a second-order nonlinear dynamical system if relative risk aversion is not constant; (ii) the sign of $g_1+f''(\cdot)\phi_4/\phi_1$ may be either positive or negative; (iii) the average per capita capital stock in the steady state may be higher under private than under public information in steady state; and (iv) at the steady state equilibrium, income distribution may assign the top income level to either rationed or unrationed producers. Finally we note that, for each example presented, there is a non-trivial stationary equilibrium that corresponds to a Nash equilibrium in pure strategies for intermediaries.

All examples have the following structure: $u(c) = \ln(c+\beta)$, and $f(k) = Ak^{\alpha}$, with $\alpha = .3$. Then the index of relative risk-aversion is increasing if, and only if $\beta>0$. Also, we consider two values of A, A=5 and A=10.

Under full information, the non-trivial steady state equilibrium depends only on technological parameters: $k^* = [A(1-\alpha)]^{1/(1-\alpha)}$, which equals 5.987 for A=5 and 16.117 for A=10.

Under private information, non-trivial steady state equilibria depend on the parameters (λ,p_H,p_L,β) s well. We illustrate some of the possibilities below.

<u>**Example 1**</u>: $A=10$, $p_H=.2$, $p_L=.9$, $\beta=.5$, $\lambda=.99$. For this configuration of parameters, $g_1+f''\phi_4/\phi_1 = .05$, when evaluated at the steady state equilibrium.

<u>**Example 2**</u>: Identical to example 1, except that $\beta=-.5$. Then $g_1+f''\phi_4/\phi_1=-.053$ when evaluated at the steady state equilibrium.

<u>**Example 3**</u>: $A=5$, $p_H=.3$, $p_L=.5$, $\beta=-1.5$, $\lambda=.25$. For this economy, $\tilde{k} = 15.025$, $\tilde{k}_L=4.443$, and $\lambda\tilde{k}+(1-\lambda)\tilde{k}_L=7.088$. Thus the average per capita capital stock is higher under private than under full information. In addition, $w(\tilde{k})=7.891$, $y^L=6.911$, and $\hat{y}^L=6.731$. Therefore $w(\tilde{k})>y^L>\hat{y}^L$, which makes H-agents the richest individuals.

<u>**Example 4**</u>: Identical to example 3, except that $\beta=-1$ and $\lambda=.4$. Here $\tilde{k}=10.329$, $\tilde{k}_L=4.302$, $\lambda\tilde{k}+(1-\lambda)\tilde{k}_L=6.713>k^*$, $w(\tilde{k})=7.052$, $y^L=7.894$, and $\hat{y}^L=5.081$. For this economy, $y^L>w(\tilde{k})>\hat{y}^L$ which makes successful L-producers the richest individuals.

Suppose that for each of these economies we linearize (33) and (35) in a neighborhood of the steady state, and let η_1 and η_2 denote the eigenvalues of the linearized system. Then for example 1, $\eta_1<-1<0<\eta_2<1$ holds, while for examples 2 through 4, we have $0<\eta_1<1<\eta_2$. For each example, the steady state is a saddle; orbits starting on the stable manifold converge to this state monotonically.

F. National Debt

Suppose again that the outstanding stock of national debt is $z_t \neq 0$. This default-free debt is perpetually refinanced while taxes and government purchases remain at zero. In this case the equilibrium condition (31) is replaced by

$$\lambda k_{t+1} + (1-\lambda) k^L_{t+1} = \lambda w(k_t) + (1-\lambda)[p_L y^L_t + (1-p_L)\hat{y}^L_t] - z_t \tag{40}$$

so that savings equal investment plus the real value of national debt at t. In addition, debt will have the same yield as sure bank deposits, that is

$$z_{t+1}/z_t = r_{t+1} = f'(k_{t+1}) \tag{41}$$

If we substitute (32) into (40), we obtain

$$\lambda k_{t+1} + (1-\lambda) k^L_{t+1} = \lambda w(k_t) + (1-\lambda)[f(k^L_t) - k_{Lt} f'(k_t)] - z_t \tag{42}$$

Equations (34), (41), and (42) constitute the complete set of equilibrium conditions which hold for all $t \geq 1$.

If relative risk aversion is constant, equation (34) gives k^L_t as a function of k_t alone. Then k^L_t can be substituted out of (42), leaving (41) and (42) as a pair of first-order difference equations. However, if $\rho'(c) \neq 0$, then (34) can be implicitly solved for k_{t+1} to yield equation (35), as before. In this event, (35), (41), and (42) constitute a system of three non-linear first-order difference equations. Thus, as in section A, private information complexities the equilibrium dynamics of the economy relative to the situation under full information.

Fortunately, stationary equilibria are not all that different. A steady state equilibrium with national debt satisfies $f'(k) = 1$. Thus, if national debt is not zero, credit rationing does not change capital accumulation by type-H agents, the steady state interest rate, or type-H agents, the steady state interest rate, or type-H agent utility.[11] Consequently, type-L agents bear all the effects of private information. Moreover, since $f'(k_L)>f'(k) = 1$, type-L producers accumulate <u>less</u> capital than under full information. Therefore, credit rationing reduces the steady average per capita capital stock. This result conforms to standard results in the development literature[12] about the consequences of credit rationing on investment.

5 CREDIT RATIONING WITH RISK-NEUTRAL AGENTS

This section explores the effect of credit rationing on capital accumulation in an economy inhabited by risk-neutral individuals. As we know from sections 3 and 4, the rationing of low-risk barrowers in this economy depends only on current business conditions, not on anticipated ones. As a result, dynamical equilibria satisfy a <u>first-order</u> difference equation in the capital-labor ratio, both for publicly and for privately informed economies.

Risk neutrality and, more generally, constant relative risk-aversion ensure that credit rationing does not lead to high-order dynamics. The purpose of the examples in this section is to explore a simpler issue: how does private information affect the existence and asymptotic stability of steady states in a neoclassical growth model? To formulate answers, we look at economies with risk-neutral agents and three benchwork technologies in which the elasticity of substitution between capital and labor is, alternatively, zero, one and plus infinity.

The class of economies we study satisfies the accumulation equation (33) and the self-selection condition (34) which is considerably simplified by the assumption of risk neutrality. To see how this is done, it is useful to go back to the self-selection constraint in equation (22) and note that

$$\begin{aligned} V(D,R,r,\hat{r};p_H) &= \hat{r}\max_{0\le k\le D}\{p_H[f(k)/p_H-RD+r(D-k)] \\ &\qquad +(1-p_H)r(D-K)\} \\ &= \hat{r}[(r-p_HR)D+\max_{0\le k\le D}(f(k)-rk)] \\ &= \hat{r}[(r-p_HR)D+f(k^H)-rk^H] \end{aligned} \tag{43}$$

where

$$k^H = \mathrm{argmax}_{0\le k\le D}(f(k)-rk) \tag{44}$$

Adding subscripts back to equation (43), we may rewrite equation (22) as

$$r_{t+1}w(k_t) = r_{t+1}[(1-p_H/p_L)r_tD_t^L+f(k_H)-r_tk_H]$$

which implies

$$w(k_t) = (1-p_H/p_L)r_tD_t^L+f(k_t^H)-r_tk_t^H \quad (45)$$

If the maximizer k_t^H is not constrained by the inequality $k_t^H \le D_t^L$, then $f'(k_t^H)=r_t$, which means $f(k_t^H)-r_tk_t^H = f(k_t)-r_tk_t \equiv w(k_t)$ and, hence, (45) is violated. Assume then that $k_t^H=D_t^L$, drop superscripts and rewrite the self-selection constraint in the form

$$w(k_t) = f(D_t)-\gamma D_t f'(k_t) \quad (46)$$

where $\gamma \equiv p_H/p_L \in [0,1]$, $D_t \equiv D_t^L$ and $r_t = f'(k_t)$. Equation (46) implicitly defines the credit ration, D_t, of the low-risk borrower as a function of the unrationed demand for capital by the high-risk borrower and, hence, as a function of the deposit yield r_t.

Equilibria consist of sequences (D_t, k_t) that simultaneously satisfy equations (33) and (46). To obtain those sequences, we first "solve" equation (46) for D_t. Given γ and k_t, it is easy to establish the existance of a well-behaved function d: $\mathbb{R}_+ \times [0,1] \to \mathbb{R}_+$ such that $D_t = d(k_t, \gamma)$ satisfies (46). In particular, one shows directly that d is continuously differentiable if the production function is continuously differentiable; it is increasing in each argument and such that

$$d(k,\gamma)\in[0,k] \quad \text{for all} \quad \gamma\in[0,1] \tag{47a}$$

$$d(k,1) = k \tag{47b}$$

Having solved equation (46), we replace D_t and D_{t+1} out of equation (33) to obtain a scalar difference equation that describes the evolution of the capital-labor ratio for the unconstrained high-risk producer, viz.,

$$\lambda k_{t+1}+(1-\lambda)d(k_{t+1},\gamma) = \lambda w(k_t) + (1-\lambda[f(d(k_t,\gamma))-f'(k_t)d(k_t,\gamma)] \tag{48}$$

For each $\gamma\in[0,1]$, this equation defines an increasing time map $k_{t+1} = G(k_t)$ with the usual trivial steady state $k_t = 0$. The map G is readily seen to satisfy

$$\lim_{k_t\to\infty}(k_{t+1}/k_t) = 0 \tag{49a}$$

A positive steady state exists if we also have

$$\lim_{k\to 0}G'(k)>1 \tag{49b}$$

Two illustrations below explore credit rationing in economies with high elasticity of substitution between capital and labor.

Example 5 (Perfect substitutes): Suppose $f(k) = ak+b$ with $a>0$ and $b>0$ being given constants. Then the wage rate is $w(k) = b$ for all $k\geq 0$, and dynamical equilibria under public information satisfy equation (14), i.e., $k_t = b$ for all t. Output per worker is $y_t = (1+a)b$ for all t.

In the case of private information, the self-selection constraint (46) is $b = aD_t+b-\gamma D_t a$ where $\gamma = p_H/p_L$. This one yields $D_t=0$ and rations L-type borrowers out of the credit market altogether. In this case, too, $k_t=b$ for all t, that is, unrationed borrowers invest b, but average capital per worker is now λb and output per worker,

$$\hat{y}_t = (1+\lambda a)b < y_t, \tag{50}$$

falls short of the public-information output.

Example 6 (Cobb-Douglas technology): Suppose $f(k) = k^{1/2}$ and $p_H/p_L \equiv \gamma \in [0,1]$. Then the public-information dynamics of this economy is described by equation (14), viz., $k_{t+1} = (1/2)k_t^{1/2}$ which possesses one asymptotically stable, positive steady state at $k^* = 1/4$. Income per worker in the steady state is $y^* = 1/2$.

Under private information, the self-selection constraint (46) becomes $(1/2)k^{1/2} = D^{1/2}-(\gamma/2)Dk^{1/2}$ which reduces to

$$\phi(D/k) \equiv 2(D/k)^{1/2}-\gamma D/k = 1 \tag{51}$$

Note that the function $\phi(x)$ is continuous and increasing in the interval [0,1], and such that $\phi(0) = 0$, $\phi(1) = 2-\gamma>1$. Therefore, for each $\gamma\in[0,1]$ there exists $x(\gamma)$ such that $\phi(x) = 1$; it is easy to check that x is increasing, with $x(0) = 1/4$ and $x(1) = 1$.

The accumulation condition (33) in this case becomes

$$k_{t+1} = [A(\gamma)/2]k_t^{1/2} \tag{52a}$$

where

$$A(\gamma) = [1-(1-\gamma)x(\gamma)]/[\lambda+(1-\lambda)x(\gamma)] \tag{52b}$$

The difference equation (52a) has one asymptotically stable, positive steady state $k = [A(\gamma)/2]^2$. Average capital per worker is

$$\tilde{k} = [\lambda+(1-\lambda)x(\gamma)]A^2(\gamma)/4 \tag{53}$$

in the steady state, and average output per worker is

$$\tilde{y} = [A(\gamma)/2][\lambda+(1-\lambda)(x(\gamma))^{1/2}] \tag{54}$$

This equation suggests that the steady-state output per worker in the private information economy is a <u>proper fraction</u> of the corresponding output in the public information economy; the size of that fraction depends on the parameter γ. In particular, let

$$\theta(\gamma) \equiv \hat{y}/y^{*} = A(\gamma)[\lambda+(1-\lambda)(x(\gamma))^{1/2}] \tag{55}$$

Then $\theta(1) = 1$: the two economies behave similarly if the adverse selection problem is slight or non-existent. At the other extreme, as $\gamma \to 0$, $x(\gamma) \to 1/4$, $A(\gamma) \to 3/(3\lambda+1)$, and

$$\theta(0) = 3(1+\lambda)/2(1+2\lambda) \tag{55}$$

For values of λ near one, $\theta(0) \approx 3/4$ while for $\lambda = 1/2$ we have $\theta(0) = 9/10$. Private information reduces steady-state output per capita anywhere from 10% to 25%. Figure 3 illustrates the comparative dynamics of public and private-information economies for large λ.

We conclude this section with a detailed example that illustrates how private information affects the accumulation of capital in an economy with risk-neutral agents and a <u>fixed-proportions</u> technology. Accordingly, suppose that the intensive production function is

$$f(k) = A\min(k,\bar{k}) \tag{56}$$

where $A>1$ and $\bar{k}>0$ are two given constants. In economies with non-smooth technologies, we may derive the factor-price frontier from the zero profit condition

$$w_t = \max_{k\geq 0}[f(k)-r_t k] \tag{57}$$

where r_t is both the interest factor and the rental rate of capital if, as we have assumed up to this point, the depreciation rate for physical capital is one.

Given the technology in equation (56), both factor prices are positive if, and only if, for each t

$$0<r_t<A \tag{58}$$

Public information equilibria are straightforward here. If $r_t>A$, then

$$k_t \equiv \operatorname{argmax}_{k\geq 0}[f(k)-r_t k] \tag{59}$$

necessarily equals zero which implies $w_t = 0$ from equation (57) and, hence, $k_{t+1} = 0$ as well. If $r_t = A$, then $w_t = 0$ for any $k_t \in [0,\bar{k}]$ and, again, $k_{t+1} = 0$.

The interesting case is $r_t \in [0,A)$ for which (59) dictates $k_t = \bar{k}$ and, therefore, $w_t = (A-r_t)\bar{k} = k_{t+1}$. Since $r_t \in [0,A)$ is a free parameter, this analysis suggests that $k_t = \bar{k}$ is followed by a k_{t+1} lying anywhere in the interval $(0,A\bar{k}]$. Figure 4(a) shows the complete phase portrait of the public-information economy with one trivial steady state and <u>two</u> positive states, $\bar{k}$ and $A\bar{k}$. Of these, 0 and $A\bar{k}$ are stable: all trajectories starting in $(0,\bar{k})$ converge instantaneously to 0 while trajectories starting at any $k_1>\bar{k}$

converge instantaneously to $A\bar{k}$.[14]

For on economy with private information, we rewrite the self-selection constraint (46) as

$$w_t = f(D_t) - \gamma D_t r_t \tag{60}$$

and note from equation (59) that k_t is any number in the interval $[0,\bar{k}]$ if $r_t = A$. In this case labor is a free input: $w_t = 0$ from equation (57) and $D_t = 0$ from the self-selection constraint (60). Hence both D_{t+1} and k_{t+1} must be zero from the accumulation relation (48). In other words, $r_t = A$ implies that the average capital stock per worker is $\tilde{k}_t \in [0, \lambda\bar{k}]$ at t and $\tilde{k}_{t+1} = 0$ at t+1. The line segment (0A) in figure 5 represents the phase portrait of a private-information economy in which $r_t = A$.

Next we look at the case $r_t = 0$. Here equations (57) and (59) lead to $k_t \geq \bar{k}$ and $w_t = A\bar{k}$; therefore the self-selection condition (60) holds at $D_t = \bar{k}$. The economywide capital-labor ratio is then $\tilde{k} \geq \bar{k}$ while $\tilde{k}_{t+1}$ necessarily equals total saving, that is,

$$k_{t+1} \equiv \lambda A\bar{k} + (1-\lambda)A\bar{k} = A\tilde{k} \tag{61}$$

The horizontal line segment to the right of point B in Figure 5 is the phase portrait of a private-information economy in which capital is a free input.

The most interesting situation arises if $r_t \in (0,A)$ and neither input is free. Here we have $k_t = \bar{k}$, $w_t = (A-r_t)\bar{k}$ and a self-selection constraint that can be solved to yield

$$D_t = \bar{k}\theta(r_t) \tag{62a}$$

where

$$\theta(r) \equiv (A-r)/(A-\gamma r)\in(0,1) \tag{62b}$$

Economywide capital-labor ratios are

$$\tilde{k}_t = [\lambda+(1-\lambda)\theta(r)]\bar{k} \tag{63a}$$

and

$$\begin{aligned}\tilde{k}_{t+1} &= \lambda w_t+(1-\lambda)[f(D_t)-r_tD_t] \\ &= \lambda(A-r_t)\bar{k}+(1-\lambda)(A-r_t)D_t \\ &= (A-r_t)\tilde{k}_t\end{aligned} \tag{63b}$$

Eliminating r_t from equations (63a) and (63b), we obtain a first-order difference equation in the economywide capital-labor ratio, i.e.,

$$\frac{\tilde{k}_{t+1}}{(1-\gamma)A\tilde{k}_t+\gamma\tilde{k}_t} = \frac{\tilde{k}_t-\lambda\bar{k}}{(1-\lambda)\bar{k}} \tag{64}$$

which should hold for any $k_t\in[\lambda\bar{k},\bar{k}]$. Equation (64) describes an upward-sloped line connecting points A and B in Figure 5: it yields $\tilde{k}_{t+1} = 0$ if $\tilde{k}_t = \lambda\bar{k}$, and $\tilde{k}_{t+1} = A\bar{k}$ if $\tilde{k}_t = \bar{k}$. The fixed point of equation (64) is

$$\hat{k} = \lambda\bar{k}+(1-\lambda)\bar{k}/[\gamma+(1-\gamma)A]<\bar{k} \tag{65}$$

The private-information economy of Figure 5 has many things in common with the public-information economy of Figure 4(a): it has three steady states $(0,\hat{k},A\bar{k})$ two of which are stable stationary equilibria of the public-information economy as well. The middle state $\hat{k}$, however, is smaller than the corresponding public-information state $\bar{k}$, and it is one that must claim our attention for it represents a competitive equilibrium in which both factor prices are positive.

One general message we draw for the risk-neutrality examples of this section is that, if idiosyncratic risks do not significantly affect the saving patterns of households, then credit rationing will have a broad tendency to lower capital formation and income per head in the steady state.

6. CONCLUSIONS

This paper has explored the influence of credit rationing on economic growth by introducing adverse selection in an otherwise standard neoclassical growth model and describing how the resulting equilibria differ from what obtains when the credit market functions in the ideal setting of public information. Our objectives parallel those of Bewley (1986) and Woodford (1988) in that we, too, seek to identify the impact of credit market imperfections on dynamical behavior even though our method is considerably different; we study a production economy with privately informed agents rather than a pure-exchange economy with liquidity constrained agents.

In broadest outline, our results confirm those of Bewley and Woodford. Private information in our model complexifies the dynamics of competitive equilibria because it adds a state variable that plays no role when the credit market is perfect: that variable is the credit ration of low-risk borrowers in our model, real currency balances in the work of Bewley and Woodford.

Credit rationing, and the underlying phenomenon of private information, appear to influence the set of competitive equilibria of a growing economy in a complex manner which we have not managed to characterize in full. Just like pure-exchange economies, neoclassical growth models without currency or national debt tend to have lower stationary equilibrium interest rates if some borrowers are rationed then they would without rationing. This reduction in interest rates allows unrationed borrowers to invest more than they otherwise would but rationed borrowers invest less; the overall impact of credit rationing on the economywide capital stock and steady-state output per worker is uncertain. Most of our examples in section 5 show that private information lowers capital formation and output.

Private information also has a pronounced effect on the distribution of income. Economies with ex ante identical agents who suffer from idiosyncratic risks will use the law of large numbers to insure individual consumption and income from these risks. If personal attributes are public information, insurance will be actuarially fair and ex ante identical agents will have identical ex post incomes. Private information, however, will result in incomplete insurance and, hence, in a distribution of income that is skewed towards individuals who are favored by the realized state of nature.

Economies with a positive amount of currency or a non-negative amount of national debt have stationary interest rates that equal the natural rate of growth <u>independently</u> of credit rationing. In these stationary outside-money equilibria, credit rationing keeps capital accumulation short of what it would have been under public information. What we don't know is how credit rationing changes the stability properties of steady states and how it is, in turn, influenced by fiscal instruments such as taxes, government purchases and the like.

APPENDIX

A. Proof of Lemma 1.

$f'[\phi(D,R,r_t,r_{t+1};p)]/p$ is a decreasing function of p iff

$$f'[\phi(\cdot)]/p > f''[\phi(\cdot)]\phi_5. \tag{A.1}$$

From equation (4), we have $f'[\phi(\cdot)/p > r_t$, so that a sufficient condition for (A.1) is

$$f''(\phi(\cdot)]\phi_5/r_t \leq 1. \tag{A.2}$$

A straightforward but rather tedious differentiation of equation (4) establishes that (A.2) indeed holds.

B. Proof of Lemma 2.

In order to prove this Lemma we need the following preliminary result. Let y_t^H and $\hat{y}_t^H$ denote the state-contingent income levels of a type-H agent who takes a type-L contract and chooses a capital stock optimally. Then

$$y_t^H = f(k_t^H)/p_H - (r_t D_t^L/p_L) + \hat{y}_t^H$$

$$\hat{y}_t^H = r_t(D_t^L - k_t^H),$$

where $k_t^H \equiv \phi(D_t^L, r_t/p_L, r_t, r_{t+1}; p_H)$. We clain that

<u>Claim 1</u>: $y_t^H > \hat{y}_t^H$. (A.3)

To prove this claim, we note that $(D_t^L, r_t/p_L)$ and $(D_t^H, r_t/p_H)$ lie on the same type-H agent indifference curve in figure 1. Therefore

$$p_H u(r_{t+1} y_t^H) + (1-p_H) u(r_{t+1} \hat{y}_t^H) = u[r_{t+1} w(k_t)] \tag{A.5}$$

Then the result claimed in (A.3) follows if we show that $\hat{y}_t^H < w(k_t)$. Now $\hat{y}_t^H = r_t[D_t^L - \phi(D_t^L, r_t/p_L, r_t, r_{t+1}; p_H)]$ and $w(k_t) = r_t[D_t^H - \phi(D_t^H, r_t/p_H, r_t, r_{t+1}; p_H)]$. Moreover, $D_t^H > D_t^L$, and $(D_t^H, r_t/p_H)$ and $(D_t^L, r_t/p_L)$ lie on the same type-H agent indifference curve. Therefore, the proof of (A.3) is complete if we show that $D-\phi(D, R, r_t, r_{t+1}; p_H)$ decreases as D and R fall along a given type-H indifference curve (compare points H and L_1 in figure 1).

As D varies along a type-H agent indifference curve, the change in $D-\phi(\cdot)$ is given by

$$1-\phi_1-\phi_2[(\frac{dR}{dD})_{dV_H=0}]$$

This term is positive (as desired) if

$$1 > \phi_1+\phi_2. \tag{A.6}$$

Using equation (4) to obtain ϕ_1 and ϕ_2, and substituting the result into (A.6) gives the equivalent condition

$$- f''(k_t^H) u'(r_{t+1} y_t^H) > 0,$$

which is obviously true. This completes the proof of Claim 1.

We are now prepared to prove Lemma 2. To begin, we note that equation (22) and the definition of the function g imply that $g(k_{t+1},k_t)$ satisfies

$$\begin{aligned} V[g(k_{t+1},k_t),f'(k_t)/p_L,f'(k_t),f'(k_{t+1});p_H] &\equiv \\ u[f'(k_{t+1})w(k_t)] &= u[r_{t+1}w(k_t)]. \end{aligned} \tag{A.7}$$

To prove part (a), implicitly differentiate (A.7) with respect to k_t to obtain

$$\begin{aligned} g_2 &= \{r_{t+1}u'[r_{t+1}w(k_t)]w'(k_t)-[V_3+(V_2/p_L)]f''(k_t)\}/V_1 = \\ &-f''(k_t)\{r_{t+1}u'[r_{t+1}w(k_t)]k_t+V_3+(V_2/p_L)\}/V_1. \end{aligned} \tag{A.8}$$

Since $V_1(D_t^L,\ r_t/p_L,r_t,r_{t+1};p_H) > 0$ (see figure 1), $g_2 > 0$ iff

$$r_{t+1}u'\,[r_{t+1}w(k_t)]k_t+V_3+V_2/p_L > 0. \tag{A.9}$$

From equation (7), however, we obtain

$$\begin{aligned} V_3+V_2/p_L = {} & p_H r_{t+1}(D_t^L-k_t^H)u'\,(r_{t+1}y_t^H) \\ & +(1-p_H)r_{t+1}(D_t^L-k_t^H)u'\,(r_{t+1}\hat{y}_t^H)-(p_H/p_L)D_t^L r_{t+1}\ \ u'\,(r_{t+1}y_t^H) \end{aligned} \tag{A.10}$$

Therefore, condition (A.9) is equivalent to

$$r_{t+1}u'[r_{t+1}w(k_t)]k_t+r_{t+1}u'(r_{t+1}y_t^H)\{p_H(D_t^L-k_t^H)-$$
$$(p_H/p_L)D_t^L+(1-p_H)(D_t^L-k_t^H)[u'(r_{t+1}\hat{y}_t^H)/u'(r_{t+1}y_t^H)]\}>0. \qquad (A.11)$$

Now, substituting (4) into (A.11), we have

$$r_{t+1}u'[r_{t+1}w(k_t)]k_t+r_{t+1}u'(r_{t+1}y_t^H)\{p_H(D_t^L-k_t^H)-$$
$$(p_H/p_L)D_t^L+(D_t^L-k_t^H)[f'(k_t^H)/r_t-p_H]\}>0 \qquad (A.12)$$

as a necessary and sufficient condition for $g_2>0$. Moreover, by claim 1, $y_t^H > \hat{y}_t^H$. Therefore, says equation (4), $f'(k_t^H) > r_t$, and the left-hand side of equation (A.12) exceeds

$$r_{t+1}u'[r_{t+1}w(k_t)]k_t+r_{t+1}u'(r_{t+1}y_t^H)[(p_L-p_H)/p_L]D_t^L$$
$$-r_{t+1}u'(r_{t+1}y_t^H)k_t^H.$$

However, this term is positive, since $y_t^H>w(k_t)$ and $k_t>k_t^H$($k_t>k_t^H$ follows from $k_t = \phi(D_t^H, r_t/p_H, r_t, r_{t+1}; p_H) > \phi(D_t^L, r_t/p_L, r_t, r_{t+1}; p_H) = k_t^H$, and the fact that ϕ_1 and ε_2 are both positive). ∎

We now prove part (b) of Lemma 2. Differentiating equation (A.7), we obtain

$$g_1=\{u'[r_{t+1}w(k_t)]w(k_t)f''(k_{t+1})-V_4(\cdot)f''(k_{t+1})\}/V_1. \qquad (A.13)$$

Since $V_1 > 0$, as before, g_1 is opposite in sign to the term

$$u'[r_{t+1}w(k_t)]w(k_t)-V_4 = w(k_t)u'[r_{t+1}w(k_t)] - p_H y_t^H u'(r_{t+1}y_t^H)-(1-p_H)\hat{y}_t^H u'(r_{t+1}\hat{y}_t^H) \equiv \zeta[y_t^H,\hat{y}_t^H,w(k_t)], \tag{A.14}$$

where V_4 is obtained by differentiating equation (7)

We now make two observations. First $\zeta[w(k_t),w(k_t),w(k_t)] = 0$, and second

$$p_H u(r_{t+1}y_t^H)+(1-p_H)u(r_{t+1}\hat{y}_t^H) \equiv u[r_{t+1}w(k_t)]. \tag{A.15}$$

If we regard (A.15) as defining $\hat{y}_t^H$ as a function of y_t^H and $w(k_t)$, say $\hat{y}_t^H = \theta[y_t^H,w(k_t)]$, then g_1 is opposite in sign to $\zeta[y_t^H,\theta(y_t^H,w(k_t)),w(k_t)] \equiv \xi[y_t^H,w(k_t)]$.

Claim 1 implies that $y_t^H > w(k_t)$. Thus g_1 is positive (respectively negative) if ξ is everywhere decreasing (increasing) in its first argument. It is straightforward to show that

$$\xi_1 = p_H u'(r_{t+1}y_t^H)[\rho(r_{t+1}y_t^H)-\rho(r_{t+1}\hat{y}_t^H)],$$

where we recall that $\rho(c) \equiv -cu''(c)/u'(c)$. Since $y_t^H > \hat{y}_t^H, \xi_1$ is positive (respectively, negative) if relative risk aversion is increasing (respectively, decreasing). Thus g_1 is positive (negative) if relative risk aversion is decreasing (increasing).■

C. <u>Proof of Proposition 1.</u>

The proof is by contradiction. We note first that $f'(\tilde{k}) = r < f'(\tilde{k}_L)$, so that $\tilde{k} > \tilde{k}_L$. Then suppose that $\tilde{k} \le k^*$. From equation

(36) and the fact that $k-w(k) \le 0$ for all $k \le k^*$ (see figure 2), we obtain

$$(1-\lambda)[f(\tilde{k}_L)-\tilde{k}_L f'(\tilde{k})-\tilde{k}_L] = \lambda[\tilde{k}-w(\tilde{k})] \le 0 \qquad (A.16)$$

But then $\tilde{k}_L \ge f(\tilde{k}_L)-\tilde{k}_L f'(\tilde{k}) > f(\tilde{k}_L)-\tilde{k}_L f'(\tilde{k}_L) \equiv w(\tilde{k}_L)$, implying that $\tilde{k}_L > k^*$. This contradicts $\tilde{k} > \tilde{k}_L$, as desired. ∎

FOOTNOTES

1. For an investigation of how measures of financial market distortions correlate with U.S. investment behavior, see Gertler, Hubbard and Kashyap (1990).
2. With the exception of the initial middle-aged generation, which is endowed with some initial capital, as described in footnote 5.
3. Recall that each self-employed agent is endowed with one unit of labor. Thus k_t is this producer's capital-labor ratio.
4. Smoothness assumptions are dropped in section 5.
5. At t=0 we must regard y_0^i and $\hat{y}_0^i$ as initial conditions. Then the time-zero equilibrium condition is $k_1 = \lambda[p_H y_0^H + (1-p_H)\hat{y}_0^H] + (1-\lambda)[p_L y_0^L + (1-p_L)\hat{y}_0^L]$. Note that this amounts to taking k_1 as an initial condition.
6. The positive steady state exists if w(k) is continuously differentiable, and satisfies $\lim_{k \to 0} w'(k) > 1$; see Galor and Ryder (1989). Uniqueness is guaranteed by assuming that w(k) is concave;see section 5 for an example of <u>multiple</u> positive steady states.
7. The stability properties of the outside-money equilibrium are discussed in Tirole (1985) and Azariadis (1991), ch. 1G.
8. We also assume that intermediary owners cannot lend to themselves. This assumption serves only to simplify notation. An alternative assumption that would do the job is that only middle-aged or old agents can establish an intermediary. This could be accomplished by requiring intermediaries to have some

capital, which young individuals cannot raise in advance.

9. Below we display examples where $\lambda \geq .25$ is sufficient at a steady state equilibrium.

10. See, for instance, Rothschild and Stiglitz (1976).

11. For example, if $f(k) = AK^{\alpha}$, the function $I(k)$ is increasing (decreasing) if α exceeds (is less than) 1/2.

12. Analogous results are obtained by Azariadis and Smith (1991) for a pure exchange economy with national debt.

13. See Beneivenga and Smith (1991) for an overview of some of this literature.

14. The phase portrait of Figure 4(a) is generic in the space of CES production functions; if the Leontieff technology is replaced with another that has a small positive elasticity of substitution, the qualitative properties of dynamical equilibria are well described by the similar-looking phase portrait in Figure 4(b).

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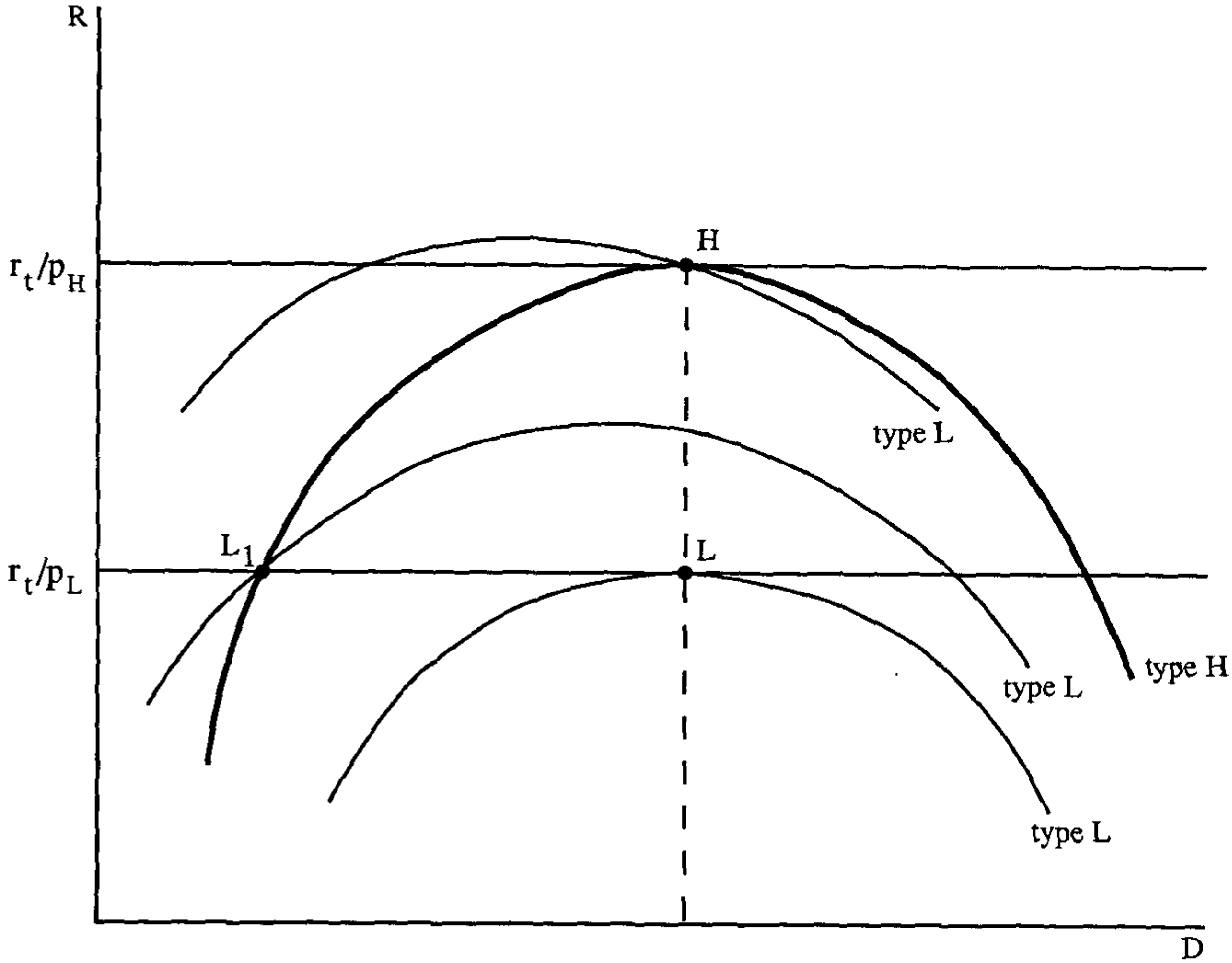

Utility increases toward the south

Figure 1

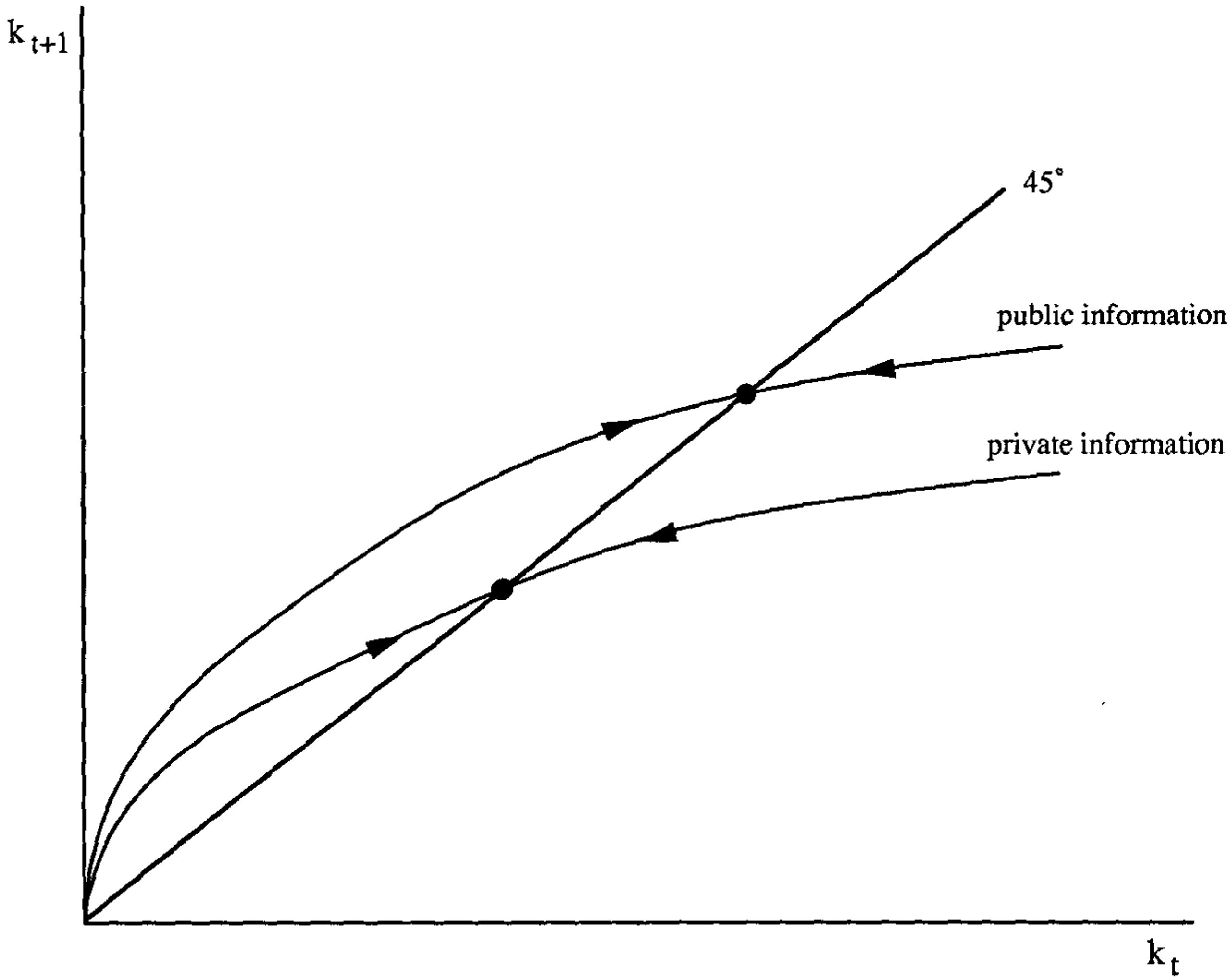

Figure 3

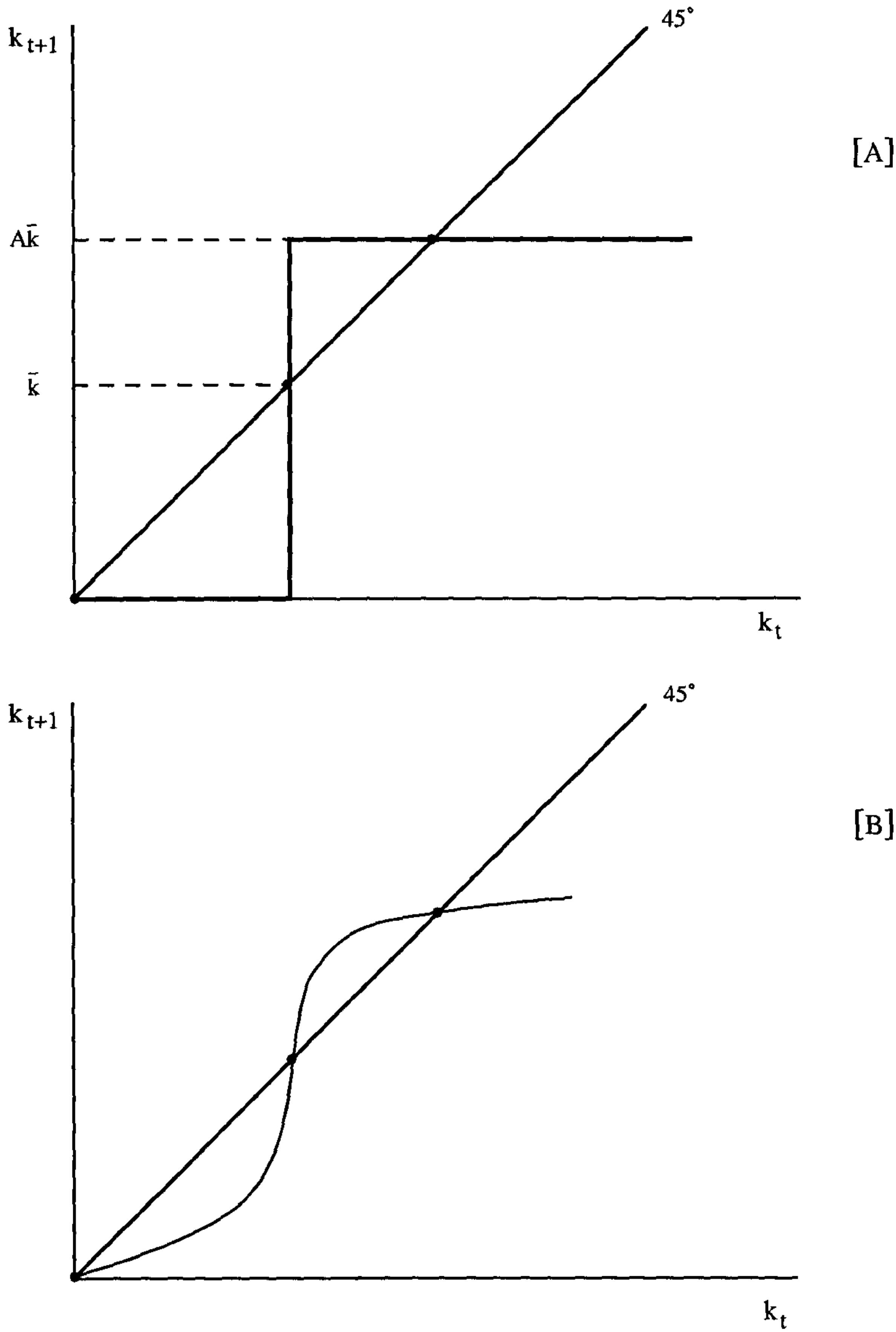

Figure 4

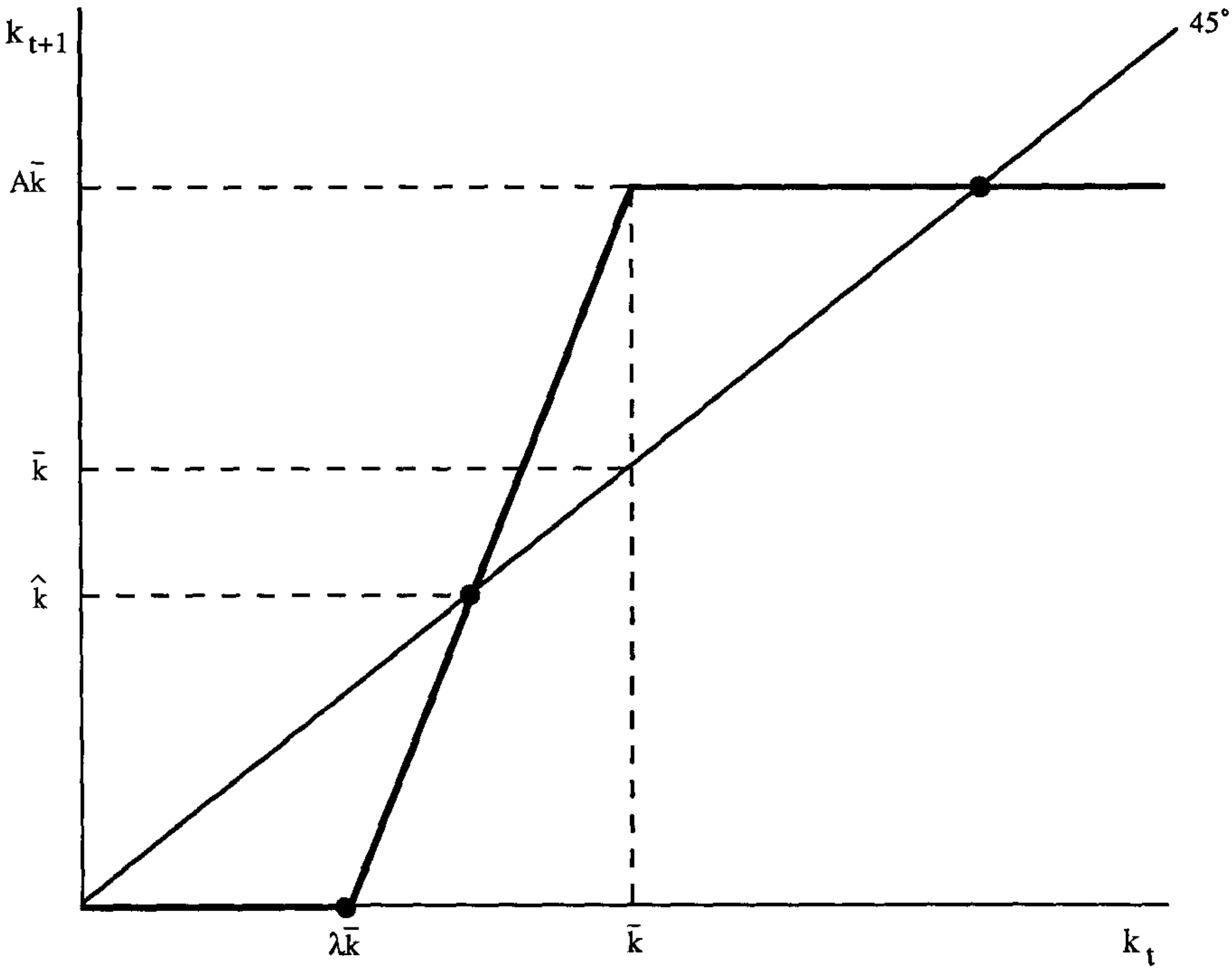

Figure 5