SEARCHING FOR INVESTMENT OPPORTUNITIES: A MICRO FOUNDATION FOR ENDOGENOUS GROWTH

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ABSTRACT

Recent literature on the phenomenon of sustained growth has emphasized the role of increasing returns to scale technologies. We suggest in this paper a microeconomic foundation for the existence of increasing returns technologies.
Production is assumed to require a combination of capital and labor in a standard, constant returns to scale technology. However, this technology is affected multiplicatively by a productivity factor. This factor is assumed to be a result of a research and development process. The R&D process is modeled as a search problem. Firms face a known distribution of productivity factors from which they can sample. Sampling is costly in terms of capital, and therefore firms, which possess a certain amount of capital, have to decide when to stop sampling, hire labor in a competitive labor market, and invest the remainder of their capital in the CRS technology multiplied by the productivity factor they have uncovered.
The paper analyzes the search problem faced by the firms, and shows that under certain assumptions about the probability distribution function which governs the behavior of the productivity factor, output is likely to display increasing returns to scale with respect to capital (in a probabilistic sense).
This result is embeded in a Diamond-like growth model. It is argued that the model can possess sustained growth paths with interesting stochstaic features. In particular, poorer economies are likely to grow faster on average but also to suffer from higher variability in their growth rate.

1. INTRODUCTION

This work tries to provide a link between observable characteristics of levels and growth rates of output per worker both within and across countries. The link that we try to establish is that countries with higher output per worker are better able to affect a given *improvement* in their productive capacity. Such improvements in production technologies, broadly interpreted, come as a result of more investment, which, in turn, is more affordable by richer countries. Conversely, sufficiently low per capita income, which amounts to low levels of savings per worker will result in small benefits from optimal reallocation of capital, resulting in maintained and relatively low per capita income levels.

Recent growth literature, for instance, Lucas (1988), Romer (1986), Rebelo (1987), Jones and Manuelli (1988), has been successful in modeling sustained growth *endogenously*, in the sense that the rate of growth in per capita output is the result of optimal decisions by utility maximizing agents. In particular, these models do not have to appeal to some *exogenous* "engine of growth". Instead, in order to achieve *sustained growth,* these models rely on some notion of increasing returns to scale, or on some lower bound on capital productivity. Little emphasis is put on the micro foundation for such a technology, although its micro underpinning might be crucial for designing policies aimed at promoting growth. A different mechanism for generating endogenous growth is provided by explicit models of financial intermediation, which exploit increasing returns in information processing, or from economizing on monitoring costs of funded investment projects, (Greenwood and

Jovanovic (1990), Williamson (1987)).

Yet, in all the aforementioned works the underlying production technology is taken as given. We suggest that production technologies, broadly interpreted, have to be *discovered* and that the discovery process is both *random* and *costly*. These two features will be shown to imply, for certain environments, a random growth process which can display increasing, maintained or decreasing conditional first moments of growth rates, depending on the beginning of period output or capital stock.

The model is based on a costly search process over potential technologies with unknown productivity rates. When searching optimally, the resulting output exhibits increasing returns in "gross" capital, while exhibiting constant or decreasing returns in "productive" capital. Search costs account for the difference between "gross" and "productive" capitals.

The increasing returns to capital in this search-investment model create a natural role for *intermediation*, modeled simply as the process of pooling individual savings to exploit these increasing returns. In a deeper analysis, the equilibrium structure and extent of intermediation of this kind is determined endogenously and jointly with the growth process itself, (as in Greenwood and Jovanovic (1990)). Here, however, we simply impose a joint investment process, which amounts to ripping the full benefits from the increasing returns from search. As in Boyd and Prescott (1986), pooled investment will economize on project evaluation costs.

This paper outlines a model which can address the issues mentioned above. In section 2, we analyze a one-period problem of searching over investment opportunities, and characterize the optimal search strategy. In section 3 we embed the sequential search process in a dynamic model. The sense in which the solution to the search problem can give rise to an aggregate production technology that exhibits increasing returns to scale is illustrated in section 4. In particular, we provide in that section an example economy which exhibits stochastic growth path with increasing expected growth rates, conditional on beginning of period capital. However, that same economy can also have an equilibrium with no search, and negative growth. Further implications of the model to the stochastic growth process are reviewed in the concluding section.

2. SEQUENTIAL SEARCH FOR INVESTMENT OPPORTUNITIES

2.1 *A single period problem*

Consider the following search problem. An investor has Q units of capital good, (hereafter, investment capital), available for investment at the current time. For a fixed price of α units of capital the investor can sample one investment project at a time, from an infinitely large population of projects, indexed by their (constant) productivity rate, θ. The cumulative probability distribution of the productivity rates is denoted by $H : \mathbb{R}^1 \longrightarrow [0\ ,\ 1]$, $H(\theta) = 0$ for $\theta < \underline{\theta}$, $H(\theta) = 1$ for $\theta \geq \overline{\theta}$, $0 \leq \underline{\theta} < \overline{\theta} \leq \infty$. This distribution is assumed to be invariant under successive sampling, (i.e., sampling with replacement). After each sampling the investor has the option of accepting or rejecting the project just examined. Rejecting the project means sampling at least one more time. Accepting means activating the project with all the remaining capital and possibly additional factors of production which can be hired at that point. Thus, the search is done "without recall" during the search process, so that a project rejected cannot be adopted later on, (it will be shown below how to modify the analysis to allow for recall within the search period).

A project with a particular θ in which k units of capital and optimal levels of other factors are utilized will generate a payoff to the searcher, denoted $\pi(k,\theta)$, where $\pi(\cdot,\cdot)$ increases monotonically in both arguments. The index π will be interpreted in the next section as the utility from the profit generated by operating the project θ with k units of capital. It is assumed that the only cost of search is the direct sampling cost, and in particular, the search activity does not take any time. The investor's goal is to maximize the expectations of the index π by specifying an optimal stopping strategy for the sequential search process.

Since resources are limited, the search will be finite in the number of observations sampled. Let $V(k,\theta)$ be the value of the objective when k units of capital remain for investment and the available productivity rate is θ, assuming that an optimal search strategy is followed thereafter. The function

$V(\cdot,\cdot)$ must satisfy the following functional equation:

$$(2.1) \qquad V(k,\theta) = \operatorname{Max} \{ \pi(k,\theta) , E_{\theta'} V(k-\alpha,\theta') \}, \qquad \alpha \le k$$

$$V(k,\theta) = \pi(k,\theta), \qquad 0 \le k \le \alpha,$$

where E is the expectations operator.

Note that if the searcher can always go back to any project sampled earlier, then equation (2.1) changes to:

$$(2.2) \qquad V(k,\theta) = \operatorname{Max} \left\{ \pi(k,\theta) , E_{\theta'} \operatorname{Max}\{V(k-\alpha,\theta) , V(k-\alpha,\theta')\} \right\},$$

$$k \ge \alpha;$$

$$V(k,\theta) = \pi(k,\theta), \qquad k < \alpha.$$

Since $\pi(k,\cdot)$ is a monotone function of θ, the optimal search strategy takes the form of a reservation productivity rates. Let $\theta^*(k)$ denote the minimal productivity rate which is acceptable when the remaining investment capital is k. Then, from (2.1), we have:

$$(2.3) \qquad V(k,\theta) = \begin{cases} \pi(k,\theta), & \text{if } \theta > \theta^*(k) \\ E_{\theta'} V(k-\alpha,\theta'), & \text{otherwise.} \end{cases}$$

If $\theta^*(k) > 0$, then it must equate the two terms in the maximand of (2.1), in which case we have:

$$(2.4) \qquad \pi(k,\theta^*(k)) = E_{\theta'} V(k-\alpha,\theta')$$

$$= H[\theta^*(k-\alpha)]\cdot\pi(k-\alpha,\theta^*(k-\alpha)) + \int_{\theta^*(k-\alpha)}^{\bar{\theta}} \pi(k-\alpha,\theta)\, dH(\theta)$$

while $\quad \theta^*(k) = \underline{\theta} \quad$ for $k \in (0,\alpha)$.

Equation (2.4) is a recursive relation from which $\theta^*(k)$ can be found for any $k > 0$. Note also that the first equality in (2.4) gives the expected value of the searcher's objective with Q units of capital prior to any project sampling as $\pi(Q,\theta^*(Q))$.

2.2 *Properties of* $\theta^*(k)$

Without additional assumptions on the index $\pi(\cdot,\cdot)$ it is difficult to characterize the optimal search strategy. We assume, hereafter, that:

$$(2.5) \qquad \pi(k,\theta) = f(k)\cdot g(\theta), \qquad\qquad k \geq 0, \quad \theta \in [\underline{\theta}\ ,\ \bar{\theta}],$$

where f and g are non-negative, monotonically increasing, f is concave and f(0)=0. These assumptions will be motivated in the context of a dynamic growth model in the next section. With assumption (2.5), equation (2.4) becomes:

$$(2.6) \qquad g(\theta^*(k)) = \text{Max}\left\{\frac{f(k-\alpha)}{f(k)} E_\theta \text{Max}\{g(\theta^*(k-\alpha)), g(\theta)\}\ ,\ g(\underline{\theta})\right\}$$

For sufficiently large k relative to α, the RHS of (2.6) equals its first argument, and this recursive relation can be examined to establish some important properties of the function $\theta^*(\cdot)$. In particular, we have:

<u>CLAIM 1</u>: (i) $\theta^*(k)$ increases monotonically in k for all $k > 0$;

(ii) $\theta^*(k) \longrightarrow \bar{\theta}$ as $k \longrightarrow \infty$;

(iii) $g(\theta^*(k)) - g(\theta^*(k-\alpha)) \longrightarrow 0$ as $k \longrightarrow \infty$.

Proof:

For sufficiently low k, at least $k \in (0,\alpha)$, we have from (2.6) that $\theta^*(k) = \underline{\theta}$, so that (i) holds weakly. Assume then that $d\theta^*(k-\alpha)/dk > 0$, and that $\theta^*(k)$ is given implicitly by:

$$(2.7) \qquad g(\theta^*(k)) = \frac{f(k-\alpha)}{f(k)} \cdot E_\theta \text{ Max } \{ g(\theta^*(k-\alpha)) , g(\theta) \}$$

Differentiating both sides of (2.7) w.r.t. k, we get

$$(2.8) \qquad g'(\theta^*(k))\frac{d\theta^*}{dk}(k) = \frac{d\left(\frac{f(k-\alpha)}{f(k)}\right)}{dk} \cdot \text{E Max } \{g(\theta^*(k-\alpha)), g(\theta)\}$$

$$+\frac{f(k-\alpha)}{f(k)}\left(H(\theta^*(k-\alpha))g'(\theta^*(k-\alpha))\frac{d\theta^*}{dk}(k-\alpha)\right).$$

Since $f(\cdot)$ is a concave increasing function, $f(k-\alpha)/f(k)$ also increases in k. It follows, since $g'(\cdot) > 0$ by assumption, that $\frac{d\theta^*}{dk}(k) > 0$ whenever $\frac{d\theta^*}{dk}(k-\alpha) \geq 0$.

To prove (ii), note that $\theta^*(\cdot)$, for sufficiently large k, strictly increases in k, and is bounded from above by $\bar{\theta}$, since the RHS of (2.7) is bounded from above by $g(\bar{\theta})$. However, for any $x < \bar{\theta}$, $E_\theta \operatorname{Max} \{ x , \theta \} > x$, while $f(k-\alpha)/f(k)$ converges to 1. Hence we cannot have $\lim_{k\to\infty} \theta^*(k) < \bar{\theta}$.

Finally, to prove (iii), use (2.7) again to write

$$(2.9) \qquad g(\theta^*(k)) - g(\theta^*(k-\alpha)) =$$

$$\frac{f(k-\alpha)}{f(k)}\left(E_\theta \operatorname{Max} \{ g(\theta^*(k-\alpha)) , g(\theta) \} - g(\theta^*(k-\alpha))\right)$$

$$- \frac{f(k) - f(k-\alpha)}{f(k)} g(\theta^*(k-\alpha)).$$

As $k \longrightarrow \infty$ and $\theta^*(k-\alpha) \longrightarrow \bar{\theta}$, the first term on the RHS of (2.9) goes to zero, while the second term is positive. Since $g(\theta^*(k))$ increases monotonically in k it must be that $g(\theta^*(k))-g(\theta^*(k-\alpha))$ converges to zero.∎

Note that the fact that first α-differences of $g(\theta^*(k))$ go to zero does not necessarily imply that first α-differences of $\theta^*(k)$ go to zero, unless g is a convex increasing function. Hence, without further restrictions on g we cannot establish in general a property like concavity of $\theta^*(\cdot)$. Nevertheless, in the next section we examine example economies in which this is the case.

3. SEARCH IN A DYNAMIC MODEL

Here we embed the one-period search-investment problem in an overlapping generations model. Each period N identical two period lived agents appear. Agents supply labor services (inelastically) when young, and allocate the wage income received between first period consumption and savings, where the latter are rented out to a competitive firms given a known distribution of the rate of return on saving. The capital income on savings provides the sole source of second period consumption, and accordingly the amount saved is determined by an expected utility maximization problem. Formally, we assume a time separable utility function, so that an agent born at period t maximizes

(3.1) $$v(c_{1t}) + E_R\, u(c_{2t})$$

subject to

(3.2) $$c_{1t} = w_t - s_{t+1}$$

(3.3) $\tilde{c}_{2t} = \tilde{R}_{t+1}s_{t+1}$

Here $v(\cdot)$ and $u(\cdot)$ are strictly concave, twice differentiable increasing functions, E_{ω} is the expectations operator with respect to a random variable ω, c_{it} is the consumption at period i of the agent's life, (i = 1,2), w_t is the wage rate the agent obtains when young, s_{t+1} is the agent's saving and $\tilde{R}_{t+1}$ is the random gross rate of return the agent faces.

The firm attracts savings by issuing equities, or equivalently, by offering a cumulative probability distribution of rates of return, $\mathcal{R}(\cdot)$. The firm generates the distribution $\mathcal{R}(\cdot)$ by searching for an acceptable production technology and when one is found, utilizing it with its remaining investment capital. The firm's profits are then distributed to share holders as returns on their savings.

Specifically, denote the underlying technology of the firm is $\tilde{\theta}F(k,\ell)$, $F(\cdot,\cdot)$ strictly concave, twice differentiable and increasing in both arguments, and homogeneous of degree one in k and ℓ. At the beginning of the period, the firm views the wage rate it will ultimately have to pay its workers, when it decides on its optimal employment level, as a random variable, $\tilde{w}$, with an exogenously known cdf $\mathcal{W}(\cdot)$. The firm has a given initial amount of investment capital, K, and has to search for an acceptable productivity rate, θ, so as to maximize the expected utility of its share owners. When an acceptable project is found, the wage rate w is realized, and the firm decides on optimal labor input, $\ell^*(\theta,k,w)$, where k is the residual investment capital net of search costs. It will then produce the profits $\theta F(k,\ell^*(\theta,k,w)) - w\ell^*(\theta,k,w)$, which, by the constant returns to scale assumption on F, is a linear function of k. Letting the profit function be denoted by $\nu(\theta,w)k$, the return on the firm's initial capital is given by $R = \nu(\theta,w)k/K$. The firm's initial investment capital, K, which is taken as given by the firm, is given by Ns, where N denotes the number of savers. Therefore, when the firm chooses an optimal search strategy which maximizes the expected utility of a representative saver, u(Rs), in effect it maximizes the expected utility from the per-saver profit, $\nu(\theta,w)k/N$. Thus, we can write that the optimal search policy of a firm which has observed θ and owns k units of capital must satisfy the following functional equation:

$$(3.4) \qquad V(k,\theta) = \mathrm{Max}\left\{E_w\ u(\nu(\theta,w)k/N)\ ,\ E_{\theta'}\ V(k-\alpha,\theta')\right\}$$

In equilibrium, the realization of the wage rate w is given by

$$(3.5) \qquad \ell^*(\theta,k,w) = N$$

where k and θ are, respectively, the final investment capital and the productivity level of the project that were accepted by the searching firm. In equilibrium, w, k and θ will all be realizations of random variables, whose distributions are interdependent: given the distribution on w, the search strategy determines the joint distribution of k and θ, which, in turn, determines via (3.5) the distribution of w. Therefore, the equilibrium distributions of w, k and θ must satisfy some fixed point property.

The intertemporal problem involves another (related) fixed point argument. The distribution of the rate of return $\mathcal{R}(\cdot)$ determines (in general) the saving of the young, s. That amount in turn determines the initial amount of capital the firm has, which determines the search strategy and hence, the distribution of k and θ (and of w). Thus, the distribution $\mathcal{R}(\cdot)$ which is taken as given by the young, is generated by their behavior, and must have a fixed point property. However, as noted above, this fixed point is determined jointly with the equilibrium relationships governing the behavior of k, θ, and w.

Finally, note that equations (3.4) and (2.1) agree with each other when $\pi(k,\theta) = E_W u[v(k,\theta)k/N]$. As assumed in the analysis of the search problem in section 2, $\pi(\cdot,\cdot)$ is strictly monotone, and is a concave function of k whenever $u(\cdot)$ is a concave function. In the next section we describe an environment in which $\pi(\cdot,\cdot)$ can be further decomposed in accordence with (2.5).

4. AN EXAMPLE

In this section we provide an example which considerably simplifies the fixed point problems discussed above. First we show that if the objective function takes a multiplicative form of a particular type, the optimal strategy is independent of the distribution of the wage rate, so that that distribution can be calculated after the optimal search strategy has been determined. Then we establish the fact that we can also sever the link between the distribution of the rate of return and the saving behavior, while maintaining the structure that unlinks the optimal search strategy from the distribution of the wage rate. Finally we calculate some lower bounds on the expected rate of growth of the economy, and show that for a particular class of probability distributions on productivity rates - the economy may have a sustained growth path (in expectation).

Claim 2: Suppose that $u(\nu(\theta,w)k/N)$ can be written as $g(N)\pi_1(w)\pi_2(k,\theta)$. Then, if the random variable $\tilde{w}$ is treated as though it is independent of the random variables $\tilde{k}$ and $\tilde{\theta}$, the optimal strategy which satisfies (3.4) depends only on $\pi_2(k,\theta)$, and is independent of the distribution of w.

Proof: Let $\pi(k,\theta)$ be given by $E_w[g(N)\pi_1(w)\pi_2(k,\theta)]$. Because of the independence assumption, we obtain $\pi(k,\theta) = g(N)\pi_2(k,\theta)E_w[\pi_1(w)]$.Using this relationship in (2.4), we get

$$g(N)\pi_2(k,\theta^*(k))E_w[\pi_1(w)] = H(\theta^*(k-\alpha))g(N)\pi_2(k-\alpha,\theta^*(k-\alpha))E_w[\pi_1(w)] + \int_{\theta^*(k-\alpha)}^{\theta} g(N)\pi_2(k-\alpha,\theta)E_w[\pi_1(w)]dH(\theta).$$

Since the element $g(N)E_w[\pi_1(w)]$ cancels out, $\theta^*(\cdot)$ is independent of the distribution of w. ∎

Suppose now that the production function is Cobb-Douglas, so that

(4.1) $F(k,\ell) = Ak^{\gamma}\ell^{(1-\gamma)}$, $A > 0$, $0 < \gamma < 1$.

Then we obtain that given the wage rate w and θ, the profit function is given by:

$$(4.2) \qquad \nu(\theta,w)k = A\gamma(1-\gamma)^{(1-\gamma)/\gamma}\theta^{1/\gamma}(1/w)^{(1-\gamma)/\gamma}k.$$

Suppose in addition that the utility function $u(\cdot)$ is CRRA, so that $u(c) = (1/\delta)c^{\delta}$, $\delta \leq 1$. Then $u(\nu(\theta,w)k/N)$ clearly satisfies the conditions of the claim, so that if the firm takes the distribution of w as given, its optimal search strategy is independent of that distribution and of N. Hence the equilibrium distribution of the wage rate is induced by the optimal search, but does not affect it.

If, in addition, we assume that preferences are logarithmic in both periods, the saving behavior is independent of the distribution of the rates of return , and the intertemporal link depends solely on the current wage rate the young obtain. As the logarithmic utility function satisfies CRRA, we do not violate the previous condition.

For the remainder of this section we assume that the production function is Cobb-Douglas, and preferences are logarithmic. For this case, we obtain

$$(4.3) \qquad u(\pi_2(k,\theta)) = \ln(\theta^{1/\gamma}k).$$

We can now establish the relationship between the objective of the firm and output. In particular, we show that what the firm is maximizing is related to output in a way that places an intuitive lower bound on expected output.

Consider a firm which starts with Q units of capital, and has not observed any θ. Suppose that it wants to maximize $E_{k,\theta}[u(\pi_2(\tilde{k},\tilde{\theta})]$, where $\tilde{k}$ and $\tilde{\theta}$ are random variables with a joint distribution determined by the optimal search strategy. Under the distribution induced by the optimal search, the value of $E_{k,\theta}[u(\pi_2(\tilde{k},\tilde{\theta})]$ is the expected value of the program. That value can be written as follows: After α units of capital have been spent and an observation θ has been obtained, the value of the optimal program is $V(Q-\alpha,\theta)$. The expected value of that expression, $E_\theta V(Q-\alpha,\theta)$, is therefore equal to $E_{k,\theta}[u(\pi_2(\tilde{k},\tilde{\theta})]$ under the distribution induced by the optimal program. From equation (2.3) we have

$$(4.4) \qquad E_{\theta'} V(Q-\alpha,\theta') = u(\pi_2(Q,\theta^*(Q)) = \ln[(\theta^*(Q))^{1/\gamma}Q]$$

Output $(\tilde{y})$ is given by

$$(4.5) \qquad \tilde{y} = \tilde{\theta}\ \tilde{k}^{\gamma} N^{(1-\gamma)}.$$

Therefore,

$$(4.6) \qquad E(\tilde{y}) = E(\tilde{\theta}\ \tilde{k}^{\gamma} N^{(1-\gamma)}) = E([\tilde{\theta}^{1/\gamma}\tilde{k}]^{\gamma} N^{(1-\gamma)}) =$$

$$= E\left\{\left[e^{\ln(\tilde{\theta}^{1/\gamma}\tilde{k})}\right]^{\gamma} N^{(1-\gamma)}\right\} >$$

$$> \left[e^{E[\ln(\tilde{\theta}^{1/\gamma}\tilde{k}]}\right]^{\gamma} N^{(1-\gamma)} =$$

$$=\left[e^{\ln[(\theta^*(Q))^{1/\gamma}Q]}\right]^{\gamma}N^{(1-\gamma)} = \theta^*(Q)Q^{\gamma}N^{(1-\gamma)}$$

Thus we have established a lower bound on the expected output (conditional on Q) in terms of Q and $\theta^*(Q)$.

We can use this result in order to obtain conditional expected rates of growth for this economy. For the logarithmic utility case we have that aggregate saving S_{t+1} is given by

$$(4.7) \qquad S_{t+1} = \beta(1-\gamma)y_t$$

where β is the saving rate and y_t is the realized output at period t. Accordingly, let $Q = S_{t+1}$, so that $E(\tilde{y}_{t+1} \backslash y_t)$ can be bounded from below, using (4.6). The lower bound on the expected conditional rate of growth is then given by

$$(4.8) \qquad \theta^*(\beta(1-\gamma)y_t)(\beta(1-\gamma)y_t)^{\gamma}N^{(1-\gamma)}/y_t - 1.$$

We show next that for an appropriate choice of the pdf of θ, the expected output can be a convex function of Q, and hence (4.8) will imply that the economy may grow at a sustained rate in expectation.

Let the cdf $H(\cdot)$ which describes the distribution of θ be given by

$$(4.9) \qquad H(\theta) = 1 - \theta^{-\lambda}, \qquad \theta \geq 1, \ \ \lambda > 0.$$

Then, equation (2.4) becomes

$$(4.10) \qquad \ln(\theta^*(k)) = \gamma\ln[(k-\alpha)/k] + [1-\theta^*(k-\alpha)]\ln[\theta^*(k-\alpha)] + + \int_{\theta^*(k-\alpha)}^{\bar{\theta}} \ln(\theta)\, \lambda\theta^{-(\lambda+1)} d\theta.$$

We approximate the solution to (4.10) by the solution to the problem of search with recall, in which $\theta^*(k-\alpha)$ on the RHS is replaced by $\theta^*(k)$. In this way we avoid the recursive nature of the problem, and we solve it as an equation. It can be shown that the approximation improves as k increases. Thus we obtain

$$(4.11) \qquad [\theta^*(k)]^{-\lambda} \ln[\theta^*(k)] = \gamma\ln[(k-\alpha)/k] + + [\theta^*(k)]^{-\lambda}[\ln[\theta^*(k) + 1/\lambda]$$

Accordingly, we get

$$(4.12) \qquad [\theta^*(k)]^{-\lambda} = -\lambda\gamma\ln(1-\alpha/k).$$

For sufficiently large values of k, we approximate

$$(4.13) \qquad \ln(1-\alpha/k) \cong -\alpha/k.$$

Using (4.13) in (4.12) we have

$$(4.14) \qquad \theta^{*}(k) \cong (k/\alpha\gamma\lambda)^{1/\lambda}.$$

Accordingly, we get

$$(4.15) \qquad E(\tilde{y}_{t+1} \backslash y_t) > \Lambda y_t^{\gamma+1/\lambda} N^{(1-\gamma)},$$

where Λ is a constant which depends on α, β, γ and λ. Thus, a sufficient condition for the economy to have a sustained growth path (in expectation) is $\gamma+1/\lambda > 1$.

Notice, however, that the probability distribution under consideration in this example has no more than the first λ moments. Larger λ implies the existence of additional moments, a faster declining upper tail of the density function, and a lower expected value for θ. This has to be matched by a larger capital share of output, γ, to satisfy the aforementioned sufficient condition for sustained expected growth. For instance, when λ=2, (so that both $E(\theta)$ and $Var(\theta)$ exist), γ has to exceed 0.5 to meet that sufficient condition.

The stochastic behavior of this economy may display several patterns. In particular, the economy may have the property that it can reach an absorbing state in which it "collapses". Other configurations of the parameters imply that the economy can never collapse.
Specifically, define $\hat{k}$ by

$$(4.16) \qquad \hat{k} + \alpha = \beta\underline{\theta}F(\hat{k},N).$$

In addition, let $\mathcal{K} = \{k : \overset{*}{\theta}(k) = \underline{\theta}\}$. Let $\underline{k}$= Sup $\{k : k \in \mathcal{K}\}$. Suppose first that $\hat{k} < \underline{k}$. Then, if the economy starts with any $Q > \underline{k}$, the worst that can happen is that the search program depletes Q to $\underline{k}$, and then draws a very low value of θ. In this case, next period's Q exceeds $\underline{k}$ by at least α units, and the economy will never do worse. In fact, if it draws a sufficiently high value of θ, it may start growing. On the other hand, if $\hat{k} > \underline{k}$, the economy may collapse. The search process may deplete Q to a level k which is below $\hat{k}$, and draw a sufficiently low value of θ such that

$$k + \alpha < \beta\theta F(k,N).$$

In this case, the economy may reach a level of Q such that $Q \leq \underline{k}$, and eventually even get to the point where $\beta \cdot \theta \cdot F(Q-\alpha,N) < \alpha$, so that no search can take place next period, and the economy collpases.

5. CONCLUSION

The framework developed in this paper was chosen not so much for its realistic features, as for its interesting behavior concerning the returns to scale. Thess returns emerge from a combination of a sequential search for investment opportunities, which display increasing returns to capital, and a standard decreasing returns technology. It was demonstrated that the increasing returns to scale property may emerge despite some assumptions which were adopted in order to make it difficult for this characteristc to appear.

In particular, it is obvious that risk aversion reduces the selectivity in optimal search, and hence, makes sustained growth harder to obtain. Likewise, the assumption of "no recall" *between* periods - makes any success in the search process in a particular period impacting on subsequent periods only through its impact on current wages and savings. This description completely ignores the cumulative nature of R&D and technological progress in general. Had we modified the search problem, such that successive generations always have the fall back option of the productivity adopted by the previous generation - we would obtain a much faster growth, or equivalently, a broader set of environments with sustained growth. The specification "with recall between periods" has the additional attractive feature in that it allows for spells of "no search" following a particular "good" discovery by any generation. It would also make the possibility of a negative growth spell, and ultimate total collapse of the economy, less likely.

On the other hand, by abstracting from the structure of investment intermediation, we have also affected the stochastic growth path in a particular way. Consider a competitive intermediation sector, a typical firm in it conducts search for investment projects each period financed by savings deposited with it. There will be scope for such competing firms to coexist, since given the possibility to split their savings between firms that offer identically distributed returns - risk averse agents will take it. However, splitting the search between many investment firms will reduce the initial capital of each firm, and will result in a worse disribution of rates of return on savings, (in fact, optimal search induces a rate of return distribution which is ordered by first degree stochastic dominance according to the initial capital). Accordingly, modifying the model to account for intermediation structure will likely reduce the conditional first *two* moments of growth rates, in addition to having implications on such things as the length and severity of downturns along the growth path.

Finally, the simplicity of the model described allows to examine its stochastic properties by simulation. We plan on utilizing this method for examining the model implications on several moments of the resulting growth rate, and on the share of the resources utilized for search out of total resources. For instance, this model may help in explaining the positive (and significant) corellation between mean growth rates over time and mean standard deviation of growth rates that seems to exist across industrialized countries over long periods, both before and after the two World Wars. Moreover, since we get that sustained (expected) growth rates depend on the level of employment, in addition to the level of capital stock, we may have another way to test the model's implications.

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