FLEXIBILITY, INVESTMENT, AND GROWTH

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ABSTRACT

This paper proposes a model of diversifiable uncertainty, irreversible investment decisions, and endogenous growth. The detailed microeconomic structure of the model makes it possible to study the general equilibrium effects of obstacles to labor mobility, due to institutional as well as technological features of the economy. Labor mobility costs reduce private returns to investment, and the resulting slower rate of endogenous growth unambiguously lowers a representative individual’s welfare. Turnover costs can have positive effects on full employment equilibrium wages when all external effects are disregarded: this may help explain why policy and institutions often tend to decrease labor mobility in reality, rather than to enhance it. Lower flexibility, however, reduces the growth rate of wages in endogenous growth equilibrium, with negative welfare effects even for agents who own only labor.

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I - Introduction

Since most employment relationships entail set-up costs and formation of match-specific human capital, labor reallocation in the face of productivity and taste shocks is far from costless. Employers pay a large share of technological mobility and retraining costs: new hires need not be paid wages so low as to compensate for their low productivity and for reduced production by the coworkers who train them. Legislation and contracts also burden firms with firing costs when business conditions make labor shedding desirable. Workers fired because of adverse business conditions (rather than because of their own incompetence) are often entitled to redundancy payments, or to advance notices at unchanged wages which, from the employer's point of view, entail a loss at the margin. When directly paid to redundant workers, these costs may in fact compensate fired employees for the loss of that portion of match-specific human capital financed by labor in the form of low entrant wages or relocation expenses. "Job security" legislation, however, often requires firms to follow costly notification and negotiation procedures, and labor shedding may be further hampered by contracts and union activity. Thus, labor relocation is often made costly or difficult for firms without providing a direct monetary benefit to fired workers.¹

The effects of hiring and firing costs on firms' labor demand are well understood. In partial equilibrium, they decrease the responsiveness of employment to cost and demand shocks without necessarily reducing the average level of employment at an existing firm: in particular, firing costs tend to increase average labor demand at given wages if the firm's discount rate is positive (Bentolila and Bertola, 1990; Bertola, 1990a,b). The positive effect of firing costs on the average wage bill paid by existing firms might explain the favor accorded to job security provision by unions and by labor's political representatives. Still, the partial-equilibrium nature of the result raises obvious questions as to its validity in more general frameworks. In particular, constraints on employment flexibility reduce production efficiency and the value of firms, especially in the presence of important sources of idiosyncratic uncertainty.² In general equilibrium, a lower value of firms decreases

¹ Emerson (1988), Piore (1986), and Bentolila and Bertola (1990) review the stringency and character of such constraints in different countries and periods.

² The variability of individual firms' business conditions is largely idiosyncratic in reality; see, for example, the empirical evidence in Davis and Haltiwanger (1990).
private incentives to invest, and reduced efficiency and slower capital accumulation might in turn have adverse effects on the level and rate of growth of product demand and wages.

To address these issues, the present paper models capital accumulation as a rational and irreversible decision to create new opportunities for production and employment (or "firms") in the context of an endogenous growth model with externalities, similar to those proposed by Romer (1986, 1987) and by Grossman and Helpman (1991). Higher labor turnover costs may well have a positive effect on the equilibrium level of wages when all external effects are taken as given. Lower labor mobility, however, unambiguously reduces the value of firms and (through external effects) the level and rate of growth of output, with negative welfare effects not only for a representative agent but, for most parameter values, of agents who own only labor as well — because the endogenously determined growth rate of wages is reduced by slower capital accumulation.

Section II presents a benchmark model of growth and its solution under perfect labor mobility. The model combines aggregate production possibilities typical of endogenous growth model with a disaggregated microeconomic structure: production of each differentiated good requires a one-time outlay irreversibly allocated to a specific variety as well as a stochastically varying amount of labor, in exogenously given aggregate supply. In the economy's dynamic general equilibrium, labor moves from old to new firms, and from firms experiencing a decrease in labor productivity towards firms experiencing the opposite transition. Section III introduces labor mobility costs, derives the forward-looking employment policy followed in steady growth equilibrium by a wage-taking firm which bears turnover costs, and characterizes the effect of imperfect flexibility on equilibrium growth. Section IV discusses the welfare effects of idiosyncratic uncertainty and labor mobility costs, Section V notes that the effects of imperfect labor mobility on income distribution may explain labor turnover costs in excess of technologically given ones, and Section VI concludes.
II - Growth with perfect labor mobility

Consider an economy populated by a continuously divisible labor force, whose total size is normalized to unity. Production takes place at a continuum of production sites ("firms"), indexed by $i \in [0, M_t]$ at time $t$. A continuum of firms of infinitesimal size simplifies derivation of equilibria in two crucial respects: first, there are no integer constraints on the equilibrium "number" of firms, $M$; second, sources of uncertainty which are independently and identically distributed across firms yield deterministic aggregate outcomes.

Let the aggregate production flow at time $t$ be defined as

$$Y_t = \left( \int_0^{M_t} x_{it}^\alpha \, di \right)^{\frac{1}{1-\alpha}}, \quad \alpha < 1,$$

(1)

where $x_{it}$ denotes an individual firm's output. Since $\alpha < 1$, individual varieties are imperfectly and symmetrically substitutable to each other; thus, marginal returns to an individual firm's production in terms of aggregate output are decreasing, and the index of aggregate output rewards variety in its composition. While $M_t$ is bounded at every point in time because introduction of new goods entails a real resource cost, $Y_t$ increases without bounds as smaller and smaller amounts of more and more varieties are produced. This may be interpreted in two equivalent ways. One may assume, as in Romer (1987), that production of a single final consumption good $Y$ uses intermediate inputs $\{x_i\}$ and becomes more efficient when many different such inputs are available. Alternatively, the CES aggregate $Y$ may be seen as a Dixit-Stiglitz (1977) subutility function, attributing a taste for variety to the final consumers themselves.

The $1/(1 - \alpha)$ exponent applied to the integral of individual firms' production reflects increasing aggregate returns to scale external to every firm and, in equilibrium, ensures that aggregate output increases linearly with the number of active firms $M_t$, making it possible for the economy to grow at an endogenously determined rate. This functional form models in a simple way the idea that capital accumulation increases production efficiency in ways that can only partially be appropriated by individual investors. Various rationales for similar assumptions have been proposed by Romer (in e. g. 1986, 1988), by Grossman and Helpman (1990), and others. For the results of this paper, it is essential that growth be endogenously determined and that labor be necessary for production of each individual variety: total production being bounded by the available supply of a non-accumulated
factor of production (labor), external effects must then be present. The exact form of the externalities, and the specific channels through which they operate, are inconsequential for the results; in particular, endogenous productivity growth might equivalently be modeled as a learning externality, letting the cost of new firms in terms of consumption be inversely related to $M_t$ (see Grossman and Helpman, 1990).

II.a - Instantaneous equilibrium

Let each existing firm employ labor in proportion to its production, and let the unit labor requirement of variety $i$ be $1/\eta_i$. If $Y$ in equation (1) is interpreted as a subutility function, $\eta_i$ may represent a variety-specific taste parameter as well as purely technological productivity. Thus, this parameter is a generic index of an individual firm’s business conditions. The price of variety $i$ is proportional to

$$\frac{\partial Y}{\partial x_i} = \frac{\alpha}{1-\alpha} \left( \int_{0}^{M_t} x_i^\sigma \, di \right)^{\frac{1}{1-\sigma}} \cdot x_i^{\sigma - 1},$$

and the term in parentheses (a function of total production) is taken as given by each infinitesimally small, independently managed firm. Defining

$$Z_t \equiv \left( \int_{0}^{M_t} x_i^\sigma \, di \right)^{\frac{1}{1-\sigma}} = Y_t^\alpha,$$

and choosing aggregate output $Y$ as the numeraire, it is easy to verify that the price of variety $i$ is $Z_t x_i^{\sigma - 1}$.

If labor mobility is perfect, firm $i$ takes as given the competitively determined wage rate $w$ and chooses employment and production to maximize its cash flow. Omitting time subscripts for now,

$$x_i = x(w, \eta_i, Z) \equiv \arg \max_{x_i} \left( Z x_i^\sigma - \frac{w x_i}{\eta_i} \right) = \left( \frac{w x_i}{\alpha \eta_i Z} \right)^{\frac{1}{\sigma-1}}.$$

For simplicity, let $\eta_i$ take only two values, $\eta_g$ and $\eta_b$, with $\eta_g > \eta_b$: “good” firms enjoy favorable business conditions, while “bad” ones experience low productivity and/or

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3 In the Romer (1987) model, conversely, only the accumulated factor is necessary for production of differentiated goods, and the imperfectly competitive market structure is by itself sufficient to sustain endogenous growth.
depressed demand; only analytic complications would be introduced if \( \eta_i \) were allowed to take a larger number of values. Denoting with \( \pi \) the proportion of good firms, the full-employment condition for the (unitary) labor force,

\[
M \left( \frac{\pi x(w, \eta_g, Z)}{\eta_g} + (1 - \pi) \frac{x(w, \eta_b, Z)}{\eta_b} \right) = 1,
\]

yields the market-clearing wage level

\[
w = \alpha Z \left( (\pi \tilde{\eta}_g + (1 - \pi) \tilde{\eta}_b) \right) M^{1 - \alpha}, \tag{4}
\]

where \( \tilde{\eta}_i \equiv \eta_i^{\frac{\alpha}{1 - \alpha}} \) for typographical convenience. When the marginal revenue product is equalized across firms at this level, using (4) and (3) in (1) yields

\[
Y = (\pi \tilde{\eta}_g + (1 - \pi) \tilde{\eta}_b) M \tag{5}
\]

and the shares of wage and profit income are, respectively, \( \alpha \) and \( 1 - \alpha \). This completes the derivation of the economy's instantaneous equilibrium under perfect labor mobility.

II.b - Dynamics

To model capital accumulation, let it be possible to exchange one unit of output with the right to produce a new differentiated commodity; let it be impossible to transform existing firms back into consumption goods;\(^6\) and let a good firm turn into a bad one with constant probability intensity \( \delta \) per unit time, the reverse transition occurring with constant intensity \( \gamma \) and these events being independent across firms, so that —subject to the technical qualifications in Judd (1985)— all uncertainty washes out in the aggregate by a law of large numbers.\(^7\) Further, it is convenient to let a new firm be good with probability \( \gamma/(\delta + \gamma) \), i.e. with the (ergodic) probability that an indefinitely old firm enjoys good business conditions: thus, the long-run probabilities of good and bad states for an

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\(^6\) The irreversibility constraint \( \dot{M}_t \geq 0 \) is not binding at the aggregate level in a steadily growing economy. It has macroeconomic relevance, however, since it implies that idiosyncratic shocks to business conditions are reflected in existing firms' value (to an extent that, in the next section, will depend on the flexibility of labor reallocation).

\(^7\) Only unessential complications would be introduced if more than two \( \eta_i \) levels were allowed for. In particular, it would not be difficult to model product obsolescence, interpreting \( \eta \) as a taste parameter and assuming it to be absorbed at zero with positive probability.
individual firm coincide at all times with the actual fractions \( \pi \) and \((1 - \pi)\) of good and bad firms in the continuum \([0, M]\).\(^8\)

Let wages and operating profits (in the form of dividends) accrue to agents with possibly different wealth, but identical objective functions

\[
U(\{c_t\}) = \int_0^\infty e^{-\rho t} \frac{c_t^{1-\sigma} - 1}{1 - \sigma} dt, \quad \sigma > 0,
\]

and let these agents interact in a perfect capital market where all idiosyncratic uncertainty can be costlessly diversified away and the instantaneous lending/borrowing rate is \( r \). In the absence of aggregate uncertainty, optimization of (6) requires \( \dot{c}_t/c_t = (r - \rho)/\sigma \), and aggregate consumption \( C_t \) similarly satisfies

\[
\frac{\dot{C}_t}{C_t} = \frac{r - \rho}{\sigma},
\]

regardless of wealth distribution.

In equilibrium, the interest rate \( r \) equals the private return to investment in an optimally diversified portfolio of existing or new firms. Diversified investment does yield a riskless return: although individual firms experience positive and negative shocks, the market portfolio contains a constant share \( \pi \) of good firms and offers a riskless, constant dividend yield. Measuring consumption in the same units as output,

\[
r = (1 - \alpha) (\pi \tilde{\eta}_g + (1 - \pi) \tilde{\eta}_b).
\]

Since the probability that a new firm is good equals the fraction of good firms among the existing ones, the expected rate of return from investment in new firms is also given by the expression in (8), which is therefore consistent with decentralized entrepreneurial innovation when viewed as the cost of capital.

Insertion of (8) in (7) yields the equilibrium growth rate of aggregate consumption,

\[
\frac{\dot{C}_t}{C_t} \equiv \vartheta = \frac{1}{\sigma} \left( (1 - \alpha) \left( \pi \tilde{\eta}_g + (1 - \pi) \tilde{\eta}_b \right) - \rho \right).
\]

\(^8\) This assumption is made for convenience only. One might prefer to let new firms to be always "good" instead: then, the proportion of good firms would tend to be larger the faster is growth, and the economy would exhibit (nonstochastic) off-steady state dynamics. These are not central to the points made in this paper, and their study is left to future research.
Noting that the evolution through time of \( \{M_t\} \) is subject to the aggregate constraints

\[
\frac{dM_t}{dt} = Y_t - C_t, \quad \frac{dM_t}{dt} > 0
\]

and that equilibrium output is proportional to \( M \) by (5), it is easy to verify that \( \dot{M}_t/M_t = \dot{Y}_t/Y_t = \vartheta \) in equilibrium if the parameters are such that \( \vartheta > 0 \) (to imply that the aggregate irreversibility constraint is not binding) and that \( (1 - \sigma)\vartheta < \rho \) (to imply that the integral in (6) converges and the consumers' transversality condition is satisfied).

For the purpose of evaluating the quantitative importance of flexibility in the economy's dynamic equilibrium, it is useful to choose a baseline set of roughly realistic parameter values. Measuring time in years, let \( \delta = \gamma = 40\% \), so that spells of good and bad business conditions last a little more than two years on average. Let \( \alpha = 2/3 \), to obtain a realistic share of labor in total output. Let \( \tilde{\eta}_g + \tilde{\eta}_b = .5 \), to obtain a capital/output ratio of about four; and let \( \eta_g/\eta_b = 5 \), to obtain positive turnover over a wide range of mobility costs and to reflect the empirical importance of idiosyncratic uncertainty noted by Davis and Haltiwanger (1990) and others. With \( \rho = 3\% \) and \( \sigma = 1 \) (logarithmic utility), these parameters yield \( r = 8.33\% \) and \( \vartheta = 5.3\% \) in the costless labor mobility case.

III - Costly labor mobility

The model proposed above is similar to existing models of endogenous growth at the aggregate level, but has enough microeconomic structure to permit a study of the effects of limited factor allocation flexibility. In the costless-mobility equilibrium of the previous section, labor moves across firms for two reasons: a firm's employment changes discretely when it is hit by a business conditions shock, and every firm's production and employment decline at exponential rate \( \vartheta \) when its business conditions do not change, since \( w \) and \( Z \) grow at rates \( \vartheta \) and \( \alpha \vartheta \) in equation (3). This steady decline releases workers to be employed by newly created firms.

As in any endogenous growth model, the economy's rate of growth depends on the private returns to investment. Reducing the value of a unit of capital or "firm," obstacles to labor mobility imply lower growth rates. This section derives the economy's dynamic equilibrium under the assumption that a real resource cost is incurred when one firm's employment decreases and another's increases. Recalling that firms' product are differen-
tiated along some dimension, these costs may represent firm-specific training or relocation expenses; over and above such technological costs, labor mobility may be hampered by the labor market institutions noted in the introduction above.

For simplicity, let firms bear all labor mobility costs, whether in the form of hiring costs (e.g. relocation allowances and training) or of firing costs (e.g. redundancy payments and court procedures), and let the portion of firing costs paid directly to the fired worker correspond exactly to the relocation costs paid by him or her, rather than by his/her next employer. Mobility across firms is then costless for workers, and in equilibrium the wage rate is conveniently equalized across firms at every point in time. 9 Also, let the cost of reallocating one unit of labor increase at the same rate as labor's average productivity. This assumption conveniently yields steady equilibrium growth, and is not unrealistic: training and relocation are labor-intensive activities, and redundancy benefits are typically proportional to wage rates.

In the previous section the model could be solved in two steps — determining first the economy's instantaneous equilibrium, then its dynamic behavior. With labor mobility costs and idiosyncratic uncertainty, conversely, labor demand policies are forward-looking and are not independent of interest and growth rates. Thus, the dynamic features of the economy matter for instantaneous equilibrium, and the model needs to be solved recursively.

III.a - Dynamic labor demand
Consider first the optimal labor demand policy of a firm which takes as given the wage \( w_t \), the productivity index \( Z_t \), the growth rate \( \vartheta \), and the interest rate \( r \). Let hiring a unit of labor at time \( t \) entail a cost \( h \, w_t \) for the firm, while firing a worker costs \( f \, w_t \) at time \( t \).

If the positive constants \( h \) and \( f \) are not so large as to prevent all turnover, then it is

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9 If workers were assumed to bear some mobility costs, conversely, wages paid by good firms would need to be higher than those paid by bad firms, to induce the necessary labor mobility: with perfect capital markets and no aggregate uncertainty, however, the expected present value of wage differentials would in equilibrium offset the real mobility cost borne by workers. The employment policy of firms would take such wage differentials into account, and the value of firms and the economy's growth path would depend on the total real cost of moving one unit of labor, not on its incidence on firms rather than on workers (see the CEPR Discussion Paper version of this paper).
shown in the Appendix that the value-maximizing employment levels of a good and bad firm are given by (respectively) $l_g = l_g e^{-\vartheta t}$ and $l_b = l_b e^{-\vartheta t}$, for $l_g$ and $l_b$ satisfying

$$\alpha Z_0 \eta_g \ell_g^{\alpha-1} = w_0 (1 + \vartheta f + \delta (h + f) + r h)$$

(9)

$$\alpha Z_0 \eta_b \ell_b^{\alpha-1} = w_0 (1 + \vartheta f - \gamma (h + f) - r f)$$

These conditions are easily interpreted. Absent turnover costs, a firm should adjust employment so that the marginal revenue products of labor on the left hand sides of (9) equal the wage rate. If $h$ and/or $f$ are larger than zero, conversely, the right hand sides of (9) include terms which capture the effects of turnover costs and of uncertainty. When business conditions are unchanging, firms are continuously shedding labor because wages are steadily bid up by new entrant firms, hence the $\vartheta f$ term.10 When hiring, firms whose business conditions have improved (and new firms) regard the annuity value of the hiring cost, $r h$, as part of the wage; conversely, a firm will shed less labor when experiencing a negative shock if $r f$ is positive. The terms $\delta (h + f)$ and $\gamma (h + f)$ reflect the cautiousness of firms’ hiring and firing policies in the presence of uncertainty. For example, $l_g$ is larger the smaller is $\delta$, i.e. the more permanent is a positive business conditions shock: firms refrain from responding fully to positive shocks because with probability $(1 - e^{-\delta dt}) = \delta dt$ the favorable developments may be immediately reversed, causing a loss of $h + f$ on the marginal employment decision.

III.b - Labor market equilibrium

If $p$ denotes the proportion of the unitary labor force employed by good firms, then

$$l_g = \frac{p}{\pi} \frac{1}{M}, \quad l_b = \frac{(1 - p)}{(1 - \pi)} \frac{1}{M}$$

(10)

in full employment equilibrium. Still taking $r$ and $\vartheta$ as given, define

$$\omega \equiv \frac{1 + \vartheta f + \delta (h + f) + r h}{1 + \vartheta f - \gamma (h + f) - r f} > 1,$$

10 It is perhaps unrealistic to let firms pay “turnover” costs as they shrink inexorably. At the cost of analytical complication, the model could be extended to allow for finite lifetimes: labor force reduction in the absence of business condition changes could then be accomplished by not replacing the retiring workers, and intertemporal equilibria could be constructed along the lines of Blanchard (1985).
and note that the relationships in (9) yield

\[
p = \frac{\pi \tilde{n}_g \omega^{\frac{1}{\alpha - 1}}}{(1 - \pi) \tilde{n}_b + \pi \tilde{n}_g \omega^{\frac{1}{\alpha - 1}}}.
\]  

(11)

Thus, the larger are turnover costs, the smaller is the proportion of labor employed by good firms, as shown in Figure 1 where the marginal revenue products are plotted as functions of \( p \). (Note that \( p < \pi \), by (10) and \( I_g > I_b \).)

By its definition (1), the equilibrium output flow can be written

\[
Y_t = M_t \left( \pi^{1 - \alpha} p^\alpha \eta_g^\alpha + (1 - \pi)^{1 - \alpha} (1 - p)^\alpha \eta_b^\alpha \right)^{\frac{1}{1 - \alpha}}.
\]  

(12)

and is easily shown to be maximized when \( \omega = 1 \). Turnover costs drive a wedge between the marginal revenue product of labor at good and bad firms, and obviously reduce the (static) efficiency of labor allocation.

To complete the derivation of instantaneous equilibrium for given \( M_t \), given \( r \), and given \( \eta \), equations (10) and (11) can be inserted in (9) to obtain an expression for the equilibrium wage rate:

\[
w_t = \alpha M_t^{1 - \alpha} Z_t \left[ (1 - \pi) \tilde{n}_b + \pi \tilde{n}_g \omega^{\frac{1}{\alpha - 1}} \right]^{1 - \alpha} \frac{1}{1 + f - \gamma (h + f) - r f}.
\]  

(13)

Recognizing from its definition in (2) that \( Z_t = Y_t^\alpha \), (12) and (11) can be used in (13) to express \( w_t \) in terms of the parameters. The wage is linear in \( M_t \), and therefore increases at the same rate as capital and consumption along a balanced growth path.

III.c - Growth

The previous subsections took as given interest and growth rates which, however, are not independent of turnover costs in the economy's dynamic equilibrium with externalities. Hiring costs increase the cost of creating a new firm, and the cash flow of existing firms is reduced by the firing costs paid to reduce employment in the face of rising wages. The rate of return on investment in new productive opportunities is thus lowered by costly labor mobility, to imply slower growth by the savings relation (7). Idiosyncratic uncertainty amplifies the effects of imperfect flexibility on private returns to investment: firms' cash flows suffer from lower operating revenues due to the relatively inefficient allocation of labor on the one hand, and from turnover cost payments on the other.
These effects can be characterized quantitatively, albeit not in closed form. An entrepreneur who contemplates creating a new firm knows that its business conditions will be good with probability $\pi$, bad with probability $(1 - \pi)$. This uncertainty is idiosyncratic, and can be costlessly diversified away; to avoid arbitrage opportunities, the total ex-ante expected cost of creating a new firm must then be equal to its expected value, and in equilibrium it must be the case that

$$1 + (\pi l_g + (1 - \pi) l_b) h w = \pi V_g + (1 - \pi) V_b,$$

where $V_g$ and $V_b$ denote the value of a firm in good and (respectively) bad business conditions. These (derived in the Appendix) are quite obviously both lowered by larger turnover costs in the presence of idiosyncratic uncertainty and rising wages: the employment policy (9) optimally trades off revenue losses and costly turnover, but cannot avoid lower average cash flows.

It can be shown that equation (14) (where $l_g$, $l_b$, $w$, $V_g$ and $V_b$ depend nonlinearly on $r$ and $\vartheta$) uniquely identifies a rate of return $r$ for any growth rate $\vartheta$, and that $r(\vartheta)$ is lower the larger are $h$ and $f$. The economy is in dynamic equilibrium at the $(r, \vartheta)$ pair that solves equation (14) (ruling out arbitrage in investment opportunities) simultaneously with equation (7) (ensuring that no arbitrage opportunities exist on the consumption/savings margin for each of the agents in the economy). The loci of points that satisfy these two equations are plotted in Figure 2, using the baseline parameters above and a variety of hiring and firing cost levels.

**IV - Representative-individual welfare, and policy**

The model provides a fully characterized equilibrium solution for different labor mobility costs, and makes it possible to study the welfare effects of low flexibility. This section discusses the welfare of the economy’s representative individual, while the next section consider agents whose wealth consists of labor and firms in proportions different from the aggregate ones.

In the costless-mobility equilibrium of Section II, the accumulation constraint $C_t = Y_t - \vartheta M_t$ and equation (5) set the level of initial consumption at $C_0 = (r - \vartheta) M_0$. Aggregate and individual consumption grow steadily at exponential rate $\vartheta$, and integration of (6)
readily yields a measure of the representative individual’s welfare. Obviously, welfare increases in $C_0$ for given $\vartheta$, and in $\vartheta$ for given $C_0$. As in other models of endogenous growth which allow for non-accumulated factors of production, decentralized supply and investment decisions which take as given the level and dynamic behavior of $\{Z_t\}$ yield too much initial consumption and too low a growth rate. The representative individual’s welfare could be increased if investment and savings subsidies were used to decrease initial consumption and increase its growth rate, internalizing the positive aggregate effects of higher levels and faster growth of $\{Z_t\}$.

Before turning to limited-flexibility equilibria, it is interesting to note that firm-level uncertainty per se need not decrease welfare. By (5), (8), and (7), the level of output and its rate of growth are linear in $\eta_g$ and $\eta_b$: thus, a mean-preserving spread in these indexes of idiosyncratic business conditions has no effect on output, consumption, or welfare. Further, all idiosyncratic uncertainty being costlessly diversified away, larger values for $\delta$ and $\gamma$ (which imply more unstable business conditions) have no effect on output, growth, and welfare as long as the proportion $\pi$ of good firms in the economy is unchanged.

Larger turnover costs, conversely, reduce the level of production at all points in time, decrease the growth rate, and unambiguously worsen the representative individual’s welfare. Even in the absence of business conditions variability ($\eta_g = \eta_b$), labor mobility costs are relevant since labor needs to be steadily reallocated from existing to new firms; if $\eta_g > \eta_b$, the growth and welfare effects of low flexibility are stronger the more variable are individual firms’ business conditions, i.e. the larger are $\gamma$ and $\delta$.

To evaluate the quantitative relevance of these results, the growth rate can be solved numerically from (14), and the level of aggregate consumption can be computed subtracting from aggregate production the resources spent on labor reallocation as well as the cost of capital accumulation. Table 1 reports the equilibrium level of output for various levels

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11 Since the level and growth rate of output are convex in $\eta_g$ and $\eta_b$ if $\alpha > \frac{1}{2}$, a mean-preserving spread of firm-level productivity may well increase expected profits per unit time, make investment more attractive, and have a positive effect on growth and welfare. The profit function of a competitive firm is always convex in prices and wages (Hartman, 1972). The firms in the economy under consideration are monopolistically competitive, and cost and demand uncertainties may or may not increase their expected profits, depending on how they are measured and on functional forms.

12 Per unit time, $\vartheta$ units of labor move from old to new firms, and $(l_{gt} - l_{bt}) \delta \pi M_t$ from
of turnover cost and idiosyncratic uncertainty parameters, fixing the other parameters at the baseline values discussed above and setting \( M_0 = 1, h = 0 \); Table 2 reports the corresponding growth rates; and Table 3 reports the implied levels of the representative individual’s welfare. For realistic levels of idiosyncratic uncertainty, the growth and level effects of labor mobility costs are far from negligible: both the level and the growth rate of output fall by about half before labor mobility is shut down. The welfare measure is also drastically reduced by the resulting downward shift of the consumption path.

For any given level of labor mobility costs, the equilibrium rate of growth is again too low from the representative individual’s point of view: since markets provide no incentives for individual agents to invest at a faster rate than \( \vartheta \), more growth and higher welfare should be obtained by government intervention (or by other forms of cooperative behavior). In equilibrium, there are no incentives to speed up labor mobility either, since decentralized market interactions between atomistic workers and firms cannot result in lower turnover costs than the technologically given ones. The representative individual’s welfare could be increased not only by direct subsidies to investment and savings, but by policy measures tending to reduce labor mobility costs and/or idiosyncratic business conditions variability. Such measures would improve firms’ cash-flows and, if financed by lump-sum or consumption taxes, would beneficially distort private choices towards faster capital accumulation. Thus, the externality that makes endogenous growth possible in the presence of a non-accumulated factor of production not only magnifies the welfare effects of imperfect flexibility, but also justifies policies tending to improve flexibility: if static and intertemporal markets were perfect and complete, conversely, the decentralized equilibrium with adjustment costs would achieve a constrained optimum, and adjustment assistance would not improve the representative individual’s welfare (a point made by Mussa 1978, Rob 1989, and others, in different contexts).

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existing firms turned bad to existing firms turned good. The cost of these flows in terms of output is \((h + f)w_t\) per unit of labor. Thus, \( C_t = Y_t - \vartheta M_t - (h + f)w_t(\vartheta + \pi M_t\delta(l_g - l_h))\). Since \( w_t, Y_t, \) and \( M_t \) grow at rate \( \vartheta \), and \( l_g \) and \( l_h \) decline at the same rate, the proportion of output spent on labor reallocation is constant in steady growth equilibrium.
V - Income distribution and excess turnover costs

The normative implications of the model should be contrasted with the institutional features noted in the Introduction above: in reality, legislation often tends to restrict labor mobility, and measures intended to reduce job security encounter fierce opposition. To rationalize such resistance to labor mobility across firms, one might want to consider features of real-life economies left out of the scope of the model (like financial market imperfections, or insider/outside interactions of Lindbeck and Snower, 1988). It is interesting to note, however, that in the context of the model proposed here labor mobility costs in excess of technologically given ones may have (locally) positive effects costs on equilibrium wages when all external effects are disregarded: thus, organized labor may have incentives to lobby against flexibility measures.

V.a - Wages and turnover costs

It is not difficult to see from equation (13) that the wage may be a decreasing or increasing function of turnover costs for given r, θ, Mt, and Zt. On the right-hand side of (13), the term in square brackets is smaller the larger are turnover costs: quite intuitively, the wage rate in terms of output tends to be lowered by decreased labor allocation efficiency (as indexed by ω, the wedge between the marginal revenue product of labor at different firms). Since r > θ, however, the last term on the right-hand side of (13) increases in h and f at an increasing rate, to imply that the wage is an increasing function of in h or f when their positive effect on the last term more than offsets their negative effect on the efficiency of labor allocation.

The relationship between the wage and turnover costs implied by equation (13) is plotted in Figure 3 for given r, θ, and Z. The wage is a U-shaped function of f for fixed h, and of h for fixed f. It is decreasing in h and f when these are low and the first two terms on the right-hand side of (13) are steeper functions of h and f than the third one. When labor mobility is already very costly, conversely, higher h and (especially) higher f are consistent with a higher wage rate in full-employment equilibrium. This somewhat unintuitive result has a straightforward explanation. Turnover costs inhibit firing as well as hiring: thus, they reduce labor demand by firms in good business conditions, but increase labor demand by firms in bad business conditions. If the latter effect dominates the first, the aggregate level of labor demand at given wage, and of the wage rate that yields full-employment of
a given labor force, are an increasing function of turnover costs.

In the Figure, which uses the baseline parameter values discussed above, $w$ starts increasing in $f$ at $f \approx 0.9$ if $h = 0$: thus, lower flexibility increases the wage if firing costs exceed 11 months of wages.\textsuperscript{13} Conversely, if $f = 0$ then $w$ starts increasing in $h$ only at $h \approx 1$. The point at which the wage effect of mobility costs changes sign depends on the parameters in complex ways. The tendency of the wage to be an increasing function of firing costs sooner than of hiring costs is general, however, and is due to the "discounting effect" discussed in Bentolila and Bertola (1990) and in Bertola (1990a,b): when hiring, firms take into account the full annuity value of hiring costs, but only the expected discounted value of firing costs from the time of the next adverse shock to business conditions; conversely, firing decisions are inhibited by the full, undiscounted firing cost and by the expected discounted value of the hiring cost. Thus, higher firing costs tend to discourage firing decisions more than hiring decisions, and to increase average employment. As to the other parameters, the net effect of more job security is more likely to be positive the larger is the difference between the interest rate and the rate of growth, the less convex is marginal revenue product as a function of employment, and the more persistent are spells of good business conditions.\textsuperscript{14}

\textbf{V.b - Workers vs. investors}

The effect of turnover costs on wage rates is relevant to policy choices if the economy is inhabited not by identical, "representative" individuals, but by agents with heterogeneous endowments who fail to recognize the endogeneity of productivity and growth. To solve the model, it was not necessary to specify the distribution and composition of wealth across

\textsuperscript{13} Firing costs this high are not unrealistic. Bentolila and Bertola (1990) provide measures of firing costs in the 1975-1986 period which range from 3 months of wages for the United Kingdom to over a year of wages for Italy. It is important to note at this point that, in the context of the model, $f$ measures mobility costs across products, which may or may not correspond to mobility costs across corporation. The Japanese system of lifetime employment and frequent retraining corresponds to low $f$, while constraints on labor redeployment inside the firm — such as those noted by Piore (1986) — would increase $f$.

\textsuperscript{14} These results are closely related to those derived in Bentolila and Bertola (1990) and Bertola (1990a) for the ergodic mean of a single firm’s labor demand. Since uncertainty is purely idiosyncratic and new firms are spread across states with the ergodic probabilities, the crosssectional average over a continuum of firms coincides with the ergodic mean over time for a representative firm.
agents: as long as every individual’s objective function is the one in equation (6) and
the output, labor, and financial markets function as assumed above, only the aggregate
budget constraint matters for equilibrium prices and quantities. In particular, the model
can accommodate agents who own only labor, or “workers.” Every unit of labor contributes
\( w_t \) to their income at time \( t \) and, recalling that any portion of \( f \) paid directly to the worker
is assumed to cover mobility costs exactly, idiosyncratic uncertainty has no effect on labor
income.\(^{15}\) Further, the rate of growth of wage income is the same as the desired rate of
growth of consumption: thus, agents who own only labor never save and never earn any
profit income, and an initial dispersion of factor incomes across agents is perpetuated by
different saving propensities out of wages and profits.\(^{16}\)

Since workers never save, they are indifferent to the decrease in interest and profit rates
caused by limited labor mobility. As long as the level and dynamic behavior of disembodied
productivity \( \{ Z_t \} \) is viewed as exogenous (as would be appropriate in the context of the
Solow (1956) model of growth), workers have apparent incentives to slow down labor
reallocation if the economy is on the upward-sloping portion of the relationships in Figure
2. In the model considered here, however, lower flexibility decreases the productivity
index \( Z \) by worsening the efficiency of labor allocation, and slows down wage growth by
discouraging capital accumulation: these effects, though external to every agent’s decision
problem, have to be considered in welfare evaluations.

The welfare of a worker is fully determined by the level and rate of growth of wages,
and is easily computed integrating (6) with \( c_t = w_t \). In theory, constraints on labor
mobility might increase this welfare measure if their effect on income distribution were
to offset the lower level of output (hence of disembodied productivity, \( Z \)) and the lower
growth rate of wages. For this to be the case, not only should the economy be on the

\(^{15}\) If the incidence of mobility costs fell (partly) on labor, conversely, workers would finance
mobility by borrowing and lending against wage differentials (see the CEPR D.P. version of this
paper for details). All agents would then participate in the capital market, and the extension
would somewhat complicate the welfare comparisons of this section. As noted in footnote 9,
however, none of the substantial results would be affected.

\(^{16}\) Such dispersion of factor endowments across agents, though realistic, is not explained by
the model. Heterogeneity of factor incomes might perhaps be corrected by redistributive policies:
still, workers do not participate in the capital market, and endowment redistribution may not be
feasible. These points are further discussed in Bertola (1990c).
upward-sloping portion in Figure 3 (indicating that mobility costs are quite high already), but a large value of $\sigma$ would be needed as well: a low elasticity of intertemporal substitution would both reduce the growth effects of lower return rates (by equation 7), and minimize the effect on a worker's welfare of lower future consumption.

For realistic parameter values, however, the welfare impact of labor mobility costs is negative for workers as well as for firm-owners and representative individuals. Tables 4 and 5 report the wage, the share of labor in income, and the welfare of a unit-sized worker, using the baseline parameters of Section II, various turnover cost, and different levels of idiosyncratic uncertainty. Even though more job security may increase the share of labor (as shown in Figure 2), its negative effect on the productivity indicator $Z$ is such as to make the wage a decreasing function of turnover costs over all the range of parameters considered. Thus, manipulation of turnover costs may make it possible for "labor" to extract a larger share of a (smaller) producer's surplus but, when all general equilibrium interactions are taken into account, stringent job security provisions yield decentralized equilibria that are worse, even for individuals who own only labor, than those implied by technological mobility costs.

17 Note, once again, that the firing cost $f$ is by assumption a net loss for the economy: as $f$ rises, workers make it more and more unpleasant for their employers to fire them but do not receive redundancy payments in a form that could increase their wealth and consumption. Firing costs should affect wage determination if they were paid to workers instead, and would be irrelevant under perfect capital markets (see footnote 9).
VI - Conclusions

This paper proposes a disaggregated growth model and uses it to study the general equilibrium effects of costly labor mobility. Labor mobility costs lower the welfare of a representative individual in the economy. The market imperfections that make endogenous growth possible in the presence of non-accumulated factors of production magnify the negative welfare effects of imperfect flexibility on the one hand, and justify subsidies to labor mobility (as an alternative to direct investment subsidies) on the other. Obstacles to labor reallocation also affect the share of wages in aggregate income, and this may induce individuals who own only labor to lobby for stringent “job security” provisions. Their welfare, however, is quite likely to decrease when the level and rate of growth of labor productivity are lowered by external effects in general equilibrium.

The results may be read as providing a new explanation for different growth rates in different countries and different historical periods. Many other features of an economic system are also relevant for growth, however, and it might be misleading to focus narrowly on any given one. Rather, the paper provides a framework in which to study the interactions between labor market institutions and industrial structure on the one hand, and capital accumulation and growth on the other. The simplifying assumptions made above could (and, for empirical work, should) be relaxed without destroying the main results of the paper. In particular, the model could (in principle) accommodate realistic business conditions processes with more than two states, search unemployment, mobility costs borne by labor, and non-steady-state dynamics due to quality differences between new and existing productive opportunities. The simple structure of the model solved above, however, yields insights which appear quite general.

Some assumptions, and their role in producing the results, deserve to be reviewed in conclusion. The model departs from standard representative-agent assumptions in two key respects: production sites are heterogeneous, and politico-economic conflicts between classes of agents with different factor endowments, though not explicitly modeled, are taken into consideration. Labor mobility constraints are binding in the model not only because the exogenously given supply of labor needs to be relocated from old to new productive activities, but also because the desirability of production at different sites varies idiosyncratically over time. Business conditions being heterogeneous, mobility costs reduce the
efficiency of labor allocation and the economy's productivity on the one hand, and affect wage determination in non-obvious ways on the other. The former effect is obviously harmful for the representative individual; yet, the latter may appear desirable to agents who own only labor, and the interaction of endogenous growth and idiosyncratic uncertainty is crucial to the results. If all firms were the same, constraints on labor mobility would still affect capital accumulation, but would have no effect on equilibrium wages; conversely, if the level and growth rate of productivity were exogenously given, then lower profit rates would not feed back into wage determination, and would be a matter of indifference to workers. Growth being endogenous, conversely, on the one hand workers never save, and may disregard the negative effect of low flexibility on profit rates and firms' values; on the other hand, wage growth depends (through external effects) on the pace of capital accumulation and on the degree of labor mobility, and low-flexibility, low-growth equilibria may be bad ones for workers as well as for capitalists and "representative" agents.
Appendix

In steady growth equilibrium, the rate of return is constant and equal to \( r \); wages and unit turnover costs grow at rate \( \vartheta \); and disembodied productivity \( Z_t \) grows at rate \( \vartheta a \). Denote with \( V(I, q, t) \) the value of a firm which employs \( I \) workers and whose business conditions index is \( q \), at time \( t \). If the firm's employment policy is chosen so as to maximize its value, \( V(.) \) must solve the pair of functional Bellman equations

\[
\begin{align*}
    r V(I, q, t) dt &= \max_{I_t} \left( Z_0 e^{\alpha q t} (\eta q I_t)^a - w_0 e^{\alpha q t} \left( I_t - h[I_t]^+ + f[I_t]^+ \right) \right) dt + \\
    &\quad + (1 - e^{-\delta dt}) \left( V(l_t+dt, \tau_t + dt) - w_{l_t+dt} \left( [h(l_t+dt - l_t]^+ + f[l_t+dt - l_t]^+] \right) \right) \\
    r V(l, q, t) dt &= \max_{l_t} \left( Z_0 e^{\alpha q t} (\eta q l_t)^a - w_0 e^{\alpha q t} \left( l_t - h[l_t]^+ + f[l_t]^+ \right) \right) dt + \\
    &\quad + (1 - e^{-\gamma dt}) \times \left( V(l_t+dt, \tau_t + dt) - w_{l_t+dt} \left( [l_t+dt - l_t]^+ + f[l_t+dt - l_t]^+] \right) \right)
\end{align*}
\]

(A1)

where \([x]^+ \equiv \max(x, 0)\), \([x]^- \equiv \min(x, 0)\), and \( \dot{x} \) denotes the left time derivative of \( x \). In words, the required return on the value of the firm over a small interval of time \( dt \) must equal the sum of the cash flow (the term multiplied by \( dt \)) and of the expected capital gain or loss per unit time, computed considering that over a time interval \( dt \) a good firm becomes bad with probability \( (1 - e^{-\delta dt}) \), and a bad one becomes good with probability \( (1 - e^{-\gamma dt}) \).

By inspection of (A1), the value of a firm is independent of time if employment decreases exponentially at rate \( \vartheta \) in the absence of business condition changes. Thus, we can write \( V(l_t, q, t) = V_g \) and \( V(l_t, \tau, t) = V_b \), understanding that \( l_t = l_g e^{-\vartheta t} \) when \( \eta_t = \eta_g \), and \( l_t = l_b e^{-\vartheta t} \) when \( \eta_t = \eta_b \). Only the case \( l_t > l_i \) will be considered in what follows. Noting that \((1 - e^{-\delta dt}) = \delta dt \) and omitting time subscripts, \( V_g, V_b, l_g, l_b \) must solve the system

\[
\begin{align*}
    r V_g &= \max_{l_g} \left\{ Z (\eta q l_g)^a - (1 + f \vartheta) l_g w + \delta (V_b - V_g - (l_g - l_b) f w) \right\} \\
    r V_b &= \max_{l_b} \left\{ Z (\eta q l_b)^a - (1 + f \vartheta) l_b w + \gamma (V_g - V_b - (l_g - l_b) h w) \right\}
\end{align*}
\]

(A2)

It is convenient to solve these equations in two steps. Taking the maximization for granted, differentiate the two equations in (A1) with respect to \( l_t \) (this is the same as
considering a perturbation that would vary employment by the same small amount at all points in time). The resulting pair of linear relationships between the two possible levels of the (detrended) marginal revenue product and the two possible marginal values of labor, denoted $V'_g$ and $V'_b$, can be solved for the latter to obtain

$$(r + \delta + \gamma)r V'_g = (r + \gamma)\left(\alpha Z\eta_g l_g^{n-1} - (1 + \vartheta f) w\right) + \delta \left(\alpha Z\eta_g l_g^{n-1} - (1 + \vartheta f) w\right)$$

$$(r + \gamma + \delta)r V'_b = (r + \delta)\left(\alpha Z\eta_b l_b^{n-1} - (1 + \vartheta f) w\right) + \gamma \left(\alpha Z\eta_b l_b^{n-1} - (1 + \vartheta f) w\right)$$

Provided that if $l_g > l_b$, optimality of the employment jumps at times when business conditions change requires $V'_g = h, V'_b = -f$. These and (A3) form a linear system which is easily solved to yield equation (9) in the text.

Using $l_g$ and $l_b$ from (9) in (A2), it is straightforward to obtain expressions for the value of firms in "good" and "bad" business conditions:

$V_g(r, \vartheta) = (r + \gamma) \frac{Z(\eta_g l_g)^{\alpha} - w(l_g(1 + \vartheta f) + \delta(l_g - l_b)f)}{r(r + \gamma + \delta)}$

$+ \delta \frac{Z(\eta_b l_b)^{\alpha} - w(l_b(1 + \vartheta f) + \gamma(l_g - l_b)h)}{r(r + \gamma + \delta)}$  

$V_b(r, \vartheta) = \gamma \frac{Z(\eta_g l_g)^{\alpha} - w(l_g(1 + \vartheta f) + \delta(l_g - l_b)f)}{r(r + \gamma + \delta)}$

$+ (r + \delta) \frac{Z(\eta_b l_b)^{\alpha} - w(l_b(1 + \vartheta f) + \gamma(l_g - l_b)h)}{r(r + \gamma + \delta)}$  

(A4)
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Bertola, Giuseppe (1990c), "Factor Shares, Saving Propensities, and Endogenous Growth," working paper


Emerson, Michael (1988), "Regulation or Deregulation of the Labor Market," *European Economic Review* 32, 775-817


Piore, Michael (1986), "Perspectives on Labor Market Flexibility," *Industrial Relations* 25, 146-166
Rob, Rafael (1989), "Labor Movements in a Two Sector Economy with Switching Costs," working paper
### TABLE 1

Growth rate, $\theta$

<table>
<thead>
<tr>
<th>f = 0</th>
<th>f = 0.2</th>
<th>f = 0.5</th>
<th>f = 1.0</th>
<th>f = 1.5</th>
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<tbody>
<tr>
<td>$\delta = \gamma = 0.1$</td>
<td>0.0533</td>
<td>0.0527</td>
<td>0.0512</td>
<td>0.0491</td>
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<td>$\delta = \gamma = 0.2$</td>
<td>0.0533</td>
<td>0.0521</td>
<td>0.0475</td>
<td>0.0397</td>
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<td>$\delta = \gamma = 0.3$</td>
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<td>0.0509</td>
<td>0.0397</td>
<td>0.0210</td>
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<td>0.0491</td>
<td>0.0268</td>
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<tr>
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<td>0.0463</td>
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### TABLE 2

Output, $Y_0$

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<tr>
<th>f = 0</th>
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<tr>
<td>$\delta = \gamma = 0.1$</td>
<td>0.250</td>
<td>0.249</td>
<td>0.247</td>
<td>0.242</td>
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<td>$\delta = \gamma = 0.2$</td>
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<td>0.248</td>
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<td>0.217</td>
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<tr>
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<td>0.215</td>
<td>0.163</td>
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<td>$\delta = \gamma = 0.4$</td>
<td>0.250</td>
<td>0.239</td>
<td>0.178</td>
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<tr>
<td>$\delta = \gamma = 0.5$</td>
<td>0.250</td>
<td>0.231</td>
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</tr>
</tbody>
</table>

### TABLE 3

Representative individual welfare

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<tr>
<th>f = 0</th>
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<th>f = 0.5</th>
<th>f = 1.0</th>
<th>f = 1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta = \gamma = 0.1$</td>
<td>5.05</td>
<td>2.46</td>
<td>-1.02</td>
<td>-4.8</td>
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<tr>
<td>$\delta = \gamma = 0.2$</td>
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<td>0.55</td>
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<td>-18.6</td>
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<td>-1.90</td>
<td>-18.7</td>
<td>-43.9</td>
</tr>
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<td>-5.15</td>
<td>-36.2</td>
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<tr>
<td>$\delta = \gamma = 0.5$</td>
<td>5.05</td>
<td>-9.49</td>
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</tbody>
</table>
### TABLE 4

Wage level, $w_0$

<table>
<thead>
<tr>
<th>$f$</th>
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<th>$f = 0.2$</th>
<th>$f = 0.5$</th>
<th>$f = 1.0$</th>
<th>$f = 1.5$</th>
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</thead>
<tbody>
<tr>
<td>$\delta = \gamma = 0.1$</td>
<td>0.167</td>
<td>0.156</td>
<td>0.146</td>
<td>0.139</td>
<td>0.132</td>
</tr>
<tr>
<td>$\delta = \gamma = 0.2$</td>
<td>0.167</td>
<td>0.149</td>
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</tr>
<tr>
<td>$\delta = \gamma = 0.3$</td>
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<td>0.143</td>
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<tr>
<td>$\delta = \gamma = 0.4$</td>
<td>0.167</td>
<td>0.137</td>
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<td>0.167</td>
<td>0.131</td>
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### TABLE 5

Share of wages in output

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<tr>
<th>$f$</th>
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<th>$f = 0.2$</th>
<th>$f = 0.5$</th>
<th>$f = 1.0$</th>
<th>$f = 1.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta = \gamma = 0.1$</td>
<td>0.667</td>
<td>0.624</td>
<td>0.591</td>
<td>0.573</td>
<td>0.562</td>
</tr>
<tr>
<td>$\delta = \gamma = 0.2$</td>
<td>0.667</td>
<td>0.602</td>
<td>0.566</td>
<td>0.564</td>
<td>0.598</td>
</tr>
<tr>
<td>$\delta = \gamma = 0.3$</td>
<td>0.667</td>
<td>0.584</td>
<td>0.566</td>
<td>0.635</td>
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</tr>
<tr>
<td>$\delta = \gamma = 0.4$</td>
<td>0.667</td>
<td>0.571</td>
<td>0.608</td>
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</tr>
<tr>
<td>$\delta = \gamma = 0.5$</td>
<td>0.667</td>
<td>0.565</td>
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</tr>
</tbody>
</table>

### TABLE 6

Welfare of unit-size worker

<table>
<thead>
<tr>
<th>$f$</th>
<th>$f = 0$</th>
<th>$f = 0.2$</th>
<th>$f = 0.5$</th>
<th>$f = 1.0$</th>
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<tbody>
<tr>
<td>$\delta = \gamma = 0.1$</td>
<td>-0.466</td>
<td>-3.42</td>
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<td>-16.5</td>
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<tr>
<td>$\delta = \gamma = 0.2$</td>
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<tr>
<td>$\delta = \gamma = 0.4$</td>
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<td>-11.8</td>
<td>-44.4</td>
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<td>$\delta = \gamma = 0.5$</td>
<td>-0.466</td>
<td>-16.4</td>
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</table>
Figure 1: labor market equilibrium

\[ \text{MRPL} = \frac{\eta (1-p)/(1-\pi)^{\alpha-1}}{\eta (p/\pi)^{\alpha-1}} \]

\[ (r+\delta+\gamma)(h+f) \]

Figure 2: no-arbitrage conditions

\[ f = 0.0, \ h = 0.0, f = 0.0 \]

\[ 1+(n_1+(1-\pi)n_2)wh = \pi \nu_0 + (1-\pi)\nu_6 \]

\[ h = 0.3, f = 0.3 \]

\[ h = 0.4, f = 0.4 \]

\[ h = 0.5, f = 0.5 \]
Figure 3: wage and turnover costs

\( w = f(x) \) for different values of \( h \):
- \( (h=0, f=x) \)
- \( (h=z, f=0) \)