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SOCIAL NORMS, SAVINGS BEHAVIOR, AND GROWTH*

by

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"To what purpose is all the toil and bustle of the world?...It is our vanity which urges us on. ...It is not wealth that men desire, but the consideration and good opinion that wait upon riches." Adam Smith, *The Theory of Moral Sentiments*.

"The boy with the cold hard cash is always mister right because we are living in a material world and I am a material girl." Madonna, *Material Girl*.

1. Introduction

Economists have difficulty explaining the differences in growth rates across countries. In recent decades a number of Asian countries - Japan, Korea, and Taiwan, for example - have grown at extremely high rates, while at the same time the U.S. has exhibited steady growth and a number of South American countries such as Argentina, Brazil and Chile have grown very slowly. Any attempt to explain cross-country differences in growth must look at the incentives people in an economy have to expend effort and to invest resources in the production of goods and services. A standard approach consists of specifying preferences for the agents and a production technology available to those agents and deriving the growth path implied by optimizing behavior on the part of the agents. Typically, a representative agent model is studied in which the agent has access to a capital-consumption technology and the agent's decision consists of period-by-period choices of the level of labor effort and the fraction of output to consume, the remainder being the investment next period.[1] This approach then seeks to explain differences across countries on the basis of differences in endowments, physical or human. These attempts have not been completely successful; there is typically a large residual that cannot be explained by such differences (see Barro (1989) or Kormendi and Meguire (1985)).

It is tempting to explain these differences by positing differences in the underlying preferences or technologies across countries. For example, one might seek to explain the high savings rates in Japan and the corresponding low rate in the United States by assuming differences in the discount rates for people in the two countries. While such assumptions can bring the predictions of the model in line with the observed facts (as such assumptions reconcile almost any observations with the theoretical predictions of the model), it leaves unexplained the basis for these differences in preferences or technologies. In particular, if we start from the belief that in the long-run, technologies are the same and that the agents

[1]This literature began with the nonoptimizing models of Solow (1956) and Swan (1956). More recent optimizing models include Cass (1965), Lucas (1988), Rebelo (1987), and Romer (1986).

populating countries are, on average, essentially identical biological organisms, it is incumbent upon the modeler to explain how such differences arise.

A way of resolving this problem is to postulate that people have identical preferences over "deep" variables and that what we typically observe in economic problems are "reduced form" preferences. For example, people in different societies may have identical utility functions over streams of consumption for themselves and their children which give rise to different reduced form, one period utility functions for consumption because fewer children survive in one environment than another. In this spirit, we investigate how different social organizations within societies can lead to different reduced form preferences even when the underlying preferences are identical. Such different social organizations will lead to different reduced form preferences, which in turn may result in different rates of growth.

The paper is structured as follows. In Section 2 we elaborate upon how societies may differ with regard to the manner in which they allocate nonmarket goods. The specific distinction in social organization on which we have chosen to focus is the allocation of mates. Section 3 presents the basic voluntary matching model. Section 4 discusses a stylized version of the model in which only savings and matching decisions are considered, and analyzes the growth trajectories for economies with different stylized social structures. In Section 5 we discuss at greater length some of the issues that our model raises.

2. Social Decisions and Economic Models

The interaction between the organization of a society and its economic performance was once considered perhaps the fundamental question of political economy. At least since Marx, it has been argued that there is an intimate relationship between the social and political organization of a society and its method of production. One could distinguish between those aspects of a society's organization that impact on the reduced form preferences and those aspects that impact on the reduced form technology. Reduced form technology could vary, for example, if societies differed in their ability to generate cooperative behavior among economic agents. In future work we plan to address precisely such a question. In this paper we restrict attention to analyzing how reduced-form preferences might be affected by social organization.

The specific question we examine is how the means by which a society makes non-market decisions can affect the market behavior of the agents. Examples of non-market decisions include the process by which decisions are made about the provision of public goods, the most attractive mate for oneself or one's child, a favorable location in church or at the dinner table, a respectful audience when one speaks, or the cemetery plot with the best view. In many cases the consumption of these goods is done within a particularly social context, which may explain why they are not allocated via a market mechanism. We assume that an agent's utility is affected by non-market decisions as well as by the goods and services he acquires in the market and directly consumes, and further, that agents have identical utility functions over these variables.

Agents will often have conflicting preferences over these non-market goods, hence a society must have some means of reconciling differences. We interpret an agent's *status* as a ranking device which determines how well he fares with respect to non-market decisions.

We are interested in examining how differing conventions for making non-market decisions interact with market decisions and which rules for conferring status on agents can be consistent with their market behavior. In order to make these conventions consistent with optimizing behavior we want agents to take into account the effect of their decisions on their status. Thus, our model will specify preferences over market goods and non-market decisions, how non-market decisions are made as a function of the status levels of the agents, and how the allocation of status will change as a function of agents' decisions.[2] An equilibrium will be a decision rule for agents such that, given these specifications, it is optimal for each agent to follow the prescribed rule when all other agents are following the rule.

By allowing for different social conventions in the allocation of non-market decisions, we have introduced a new dimension of indeterminacy of equilibria. If differing social conventions alter agents' incentives in making market decisions, there can be differing levels of economic activity. More specifically, if the relation between wealth and status differs across societies, we should expect this to be reflected in different levels of economic activity. We interpret a society in which higher wealth confers higher status as a society that exhibits greater social mobility than a society in which status depends primarily upon ancestry. Here social mobility refers to the nonmarket benefits of wealth rather than the equality of market opportunities. To the extent that one society is more socially mobile than another, agents in that economy have a greater incentive to acquire wealth.

As we have suggested, we believe that there are a number of non-market decisions that we could employ to construct the sort of model in which we are interested. The specific decision we will use is a matching decision: our model will have men and women who will match and have preferences over the

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[2]Status enhancing actions can vary widely across cultures. Examples might include graduating from Harvard, having a title, possessing good table manners or witty conversational skills, owning jewelry, a stylish wardrobe, a Ferrari, anything with Gucci on it, or, within the inner city, a pair of "Air Jordans" and a large boom box.

matches they will enter into.[3] Roughly speaking, there may be different social norms as to how people are "supposed" to match. An equilibrium will include a description of a social norm that agents have an incentive to follow. An agent's decision about whether or not to follow the prescriptions of a social norm will be complicated by the fact that deviating from the prescription may have consequences for the agent's offspring, about whom the agent cares.

Agents will have common preferences over potential matches. Relative success in the matching process will be determined by agents' status. We will be particularly interested in two stylized social norms: one in which agents' status is inherited and another in which status is determined solely by relative wealth.

This paper departs sufficiently from past models that it is worth making a few points clear before presenting our model. First, we don't "put social status into the utility function". If we were to say that an agent directly cared about his position in society and that his position depended upon his relative wealth, we would obviously have savings behavior that differed from the case in which he cared only about his consumption. In our model, people only care about status because in equilibrium it may affect variables that enter the utility function. We emphasize that even when agents don't care directly about their position, there may be an "indirect utility" to status because in equilibrium it affects real variables. Note that a model in which one put status into the utility function would be silent on exactly the sort of cross cultural comparisons that one would like to make.

We should also emphasize that the fundamental ideas about which we are writing are not new to us. As the quote at the beginning of this section indicates, the notion that status is important in understanding economic decision-making goes back (at least) to Adam Smith.[4] We see our contribution not as the introduction of the concern about status into the analysis of growth but as reintroducing the treatment of social organization into the analysis of economic growth in a way that is compatible with the more formal models of growth that have been the prevailing mode of analysis in the past several decades. Given this view of our contribution, we have tried wherever possible to maintain the form and substance of more traditional models so as to highlight the differences that arise from integrating social organizations with these prevailing models. We make no claim that all our choices in modelling are obvious, or even best. Rather, our model is constructed to get us to our goal: to incorporate status into

[3] The advantage of this particular decision is it requires few ad hoc assumptions (about such things as property rights). See also Section 5.1.

[4] More recently, Akerlof (1976) and Basu (1989) have presented models in which an agent's caste (or status) has economic implications and moreover, the social norms supporting the caste structure form an equilibrium.

a growth model that has testable implications and fits, in at least broad terms, the stylized facts as we see them.

3. The Basic Model

We focus on a simple multi-generational society in which at each period there is a continuum of each of two different types of one period lived agents, men and women. The agents are matched into pairs. Each pair will produce two offspring, one male and one female. Besides the matching decision, agents make standard economic decisions: what to consume and/or what to invest. The agents in this model differ from neoclassical agents in that they care about the nature of the mate with whom they pair.

The model is asymmetric with respect to men and women in two important respects. First, women are endowed with a non-traded, non-storable good. (For concreteness, the reader can think of some ability with which women are exogenously endowed.) The distribution of endowments of women is uniform on [0,1] in each period. The endowment of a daughter is independent of the mother's endowment. We use endowments to index the women so that woman j is endowed with an amount j of the non-traded good. The men will be indexed by $i \in [0,1]$.

The second asymmetry is that we assume that only the welfare of the male offspring enters the pair's utility function. We normalize so that a male offspring inherits his father's index, and we will refer to man i, his son, his son's son, and so on, as *family line* i.

Men and women have identical utility functions defined over *joint* consumption c. (This is done to avoid distributional issues.) We assume that the utility derived from joint consumption is given by a constant relative risk aversion utility function with degree of risk aversion γ, $u(c) = (1-\gamma)^{-1}c^{1-\gamma}$. The level of the woman's endowment, j, enters linearly into current utility.[5] Finally, the utility level of their male descendants also enters linearly into each parent's utility function, discounted by $\beta \in (0,1)$.

The problem facing a couple is, given their capital (the bequest from the male's parents), how much to consume and how much to bequeath to their son. Their son receives utility from the bequest in two distinct ways. First, it may affect the quality of his mate. Second, it affects the amount he and his male descendants can consume (and their mates, but the couple only cares about the male offspring).

[5] The model is clearly well defined for more general utility functions and value of the woman's endowment. The existence theorem for one type of equilibrium is stated in the Appendix for the general case. We believe that most of the results of interest carry over to much broader class of utility function than CRRA. We have chosen for expository purposes to keep the discussion in the text confined to this case.

Parents will be willing to reduce current consumption if it sufficiently increases the quality of their son's mate.

In order to describe equilibria, it is convenient to introduce the notion of status. The role of status in our model will be to rank the men. This ranking determines the outcome of the matching stage. A man's status can depend upon, among other things, his own wealth and the status and match of his parents. A *social norm* is a status assignment rule indicating how a man's status is determined and a prescription of status-dependent matching behavior. An *equilibrium* is a social norm together with a specification of market decisions such that no agent wishes to deviate from the social norm or specified market decisions.

We assume that matching is voluntary and takes place based upon complete current information. Let P_i and P_j denote the (reduced form) preferences of men and women over mates.[6] Men prefer women with higher values of j, all else equal. Of course, being successfully matched with a woman with a high j may (depending upon the prevailing social norm) result in a loss of status for the offspring, which P_i will reflect. Women prefer men who can provide high current consumption (through their inheritance) or whose offspring will have high status or consumption. Here too, being successfully matched with a man who can provide high current consumption may result in a loss of status for the offspring, which P_j will reflect.

We say that $m:[0,1] \rightarrow [0,1]$ describes a *voluntary matching*, where $m(i) = j \in [0,1]$ is i's match, if the following two conditions hold:

(i) there does not exist $i \neq i' \in [0,1]$ such that $m(i')P_i m(i)$ and $iP_{m(i')}i'$, with both preferences holding strictly;

(ii) for any (measurable) set $B \subset m^{-1}[0,1]$, m(B) has the same Lebesgue measure (size) as B.

Condition (i) says that no pair prefers to be matched to each other rather than with their matches. Condition (ii) requires that any fraction of matched men are matched with that fraction of matched women (this condition is implied by one-to-oneness when there is a finite numbers of agents). In a voluntary matching, all men and women receive a match (except possibly for a zero measure set of women).

We will not describe the details of the matching process. What is important is that each man can make an offer to match with any woman. Women will choose the most attractive offer if they receive more than one proposal. This concludes the matching round; in particular there are no counter proposals by women to men and no offers from men reacting to the offers of other men. If only one proposal is

[6]That is, these preferences take as fixed the behavior of all other agents.

received by a woman, then that proposal is accepted. If a man's proposal is unsuccessful, then that man is matched with a zero endowment woman.[7]

4. The Capital Accumulation Model

4.1 A Two Period Example

Before considering the general infinite horizon model, we will present a two period example that will illustrate some of the characteristics in which we are interested. While the finite horizon eliminates the multiplicity of equilibria that is at the heart of our model, the example is nevertheless of some independent interest.

Agents use capital for current consumption and savings. Output is produced according to:

$$c = Ak - k', \tag{1}$$

where k is the initial endowment capital, c is first period consumption, k' is second period capital, and $A > 1$ is a constant.

Matching takes place in the second period only.[8] Each man must choose in the first period how much to consume out of first period output with the remainder being saved; in the second period all output is consumed jointly by the son and his mate.[9] Initially, suppose that all men have the same initial endowment of capital in the first period, K. Each man's utility function is given by:

$$u(c_0) + \beta[u(c_1) + j],$$

where c_1 is the offspring pair's joint consumption in the second period, j refers to the endowment level of the man's mate, and $u(\cdot)$ is CRRA with degree of relative risk aversion γ. The utility function of

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[7]This is well defined as long as only a zero measure set of men are unsuccessful. If there is a set of positive measure, then they receive no match. Note that in checking for equilibrium, we need only be concerned with unilateral deviations in the matching stage, which have zero measure. In a model with a finite number of agents, we can simply assign the woman with the lowest endowment to the unsuccessful man and reassign the other women, maintaining the order.

[8]We have suppressed matching considerations in the first period since including first period matching would have no effect upon savings decisions, and when all agents have equal wealth, any first period allocation is an equilibrium.

[9]Note that the only decision makers in this example are the men alive in the first period and the women; the sons affect the analysis only through the utility they provide their fathers.

the woman depends only on the pair's joint consumption in the second period and on its endowment level: $u(c_1) + j$. Note that $0 \leq c_0 \leq AK$ and $c_1 = A^2K - Ac_0$.

Since the second period is the last period, if a woman receives multiple proposals, then the proposal from the wealthiest man is accepted (we can resolve any ties by assuming that the woman flips a coin). A man's status, then, is determined precisely by his capital relative to other men's capital, higher capital yielding higher status. The equilibrium matching thus depends only on the men's capital endowments of that period (which are determined by their fathers' bequests), matching the wealthiest man with the woman of highest endowment, and so on. A man's match in period 2 depends, then, only on his relative position in the capital distribution of period 2. An equilibrium is a description of consumption-savings decisions and matching behavior for the agents such that no agent has an incentive to deviate from the described behavior.

Since all of the men have the same initial capital, and can therefore imitate the behavior of any other man, all men must have the same utility level in equilibrium. At the same time, their sons will be matching with women of different endowment levels. Consider the problem of the man whose son matches with the woman with lowest endowment, 0. His savings behavior cannot be distorted (from the level that is optimal when matching considerations are ignored) since any deviation in savings cannot reduce the quality of his match. Thus his maximized utility is given by:

$$(2) \qquad V(0) \equiv \max_\lambda u(AK\lambda) + \beta u(A^2K(1-\lambda)).$$

Denote the optimal value of λ by $\lambda(0) = [1+(\beta A^{1-\gamma})^{1/\gamma}]^{-1}$. Note that the sign of $\partial\lambda(0)/\partial A$ is the same as the sign of γ-1, so that $\gamma > 1$ corresponds to the case where the income effect dominates the substitution effect when there is a fall in the relative price of future consumption, while $\gamma < 1$ corresponds to the case where the substitution effect dominates.

Now consider the equilibrium behavior of the man whose son matches with a woman whose endowment level is j. It must be the case that his welfare level, $V(j)$, is the same as $V(0)$. This implies that his consumption fraction in the first period, $\lambda(j)$, must be such that

$$V(j) \equiv u(AK\lambda(j)) + \beta[u(A^2K(1-\lambda(j))) + j] = V(0),$$

and $\lambda(j) < \lambda(0)$. This implies that $\lambda(j)$ must be such that

$$\lambda(j)^{1-\gamma} + \beta A^{1-\gamma}(1-\lambda(j))^{1-\gamma} = [V(0) - \beta j](1-\gamma)(AK)^{\gamma-1}.$$

Implicitly differentiating the above expression yields

$$\frac{\partial\lambda(j)}{\partial j} = \frac{-\beta(AK)^{\gamma-1}}{\lambda(j)^{-\gamma}-\beta A^{1-\gamma}(1-\lambda(j))^{-\gamma}}.$$

Since $\lambda(j) < \lambda(0)$, the denominator is positive and so $\partial\lambda/\partial j < 0$.

This example illustrates several points. First, matching considerations cause agents to increase their savings levels relative to their nonmatching or involuntary matching levels.[10] Second, after the first period the income distribution is unequal. Note also that bequests as a function of j are continuous and strictly increasing. Further, since we began with a degenerate income distribution, the men's savings behavior, conditional on their son's match, is being distorted. This follows from the fact that in equilibrium each of the men is indifferent between his equilibrium choice and the savings level he would choose if his son were to be exogenously matched with the woman of zero endowment.

It is interesting to note the relationship between the initial endowment level, K, the curvature of agents' utility function for consumption, γ, and the degree to which matching considerations increase savings rates. If $\gamma > 1$, then the slope of bequests as a function of j is increasing in K, and the competition for mates becomes more intense. The reverse occurs if $\gamma < 1$.

Suppose now that men's initial wealth is not constant but that man $i \in [0,½)$ has initial capital level $K-\epsilon$ and $i \in [½,1]$ has initial capital $K+\epsilon$, where $\epsilon > 0$. Note that the average level of capital is the same as before, but that the distribution is now unequal.

The sons of those agents with the lower capital endowment will be matching in equilibrium with the lower half of the distribution of the women. Their equilibrium consumption fractions can be derived just as before. The welfare level of the agent who is in the lower half of the capital distribution and whose son matches in equilibrium with the zero endowment woman has welfare level V(0) given by (2) with $K-\epsilon$ replacing K. Let V(j) be the welfare level of the man whose son, in equilibrium, matches with a woman of endowment level j. As before, we must have $V(j) = V(0)$ for $j < ½$. This in turn determines this agent's consumption rate and hence his son's capital level, k(j).

Now consider the man who receives an initial endowment of $K+\epsilon$ and whose son's mate will, in equilibrium, have the lowest endowment of those in the top half of the distribution, i.e., ½. His equilibrium welfare level, V(½), is the value of the following maximization problem (where $k^-(½) = \lim_{i\uparrow ½} k(i)$):

[10]The man at the bottom of the matching hierarchy who matches with the least endowed woman is the only exception; his savings level is undistorted.

$$\max_{\lambda}\ u(A(K+\epsilon)\lambda) + \beta u(A^2(K+\epsilon)(1-\lambda)) + \beta ½$$

$$\text{subject to } \lambda \leq 1 - \frac{k^{-}(½)}{(K+\epsilon)A}.$$

This last restriction requires that the savings rate for this man be sufficiently high that his son's capital level is at least as large as that of the son of any man in the lower half of the distribution. This guarantees that no man in the lower half of the capital distribution finds it profitable to reduce first period consumption so that his son can match with a woman with endowment $j \geq ½$. Notice that for ϵ large, this restriction will not bind.

The welfare level of any arbitrary man in the upper half of the initial capital distribution whose son mates in equilibrium with a woman of endowment level j, where $j \geq ½$, must be such that $V(j) = V(½)$, where his initial consumption rate, $\lambda(j)$, is adjusted so that this equality holds. Notice that the lower is $V(½)$, the lower must be $V(j)$, and hence the lower $\lambda(j)$ must be also.

As ϵ gets larger, and the restriction that the son with mate $j = ½$ be at least as wealthy as any son with mate $j < ½$ becomes less severe, the savings rates for men in the top half of the distribution fall. At the same time, this subset of men is wealthier (while those in the bottom half of the distribution are poorer), and this will tend to increase or decrease (decrease or increase) the extent of their son's competition for mates depending on whether γ is greater or less than one, respectively.

When we compare the two cases above, we see that men's savings behavior depends not just on initial capital levels, but also on the compactness of the distribution of wealth in the economy. This can be seen most simply by choosing initial distributions in the two cases such that the wealthier agents in the two point distribution are just as well off as the agents in the one point distribution. The average savings rate in any section of the top half of distribution of second period capital must be lower than the comparable fraction from the one point initial capital level economy. This illustrates the general point that if we compare two men with the same initial income level in two different economies where the equilibrium assignment rule for mates is based on wealth, then the agent from the economy with the more compact income distribution will tend to have a higher savings rate, all other things being equal.

4.2 The General Case

As above, agents use capital to fund current consumption and bequests to their male offspring.[11] Output is again produced according to:

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[11]Since it is assumed that only the welfare of the type I offspring enters the parents' utility function, parents will only leave bequests to their type I offspring.

(1) $$c = Ak - k',$$

where k is the amount of capital that the man brings to the match, k' is the bequest level, c is the amount of current joint consumption, and $A > 1$ is a constant.

Before studying the model with endogenous matching, it is useful to analyze the case of exogenous matching. Suppose that $\{j_t\}_{t=0}^{\infty}$ is the sequence of matches for family line i. Due to the recursive nature of the agents' utility functions, the problem facing the first member of family line i is effectively to solve:[12]

(3) $$\max_{\{k_t\}} \Sigma_{t=0}^{\infty} \beta^t\{(1-\gamma)^{-1}[Ak_t-k_{t+1}]^{1-\gamma} + j_t\}$$
$$\text{s.t. } k_0 = k_0(i) \text{ and } Ak_t \geq k_{t+1},$$

where $k_0(i)$ is i's capital endowment in period $t = 0$. Recall that $u(c) = (1-\gamma)^{-1}c^{1-\gamma}$. This is just the standard deterministic growth problem with an exogenous term to reflect the value of the matched woman. The first order condition is:

(4) $$(Ak_t-k_{t+1})^{-\gamma} = A\beta(Ak_{t+1}-k_{t+2})^{-\gamma}.$$

Note that $A\beta > 1$ implies that consumption, and hence the capital stock, is increasing over time. In what follows, we assume $1 < A\beta < A^{\gamma}$ (the second inequality is needed to ensure that the discounted value of utility from consumption is finite).

Let λ_t be the fraction of output that is consumed in period t, c_t/Ak_t. Then λ_t satisfies:

(5) $$(\lambda_t Ak_t)^{-\gamma} = A\beta((1-\lambda_t)\lambda_{t+1}A^2k_t)^{-\gamma}.$$

This expression reduces to an expression which is independent of k:

[12]We are assuming here that the first member of a family line believes that the descendants will make the consumption-investment decisions that that member would choose if it was possible to dictate their choices. Since Bellman's principle of optimality applies, we have described a subgame perfect equilibrium of the extensive form game of consumption-investment choices with an infinite number of players (one player for each generation). It is possible that there are other equilibria, which we will not discuss.

(6)
$$\lambda_{t+1}^{\gamma} = \frac{\lambda_t^{\gamma}}{(1-\lambda_t)^{\gamma}} A^{1-\gamma}\beta.$$

It is then a standard exercise to show that, in the solution to (3), the fraction of output consumed is time invariant and given by $\lambda^* = 1-(A^{1-\gamma}\beta)^{1/\gamma}$. Note that λ^* is independent of k_t, depending only on γ, β, and A. The evolution of capital is governed by:

(7)
$$k_{t+n} = (1-\lambda^*)^n A^n k_t, \quad \text{for } n \geq 0.$$

The fact that the capital stock is increasing over time follows from our assumption that $A\beta$ is greater than one, which implies that $(1-\lambda^*)A > 1$.

The discounted value of utility from consumption is

(8)
$$\sum_{t=0}^{\infty} \beta^t u(c_t) = (1-\gamma)^{-1} \sum_{t=0}^{\infty} \beta^t [\lambda^* (1-\lambda^*)^t A^{t+1} k_0]^{1-\gamma}$$
$$= \frac{(\lambda^* A k_0)^{1-\gamma}}{((1-\gamma)\{1-\beta((1-\lambda^*)A)^{1-\gamma}\}} = \frac{(\lambda^* A k_0)^{1-\gamma}}{(1-\gamma)\lambda^*} \equiv V^*(k_0).$$

Note that the sign of $\partial\lambda^*/\partial A$ is the same as the sign of γ-1, so that $\gamma > 1$ corresponds to the case where the income effect dominates the substitution effect when there is a fall in the relative price of future consumption, while $\gamma < 1$ corresponds to the case where the substitution effect dominates.

4.3 The Wealth-is-Status Equilibrium

We now investigate the case in which a man's status is determined as in the two period example by his capital relative to other men's capital, higher capital yielding higher status. If a woman receives multiple proposals, then the proposal from the wealthiest man is accepted (if there is a tie, then the woman flips a coin). The equilibrium matching thus depends only on the men's capital endowments of that period (which are determined by their parents' bequests), matching the wealthiest man with the woman of highest endowment, and so on. A man's match in period t depends, then, only on his relative position in the capital distribution of period t.

A *wealth-is-status equilibrium* is a description of consumption-bequest decisions and matching behavior for the agents such that no agent has an incentive to deviate from the described behavior in any situation and whenever a woman receives multiple proposals from men, then the proposal from the wealthiest man is accepted (if there is a tie then the woman flips a coin).

We will assume that the initial endowment of capital $k_0:[0,1] \rightarrow \Re_+$ is a nondecreasing function of i. This assumption will be maintained throughout the remainder of the paper.[13] Suppose that the bequest of capital in period t is given by $k_t:[0,1] \rightarrow \Re_+$, where k_t is a strictly increasing function. Further, suppose that in equilibrium all agents consume in such a way that the next period capital bequest is given by $k_{t+1}(.)$, also strictly increasing and differentiable on (0,1), so that the match in period t+1 is m(i) = i. Consider family line i's choice of bequest in period t. By increasing the bequest from $k_{t+1}(i)$ to an amount k in the next period, the man of index i will succeed in matching with any woman such that $j < k_{t+1}^{-1}(k)$. Letting $m_{t+1}(k) = k_{t+1}^{-1}(k)$ denote the supremum of i's match as a function of the bequest received, the problem facing an agent i in period t who inherits k_t amount of capital is:

$$(9) \qquad \max_{0 \le k_{t+1} \le Ak_t} u(Ak_t - k_{t+1}) + m_t(k_t) + \beta V_{t+1}(k_{t+1}) \equiv V_t(k_t).$$

If V_{t+1} is differentiable, the first order condition on bequests is given by $u'(Ak_t - k_{t+1}) = \beta V'_{t+1}(k_{t+1})$. From the envelope condition, $V'_{t+1}(k_{t+1}) = Au'(Ak_{t+1} - k_{t+2}) + m'_{t+1}(k_{t+1})$. Thus, we get the following difference equation which describes a family line's optimal capital accumulation decision:

$$(10) \qquad u'(Ak_t - k_{t+1}) = \beta\{Au'(Ak_{t+1} - k_{t+2}) + m'_{t+1}(k_{t+1})\}.$$

This condition differs from (4), the corresponding condition for the case of exogenous matching, by the presence of a term reflecting matching. We can use (10) to illustrate the role of matching in the choice of bequest. Fix k_t and k_{t+2} and suppose k_{t+1} solves (10). Suppose now that the son's match is more sensitive to the level of capital, i.e., suppose that m'_{t+1} were greater (equivalently, k'_{t+1} decreases). Then in order for (10) still to hold, u' must be smaller, which implies that period t+1 capital must be larger. Thus, we see that bequests will be larger when a son's match is more sensitive to the level of capital on the margin. This will imply, all else being equal, a more equal distribution of capital leads to a smaller share of current output devoted to current consumption, since a more equal distribution will be reflected in a larger m'_{t+1}.

If we reformulate (10) in terms of the fraction of current output consumed, λ_t, we have:

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[13]This assumption is without loss of generality in this section. The results in later sections in which we identify status with a person's index might differ if this assumption is violated.

(11) $$(\lambda_t A k_t)^{-\gamma} = A\beta((1-\lambda_t)\lambda_{t+1}A^2k_t)^{-\gamma} + \beta m'_{t+1}((1-\lambda_t)Ak_t).$$

This expression is in general not compatible with a steady state rate of growth, i.e., with $\lambda_t = \lambda_{t+1}$. Notice also that since a pair's capital stock enters into the above expression, a difference in capital stocks could result in different pairs choosing different fractions of their output to consume.

We show in the Appendix that wealth-is-status equilibria exist. In fact, the argument given there applies to a broader class of technologies and utility functions than considered here (all that is required is that the dynamic programming problem with exogenous matching be well behaved).

PROPOSITION I: *Wealth-is-status equilibria exist.*

PROOF: See the Appendix, Section A.1. □

We next describe several properties of a family line's optimal consumption-bequest decisions and wealth-is-status equilibria. In what follows, $\{m_t\}$ will denote the sequence of matching functions that the family line will face given the choices of other family lines. Family line i's optimal consumption ratio sequence is $\{\lambda_t(i)\}$, yielding a sequence of capital bequests $\{k_t(i)\}$; sometimes we will write λ_t and k_t. The proofs of the properties can be found in Section A.2 of the Appendix.

PROPERTY 1: *In an equilibrium, if* $k_0(i) > k_0(i')$, *then for all* t, *the optimal level of capital in period* t *for agent* i *and* i' *satisfies* $k_{t+1}(i) > k_{t+1}(i')$.

This property states that in an equilibrium, no family's relative position changes. Despite the fact that all family lines are taking into account how their savings behavior affects their relative ranking and adjusting their behavior accordingly, the net effect of their decisions is that no family moves up or down in the ranking over time.

PROPERTY 2: *For all family lines and all* $t \geq 0$, $\lambda_t \leq \lambda^*$.

In equilibrium, all families are consuming less than or equal to what they would have consumed in the absence of matching concerns. That is, matching considerations cause agents to (weakly) increase their savings. In fact, the inequality is strict for all but a set of families of measure 0.[14] Note that, as in the two period example, the family line matching with the zero endowment woman will not be distorted, i.e., $\lambda_t = \lambda^*$ for all t.

PROPERTY 3: *In any equilibrium, in every period, each bequest level is chosen by a zero measure of agents.*

The third property states that there are no atoms in the income distribution; at every period, the function relating family lines to capital, $k_t(.)$, is strictly increasing almost everywhere. Moreover, if $k_0(.)$ is strictly increasing, then the almost everywhere caveat can be dropped. This is a generalization of what we saw in the two period example above. If there were an interval of families with equal wealth, that family matched with the least endowed woman could have obtained a positive increase in utility from matching with an arbitrarily small increase in savings.

PROPERTY 4: *Suppose* $k_0(.)$ *is a continuous function of* i. *Given an equilibrium and any period* t, *the capital distribution is strictly increasing, so that* $k_t(.)$ *is a continuous function of* i.

PROPERTY 5: *If* $\gamma < 1$, *then for all* $i \in [0,1]$, $\lambda_t(i) \to \lambda^*$ *as* $t \to \infty$.

This property and the next describe the limiting behavior of family lines as time goes to infinity. Property 5 says that for the case of $\gamma < 1$ (recall that this is the case when the substitution effect dominates), asymptotically, consumption-savings behavior is the same with and without matching considerations. In other word, in this case, the over-saving that results from the concern about relative positions asymptotically vanishes. On the other hand, if $\gamma > 1$, Property 6 states that the effect of the concern for relative position increases as time goes on. It states that for any two family lines, either the poorer is saving a fraction of wealth that is going to 1, or the ratio of the wealth of the richer family to that of the poorer family is going to infinity.

PROPERTY 6: *If* $\gamma > 1$ *then either* $\lambda_t(i') \to 0$ *or* $k_t(i')/k_t(i) \to \infty$ *for all* $i < i'$ *for all* $i' \in (0,1]$ *as* $t \to \infty$.

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[14]For suppose not. Then, since time is discrete so that the set of all dates is countable, there is a time t, a subset $(i', i'+\epsilon)$ of family lines with a dense subset D, such that $\lambda_t = \lambda^*$ holding for all $i \in D$. But this contradicts (11).

4.4 The Aristocratic Equilibrium

In this section, we examine a second type of equilibrium. In this equilibrium, which we call aristocratic, a man's status is inherited. That is, his status is the same as his father's as long as his father matched "appropriately." In the initial period, there will be an exogenously given status assignment. An *aristocratic equilibrium* consists of the above status assignment rule, consumption-bequest decisions and matching behavior for the agents such that each man is voluntarily matched with the woman whose endowment equals the man's status; if a man matches with a woman with an endowment not equal to his status, the family line from that point on has zero status; and no woman of positive endowment will match with a zero status man when that status is newly acquired.[15]

Assume that initially the status of male i is given by i. We could in general allow status to be allocated somewhat independently of the distribution of capital. Let s denote the status of the father. The evolution of status is governed by:

$$n(s,j) = \begin{cases} s, & \text{if } s = j, \\ 0, & \text{otherwise.} \end{cases} \tag{12}$$

If everyone obeys the aristocratic social norm (i.e., the status assignment rule and matching contingent on status), then a male from family line i will always match with the female endowed with $j = i$, irrespective of his wealth. A family line is punished by having its status and hence match set to zero forever if one of the men in the line marries inappropriately.

If men match appropriately, then the value function is given by:

$$V(k,s) = \max_{c,k'} u(c) + m(s) + \beta V(k',s) \text{ s.t. } (1).$$

The first order condition for bequests is given by:

$$u'(Ak\text{-}k') = \beta V'.$$

[15]We are only requiring that it be optimal for a type II to follow status on the equilibrium path and for one generation off the equilibrium path. We will return to this issue later.

From the envelope condition, $V' = u'(Ak-k')A$, and hence we have the following difference equation which describes a family's optimal capital accumulation decision, where family denotes the family line through the son:

$$u'(Ak_t-k_{t+1}) = \beta u'(Ak_{t+1}-k_{t+2})A.$$

An important feature of this equilibrium is that the path of the capital stock is identical to that in the involuntary matching model. In particular, the value function for a man of status level s is given by:

$$V(k,s) = V^*(k) + (1-\beta)^{-1}s.$$

The aristocratic status rule is consistent with equilibrium if at no time does family line i wish to bequeath sufficient capital to their son so that he can induce a woman of endowment greater than i to match. Let $L_t(j)$ be the capital level which leaves the woman indifferent between matching with the man of status j with capital $k_t(j)$ and maintaining status for her son, and matching with a man with capital $L_t(j)$ and having a zero status son. If the woman does match with the latter, the status of the man that the woman deviates to is irrelevant since offspring will have zero status; thus, the capital level $L_t(j)$ is described by:

$$V^*(L_t(j)) + j = V^*(k_t(j)) + \frac{1}{(1-\beta)} j.$$

This implies that:

$$L_t(j) = \left[k_t(j)^{1-\gamma} + \frac{\beta}{(1-\beta)} j(1-\gamma)(\lambda^*)^\gamma A^{\gamma-1}\right]^{1/(1-\gamma)}.$$

Note that $L_t(j)$ exceeds $k_t(j)$ by an amount that compensates for the value of lost status. It is worth noting here that $L_t(j)$ is close to $k_t(j)$ for j close to zero.

A man i with capital k will not match with a woman j if $V^*(k) + j \le V^*(k) + i/(1-\beta)$, that is,

(13) $$(1-\beta)j \le i.$$

Moreover, for any β, there are men of sufficiently low status to make them willing to match with women that are higher than prescribed.

Consider the bequest decision of family line i in period t - 1. Bequeathing $L_t(j)$ will improve their son's match, but at a cost of reduced consumption today as well as lower status after two generations. Bequeathing $k_t(i)$ yields a higher payoff than $L_t(j)$ if:

$$(14) \qquad V^*(k_{t-1}(i)) + i/(1-\beta) \geq u(Ak_{t-1}(i)-L_t(j)) + i + \beta j + \beta V^*(L_t(j)).$$

Fix i' $\in$ (0,1). The next lemma shows that for sufficiently high β, neither the men nor the women with indexes above i' will deviate from the aristocratic social norm (it is not enough to ensure that the men with indexes above i' will follow the social norm; we need to ensure that women with endowments above i' will also reject offers from men with indexes below i').

LEMMA: *Suppose* $\gamma > 1$. *Fix* i' $\in(0,1)$. *There exists* β(i') *such that for* $\beta > \beta$(i'), (13) *holds and any woman with endowment* $j > i'$ *will not accept any offer from a man with index* $i \neq j$.

PROOF: Clearly (13) holds for β sufficiently close to 1. If woman j accepted an offer from man $i \neq j$, then j's son has status zero, the per period cost of which is greater than or equal to i'. Since utility is bounded, there exists $\hat{c}$ such that for all $c > \hat{c}$, $\sup_{c'} u(c') - u(c) < i'/2$. When family lines save in an aristocratic social norm, $c_t(i) \uparrow \infty$. Moreover, $c_t(i)$ is increasing in i, for all t. Therefore, there exists T such that $\forall t > T$, $c_t(i) > \hat{c}$ $\forall i \geq i'$.

The utility to woman j from following the aristocratic social norm is

$$V^*(j) + j/(1-\beta).$$

The utility to woman j of accepting an offer from $i \neq j$ is less than

$$T \sup_{\tilde{c}} u(\tilde{c}) + j + \sum_{\tau=T+1}^{\infty} \beta^{\tau} \sup_{\tilde{c}} u(\tilde{c}).$$

Thus, the gain from the deviation from the aristocratic social norm is less than

$$T \sup_{\tilde{c}} u(\tilde{c}) - \sum_{\tau=0}^{T} \beta^{\tau} u(c_t(j)) + j + \sum_{\tau=T+1}^{\infty} \beta^{\tau} [\sup_{\tilde{c}} u(\tilde{c}) - u(c_t(j))] - \frac{j}{(1-\beta)}$$

$$\leq T \sup_{\tilde{c}} u(\tilde{c}) - \frac{(1-\beta^{T+1})}{(1-\beta)} u(c_0(j)) - \frac{(1-\beta^{T})}{(1-\beta)} \beta j - \frac{\beta^{T+1}}{(1-\beta)} (j - \frac{i'}{2}).$$

For β sufficiently close to 1, this last expression is negative. □

However, since low status men and low endowment women have very little to lose from deviations, the threat of a loss of status is not sufficient to maintain equilibrium: Since $k_t(0) = L_t(0)$, the inequality (14) holds as an equality for $i = j = 0$. While the left hand side is independent of j, the partial derivative of the right hand side with respect to j is given by $\beta + (\beta V'-u')L' = \beta$ at $i = j = 0$. Thus (14) must fail for $i = 0$ and j near i.[16] However, if capital is sufficiently "spread out" at the lower tail and $k_0(0) = 0$, then it is impossible for sufficient wealth to be bequeathed: A man cannot save enough to induce a deviation next period by any woman with an endowment greater or equal to his status level, i.e., $L_t(j) \geq Ak_{t-1}(j)$.

PROPOSITION II: *Suppose* $k_0(0) = 0$. *Fix* $i' \in (0,1)$ *and suppose* $\beta \in (\beta(i'),1)$. *An aristocratic equilibrium exists if* $\gamma > 1$ *and*

$$k_0(i)^{-\gamma}k_0'(i) < A^{2(\gamma-1)}(1-\lambda^*)^{\gamma}(\lambda^*)^{\gamma-1}\beta/(1-\beta)$$

for all $i \leq i'$.

PROOF: First observe that $k_0(0) = 0$ implies $L_t(0) = k_t(0) = Ak_{t-1}(0) = 0$. It is enough to show that $dL_t(i)/di > d(Ak_{t-1}(i))/di$ for all $i < i'$ for all t. Note that as $t \to \infty$, $k_t(i) \to \infty$ and $L_t(i)/k_t(i) \to 1$ for $i \neq 0$. Differentiating $L_t(i)$ yields

$$\begin{aligned} dL_t(i)/di &= L_t(i)^{\gamma}\{k_t(i)^{-\gamma}(1-\lambda^*)d(Ak_{t-1}(i))/di + \beta(\lambda^*)^{\gamma}A^{\gamma-1}/(1-\beta)\} \\ &\geq (1-\lambda^*)d(Ak_{t-1}(i))/di + L_t(i)^{\gamma}\beta(\lambda^*)^{\gamma}A^{\gamma-1}/(1-\beta). \end{aligned} \tag{15}$$

We now argue that the second term is larger than $\lambda^* d(Ak_{t-1}(i))/di$. By hypothesis, $k_0(i)^{-\gamma}k_0'(i) < A^{2(\gamma-1)}(1-\lambda^*)^{\gamma}(\lambda^*)^{\gamma-1}\beta/(1-\beta)$. Since $k_t(i) = (A(1-\lambda^*))^t k_0(i)$, this implies $A^{2(\gamma-1)}(1-\lambda^*)^{\gamma}(\lambda^*)^{\gamma-1}\beta/(1-\beta) > (A(1-\lambda^*))^{(t-1)(\gamma-1)}k_{t-1}(i)^{-\gamma}k_{t-1}'(i) > k_{t-1}(i)^{-\gamma}k_{t-1}'(i)$, since $A(1-\lambda^*) > 1$ and $\gamma > 1$. Note that the second term in (15) is larger than $k_t(i)^{\gamma}\beta(\lambda^*)^{\gamma}A^{\gamma-1}/(1-\beta) = k_{t-1}(i)^{\gamma}(1-\lambda^*)^{\gamma}\beta(\lambda^*)^{\gamma}A^{2\gamma-1}/(1-\beta) = \lambda^* A\{k_{t-1}(i)^{\gamma}A^{2(\gamma-1)}(1-\lambda^*)^{\gamma}\beta(\lambda^*)^{\gamma-1}/(1-\beta)\} \geq \lambda^* Ak_{t-1}'(i)$, and we are done. □

Before going on, we will discuss an important difference between the aristocratic equilibrium and the wealth-is-status equilibrium. The proposition above shows that for some distributions of initial

[16] Note that if instead the value of match j is v(j) and $v'(0) = 0$, then the argument fails. In fact, one can write down conditions for the general model (i.e., general u(.), v(.), and technologies)) which yield the existence of aristocratic equilibria.

capital, there will be an aristocratic equilibrium. The possibility that there might not be an aristocratic equilibrium stems from the fact that for some distributions of wealth and status, a low status, high wealth man might find it worthwhile deviating from the social norm and make an offer to match with a woman of higher endowment than the social norm prescribed. By assumption about the matching process, the woman accepts the most desirable offer to match in the event that multiple offers are made. Sufficiently high wealth could more than compensate for the (assumed) loss of status that results from a woman's deviation from the social norm. The difficulty in establishing an aristocratic equilibrium lies in assuring that when a man bequeaths to his son sufficiently high capital to induce a woman with a higher j to match, the benefits to the offspring do not exceed the cost to the parent. As the capital distribution becomes more "spread out" the extra savings needed to induce a specific woman to deviate get larger. The inequality in the proposition above that provides a sufficient condition for an aristocratic equilibrium to exist is essentially that the capital distribution is sufficiently "spread out".

This discussion points out why the inequality in the proposition suffices to guarantee that a man will not have an incentive to deviate from the prescribed savings behavior, given that there have been no previous deviations. It should be noted that if the generations preceding a man have saved more than that prescribed, the given man may *not* find it in his interest to follow the social norm. The question here is essentially the difference between a Nash equilibrium of a game and a subgame perfect Nash equilibrium. An aristocratic equilibrium is essentially a Nash equilibrium in which each woman responds optimally to any first round deviation by a man, but not necessarily a subgame perfect Nash equilibrium.

The reason that an aristocratic equilibrium may not be subgame perfect is that given a distribution of capital, if each generation of a given line saves enough, eventually a man in the line might have sufficient capital that he will find it optimal to deviate from the social norm. On the other hand, given any positive integer N, there always exists a distribution of capital such that the number of agents that must deviate from equilibrium behavior before any agent finds it nonoptimal to follow the social norm exceeds N. Thus, while the aristocratic equilibria may not be subgame perfect, we still find them plausible.[17]

Before closing this section we would like to discuss the aristocratic status assignment rule. In that rule, we have assumed that the son of a man who has deviated from his prescribed match was assigned status 0. It might seem that what drives the behavior of the agents is this "extreme" punishment. It is important to note that the existence of aristocratic equilibria does *not* depend upon adjusting deviating men's status levels to 0. Suppose we altered the status assignment rule as follows. Following a deviation

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[17]See also the discussion of this issue in Kalai and Neme (1988) and Okuno-Fujiwara and Postlewaite (1990).

from the prescribed match, the male offspring in the deviating male's line have their status reduced by a fixed positive proportion. Under this status assignment rule, for any $i' \in (0,1)$, a deviation in prescribed matching results in a proportionate reduction from matching each period equal to the proportionate reduction in status. As before, if wealth is sufficiently high and the discount factor is sufficiently close to 1, no woman with index higher than i could obtain an increase in utility from consumption that would offset the future decrease in matching utility. Thus, no man of index at least equal to i' would wish to deviate from the social norm. But if the income distribution is sufficiently flat (as in Proposition II), no man with index below i' will have sufficient wealth to induce a deviation from a woman with higher index.

Thus, as long as the status assignment rule reduces the status of future generations by some positive amount following a deviation, there will still be equilibria of this modified aristocratic type. Of course, it is clear that if the amount that status is reduce is very small, the discount factor must be very close to 1 to assure that the annuity value of the loss of status is greater than the utility gain from increased consumption. In addition, the capital distribution must be very spread out at the lower tail.

4.5 Hybrid Equilibria

In the two previous sections we have analyzed two types of equilibria. These classes of equilibria are of interest not simply because they show that there can be multiple equilibria, but because the qualitative features of the status assignment rules in the two cases correspond (at least roughly) to those observed in some societies. In general, we may expect that there would be other status assignment rules that are consistent with equilibrium and lead to savings behavior that differs from either of these two cases. In our view, it is of questionable interest to analyze mechanically defined status rules that do not correspond, at least in a stylized way, with rules in actual societies, past or present.

There is at least one additional class of rules that we think of interest, however. One can imagine that people in a society are divided into two groups, which are governed by different rules prescribing behavior. For example, it is interesting to know whether the agents can be divided into two groups such that the bottom group is governed by the wealth-is-status social norm while the upper group is governed by an aristocratic rule. We will consider a slight variant of this in which agents in the upper group are to save and consume as they would in the exogenous matching case (and *a fortiori* as they would under aristocratic social norm) but that they are to match randomly. Formally we define a ***hybrid status assignment rule*** as follows. Given a designated index $i \in (0,1)$, we divide both men and women into two groups, $[0,i)$ and $[i,1]$ called respectively the *upper group* and *lower group*. Status for men in the lower group is determined by wealth while all males in the upper group have identical status (as long as the men in that family line have followed the prescribed matching and consumption patterns). Matching between the lower groups of men and women is as in the wealth-is-status social norm: the woman with index i marries the man with status i. For men and women in the upper group, matching is to be random; that is, for any man in [i,1] is equally likely to be matched with any woman in [i,1]. Deviations from prescribed behavior for either men or women in the upper group results in their being put into the lower group, as will be all descendants in the line. In essence, deviation from the prescribed rules in the upper group results in the family line being banished from the upper group. Since the expected value of a match for any male in the upper group is (1-i)/2, being dropped into the lower group results in a loss of at least (1-i)/2, since the best match possible if one is in the lower group yields less than i.

Simple conditions guaranteeing that a hybrid equilibrium will exist can be derived with the aid of the analysis of the two previous classes of equilibria. For the aristocratic equilibrium case we showed that if $\gamma > 1$, the discount factor was sufficiently close to 1 and the distribution of income was sufficiently "flat" at the bottom, an aristocratic equilibrium existed. The proof consisted of showing that if we chose an arbitrary index i' $>$ 0, no man or woman with index at least i' would deviate from the prescribed matching if β was sufficiently close to 1. This is because for $\gamma > 1$ the consumption utility function is bounded, hence any possible increase in utility from greater consumption that might arise from deviating from a prescribed match is bounded while the utility loss caused by the loss of status that accompanies any deviation goes to infinity as β goes to 1. The "flatness" condition was used to assure that men below the prespecified i' had no incentive to deviate.

In the case of the hybrid status assignment rule for the upper group described above, similar logic can be applied. For any agent i, there exists k(i) such that the optimal consumption-savings decisions for i's family line is such that the maximal additional utility that any generation could receive from additional consumption is less than (1-i)/2. This means that any deviation from prescribed behavior will yield a loss per period from matching that outweighs any possible offsetting consumption gain. Hence, neither the man nor the woman with index i would ever be willing to deviate from prescribed behavior. If $k(j) > k(i)$ for $j > i$, the same will hold true for all people, both men and women, whose indices are at least i.

Suppose now that the man with index i and all men above him are governed by the status assignment rule described above while men below are governed by the wealth-is-status rule and that $k(j) > k(i)$ for all $j \geq i$. Given that all agents with indices greater than or equal to i will not deviate from the social norm under any circumstances, those agents below can be treated as a society by themselves; nothing they do will affect the behavior of the agents in the upper group and nothing the upper group does will impact on the lower group. But from Proposition I, there will exist a wealth-is-status equilibrium for the agents in the lower group, considered as a distinct society. If we concatenate the wealth-is-status equilibrium for the lower group with the aristocratic equilibrium for the upper group, this will constitute a hybrid equilibrium.

To summarize, we have shown the following.

PROPOSITION III: *Suppose* $\gamma > 1$. *Fix* $i \in (0,1)$. *There exists* $k_0(i)$ *such that if* $k_0(j) \geq k_0(i)$ *for all* $j \geq i$, *there exists a hybrid equilibrium in which men with index* $j \geq i$ *are governed by the (random) aristocratic status assignment rule and men with index* $j < i$ *are governed by the wealth-is-status rule.*

In closing this section we will make several comments about hybrid equilibria. First, it is worth noting that the stringent flatness constraint we imposed on the wealth distribution in order to prove existence of aristocratic equilibrium is unnecessary here. That condition on the income distribution was imposed to assure that men with status near the bottom would not have an incentive to increase savings to improve their match. What occurs in the hybrid equilibria identified in Proposition III, is that men with low status compete on the basis of wealth while men with high status compete on the basis of birth. While our maintained assumption is that initial wealth is a nondecreasing function of i, we note it is *not* necessary that the men of higher status be the wealthiest men for the argument above, only that their wealth be sufficiently high. In fact, as shown in Proposition I, in the wealth-is-status equilibrium for the case of $\gamma > 1$, the proportion of wealth that is consumed in any period is always below that consumed in the aristocratic equilibrium and goes to 0 over time. Thus, every family line in the lower status group with positive wealth will eventually accumulate wealth greater than that accumulated by family lines in the higher status group.

The second comment about hybrid equilibria concerns the argument showing that those agents in the upper group, that is, the group with the relatively high status and sufficiently high wealth, will never deviate from prescribed behavior. Since what is used in the argument is that the consumption utility function is bounded, one would expect such equilibria to exist for a broad class of utility functions. To complete the argument, it would be necessary to show that for a class of utility functions, the wealth-is-status equilibria existed to assure that the lower status group followed their part of the proposed hybrid equilibrium.

The last comment concerns the comparison of the utility levels people would have in the first period in the wealth-is-status equilibrium and the hybrid equilibrium. For the agents at the bottom, there is no difference since they are in a wealth-is-status equilibrium relative to the bottom group in either case.

The agents in the top are not indifferent, however. Those above the average for the upper group, (i+1)/2, are receiving lower expected value from their match in the hybrid equilibrium than in the wealth-is-status equilibrium while those below the average are receiving higher expected utility from the matches. *All* agents in the upper group are better off with respect to utility from consumption in the hybrid equilibrium since they no longer have the distortion that results from the competition in the wealth-is-status equilibrium. Thus, those males below (i+1)/2 are clearly better off in the hybrid equilibrium since they are better off both with respect to matching and to consumption.

What about the welfare of males above (i+1)/2? For the man with index 1, his loss in utility from matching is (1-i)/2. But the condition that ensured that a hybrid equilibrium existed was that this be greater than any possible utility increase from consumption. Hence, this individual is strictly worse off in the hybrid equilibrium than in the wealth-is-status equilibrium. It is clear from this discussion that there is a clear majority of males who are better off in the hybrid equilibrium, however.

It should be emphasized that the argument in the preceding paragraph is for the first period. If we change from one equilibrium to the other, we change savings-consumption behavior and, hence, we change the wealth distributions of the next generations.

A similar, but not precisely the same, comparison can be made for the women.

5. Discussion

5.1 The role of matching

Since the primary departure of our model from the classical growth model is our introduction of the matching process, it is worthwhile to go into some more detail as to precisely how the matching process alters the equilibrium growth path for a particular society. In our model, the matching process may provide an incentive to save beyond that captured by standard models. The additional reward is the increased value of the matches of future male generations. A few comments on this matching process are in order.

First, as we emphasized in the introduction, our goal has been to write down a model that captures the idea that people have rational reasons to care about their relative position in an economy. The particular matching process we introduced is one of many ways the idea can be implemented. We believe that the incentives captured by the model are not unrealistic. One key advantage of our method of inducing a concern about relative position is that it does not require agents to be concerned with anything other than their family's outcome. As Duesenberry (1949) pointed out, introducing status directly into an agent's utility function seems to require a psychological theory to underpin the preference assumptions. The plausibility of the model is then tied to the plausibility of the psychological preference theory.

There is a second point regarding the model presented in this paper. One might think that it could be altered slightly to give a one type model with similar conclusions. For example, one can imagine a one type model in which there are some goods that are allocated according to status rather than by markets (e.g., invitations to the White House, church pews, seats on university boards of trustees, etc.). If agents care about the goods to be allocated according to status, then whether or not the status of an individual depends upon his wealth will affect his savings decisions. The difficulty with a model of this sort is that one would have to account for the person allocating the nonmarket goods: why is he giving away goods that people care about rather than selling them? To incorporate such decisions into a model would seriously increase its complexity. Our matching model captures these incentives while avoiding these difficulties.

5.2 The implications of relative position mattering

In the past several decades, an important theme in economics has been the importance of providing microeconomic foundations for macroeconomics. The argument has been that many (if not most) macroeconomic questions of interest can only be analyzed and understood if the individual agents in the economy were carefully modelled and there behavior rooted in rational decision making. An important theme of this paper is to turn this argument around. Man is a social creature. It is impossible to talk about an individual's preferences and decisions making in isolation; his effort choices, savings behavior and buying habits depend on his environment. This paper provides a specific model in which an agent's environment matters in describing his behavior. In a sense, this can be thought of as an initial attempt to provide "macroeconomic foundations for microeconomics."

We think such foundations are important in understanding several important economic questions. Consider, for example, standard consumption-savings models. In these models, agents' incentives to save (invest) stem from the increased future consumption that would result. This is clearly an important - possibly the most important - incentive to save. But think for a moment about an already very rich agent like Donald Trump.[18] Why does he continue to work long days, endure substantial amounts of stress and take enormous risks? Surely it can't be because he is savoring the prospect of going to the grocery store with a looser budget constraint next year. He seems to have more money than he could spend in several lifetimes. Even if we're wrong about Donald Trump's net worth, there clearly seem to be

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[18]This section was written prior to Mr. Trump's recent financial difficulties. We have chosen to leave the example in since it underscores the risks we claim he took.

wealthy individuals that continue to work very hard and take large risks to increase their net worth. It is hard to reconcile such behavior with the underlying decision-making in traditional growth models.

We propose that people like Donald Trump continue to care about increasing their net worth because their utility depends not only on the absolute level of their wealth, but also on their wealth relative to other very rich people: everyone would like to be the richest person in the world, even if the second richest has more money than can be sensibly spent. Taking account of this additional incentive to save could well change qualitatively the nature of optimizing behavior in a growth model. This paper has presented a model in which people may care about their relative position in society as well as their level of wealth. The model allows for societies to differ in the degree to which people may care about their relative position and thus, differ in the severity of the "rat race of the rich" that results.

5.3 The role of non-market decisions

In the models presented, the primary departure from more standard models was the inclusion of a matching decision about which the agents cared. We have stressed that this is just one (and perhaps not the most important) non-market decision that we could have incorporated into our model in order to have such non-market decisions affecting economic variables. Although the resulting model would be less tractable, there is no reason to believe that other non-market decisions wouldn't play the same role.

The phenomena we have identified and analyzed - the existence of multiple equilibria that differ according to how a society allocates status - depend upon there being some non-market decisions that matter. Further, if the decisions matter only slightly, they can have only a slight effect on the outcome. In the model as presented, we took as fixed and exogenously given the degree to which the non-market decision - matching - mattered. The match a man achieved could change his current period utility by at most one unit, the maximum difference in utility across all possible current matches. To the extent that future matches might be affected by current choices, his utility could be changed by at most $1/(1-\beta)$, the discounted value of the maximum per-period change. We can easily determine the effect of changing the value of possible matches, say by multiplying the utility of all matches by some constant. If we did multiply the value of a match by a sufficiently small constant, it's clear that all equilibria give approximately the same choices as in the exogenous matching case, since anything more than minor deviations will be more costly than the maximum benefit that could result from such a deviation. In summary, if the value of the non-market decisions that are affected by status is small, there may still be many equilibria, but they will all be approximated by the case in which social norms were ignored.

5.4 Income distribution

Two remarks are in order regarding income distribution. First, as the wealth-is-status savings model makes clear, if relative position matters, this gives a greater incentive to save in equilibrium. The difference in savings behavior from the case in which matching considerations were ignored is in some (ill-defined) sense, greater for higher incomes, all other things equal. In particular, there is no effect at all for the person with the lowest level of capital. Thus, the effect of the introduction of matching into the consumption-savings decision will aggravate unequal distributions of income. The most extreme case of this arises when all agents begin with identical incomes. It is clear that even in this case, the only equilibrium paths give rise to unequal distributions of capital, and that the inequality will increase over time.

The second remark is that in the context of our model, income distribution is of interest beyond the standard moral/ethical reasons. Here, the distribution of income is not just an end product of the economy. Rather, it feeds back into the economic analysis, changing agents' behavior in the future. As seen in the wealth-as-status savings model with $\gamma < 1$, when the capital distribution becomes sufficiently dispersed, the increased incentive to save disappears. Thus, exogenous (or even endogenous, perhaps) changes in the income distribution may have an important effect on future economic performance.[19]

5.5 Reduced forms of the model

As we have repeatedly stressed, we believe the above model to be of interest beyond the literal application to the particular matching context. We believe people have a deep and general concern with their relative position within society and that their position sometimes depends upon their economic decisions. Our model is one attempt to formalize and analyze how the relationship between economic decisions and non-market decisions might arise. Having provided foundations for such a relationship, we may consider a reduced form of the model.

Suppose the relevant equilibrium is the wealth-is-status equilibrium. If we were to ignore matching considerations and consider only the male population, we could consider the induced, reduced form utility functions for men. A given man's utility function would then be $\sum_{t=1,\ldots,\infty}\beta^t[u(c_t)+r_t]$, where r_t is the man's ranking in the society in period t. To the extent that one believes that our model captures real phenomena then, it provides the foundation for analyzing the reduced form in which the agents directly care about their ranking. This differs from the situation in which ranking enters into the

[19]Persson and Tabellini (1990) investigate a model in which the distribution of income affects the growth rate of an economy; growth rates depend upon income distribution for different reasons in their paper, however.

"true" utility function in an important way: since the reduced form is derived from an equilibrium of the fully specified model, we can understand how different agents in different societies can look and behave differently in the reduced form by examining the fully specified model. We expect that such reduced form models will be useful in reexamining questions of predicting agents' responses to shocks in the labor market or to exogenous changes in government policy such as income tax changes.

5.6 The determination of society's norms

A fundamental question not addressed by this paper is the origin of the social norms governing a society. The model presented here shows the possibility of multiple equilibria when social norms are taken into account. It has nothing to say about which of the differing social norms that might exist actually arises. One would like a model that endogenizes the social norm. Such a model would help understand which social norms are likely to arise and under what circumstances. While we have no firm ideas about the form of such models, recent work in evolutionary models may provide the tools necessary for this task.

Appendix

A.1 Existence of the wealth-is-status equilibrium

PROPOSITION I: *Suppose the technology is given by* $c_t + k_{t+1} \leq f(k_t)$, *where* f *is continuous, increasing, and* $f(k) > k$ $\forall k$. *Suppose that current utility is given by* $u(c) + v(j)$, *where* c *is consumption and* j *is the woman's endowment. Assume* u *is concave and continuous,* $u'(c) > 0$ $\forall c$, *either* $u(c) \geq 0$ $\forall c$ *or* $u(c) \leq 0$ $\forall c$, *and that* v *is continuous and strictly increasing. If* $\sup_{\{k_t\}} \sum \beta^t u(f(k_t)-k_{t+1}) < \infty$ $\forall k_0$, *wealth-is-status equilibria exist.*

PROOF: Following footnote 12, we will treat a family line as an infinitely lived player choosing a sequence $\kappa \equiv \{k_t\}_{t\geq 1}$. Thus the game consists of a continuum of players, indexed by $i \in [0,1]$. Let $K = \sup k_0(i)$. The set of actions is X, the set of all sequences $\{k_t\}$ such that $k_{t+1} \leq f(k_t)$, $t \geq 1$, and $k_1 \leq f(K)$. This is a subset of $\Pi_{t=1}^{\infty}[0,K_t]$, where $K_t = f^t(K)$. Define $\bar{u}(c) = u(1) + (c-1)u'(1)$ if $c \leq 1$ and $u(c)$ if $c > 1$. Let $d(\kappa,\kappa')$ denote the metric on X given by $d(\kappa,\kappa') = \sum_{t\geq 0}\beta^t|\bar{u}(f(k_t)-k_{t+1}) - \bar{u}(f(k'_t)-k'_{t+1})|$. If $\{\kappa^m\}$ is a Cauchy sequence, then so is $\{k_t^m\}$ for all t (this follows from $u'(c_t) \geq u'(f^t(K)) > 0$, since $c_t \leq f^t(K)$). Hence, the metric space (X,d) is complete. Let $M = \sup_{\{k_t\}} \sum \beta^t u(f(k_t)-k_{t+1})$ for $k_0 = K$. Then, since $\sup_{\{k_t\}} \sum \beta^t u(f(k_t)-k_{t+1}) \leq M$ $\forall k_0 \leq K$, $d(\kappa,\kappa') \leq 2\max\{M, |u(1)-u'(1)|\}$,[20] so that (X,d) is also bounded and so compact. Let $\Delta(X)$ be the space of Borel probability measures on (X,d) endowed with the weak convergence topology, with typical element ν. Note that $\Delta(X)$ is metrizable and compact. Let Ψ denote the space of continuous utility functions $U: X \times \Delta(X) \rightarrow \Re$ endowed with the sup norm.

In order to apply Mas-Colell (1984), we need payoffs to be continuous functions of κ and ν, i.e., elements of Ψ. Given ν, let F_t be the unconditional distribution of t^{th} period capital choices, $F_t(k) = \nu(\{(k_1,k_2,\ldots):k_t \leq k\})$. The difficulty is that payoffs are *not* continuous in κ when F_t has an atom. The proof proceeds by approximating agents' utility functions by continuous utility functions. The game with these continuous utility functions has an equilibrium. Taking limits and extracting a convergent subsequence yields a distribution, which by the Maximum Theorem will be an equilibrium of the original game.

If the utility from consumption is unbounded below (and so bounded above) and $k_0 > 0$, then there exists c' such that, for all $0 \leq c < c'$, $u(c) + v(1)/(1-\beta) + \sup_{c''} \beta u(c'')/(1-\beta) < u(\tilde{c})/(1-\beta)$, where $\tilde{c} = \sup\{c: f(k_0) - c > k_0\}$. Thus, in this case, any $c < c'$ is strictly dominated and replacing u by $u(c') + (c-c')u'(c')$ for $c \leq c'$ does not alter this. Moreover, with this interpretation, u is a

[20]If u rather than $\bar{u}$ is used to define the metric d, then for the case of CRRA preferences with $\gamma > 1$, (X,d) is not bounded (since utility is not bounded below).

continuous function defined on negative consumptions. Similarly, in general we can extend u to allow for negative consumption in a way which preserves continuity, but which does not alter optimal choices.

Define $U_m(\kappa,\nu) \equiv \sum_{t\geq 0}\beta^t\{u(f(k_t)-k_{t+1}) + v(\frac{1}{2}m \int_{-1/m}^{1/m}F_t(k_t+x)dx)\}$. The second term is the *matching value of the convolution of* F_t *with the distribution function of a uniform random variable on* $[-m^{-1},m^{-1}]$ (i.e., we are effectively smoothing by adding a small random component to the agents k choice).[21] Thus, for each t, the second term is a continuous function of k (uniformly in F_t). Moreover, if $F'_t(k) \to F_t(k)$ at all continuity points of F_t, then (since F_t is monotone) the convergence is a.e. and by Lebesgue's dominated convergence theorem, the integral converges. In order to show that U_m is a continuous function on $X \times \Delta(X)$, it remains to consider κ and κ' and the utility due to consumption. Now, $|\sum_{t\geq 0}\beta^t u(f(k_t)-k_{t+1}) - \sum_{t\geq 0}\beta^t u(f(k'_t)-k'_{t+1})| \leq \sum_{t\geq 0}\beta^t|u(f(k_t)-k_{t+1}) - u(f(k'_t)-k'_{t+1})|$, which can be made arbitrarily small by making $d(\kappa,\kappa')$ small.

Let $\Psi_m \equiv \{U\in\Psi\colon U(\kappa,\nu) = \sum_{t\geq 0}\beta^t\{u(Ak_t-k_{t+1}) + \frac{1}{2}m \int_{-1/m}^{1/m}F_t(k_t+x)dx\}$ for some $k_0\in[0,K]\}$. Note that the only difference between utility functions in Ψ_m is the initial capital stock (and, if necessary, the modification that is a function of k_0 described above). That is, different initial capital stocks, $k_0(i)$, are reflected in different utility functions, *not* different constraint sets. The single period utility function is such that it is never optimal to choose $k_1 > Ak_0$ (this is why it was necessary to extend u to negative consumptions).

Any probability measure on Ψ_m induces an initial distribution over capital stocks and vice versa. Let μ_m be the measure reflecting $k_0(.)$, the initial capital distribution. By Theorem 1 (Mas-Colell (1984)), this game has a Nash equilibrium. That is, a measure τ_m on $\Psi\times X$ such that (where $\tau_{m,\Psi}$ and $\tau_{m,X}$ are the marginals on Ψ and X respectively) $\tau_{m,\Psi} = \mu_m$ and $\tau_m(\{(U,\kappa)\colon U(\kappa,\tau_{m,X}) \geq U(X,\tau_{m,X})\}) = 1$. Denote by $F_{t,m}$ the unconditional distribution of t^{th} period capital choices induced by τ_m.

It is a useful property of convolutions that if F has an atom at k of size δ, then for all σ, $(2\sigma)^{-1}\int_{-\sigma}^{\sigma}F(k+x)dx < F(k+\sigma) - \delta/4$. This implies that, for fixed t, the sup over the size of all atoms in $F_{t,m}$ goes to zero as $m \to \infty$. To see this, first note that the utility due to consumption is uniformly continuous on X. Second, if $F_{t,m}$ has an atom of size δ at k, then $\frac{1}{2}m\int_{-1/m}^{1/m}F_{t,m}(k+2m^{-1}+x)dx - \frac{1}{2}m\int_{-1/m}^{1/m}F_{t,m}(k+x)dx > \delta/4$, so that for large m, $k + 2m^{-1}$ yields almost the same utility from consumption and the value of the gain in the match is bounded away from zero.

Given the sequence $\{\tau_m\}$, there is a subsequence such that $\tau_{m,X}$ converges. Denote the limit τ^*_X and the sequence of capital distributions $\{F^*_t\}$. Suppose F^*_t has an atom at k for some t of size $\delta > 0$.

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[21]This is the payoff that results when each agents' choice is adjusted by an amount which is determined as an independent draw from some interval. Specifying payoffs in this way avoids the technical issues of a continuum of independent random variables.

Then $\forall\epsilon>0$ $\exists M$ such that $\forall m>M$, $F_{t,m}(k+\epsilon) - F_{t,m}(k-\epsilon) > \delta/2$. Then $\exists k'$, k'' in the support of $F_{t,m}$ such that $F_{t,m}(k'') - F_{t,m}(k') \geq \delta/2$ and $2\epsilon > k''-k' > 0$. Now, $\frac{1}{2}m\{\int_{-1/m}^{1/m} F_{t,m}(k''+2m^{-1}+x)dx - \int_{-1/m}^{1/m} F_{t,m}(k'+x)dx\} \geq F_{t,m}(k''+m^{-1}) - \frac{1}{2}\{F_{t,m}(k') + F_{t,m}(k'+m^{-1})\} \geq \delta/4$. By choosing ϵ sufficiently small and $m > \epsilon^{-1}$, the consumption utility loss from choosing $k''+2m^{-1}$ rather than k' can be made arbitrarily small. Since the matching gain is bounded away from zero, k' cannot be an optimal choice for any agent, a contradiction. Thus F_t has no atoms.

It remains to argue that $\{F_t^*\}$ describes an equilibrium of the original game. We claim that, since the F_t are continuous for all t, the convergence of $U_m(.,\tau_X^*)$ to $U(.,\tau_X^*)$ is uniform. In order to show this, fix $\epsilon > 0$ and T such that $\sum_{t\geq T}\beta^t v(1) < \epsilon/4$. We demonstrate the uniformity of convergence for any finite number of periods, which is sufficient since the total discounted value of matching for the periods after T is less than $\epsilon/4$. So suppose $t < T$. Since F_t^* is continuous on $[0,K_t]$, there exists $\delta > 0$ such that $|F_t^*(k'') - F_t^*(k')| < \epsilon/4$ whenever $|k''-k'| \leq \delta$. Now, $\exists M_n$ such that if $m > M_n$ then $|F_t^*(n\delta)- F_{t,m}(n\delta)| < \epsilon/4$, for $n = 0,\ldots,\delta^{-1}K_t$. Let $M = \max\{M_n : n\leq\delta^{-1}K_t\}$. Now, suppose $m > M$ and $k \in [n\delta^{-1},(n+1)\delta^{-1}]$. Note that $F_t^*(n\delta) + \epsilon/4 > F_t^*((n+1)\delta) \geq F_t^*(k) \geq F_t^*(n\delta) > F_t^*((n+1)\delta) - \epsilon/4$ and $F_t^*((n+1)\delta) + \epsilon/4 > F_{t,m}((n+1)\delta) \geq F_{t,m}(k) \geq F_{t,m}(n\delta) > F_t^*(n\delta) - \epsilon/4$. Then $|F_t^*(k)-F_{t,m}(k)| \leq \epsilon/2$ for all k. Since payoff functions are converging uniformly, the limit of best reply capital choices to $\{F_t^*\}$ is a best reply in the limit, completing the argument. □

A.2 Properties of wealth-is-status equilibria

PROPERTY 1: *In an equilibrium,* if $k_0(i) > k_0(i')$, *then for all* t, *the optimal level of capital in period* t *for agent* i *and* i' *satisfies* $k_{t+1}(i) > k_{t+1}(i')$.

PROOF: Suppose not. Let i and i' be two family lines in which a switch occurs and let t be the first period in which it occurs. The optimality of the agents' choices implies that:

$$\text{(i)}\qquad u(Ak_t(i)-k_{t+1}(i))+m_t(k_t(i))+\beta V(k_{t+1}(i)) \geq u(Ak_t(i)-k_{t+1}(i'))+m_t(k_t(i))+\beta V(k_{t+1}(i')), \text{ and}$$

$$\text{(ii)}\qquad u(Ak_t(i')-k_{t+1}(i'))+m_t(k_t(i'))+\beta V(k_{t+1}(i')) \geq u(Ak_t(i')-k_{t+1}(i))+m_t(k_t(i'))+\beta V(k_{t+1}(i)).$$

Note that $k_{t+1}(i')$ is feasible for i since it was feasible for i', and that $k_{t+1}(i)$ is feasible for i' since it is less than the actual choice.

Conditions (i) and (ii) imply that $u(Ak_t(i')-k_{t+1}(i')) + u(Ak_t(i)-k_{t+1}(i)) \geq u(Ak_t(i')-k_{t+1}(i)) + u(Ak_t(i)-k_{t+1}(i'))$. This contradicts our assumption that $u(\cdot)$ is a monotonically increasing strictly concave function and, hence, switching is inconsistent with optimizing behavior. □

PROPERTY 2: *For all family lines and all* $t \geq 0$, $\lambda_t \leq \lambda^*$.

PROOF: First observe that for any family line, it is not optimal for there to exist a T such that for all $t \geq T$, $\lambda_t \geq \lambda^*$, with $\lambda_t > \lambda^*$ for at least one period. Suppose otherwise. Then a unilateral deviation by a family line lowering the consumption ratio from λ_t to λ^* for all $t \geq T$ increases the family line's utility from consumption, while their utility from matching cannot be lower since subsequent capital sequence is at least as large as it was originally.

Now suppose Property 2 did not hold. From the previous paragraph, if $\lambda_t > \lambda^*$, then there exists a period s such that $\lambda_s > \lambda^*$, and $\lambda_{s+1} \leq \lambda^*$. But this is inconsistent with equation (11), since $u'(\lambda_s Ak_s) < u'(\lambda^* Ak_s) = \beta u'((1-\lambda^*)\lambda^* Ak_s) < \beta u'((1-\lambda_s)\lambda_{s+1} Ak_s)$, and $m'_{s+1}(.) \geq 0$. □

PROPERTY 3: *In any equilibrium, in every period, each bequest level is chosen by a zero measure of agents.*

PROOF: Suppose not. Assume that at some time $t \geq 1$ there is a set A of positive measure of men who inherit the same level of capital. Let B denote the set of women who match in equilibrium with a man from set A. Note that $\sup(B) > \int_B j d\mu$, so that the woman in B with the highest endowment has a strictly greater endowment than the average woman in B. Then the family line in A matched with a B of lower endowment than the maximum could have achieved a discretely better match at time t with an arbitrarily small increase in its bequest. □

PROPERTY 4: *Suppose* $k_0(.)$ *is a continuous function of* i. *Given an equilibrium and any period* t, *the capital distribution is strictly increasing, so that* $k_t(.)$ *is a continuous function of* i.

PROOF: Let t be the first period for which the equilibrium capital distribution is not strictly increasing. If F_t as defined as in the proof of the proposition, then there is an interval (k',k'') where F_t is a constant, $F_t(k) > F_t(k'')\ \forall k > k''$, and $F_t(k) < F_t(k')\ \forall k < k'$. From Property 1, there is no switching, so that the capital stock in period t-1 of the family line that has stock just below k' is just below the capital stock in period t-1 of the family line with capital stock above k". A small change in capital choice by either family line which does not violate the ordering cannot have a large impact on their matches. Note that if capital stocks are close, then the discounted value of consumption must be close. Then since k" is not close to k', one of the two family lines has a profitable deviation. □

PROPERTY 5: *If* $\gamma < 1$, *then for all* $i \in [0,1]$, $\lambda_t(i) \to \lambda^*$ *as* $t \to \infty$.

PROOF: In equilibrium the quality of family line i's match is unchanging and equal to i. The cost to a family line in choosing λ^*, rather than λ_t is the reduction in quality of future matches. This cost is less than $i/(1-\beta)$; thus, the gain in utility in any period resulting from choosing λ^* rather than λ_t must also be less than $i/(1-\beta)$. The consumption gain in period t from setting $\lambda_t = \lambda^*$ is at least:

$$k_t^{1-\gamma}\{u(\lambda^* A) + V^*((1-\lambda^*)A) \ - \ [u(\lambda_t A) + V^*((1-\lambda_t)A)]\},$$

where V^* is defined in equation (8). The term $\{u(\lambda^* A) + V^*((1-\lambda^*)A) - [u(\lambda_t A) + V^*((1-\lambda_t)A)]\}$ is strictly positive. (Recall that the stationary consumption ratio λ^* is the unique solution to the exogenous matching--equivalently pure consumption/investment--problem.) Since k_t is growing at least at the rate $(1-\lambda^*)A$, the only way the consumption gain can always be less than $i/(1-\beta)$ is if the second term converges to zero. This implies that $\lambda_t \to \lambda^*$. □

PROPERTY 6: *If* $\gamma > 1$ *then either* $\lambda_t(i') \to 0$ *or* $k_t(i')/k_t(i) \to \infty$ *for all* $i < i'$ *for all* $i' \in (0,1]$ *as* $t \to \infty$.

PROOF: Note that the consumption cost to a family line i of choosing an arbitrarily low consumption fraction, $\epsilon > 0$, at time t is:

$$(1-\gamma)^{-1}(Ak_t(i))^{1-\gamma}[\lambda_t(i)^{1-\gamma} - \epsilon^{1-\gamma}].$$

Fix i and i' with $i < i'$. Since $k_t \to \infty$ as $t \to \infty$, for the case of $\gamma > 1$, for any $\epsilon > 0$ there exists a time T_ϵ such that for all $t > T_\epsilon$, the loss in utility from lowering consumption to ϵ is less than i'-i. Since there can be no switching in equilibrium, we must have $A(1-\epsilon)k_t(i) < A(1-\lambda_t(i'))k_t(i')$ for all $t > T_\epsilon$. This implies

$$\lim_{t \to \infty} (1-\lambda_t(i'))k_t(i')/k_t(i) \geq 1.$$

That is, for any two family lines i and i' with $i' > i$, it must be the case that it eventually becomes infeasible for the family line i to achieve the bequest level of the family line i'. Since the capital distribution is continuous, if $\lim_{t\to\infty} k_t(i')/k_t(i)$ is finite, then there exists a $\delta > 0$ such that $\lim_{t\to\infty} k_t(i')/k_t(i'-\delta) < 1 + \mu$ for any $\mu > 0$. If $\lambda_t(i')$ does not converge to zero, then the above condition will be violated and it will be optimal for family line $i' - \delta$ to acquire more capital than family line i at some point in time. □

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