Innovation, Imitation and Economic Growth

by

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Abstract: This paper develops a dynamic general equilibrium model of economic growth. The model has a steady state equilibrium in which some firms devote resources to discovering qualitatively improved products and other firms devote resources to copying these products. Rates of both innovation and imitation are endogenously determined based on the outcomes of R&D races between firms. Innovation subsidies are shown to unambiguously promote economic growth. Welfare is only enhanced however if the steady state intensity of innovative effort exceeds a critical level.

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1. Introduction 1

Economic growth is fueled both by innovation and imitation. Firms invest significant resources in research and development (R&D) activities to discover qualitatively improved products and capture associated profits. When they are successful, other firms, attracted by these profits, imitate and thus they accelerate the development and production of new products. While the innovation-imitation debate may be the focus of boardroom discussions, the theoretical literature on R&D races between firms focuses almost exclusively on the development of new products or processes.² Failing to address the incentives that firms have to engage in costly imitative activities, these models ignore the obvious feedback effect imitation has on the incentive to innovate. The rate of imitation in this literature is typically exogenously determined based on perfectly enforced finite patents, or the possibilities for imitation are ignored altogether.³ This contrasts sharply with the empirical evidence. Tilton (1971), for example, found that the lag time between the initial discovery of semiconductor innovations by American firms and the first commercial production by Japanese firms averaged just one year. Mansfield, Schwartz and Wagner (1981) found that 60% of the patented innovations they studied were imitated within four years.

In this paper, I present a dynamic general equilibrium model of economic growth. In this model, throughout time, firms can enter into both innovative and imitative R&D races in each industry. The winner of each innovative R&D race discovers how to produce a new superior product and the winner of each imitative R&D race discovers how to produce the state-of-the-art quality product. Both the pace and the direction of R&D activities are determined based on expected discounted profit maximizing considerations. Consumers maximize their discounted utilities and all markets clear throughout time.

¹ I thank Carl Davidson, Elias Dinopoulos, James Oehmke, Susan Linz and participants in the Midwest International Economics Conference (May 11-13, Indiana University) for insightful comments on the preliminary version of this paper. Of course, all errors are my own responsibility.

² The reader is referred to Dasgupta (1988), Dasgupta and Stiglitz (1980), Katz and Shapiro (1987), Lee and Wilde (1980), Loury (1979), Reinganum (1982), and Spence (1984) for further references.

³ Partial equilibrium models in which firms can choose between innovating and imitating have been developed by Baldwin and Childs (1969), Cheng (1989), and Jovanovic and MacDonald (1988).

I show that this dynamic general equilibrium model has a steady state equilibrium in which the rate of economic growth is constant over time. In this steady state equilibrium, firms engage in both costly innovative and costly imitative activities, however, not in the same industry at the same time. Firms find it more profitable to imitate in industries with a single leader and firms find it more profitable to innovate in industries with two leaders. Thus, at any point in time, some industries are targeted by potential innovators whereas other industries are targeted by potential imitators.

This targeting of industries, together with the general equilibrium labor market effects, helps generate some surprising comparative steady state results. I show that a once-and-for-all increase in the government subsidy to innovation unambiguously increases the steady state intensity of imitative effort in each industry where firms engage in imitative R&D (that is, cheaper innovation implies a faster rate of imitation). I also show that a once-and-for-all exogenous increase in the government subsidy to imitation increases the steady state intensity of innovative effort in each industry where firms engage in innovative R&D (that is, cheaper imitation implies a faster rate of innovation). Perhaps the most important result in this paper concerns the effect of the innovation subsidy on economic welfare. I show that although a marginal increase in the innovation subsidy always increases economic growth, it only enhances the discounted utility for the representative consumer if the steady state intensity of innovative effort exceeds a critical value (is sufficiently high to begin with).

This paper builds on the dynamic general equilibrium model of product innovation by Grossman and Helpman (1989a)⁴. I use their framework but expand upon it by allowing firms to undertake costly imitative R&D. Grossman and Helpman solved for a steady state equilibrium in their model in which the rate of innovative R&D investment is the same in all industries throughout time. In their model, the only threat to successful innovators comes from further innovation and throughout time, only one firm produces output in each industry. My analysis is richer in nature since I solve for a steady state equilibrium in which firms engage in both

⁴ Grossman and Helpman (1989a) build on an earlier North-South trade model by Segerstrom, Anant and Dinopoulos (1987), which was the first to study sequences of innovative R&D races in a dynamic general equilibrium context. Both models have similar qualitative properties but Grossman and Helpman's model is mathematically more tractable because of their "continuum of industries" assumption.

innovative and imitative activities. My equilibrium also has the property that market concentration levels fluctuate over time in each industry.

Grossman and Helpman (1989b) have recently extended their earlier analysis to include costly imitative R&D. Indeed, the present paper shares the same assumptions about consumer preferences and the structure of R&D races. There are important differences in analysis, however. Grossman and Helpman use the static Bertrand equilibrium concept to analyze dynamic product market competition and thus they implicitly assume that a perfectly competitive outcome arises whenever two identical cost firms are active in any market. In contrast, in this paper, whenever firms are in a position to collude, with any possible cheating deterred by subgame perfect equilibrium punishment threats, firms are assumed to take advantage of the opportunity. And in the highly concentrated markets that result from innovation and imitation, I show that mutually beneficial collusion between firms is feasible.

In Grossman and Helpman (1989b), imitative R&D is driven by factor price differences across countries. In their steady state equilibrium, imitation always involves firms in the South copying products developed in the North. Although factor price differences across countries are clearly important, by ignoring these differences, this paper shows that they are not necessary to explain why firms engage in costly imitative activities. Thus this paper provides an explanation for an important empirical phenomenon: firms in the North copying products developed by other firms in the North.

The remainder of this paper is organized as follows. The dynamic general equilibrium model of innovation and imitation is presented in section 2. In section 3, I show that this model has a steady state equilibrium in which some firms engage in costly innovative activities and other firms engage in costly imitative activities. Section 4 explores the comparative steady state equilibrium properties of this model and section 5 contains some concluding remarks.

2. The Model

I consider an economy with a continuum of industries indexed by $\omega \in [0,1]$. Products in each industry can be supplied in a countable number of qualities j=0,1,2,3,... All of the

⁵ Differences in implications will be discussed in section 4.

consumers live forever and have identical preferences. The intertemporal utility function for the representative consumer is given by

(1)
$$U \equiv \int_0^\infty e^{-\rho t} u(t) dt ,$$

where ρ is the subjective discount rate, and u(t) is the consumer's instantaneous utility at time t. This instantaneous utility is given by

(2)
$$u(t) = \int_0^1 \ln\left[\sum_{j=0}^{\infty} \lambda^j d_{ji}(\omega)\right] d\omega,$$

where $d_{ji}(\omega)$ denotes the quantity consumed of a product of quality j produced by industry ω at time t, and $\lambda > 1$ represents the extent to which higher quality products improve upon lower quality products δ . Every consumer maximizes discounted utility subject to an intertemporal budget constraint

(3)
$$\int_0^\infty e^{-R(t)} E(t) dt = A(0),$$

where R(t) is the cumulative interest factor up to time t, A(0) is the value of asset holdings at time t=0 plus the present value of future factor income, and E(t) is the consumer's expenditure flow at time t. Of course, the consumer's expenditure flow at time t is given by

(4)
$$E(t) = \int_0^1 \sum_{i=0}^{\infty} p_{it}(\omega) d_{it}(\omega) d\omega,$$

where $p_{ji}(\omega)$ is the price of a product of quality j produced by industry ω at time t.7

⁶ This is the same instantaneous utility function as was used in Grossman and Helpman (1989a). The "finite number of industries" version of this CDP (Cobb-Douglas with Perfect Substitutes) utility function was introduced in Segerstrom, Anant and Dinopoulos (1987) to study product innovation and replacement.

⁷ In this model, price competition between firms forces all firms producing the same quality product to charge the

Labor is homogeneous, the only factor of production, and the economy-wide endowment of labor L is constant over time. For each industry and each quality product, constant returns to scale prevail with one unit of labor producing one unit of output. However, firms cannot produce products that have not yet been discovered. At time t=0, firms only know how to produce products of quality j=0 in each industry $\omega \in [0,1]$. Time t=0 represents the beginning of a sequence of innovative R&D races between firms in each industry. The winner of the first innovative R&D race in industry $\omega \in [0,1]$ becomes the sole producer of a product of quality j=1 in this industry. In general, if the state-of-the-art quality product in industry $\omega \in [0,1]$ is of quality j at time t, then the next winner of an innovative R&D race in that industry becomes the sole producer of a product of quality j+1. There is free entry into innovative R&D races in each industry and firms are free to choose how much labor to hire to engage in innovative R&D at each instant in time. For each firm, a unit of innovative R&D activity requires a, units of labor per unit of time ("I" stands for "Innovating"). By undertaking I units of innovative R&D in industry $\omega \in [0,1]$ at time t, a firm is successful in discovering the next higher quality product with instantaneous probability I (that is, Idt is the constant probability that if the innovation is not ready at time t, it will be ready at time t+dt, where dtis an infinitesimal increment of time). The returns to engaging in innovative R&D are assumed to be independently distributed across firms, across industries, and over time.⁸ As described, each innovative R&D race has the same structure as in Lee and Wilde (1980).9

A firm that discovers a new superior quality product becomes the sole producer of this product. But this monopoly position does not necessarily last forever. Other firms can engage in imitative R&D activities to learn how to produce state-of-the-art quality products in each industry 10 . For each firm, a unit of imitative R&D activity requires a_c units of labor per unit of

same price.

⁸ Thus, if firms chose constant R&D expenditures over time, the time duration of each R&D race will be exponentially distributed.

⁹ The Lee-Wilde R&D structure has been used to study R&D subsidies and taxes in an international setting by Dixit (1988).

¹⁰ The licensing of innovations is not explicitly considered, in this paper, as a viable option for firms. However, the transaction costs and asymmetric information problems associated with licensing limit firms' abilities to achieve the licensing gains from trade. In the Mansfield et. al. (1981) empirical study of imitation costs, only 1 out of 48

time ("C" stands for "Copying"). By undertaking C units of imitative R&D in industry $\omega \in [0,1]$ at time t, a firm is successful in discovering how to produce the state-of-the-art quality product in that industry with instantaneous probability C. The returns to engaging in imitative R&D are assumed to be independently distributed across firms, across industries, and across time. As described, each imitative R&D race has the same structure as each innovative R&D race. The idea that firms can imitate more easily than innovate is captured by assuming 11

A1
$$a_1 > a_C > 0$$
.

The labor market in this economy is assumed to be perfectly competitive and all workers earn the same equilibrium wage which I normalize to equal one throughout time. In each industry, firms set prices and face no capacity constraints. ¹² Since all the firms in an industry $\omega \in [0,1]$ producing a quality j product are producing identical products, consumers will only (possibly) buy from the firms that sell the quality j product for the lowest price. At each moment in time, all firms set prices simultaneously and independently. However, it takes time for firms to learn about and react to the prices being charged by other firms and I assume that this time lag $\ell > 0$ is the same for all firms throughout time. ¹³ Thus a decision by any firm to change its price at time t does not go into effect until time $t + \ell$.

Finally, I assume the existence of a capital market which supplies the savings of consumers to firms engaged in R&D. The equilibrium interest rate $r(t) \equiv dR(t)/dt$ clears this market at each moment in time. Firms borrow funds from this market to pay workers as the R&D is done. Each firm issues a risky security that yields a positive return if it wins an R&D race and a negative return if it loses. Since there is a continuum of industries and the returns to engaging in R&D are independently distributed both across firms and across industries, by

cases involved an innovator licensing to the imitating firm.

¹¹ In their empirical study of imitation costs, Mansfield, et. al. (1981) found that the ratio of imitation costs to innovation costs was .65 on average.

¹² By assuming that price (instead of quantity) is the strategic choice of firms, I am implicitly assuming that each firm can adjust its production level more easily than its price. See Friedman (1983, p. 46-48).

¹³ This time lag plays the same role in the present paper as the length of a time period in the literature on price-setting supergames. See, for example, Lambson (1987).

holding a diversified portfolio of securities, investors are able to completely diversify away risk. Thus free entry into R&D implies that firms keep on entering each R&D race until expected discounted profits are driven to zero. This completes the description of the model.

3. The Steady State Equilibrium

In this section, I solve for a steady state equilibrium for the previously described model with the following basic properties: (i) both individual and aggregate consumer expenditure are constant over time, (ii) the market rate of interest is constant over time and equal to each consumer's subjective discount rate, (iii) in industries with a single firm producing the state-of-the-art quality product (a single quality leader), other firms engage in imitative but not innovative R&D, (iv) in industries with two firms producing the state-of-the-art quality product (two quality leaders), other firms engage in innovative but not imitative R&D, (v) in each industry, quality leaders do not engage in innovative activities, and firms that are one step down in the quality ladder do not engage in imitative activities, (vi) no industry has more than two quality leaders at any moment in time, (vii) each industry with a single quality leader has the same aggregate imitative R&D expenditure flow, (viii) each industry with two quality leaders has the same aggregate innovative R&D expenditure flow, (ix) in industries with a single quality leader, this firm earns dominant firm profit flows, (x) in industries with two quality leaders, these two firms collude and share equally dominant firm profit flows, and (xi) the proportion of industries with a single quality leader is a constant over time.

Property (i) is the defining characteristic of a steady state equilibrium. I will initially assume that the other properties [(ii)-(xi)] also hold and use them to uniquely characterize aggregate innovative and imitative R&D expenditures in each industry. Later on in this section and in the appendix, given some basic assumptions about underlying parameter values, I show that this steady state behavior constitutes a subgame perfect equilibrium, that is, no single consumer or firm ever has any unilateral incentive to deviate from the above described behavior. Thus these steady state properties are consistent with maximization on the part of all economic agents throughout time.

I begin by analyzing the behavior of the representative consumer. This consumer allocates expenditure E(t) to maximize her instantaneous utility u(t) given prices at time t. Then she choses the time pattern of spending to maximize U. The static problem involves maximizing (2) subject to the constraint (4) and the obvious restriction that this consumer cannot buy products that have not been discovered by time t. The Euler equation for this calculus of variations problem yields

(5)
$$\sum_{j \in J_t(\omega)} \lambda^j d_{jt}(\omega) = \frac{E(t)\lambda^h}{p_{ht}(\omega)},$$

where $J_i(\omega)$ is the set of available quality levels with the lowest quality adjusted prices $(p_{ji}(\omega)/\lambda^j)$, and $h = h_i(\omega)$ is the highest quality level in $J_i(\omega)$. I assume that, among the firms charging the lowest quality adjusted prices, consumers only buy from the firms which sell the highest quality products. Then (5) yields static demand functions

(6)
$$d_{ji}(\omega) = \begin{cases} E(t)/p_{ji}(\omega) & \text{for } j = h_i(\omega) \\ 0 & \text{otherwise} \end{cases}$$

Solving for the time path of spending E(t) that maximizes discounted utility U for the representative consumer involves substituting (2) and (6) into (1) and maximizing this expression subject to the constraint (3). The Euler equation for this calculus of variations problem yields

(7)
$$\frac{dE(t)}{dt}/E(t) = r(t) - \rho.$$

(7) implies that any steady state equilibrium in consumer expenditure must involve a constant market interest rate r(t) over time which is equal to the consumer's subjective discount rate. Thus, given the basic property of steady state equilibria (i), property (ii) follows from utility maximization. The actual level of expenditure E is determined by the consumer's steady state assets A. Since all consumers have identical homothetic preferences, in the rest of this paper, I

will let E denote aggregate steady state expenditure and let (6) represent aggregate demand functions.

Turning to the production side of the economy, since constant returns to scale prevail in the production of every product, with one unit of labor producing one unit of output, firms that produce state-of-the-art quality products have an advantage over all other firms. When all other firms are charging a price of one, the lowest price such that they do not make losses, a firm which is a single quality leader earns instantaneous profits

(8)
$$\pi(p) = \begin{cases} (p-1)E/p, & p \le \lambda \\ 0 & p > \lambda \end{cases}$$

where p is the quality leader's price. These profits are obviously maximized by choosing

$$(9) p = \lambda.$$

Thus when a single quality leader chooses the price $p = \lambda > 1$, this firm earns the positive dominant firm profit flow

(10)
$$\pi^L \equiv (1 - \frac{1}{\lambda})E,$$

and none of the other firms in the industry can do better than break even (by selling nothing at all). In this paper, I solve for a steady state equilibrium where each quality leader in any industry with a single quality leader earns this profit flow π^{L} , and by colluding, each quality leader in any industry with two quality leaders earns the collusive profit flow 14

(11)
$$\pi^c = \left(1 - \frac{1}{\lambda}\right) \frac{E}{2}.$$

¹⁴ This paper focuses on symmetric collusion. Of course, other forms of collusion can be supported by subgame perfect equilibrium strategies.

In both cases, the market price of a quality leader is given by (10), and no other firms sell anything in equilibrium. Firms that can (but do not, in equilibrium) produce lower quality products serve to constrain quality leaders from charging prices higher than λ .

To justify (11), I must show that, in an industry with two quality leaders, these firms are in a position to collude and each earn the profit flow π^{C} . Given property (iv) of the steady state equilibrium, these collusive profit flows are only earned until some other firm succeeds in innovating, and discovering a still higher quality product. Letting I denote the aggregate level of innovative activity in this industry (in the steady state equilibrium), and using the fact that the time lag τ until a higher quality product is discovered is exponentially distributed, the expected discounted profits from colluding are

(12)
$$\int_0^{\infty} \left\{ \int_0^{\tau} \pi^C e^{-\rho t} dt \right\} I e^{-t\tau} d\tau = \frac{\pi^C}{\rho + I}.$$

Alternatively, one of these quality leaders could cheat on the collusive agreement. Letting π^{CH} denote the profit flows that a quality leader earns from cheating, and recognizing that cheating is detected after a time lag ℓ , the expected discounted profits from cheating are

(13)
$$\int_0^{\ell} \left\{ \int_0^{\ell} \pi^{CH} e^{-\rho s} ds \right\} I e^{-It} dt + \int_{\ell}^{\infty} \left\{ \int_0^{\ell} \pi^{CH} e^{-\rho s} ds \right\} I e^{-It} dt = \frac{\pi^{CH}}{\rho + I} (1 - e^{-(\rho + I)\ell}).$$

The first bracketed expression on the LHS of (13) is the discounted profits from cheating given that a new higher quality product is discovered before cheating is detected. Since the expected discounted profits from engaging in R&D equal zero, a cheater can expect to earn nothing after another firm innovates. These discounted profits are weighted by the probability that further innovation occurs before the cheating in detected. The second expression on the LHS of (13) is the discounted profits from cheating given that this cheating is detected before another firm successfully innovates, weighted by the probability that further innovation does not occur before the cheating is detected. Then the firm that was cheated on can punish the cheater by charging a price of one for its product. By charging such a price, the punisher guarantees that the cheater

earns nonpositive profit flows. The punisher can carry out this punishment indefinitely (as is the case with Friedman's (1971) trigger strategies), or carry out this punishment for a period of length ℓ if the cheater "repents" by charging price below one during this time period and then return to behaving collusively (as is the case with Segerstrom's (1988) repentance strategies). In either case, the cheater earns expected discounted equilibrium profits of zero subsequent to being detected for cheating. Since cheating profit flows are maximized by charging a price infinitesimally below λ and undercutting the other quality leader, $\pi^{CH} \leq 2\pi^{C}$. Now putting (12) and (13) together, I conclude that neither of the two quality leaders has an incentive to cheat on collusive behavior if

$$(14) \qquad (\rho + I^*)\ell < \ln(2),$$

where I^* is the steady state level of innovative R&D.

In the appendix, I introduce some additional parameter value restrictions which guarantee that (14) is satisfied. I essentially assume that the labor force L is sufficiently small and thus the pace of innovative activity is positive but sufficiently slow so that two quality leaders find it attractive to collude. I also assume that steps up in the quality ladder are sufficiently large so that firms producing different quality products in an industry are not able to successfully collude (using subgame perfect equilibrium strategies), thus justifying (10) and (11). In industries with two quality leaders, I rule out the possibility that these firms might charge prices higher than λ and collude with firms producing lower quality products. In industries with a single quality leader, I rule out the possibility that this quality leader might charge a price higher than λ and collude with firms producing lower quality products. And in industries with two quality leaders, I show that no firm has any incentive to engage in imitative R&D, from which it follows that there never are any industries with three or more quality leaders.

Property (xi) states that the proportion of industries with a single quality leader is a constant over time. I will denote this proportion α . Since no industry has more than two quality

¹⁵ These assumptions are made to keep the discussion from becoming taxonomic.

leaders at any moment in time (vi), the proportion of industries with two quality leaders β equals $1-\alpha$. Firms only engage in imitative R&D in α industries (iii) and firms only engage in innovative R&D in β industries (iv). Thus, when innovation occurs in an industry, this industry switches from being a " β industry" to being an " α industry", and when imitation occurs in an industry, this industry switches from being an " α industry" to being a " β industry". This structure of the steady state equilibrium is summarized in the following figure:

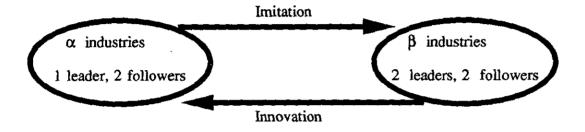


Figure 1

To maintain a steady state equilibrium, every time a new superior quality product is discovered in some industry, imitation must occur in some other industry. That is,

(15)
$$\alpha Cdt = \beta Idt.$$

(15) states that the rate at which industries leave the α group must equal the rate at which industries leave the β group. It follows that the steady state proportion of " α industries" is $\alpha = I/(C+I)$ and the steady state proportion of " β industries" is $\beta = C/(C+I)$.

In all industries, the price $\lambda > 1$ is charged by all quality leaders and firms producing lower quality products sell nothing. From (6), I can conclude that the steady state aggregate demand for production workers in these industries is E/λ . Since innovation only takes place in " β industries" and imitation only takes place in " α industries", the labor market (or full employment) condition is

(16)
$$E/\lambda + \beta a_i I + \alpha a_c C = L.$$

Substituting in for α and β , the steady state labor market condition can be written as

(17)
$$\frac{E}{\lambda} + \frac{IC(a_I + a_C)}{I + C} = L.$$

To explore the incentives firms have to engage in imitative activities, let v_c denote the expected discounted reward for winning an imitative R&D race and let τ denote the random time duration of an imitative R&D race. Since τ is exponentially distributed, the expected benefit from engaging in imitative R&D in an industry is

(18)
$$\int_0^{\infty} \Pr(\tau = s) v_C e^{-\rho s} ds = \int_0^{\infty} (Ce^{-Cs}) v_C e^{-\rho s} ds = v_C C / (\rho + C).$$

The expected cost from engaging in imitative R&D in an industry is

(19)
$$\int_0^\infty \left\{ \int_0^t a_c C e^{-\rho s} ds \right\} C e^{-ct} dt = a_c C / (\rho + C).$$

Thus the expected profit from engaging in imitative R&D is

(20)
$$\frac{C(v_C - a_C)}{\rho + C}.$$

Free entry into imitative R&D races then implies that $v_C = a_C$ in each imitative R&D race. But what does v_C equal? A successful imitator in an industry earns the collusive profit flow π^C until some other firm succeeds in innovating and starts producing a higher quality product. Since the time duration of innovative R&D races is exponentially distibuted,

(21)
$$v_C = \int_0^{\infty} \left\{ \int_0^t \pi^C e^{-\rho s} ds \right\} I e^{-it} dt = \pi^C / (\rho + I) = a_C.$$

Substituting for E in (17) using (11) and (21) yields a labor market condition in (C,I) space:

(22)
$$\frac{2a_C(\rho+I)}{\lambda-1} + \frac{IC(a_I + a_C)}{I+C} = L.$$

(22) can be rearranged to define the function

(23)
$$C = C_L(I) = \frac{I\left(L - \frac{2a_C\rho}{\lambda - 1}\right) - I^2\left(\frac{2a_C}{\lambda - 1}\right)}{\left(\frac{2a_C\rho}{\lambda - 1} - L\right) + I\left(a_I + a_C + \frac{2a_C}{\lambda - 1}\right)}.$$

The function $C_L(I)$ defines, for each level of innovative investment I, how much imitative investment is consistent with full employment of labor in the economy, and zero discounted profits in each imitative R&D race. Thus the $C_L(I)$ function can be interpreted as a steady state equilibrium labor market constraint. Note that $C_L(I)$ is unambiguously downward sloping in the positive orthant. Given a fixed endowment of labor in the world, more innovative R&D in " β industries" means less labor is available to do imitative R&D in " α industries".

To explore the incentives firms have to engage in innovative activities, let v_i denote the expected discounted reward for winning an innovative R&D race and let τ denote the random time duration of an innovative R&D race. Since τ is exponentially distributed, the expected benefit from engaging in innovative R&D in an industry is

(24)
$$\int_0^{\infty} \Pr(\tau = s) v_l e^{-\rho s} ds = \int_0^{\infty} (I e^{-ls}) v_l e^{-\rho s} ds = v_l I / (\rho + I).$$

The expected cost from engaging in innovative R&D in an industry is

(25)
$$\int_0^{\infty} \left\{ \int_0^t a_I l e^{-\rho s} ds \right\} l e^{-lt} dt = a_I I / (\rho + I).$$

Thus the expected profit from engaging in innovative R&D is

(26)
$$\frac{I(v_I - a_I)}{\rho + I}.$$

Free entry into innovative R&D races then implies that $v_1 = a_1$ in each innovative R&D race. But what does v_1 equal? A successful innovator in an industry earns the dominant firm profit flow π^L until some other firm succeeds in imitating, and then it has to share these profits with the imitator. Since the time duration of imitative R&D races is exponentially distributed,

(27)
$$v_{I} = \int_{0}^{\infty} \left\{ \int_{0}^{s} \pi^{L} e^{-\rho t} dt + e^{-\rho s} v_{C} \right\} C e^{-Cs} ds = \frac{\pi^{L} + C v_{C}}{\rho + C} = a_{I}.$$

I am now in a position to solve for the levels of innovative and imitative R&D consistent with zero expected discounted profits in both types of R&D races. Combining (10), (11), (21) and (27) yields

(28)
$$C = C_{I}(I) = \frac{(2a_{C} - a_{I})\rho + 2a_{C}I}{a_{I} - a_{C}}.$$

This $C_I(I)$ function defines, for every level of innovative investment I, how much imitative investment is consistent with zero expected discounted profits in each R&D race (both innovative and imitative). Thus I can interpret $C_I(I)$ as a steady state equilibrium zero profit in R&D constraint.

Given A1, $C_l(I)$ is unambiguously positively sloped. The intuition behind this positive slope is somewhat involved: Increasing C means that innovative firms enjoy earning dominant firm profits for a shorter expected period of time. Since firms earn zero expected discounted profits from engaging in innovative R&D, they must be able to earn higher dominant firm profit flows during this shorter expected period of time. But higher dominant firm profit flows imply higher collusive profit flows. It follows that to satisfy the zero profit in imitative R&D condition

(21), there must be higher innovative R&D I. Then these imitators earn the higher collusive profits for a shorter expected period of time. To summarize, to maintain zero expected discounted profits in both innovative and imitative R&D races, when imitative R&D C increases, innovative R&D I must also increase.

The $C_I(I)$ and $C_L(I)$ functions are illustrated in Figure 2. As illustrated, the $C_L(I)$ function is not well-defined at $I=I_1$ (where the denominator in (23) equals zero), and the $C_L(I)$ function intersects the horizontal axis at I=0 and $I=I_2\equiv (\lambda-1)L/2a_C-\rho$. As illustrated, the $C_I(I)$ function intersects the horizontal axis at $I=I_3\equiv (a_I-2a_C)\rho/2a_C$. I will assume

A2
$$L > \max \left\{ \frac{2a_c \rho}{\lambda - 1}, \frac{a_t \rho}{\lambda - 1} \right\}.$$

Assumption A2 essentially guarantees that the economy-wide labor force is sufficiently large to sustain both innovative and imitative activities in a steady state equilibium. A1 and A2 together imply that $I_2 > I_1 > 0$ and $I_3 < I_2$, from which it follows that there exists a unique intersection of the $C_I(I)$ and $C_L(I)$ functions in the positive orthant, as illustrated in Figure 2.¹⁶ The steady state imitative investment flow C^* in each " α industry" and the steady state innovative investment flow I^* in each " β industry" are pinned down by the intersection of these two functions.

At this steady state equilibrium, I can calculate the growth rate of static utility (2) for the representative consumer. This gives me a perfect measure of economic growth. Substituting (6) and (9) into (2) yields

(29)
$$u(t) = \ln E - \ln \lambda + \int_0^1 \ln \lambda^h d\omega,$$

where $h = h_i(\omega)$ is the state-of-the-art quality level in industry ω at time t. h only changes when firms are successful in innovating, and firms only engage in innovative R&D in $\beta = C/(C+I)$ industries at a time. For any ω in the β industries, the probability of exactly m

¹⁶ As illustrated in Figure 2, there also exists another intersection of the two functions, but since it is associated with negative imitative investment, it is economically meaningless and can be ignored.

improvements in a time interval of length τ is

(30)
$$f(m,\tau) = \frac{(I\tau)^m e^{-k}}{m!}.$$

Thus $\beta f(m,\tau)$ represents the measure of products that are improved exactly m times in an interval of length τ . Using properties of the Poisson distribution (see Hoel, Port and Stone (1971), page 84), one can derive

(31)
$$\int_0^1 \ln \lambda^h d\omega = \sum_{m=0}^{\infty} \beta f(m,t) \ln \lambda^m = \beta t I \ln \lambda.$$

Using (29) and (31), the steady state utility growth rate is

(32)
$$g = \frac{du(t)}{dt} = \frac{CI\ln\lambda}{C+I}.$$

Thus, not only does higher I and higher λ (bigger steps in the quality ladder) lead to higher steady state economic growth, but also higher C has the same effect. This latter conclusion may appear puzzling since imitative R&D activities just lead to a redistribution of profits and do not directly benefit consumers in the steady state equilibrium. Successful imitators collude with previous quality leaders and thus consumers do not see any drop in the market price when a state-of-the-art quality product is copied. However higher rates of imitation C (holding I fixed) also means that firms engage in innovative R&D I in a larger proportion of industries and this is what drives the higher economic growth.

Treating λ as fixed, I can use (32) to define iso-growth curves in (C,I) space. These iso-growth curves are illustrated in Figure 3. As drawn, movements in the northwest direction along the labor market constraint $C = C_L(I)$ are associated with higher economic growth. This property follows directly from (22) and plays an important role in the comparative steady state analysis which follows.

I can also make welfare statements by calculating the steady state discounted utility of the representative consumer under alternative public policy regimes, starting at t=0 where $h_0(\omega) = 0$ for all ω . Substituting (31) and (29) into (1) yields

(33)
$$U = \frac{1}{\rho} \left\{ \ln \left(\frac{E}{\lambda} \right) + \frac{g}{\rho} \right\}.$$

Thus, there is a trade-off between current expenditure and economic growth. As we will see, policy changes that increase growth can decrease welfare if they have a sufficiently negative effect on current expenditure.

4. Comparative Steady State Analysis

In this section, I explore the steady state equilibrium effects of government subsidies to innovation and imitation. I also look at the effects of entry taxes imposed on successful imitators as well as the effects of exogenous changes in basic parameters of the model $(a_P, a_C, \rho, L, \text{ and } \lambda)$.

What happens when the government subsidizes innovative R&D? Let s_I denote the fraction of innovative R&D costs borne by the government. Then I must multiply the right-hand side of (27) by $1-s_I$. Innovative R&D subsidies have no effect on the labor market constraint (23) but they do change the zero profit in R&D constraint. (28) must be rewritten as

(34)
$$C = C_{I}(I) = \frac{(2a_{C} - a_{I}) + 2a_{C}I + s_{I}\rho a_{I}}{a_{I} - a_{C} - s_{I}a_{I}}.$$

Clearly, an increase in s_I shifts the $C_I(I)$ function up, and from Figure 3, it is easy to see that

Proposition 1: An increase in the government subsidy to innovative R&D (s_l) increases the steady state intensity of imitative R&D (C^*) , decreases the steady state intensity of innovative R&D (I^*) , and increases the steady state rate of economic growth (g^*) .

The intuition behind Proposition 1 is straightforward to explain. When the government subsidy to innovative R&D s_1 is increased, the cost of engaging in innovative R&D is reduced. Since firms earn zero expected discounted profits from engaging in innovative R&D, the benefit from winning an innovative R&D race must go down also. Firms that are successful in innovating must earn dominant firm profits for a shorter expected period of time, that is, there must be a faster rate of imitation. But given a fixed economy wide labor force L, any increase in the intensity of imitative R&D C^* must be offset by a decrease in the intensity of innovative R&D I^* . Thus, innovative R&D subsidies must lower the steady state intensity of innovative R&D in each industry where firms engage in innovative R&D races. I also know that with a higher rate of imitation, firms find it profitable to engage in innovative R&D races in a larger proportion of industries. Thus, even though innovative R&D subsidies lower the intensity of innovative effort, since this effort applies to a larger proportion of industries, the steady state rate of economic growth unambiguously increases.

Even though innovation subsidies increase economic growth, it does not follow that they are good for society. From (33), I know that steady state discounted utility for the representative consumer is an increasing function of both economic growth and steady state expenditure. Since innovation subsidies decrease I^* , it follows from (11) and (21) that they also decrease steady state expenditure. Thus the effect of innovation subsidies on economic welfare is, on the surface, ambiguous.

To resolve this issue, I must explore how movements along the labor market constraint (23) change welfare. Substituting (11) and (21) into (33), I can solve for the slope of iso-welfare curves in (C,I) space. Totally differentiating (22), I can solve for the slope of the labor market constraint. Comparing these slopes reveals that there is a critical value of I,

(35)
$$\hat{I} = \rho \left\{ \frac{(\lambda - 1)(a_I + a_C)}{2a_C \ln \lambda} - 1 \right\}.$$

For $I < \hat{I}$, the iso-welfare curves are steeper than the $C_L(I)$ curve; for $I > \hat{I}$, the iso-welfare curves are flatter than the $C_L(I)$ curve; and for $I = \hat{I}$, the curves are tangent. Thus movements in

the northwest direction along the $C_L(I)$ curve lower welfare if and only if $I^* < \hat{I}$. It follows immediately that

Proposition 2: A marginal increase in the government subsidy to innovative R&D (s_I) (i) increases steady state economic welfare U if the steady state intensity of innovative effort I^* exceeds \hat{I} , and (ii) decreases steady state economic welfare U if the steady state intensity of innovative effort I^* is less than \hat{I} .

Proposition 2 is rather surprising. It essentially states that innovation subsidies, on the margin, are harmful if the intensity of innovative effort is low to begin with. Only when the intensity of innovative effort is relatively high do innovation subsidies raise welfare. There is an intuitive explanation for this conclusion, however. When the economy-wide labor force is relatively small, both steady state intensities of innovative and imitative investment (I^* and C^*) are relatively low, in particular, $I^* < \hat{I}$. Under these circumstances, firms that are successful in innovating can expect to earn dominant firm profits for a long period of time, and there is a market bias towards over-investment in innovative R&D. On the other hand, when the economy-wide labor force is relatively large, both steady state intensities of innovative and imitative investment (I^* and C^*) are relatively high, in particular, $I^* > \hat{I}$. Under these circumstances, firms that are successful in innovating can expect to earn dominant firm profits for a short period of time. Since the benefits to society from an innovation last forever, there is a market bias towards under-investment in innovative R&D. Because innovative R&D subsidies increase the resources used by the R&D sector, when there is a market bias toward overinvestment, a marginal increase in the innovation subsidy lowers welfare, and when there is a market bias toward under-investment in innovative R&D, a marginal increase in the innovation subsidy raises welfare.

What happens when the government subsidizes imitative R&D? Let s_C denote the fraction of imitative R&D costs borne by the government. Then I must multiply the right-hand side of (21) by $1-s_C$. This changes both the zero profit in R&D constraint and the labor market

constraint. The labor market constraint (22) becomes

(36)
$$\frac{2a_{c}(\rho+I)(1-s_{c})}{\lambda-1} + \frac{IC(a_{I}+a_{c})}{I+C} = L,$$

and the zero profit in R&D constraint can be rewritten as

(37)
$$(\rho + I)(1 - s_C) = \frac{a_I \rho + C(a_I - a_C + s_C a_C)}{2a_C}.$$

It is easily verified that a marginal increase in s_c shifts both the "zero profit in R&D constraint" $C_I(I)$ and the "labor market constraint" $C_L(I)$ to the right. Thus, a marginal increase in s_c unambiguously increases the steady state intensity of innovative R&D I^* . Suppose that C^* does not decrease. Then g^* increases; from (36), $(p + I^*)(1 - s_c)$ decreases; and from (37), $(p + I^*)(1 - s_c)$ increases. Thus, to avoid a contradiction, I must conclude that

Proposition 3: An increase in the government subsidy to imitative R&D (s_c) decreases the steady state intensity of imitative R&D (C^*) and increases the steady state intensity of innovative R&D (I^*) .

An increase in s_C has theoretically ambiguous effects on both steady state economic growth and welfare. Indeed, numerical calculations reveal that the effects can go either way. When $a_I = 1$, $a_C = .7$, $\rho = 1$, $\lambda = 4$, $\ell = .3$ and L = .5, an increase in s_C from 0 to .01 raises growth but lowers welfare. When $a_I = 1$, $a_C = .3$, $\rho = 1$, $\lambda = 4$, $\ell = .3$ and L = .34, an increase in s_C from 0 to .01 lowers growth but raises welfare. And it is not always the case that growth and welfare move in opposite directions. When $a_I = 1$, $a_C = .3$, $\rho = 1$, $\lambda = 4$, $\ell = .3$ and L = .9, an increase in s_C from 0 to .01 raises both growth and welfare.

It is interesting to contrast the results concerning innovation and imitation subsidies with those derived by Grossman and Helpman (1989b) in their North-South trade model. When followers are relatively efficient at engaging in innovative R&D compared to leaders in the North

(this paper assumes that followers are equally efficient), Grossman and Helpman proved that innovation subsidies increase steady state economic growth but reduce the aggregate rate of imitation in the South. They also proved that, with relatively efficient followers, imitation subsidies decrease steady state economic growth but increase the aggregate rate of imitation in the South. Thus they found an inverse relationship between policies that support learning in one R&D sector and the equilibrium rate of learning in the other. In this paper, innovation subsidies also unambiguously increase economic growth (Proposition 1), but the effect of imitation subsidies on economic growth was shown to be ambiguous. Furthermore, I found a *direct* relationship between policies that support learning in one R&D sector and the equilibrium rate of learning in the other. The rate at which "α industries" become "β industries" (the aggregate rate of imitation) is exactly balanced by the rate at which "β industries" become "α industries" (the aggregate rate of economic growth) in the steady state equilibrium. Thus policies that support learning in one R&D sector automatically support learning in the other R&D sector.

What are the steady state effects of a once-and-for-all increase in the unit labor requirement of innovative R&D a_I ? From (28), the $C_I(I)$ function unambiguously shifts down and from (23), the $C_L(I)$ function also unambiguously shifts down. These curve shifts are illustrated in Figure 4. The movement from point A to point B in Figure 4 shows that

Proposition 4: An increase in the unit labor requirement of innovative R&D (a_1) decreases the steady state intensity of imitative R&D (C^*) and decreases steady state economic growth (g^*) .

The effect of an increase in a_I on steady state innovative R&D I^* is unclear since it depends on the magnitudes of the two downward shifts. The intuition behind this ambiguous effect can be understood by breaking up the movement from point A to point B into two parts: (i) the movement from A to C and (ii) the movement from C to B (in Figure 4). When a_I increases, this increases the costs of engaging in innovative R&D. Since firms earn zero discounted profits from engaging in innovative R&D must go up also. Thus firms that are successful in innovating must earn dominant firm profits for a longer

period of time, i.e., a slower rate of imitation (the $C_I(I)$ curve shifts down). But given the labor market constraint, lower C^* corresponds to higher I^* . This explains why an increase in a_I can lead to a higher innovation rate I^* (and is captured by the movement from A to C along the $C_L(I)$ curve). At the same time, when a_I increases, innovative R&D becomes more labor intensive with no change in the labor intensiveness of imitative R&D. With a fixed economywide labor force L, the intensities of both innovative and imitative R&D activities that the labor force can sustain decline. This explains why an increase in a_I can lead to a decrease in innovative R&D I^* (and is captured by the movement from C to B along the $C_I(I)$ curve). Thus when a_I increases, there are two effects on I^* going in opposite directions: (i) a substitution effect from C^* into I^* since the benefits of innovative activities must rise to balance the increased costs and (ii) a labor market effect which reduces both I^* and C^* due to the increased labor cost of R&D and the fixed economy-wide labor endowment. No general statements can be made about which effect dominates. Indeed, numerical calculations reveal that when $a_C = .7$, $\rho = 1$, $\lambda = 4$, L = .5 and $\ell = .3$, an increase in a_I from 1 to 1.1 lowers I^* , whereas when L = 2.8, the same increase in a_I raises I^* .

However, by breaking up the steady state effect of an increase a_1 into a substitution effect and a labor market effect, it is clear why both steady state imitative R&D C^* and economic growth g^* decline. When a_1 increases, to maintain zero profits in innovative R&D, the reward for winning an innovative R&D race must rise. Less imitative R&D C^* makes innovating more attractive since successful innovators earn dominant firm profits for a longer expected period of time. Thus, an increase in a_1 causes a substitution effect which reduces imitative R&D. The substitution effect from C^* into I^* also reduces economic growth since lower imitative R&D implies that firms are engaging in innovative R&D in a smaller proportion of industries. At the same time, an increase in a_1 causes a labor market effect which also reduces both imitative R&D and economic growth. Given the fixed economy-wide labor endowment L, an increase in the resource cost of innovative R&D means that the economy is able to support less innovative and imitative R&D. With both kinds of R&D reduced, obviously economic growth suffers as well. Thus both substitution and labor market effects of an increase in a_1 work together to reduce both C^* and g^* .

A once-and-for-all increase in the unit labor requirement of imitative R&D a_c unambiguously shifts the $C_L(I)$ function down and the $C_I(I)$ function up. It follows imediately that

Proposition 5: An increase in the unit labor requirement of imitative R&D (a_c) decreases the steady state intensity of innovative R&D (I^*) .

An increase in a_C has unclear effects on both steady state imitative R&D C^* and economic growth g^* . It seems strange that firms could respond to an increase in the unit labor requirement of imitative R&D by doing more on the imitative front. But one must remember that firms earn zero discounted profits in imitative R&D, and when the costs go up, the benefits must go up also. The benefits of imitating go up if successful imitators earn collusive profits for a longer period of time, i.e., if innovative R&D I^* decreases. And with less innovative R&D I^* and a fixed endowment of labor L, more imitative R&D C^* is sustainable. Numerical calculations reveal that the effect of an increase in a_C on C^* can go either way. When $a_I = 1$, $\rho = 1$, $\lambda = 4$, L = .5 and $\ell = .3$, an increase in a_C from .6 to .7 raises C^* , whereas when $a_I = 1$, $\rho = 1$, $\lambda = 4$, $\lambda = .34$ and λ

By breaking up the steady state effect of an increase in a_C into a substitution effect and a labor market effect, it is clear why steady state innovative R&D I^* unambiguously declines. When a_C increases, to maintain zero profits in imitative R&D, the reward for winning an imitative R&D race must rise. Less innovative R&D I^* makes imitating more attractive since successful imitators earn collusive profits for a longer expected period of time. Thus, an increase in a_I causes a substitution effect from I^* into C^* which reduces innovative R&D. At the same time, an increase in a_C causes a labor market effect. Given the fixed economy-wide labor endowment L, an increase in the unit labor requirement of imitative R&D means that the economy is able to sustain less of both innovative and imitative R&D. Thus both substitution and labor market effects of an increase in a_C work together to reduce I^* .

A once and for all increase in the representative consumer's subjective discount rate p

(the future becomes more heavily discounted or less important) unambiguously shifts the $C_L(I)$ function down, shifts the $C_I(I)$ function down if $a_I > 2a_C$, and shifts the $C_I(I)$ function up if $2a_C > a_I$. If $a_I > 2a_C$, then an increase in ρ unambiguously decreases steady state economic growth g^* and imitative R&D C^* , but has an unclear effect on steady state innovative R&D I^* . If $2a_C > a_I$, then an increase in ρ unambiguously decreases steady state innovative R&D I^* , but has a unclear effects on both steady state economic growth g^* and imitative R&D C^* .

A once and for all increase in the world endowment of labor L has no effect on the "zero profit in R&D constraint" $C_L(I)$ but unambiguously shifts up the "labor market constraint" $C_I(I)$. An increase in L unambiguously increases steady state innovative R&D I^* , imitative R&D C^* and economic growth g^* . This anti-Malthusian conclusion, that population growth spurs on per capita economic growth, is shared with other recent models of economic growth, including Grossman and Helpman (1989a) and Romer (1988) 17 . The intuition behind this property of the model is easy to explain. A larger world population means not only that there are more workers but also that there are more consumers and increased demand for the new products that firms discover. Since firms have more to gain from both innovative and imitative R&D, it should not be surprising that a larger resource base generates faster economic growth.

A once and for all increase in the "significance of innovations" parameter λ (the extent to which new products improve upon old products) has no effect on the "zero profit in R&D constraint" $C_I(I)$ but unambiguously shifts up the "labor market constraint" $C_L(I)$. Thus, an increase in λ unambiguously increases steady state innovative R&D I^* , imitative R&D C^* and economic growth g^* . Increases in the economy-wide labor endowment L and increases in the "significance of innovations" parameter λ have the same qualitative effects because they both generate higher dominant firm profits.

I now turn to analyzing an explicit public policy intervention: a one time lump sum entry tax T payed by a successful imitator to the quality leader that it imitated. I want to explore

¹⁷ Strictly speaking, labor in this model corresponds to human capital in Romer's multi-factor model since Romer assumes that labor is not an input in the R&D process. Although Romer's growth model and the model in this paper are very different, they do share some similar properties. In Romer's model, new designs can be used by any firm to create still newer designs. In this paper, all firms can engage in innovative R&D and no firm is handicapped in these races by not being a quality leader.

whether such a tax promotes economic growth by enhancing the incentives firms have to innovate. The presence of this entry tax T changes the incentives firms have to engage in imitative R&D. (21) becomes

$$(38) \qquad \frac{\pi^c}{\rho + I} - T = a_c.$$

Substituting (38) and (11) into (17) yields a new labor market constraint which is slightly different from (22):

(39)
$$\frac{2(a_C + T)(\rho + I)}{\lambda - 1} + \frac{IC(a_C + a_I)}{I + C} = L.$$

By changing the reward to being imitated, this entry tax also changes the incentives firms have to engage in innovative R&D. (27) becomes

(40)
$$\frac{\pi^{L} + C\left(T + \frac{\pi^{C}}{\rho + I}\right)}{\rho + C} = a_{I}.$$

Substituting (38), (10) and (11) into (40) yields a new zero profit in R&D constraint which is slightly different from (28):

(41)
$$C_{I}(I) = \frac{2(a_{C} + T)I + \rho(2T + 2a_{C} - a_{I})}{a_{I} - a_{C} - 2T}.$$

I am now in a position to analyze the steady state equilibrium effects of an increase in the entry tax T. Increasing T shifts the $C_l(I)$ function unambiguously up and shifts the $C_l(I)$ function (defined by (39)) unambiguously down. It follows that an increase in T unambiguously reduces steady state innovative R&D I^* but has ambiguous effects on both steady state imitative R&D C^* and economic growth g^* . Thus it is unclear whether such an entry tax promotes economic growth.

But what happens when the government combines this entry tax with a policy which makes it easier for firms to imitate new technological developments? The government can reduce the unit labor requirement of imitative R&D a_c by, for example, weaker enforcement of the patent laws. Then firms do not have to spend so much time and resources inventing their way around patents. But after they succeed in imitating a quality leader, I will suppose that they have to pay an entry tax to the quality leader (to compensate this firm for the damages caused by being imitated). I will look at the case where $\Delta T = -\Delta a_c$, that is, where any increase in the entry tax T is offset one for one with a decrease in the resource cost of imitative R&D a_c . Under these circumstances, an increase in T shifts up both the zero profit in R&D constraint $C_l(I)$ and the labor market constraint $C_l(I)$ (defined by (39)) unambiguously up. It follows that the " $\Delta T = +1$, $\Delta a_c = -1$ " policy unambiguously increases steady state economic growth g^* .

This unambiguous growth effect stands in sharp contrast to the ambiguous effect of a_1 , a_C , and T increases on steady state economic growth. The intuition behind this result is somewhat involved. From (38), it is clear that the " $\Delta T = +1$, $\Delta a_C = -1$ " policy has no direct effect on the incentives firms have to imitate. The " $\Delta T = +1$, $\Delta a_C = -1$ " policy essentially drives the resource costs of imitative activities down and the monetary costs of imitative activities up. This means that for the same amount of imitative activity, the " $\Delta T = +1$, $\Delta a_C = -1$ " policy frees up some labor in the world economy that had been employed in the imitative R&D sector to do other things, and this freed up labor represents one reason why the " $\Delta T = +1$, $\Delta a_C = -1$ " policy generates higher economic growth. The " $\Delta T = +1$, $\Delta a_C = -1$ " policy also makes innovative R&D more attractive for firms since successful innovators do not lose as much when imitation occurs. But since firms earn zero profits in innovative R&D, the " $\Delta T = +1$, $\Delta a_C = -1$ " policy must also generate higher imitative R&D C^* to counterbalance this benefit. And the higher C^* represents the other reason why the " $\Delta T = +1$, $\Delta a_C = -1$ " policy generates higher economic growth. The higher C^* essentially means that there are more industries in the economy where firms are engaging in innovative activities.

5. Concluding Remarks

In this paper, I have developed a dynamic general equilibrium model of economic growth. Firms devote resources to discovering new superior products and new products are periodically discovered. Other firms devote resources to imitating new superior products and successful innovators cannot count on earning dominant firm profits forever. Because of product innovation, individual products eventually become obsolete. Discovering new products is costly, takes time and involves uncertainty. Copying the state-of-the-art quality products of other firms is also costly, takes time and involves uncertainty. Firms can expect to run into technical difficulties any time they try to do something that they have never done before. The ease with which firms can imitate new products adversely affects the incentives firms have to innovate. All these features of technological change have been incorporated into the model.

The model is also technically quite manageable. Steady state equilibrium behavior is summarized by the intersection of a downward sloping "labor market constraint" and an upward sloping "zero profit in R&D constraint" in Figure 3. Yet the model is rich in its predictions. For example, I showed that when it becomes more costly to innovate, firms respond by doing unambiguously less imitative R&D (in industries where they engage in imitative R&D). I believe that this model can be fruitfully extended to study a variety of issues in international trade. Can national governments play a positive role in stimulating economic growth by taxing or subsidizing their R&D sectors? How do relative factor endowments affect the incentives of countries to specialize in either innovative or imitative activities? Can countries effectively use tariffs, quotas and other commercial policy instruments to bolster their R&D sector? Much research remains to be done on these issues.

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Appendix

I will begin by examining whether firms producing different quality products in an industry can effectively collude. The most favorable scenerio for collusion occurs when the industry in question has only one firm producing the state-of-the-art quality product (firm 0) and only one firm producing the next lower quality product (firm 1). Let (p_0, p_1, d_0, d_1) define a collusive agreement between these two firms, where p_i is the price firm i agrees to charge, and d_i is the quantity that firm i agrees to sell at price p_i $(i=0,1,)^{18}$. $p_i > 1$ and $d_i > 0$ for all i, and $p_0 = \lambda p_1$ must be satisfied, because otherwise one of the firms would have nothing to gain by colluding. Furthermore, the quantities produced must exactly satisfy demand, that is, $E/p_1 = d_1 + \lambda d_0$. Let $x = d_1/(d_1 + \lambda d_0)$. The collusive profits of firm 0,

(A1)
$$\pi_0^{Coll} = (p_0 - 1)d_0 = E\left(1 - \frac{1}{\lambda p_1}\right)(1 - x),$$

must be greater than the profits π^L (given by (10)) that firm 0 could earn by not colluding. It follows that $x < 1/\lambda$. The collusive profits of firm 1 satisfy

$$(A2) \pi_1^{Coil} = \left(1 - \frac{1}{p_1}\right) xE.$$

By cheating on this collusive agreement and infinitesimally undercutting the other firm, firm 1 can earn cheating profits of

(A3)
$$\pi_1^{Cheat} = \left(1 - \frac{1}{p_1}\right)E.$$

¹⁸ I will temporarily relax the assumption that all indifferent consumers buy from the highest quality firm to allow for the whole range of possible rationing rules.

Since cheating is detected with a time lag ℓ , this collusive agreement can only be supported by subgame perfect equilibrium strategies if

(A4)
$$\int_0^\infty \pi_1^{Coil} e^{-\rho t} dt \ge \int_0^t \pi_1^{Cheat} e^{-\rho t} dt.$$

Since $\pi_1^{Coll} = \pi_1^{Cheat} x < \pi_1^{Cheat} / \lambda$, from (A4), the following assumption

A3
$$\lambda > \max\left\{\frac{1}{(1-e^{-pt})}, 3, \frac{a_c}{a_i - a_c}\right\},$$

guarantees that firms producing different quality products is an industry cannot effectively collude in the steady state equilibrium. I will assume that the innovations under consideration are of a sufficiently large magnitude so that A3 is satisfied. Thus this paper focuses in on the special case of "big innovations".

To justify (11), I must show that two firms producing the state-of-the-art quality product can collude, that is, that (14) is satisfied. It is helpful to consider the following inequalities,

(A5)
$$\frac{-\ln\left(1-\frac{1}{\lambda}\right)}{\rho} < \ell < \frac{-\ln(1/2)}{\rho + I^*},$$

where the first inequality follows from A3 and the second inequality is (14) rearranged. Trivially, (A5) can only hold if

(A6)
$$\frac{-\ln\left(1-\frac{1}{\lambda}\right)}{\rho} < \frac{-\ln(1/2)}{\rho + I^*}.$$

To establish (A5), I will consider each of the two cases: (i) $2a_c \ge a_1$, and (ii) $a_1 > 2a_c$, in turn. Suppose that $2a_c \ge a_1$. Since A3 implies that $\lambda > 3$, clearly there exists a range of ℓ

values satisfying (A5) if the steady state level of innovative R&D I^* is not too large. But as L converges to $2a_c\rho/(\lambda-1)$ from above, I^* converges to 0. Thus there clearly exists a range of L values (in a corresponding range of ℓ values) satisfying (A5).

Now suppose that $a_1 > 2a_C$. For both C^* and I^* to be positive, $I^* > I_3$ must be satisfied. Thus a necessary condition for (A6) to hold is that

A4
$$a_{I} < 2a_{C} \frac{\ln(1/2)}{\ln(1-\frac{1}{\lambda})} \text{ if } a_{I} > 2a_{C}.$$

I will assume that assumption A4 holds. Given A4, clearly there exists a range of ℓ values satisfying (A5) if the steady state level of innovative R&D I^* is not too large. Otherwise stated, two quality leaders can effectively collude if the threat of further innovation is not too large. But as L converges to $a_I p / (\lambda - 1)$ from above, I^* converges to I_3 . Thus there clearly exists a range of L values (in a corresponding range of ℓ values) satisfying (A5).

I will assume that the labor endowment is in this range, that is,

A5 L is sufficiently small so that (A6) holds, and ℓ satisfies (A5).

A2 and A5 together state that the economy-wide labor endowment is large enough so that both innovative and imitative investment activities are profitable, but not so large that two quality leaders cannot collude (because of a very rapid rate of innovation generated by a very large labor force).

In the steady state equilibrium, an outside fringe of competitive firms 19 engages in I^* units of innovative R&D in each industry with 2 (or more) leaders and engages in C^* units of imitative R&D in each industry with a single quality leader. These firms earn expected discounted equilibrium profits of zero from engaging in R&D. Given (21), (20) implies that no fringe firm has anything to gain by doing more (or less) imitative R&D in any " α industry" and

¹⁹ Neither the quality leader(s) or any follower one step down in the quality ladder engages in any R&D in the steady state equilibrium.

given (27), (26) implies that no fringe firm has anything to gain by doing more (or less) innovative R&D in any " β industry".

Do any other firms have anything to gain by engaging in innovative or imitative R&D? I begin by exploring whether a quality leader in an " α industry" has anything to gain by doing innovative R&D. If the quality leader does innovate, it is in a position to earn profits $\pi = (1 - (1/\lambda^2))E$ for some period of time since it is now price constrained by two followers two quality levels down. Since a competitive fringe of firms is going to be trying to imitate it, at best, its profits are threatened by a rate C^* of imitative R&D. However, firms have more to gain by imitating it than any leader in an " α industry" since successful imitators can now earn the profit flow $\pi = (1 - (1/\lambda^2))E/2 > (1 - (1/\lambda))E/2$. Thus the competitive fringe of firms have an incentive to choose $C^* \cong +\infty$ in this industry and the innovator earns the profits $\pi = (1 - (1/\lambda^2))E$ for a negligable period of time before imitation occurs. After imitation occurs by some firm in the competitive fringe, the two quality leaders at best earn the collusive profits $\pi = (1 - (1/\lambda^2))E/2$ until the innovative R&D I^* eliminates these profits (these two firms cannot benefit from further imitation since A3 implies that $\pi = (1 - (1/\lambda^2))E/2 > E/3$). Thus the relevant (Deviant) payoff for an industry leader in an " α industry" engaging in innovative R&D I is

(A7)
$$v_D = \frac{-a_I I + \pi^L + C * a_C + I \left(\frac{(1 - (1/\lambda^2))E/2}{\rho + I *} \right)}{\rho + C * + I}.$$

Substituting into (A7) using (21) and (27), and then differentiating with respect to I yields

(A8)
$$\frac{dv_{D}}{dI} = \frac{(\rho + C^{*}) \left(a_{C} \frac{\lambda + 1}{\lambda} - 2a_{I} \right)}{(\rho + C^{*} + I)^{2}} < 0.$$

The inequality follows from A3. Thus a quality leader in an " α industry" has no incentive to engage in innovative R&D.

Does a quality leader in a " β industry" have anything to gain by doing innovative R&D? A competitive fringe of firms engage in I* units of innovative R&D in each " β industry" and earn zero expected discounted profits from doing so. Thus a quality leader can only gain if the reward for successfully innovating is greater than for these firms in the competitive fringe. But the profit flow for this quality leader after further innovation is given by (10) and since it is price constrained by only one follower firm, this follower (one quality level down) has more to gain by imitating $[\pi = (1 - (1/\lambda^2))E/2]$ than any other firm. Thus the follower has an incentive to devote arbitrarily large resources to imitative R&D and the leader's profits (10) are earned for a negligible period of time before imitation occurs. After this imitation occurs, the quality leader earns profits $\pi = (1 - (1/\lambda^2))E/2$ and the best that it can hope for is to earn these collusive profits until a competitive fringe firm innovates again $[\pi = (1 - (1/\lambda^2))E/2 > E/3$ for $\lambda > 2$]. Thus the relevant payoff for an industry leader in an " β industry" engaging in innovative R&D I is

(A9)
$$v_D = \frac{\left(1 - \frac{1}{\lambda}\right)\frac{E}{2} - a_I I + I\left(\frac{\left(1 - \left(1/\lambda^2\right)\right)E/2}{\rho + I^*}\right)}{\rho + I^* + I}.$$

Substituting into (A9) using (11) and (21), and then differentiating with respect to I yields

(A10)
$$\frac{dv_D}{dI} = \frac{(\rho + I^*) \left(\frac{a_C}{\lambda} - a_I\right)}{(\rho + I^* + I)^2} < 0.$$

The inequality follows from A1 and A3. Thus a quality leader in an " β industry" has no incentive to engage in innovative R&D.

Does a firm one quality level down in an " α industry" have anything to gain by engaging in innovative R&D? If if succeeds, it earns profits (10) until it is imitated. But the previous quality leader, the firm that if now one quality level down, has more to gain by imitating than any other firm $[\pi = (1 - (1/\lambda^2))E/2 > (1 - (1/\lambda))E/2]$ and thus it has an incentive

to devote arbitrarily large resources to imitative R&D.²⁰ As a result, the quality leader can expect to earn the profits (10) for a negligible period of time before imitation occurs. After imitation occurs, it earns the profits $\pi = (1 - (1/\lambda^2))E/2$ until, in the most favorable scenario, a competitive fringe firm innovates. Thus the relevant payoff for an industry follower in an " α industry" engaging in innovative R&D I is

(A11)
$$v_p = \frac{-a_I I + I \left(\frac{(1 - (1/\lambda^2))E/2}{\rho + I^*} \right)}{\rho + C^* + I} = \frac{I \left(a_C \frac{\lambda + 1}{\lambda} - a_I \right)}{\rho + C^* + I} < 0 \text{ for } I > 0,$$

where the second equality follows from (21) and the inequality follows from A3. Thus a quality follower in an " α industry" has no incentive to engage in innovative R&D.²¹

Does a firm one quality level down in a " β industry" have anything to gain by engaging in innovative R&D? If it succeeds, it earns the profits (10) until a competitive fringe firm imitates it and then it earns the profits (11) until a competitive fringe firm innovates. Thus the relevant payoff for an industry follower in an " β industry" engaging in innovative R&D I is

(A12)
$$v_D = \frac{-a_I I + I \left(\frac{\pi^L + C * a_C}{\rho + C *} \right)}{\rho + I * + I} = 0,$$

where the last equality follows from (27). Thus, with the competitive fringe of firms engaging in I* units of innovative R&D, a firm one quality level down has nothing to gain by also doing innovative R&D.

Does a firm one quality level down in an "α industry" have anything to gain by engaging in imitative R&D? A competitive fringe of firms is already engaging in C* units of

²⁰ The same condition (14) that guarantees that firms one step above followers in the quality ladder can collude also guarantees that firms two steps above followers can collude.

²¹ Essentially, follower firms in industries with a single quality leader do not try to innovate because, if successful, they are vulnerable to quick imitation by the previous quality leader.

imitative R&D, in expectation that if they are successful, they will earn collusive profits (11) until further innovation occurs. Since these competitive fringe firms earn zero expected discounted profits from engaging in imitative R&D, a follower firm can only possibly gain by engaging in imitative R&D if it expects to earn higher profits upon successfully imitating, and this can only possibly happen if its product is copied by the sole firm one quality level down. Then each quality leader would earn (at most) profits E/3 until a competitive fringe firm innovates. But A3 implies that $E/3 < \pi^{C}$. Thus imitation does not help the firm being imitated and followers have no incentive to engage in imitative R&D in " α industries".

Does a firm one quality level down in a " β industry" have anything to gain by engaging in imitative R&D? A competitive fringe of firms is already engaged in I* units of innovative R&D in such an industry. Thus, by successfully imitating, this follower firm will become one of three leaders and (at best) earn collusive profits $\pi = (1 - (1/\lambda))E/3$ until either further innovation or further imitation occurs. If further innovation occurs, it wipes out the imitator's profits and if further imitation occurs, at best, these collusive profits become $\pi = E/4$. In the first case, the relevant payoff for an industry follower in an " β industry" engaging in imitative R&D C is

(A13)
$$v_D = \frac{-a_C C + C \left(\frac{(1 - (1/\lambda))E/3}{\rho + I^* + C} \right)}{\rho + I^* + C} = \frac{Ca_C (-1/3)}{\rho + I^* + C} < 0 \text{ for } C > 0,$$

and in the second case.

(A14)
$$v_D = \frac{-a_C C + C\left(\frac{E/4}{\rho + I^*}\right)}{\rho + I^* + C} = \frac{a_C C\left(\frac{1}{2}\frac{\lambda}{\lambda - 1} - 1\right)}{\rho + I^* + C} < 0 \text{ for } C > 0.$$

Thus a firm one quality level down in a " β industry" has nothing to gain by engaging in imitative R&D.²²

²² The absence of imitation in industries with two quality leaders rests on the assumption that the cost of imitation for a third firm is just as high as it is for the second. It may be fruitful to relax this assumption in future research.

What happens if a competitive fringe firm engages in innovative R&D in an " α industry"? If it succeeds in innovating, it earns profits (10) until imitation occurs. The previous quality leader has the most to gain by imitating since $\pi = (1 - (1/\lambda^2))E/2 > (1 - (1/\lambda))E/2$ and since this firm has an incentive to devote arbitrarily large resources to imitative R&D, imitation can be expected to take negligible time. Since A3 guarantees that $\pi = (1 - (1/\lambda^2))E/2 > E/3$, the best scenario for the innovator is that it earns the collusive profits $\pi = (1 - (1/\lambda^2))E/2$ until further innovation occurs. Then the relevant payoff for an competitive fringe firm engaging in innovative R&D I in an " α industry" is given by (A11) and I can conclude that a competitive fringe firm does not have any incentive to engage in innovative R&D in an " α industry".

Finally, what happens when a competitive fringe firm engages in imitative R&D in a " β industry"? If it succeeds in imitating, it earns at best collusive profits $\pi = (1 - (1/\lambda))E/3$ until further innovation occurs since A3 guarantees that $\pi = (1 - (1/\lambda))E/3 > E/5$. Thus the relevant payoff for a competitive fringe firm in a " β industry" engaging in imitative R&D C is given by (A13) and this competitive fringe firm does not have any incentive to engage in imitative R&D in a " β industry".

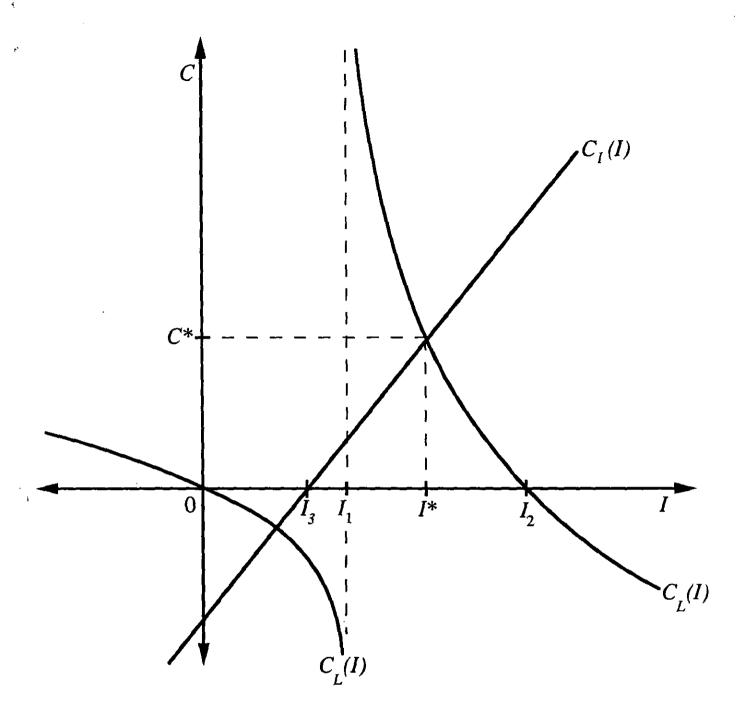


Figure 2

