

Current Draft
July 22, 1992
Comments Welcome

POLITICAL PARTY NEGOTIATIONS, INCOME DISTRIBUTION, AND ENDOGENOUS GROWTH*

Roberto CHANG

New York University, New York, NY 10003, USA

Abstract

This paper examines the determination of the rate of growth in an economy in which two political parties, each representing a different social class, negotiate the magnitude and allocation of taxes. Taxes may increase growth if they finance public services, but reduce growth when used to redistribute income between classes. The different social classes have different preferences about growth and redistribution. The resulting conflict is resolved through the tax negotiations between the political parties. I use the model to obtain empirical predictions and policy lessons about the relationship between economic growth and income inequality. In particular, I show that, although differences in growth rates across countries may be negatively related to income inequality, redistributing wealth does not enhance growth.

Running Title: Political Party Negotiations, Income Distribution, and
Endogenous Growth

* This paper originated in conversations with Jess Benhabib and Rao Aiyagari. I also thank Charles De Bartolome for suggestions. Of course, all the errors and shortcomings are mine. This research was supported by NSF Grant SES-9122454 and by the C.V. Center for Applied Economics at NYU.

Send Correspondence to:
Professor Roberto Chang
Department of Economics
New York University
269 Mercer Street, 7th Floor
New York, NY 10003
(212) 998 8958

1. Introduction

This paper examines the determination of the rate of growth in an economy in which two political parties, each representing a different social class, negotiate the magnitude and allocation of taxes. Taxes may increase growth if they finance public services, but reduce growth when used to redistribute income between classes. The different social classes have different preferences about growth and redistribution. The resulting conflict is resolved through the tax negotiations between the political parties. I use the model to obtain empirical predictions and policy lessons about the relationship between economic growth and income inequality.

A main implication of the analysis is that, in equilibrium, differences in growth rates across countries may be negatively related to measures of income inequality. This is consistent with the recent empirical findings of Persson and Tabellini (1991) and Alesina and Rodrik (1991). However, in the model studied below redistributing wealth would not enhance growth. This surprising result is possible because growth and income inequality are endogenously determined by the outcome of tax negotiations, which in turn does not depend on the initial allocation of assets.

In addition to the above policy message, this paper presents a bargaining approach that may be of interest for students of the positive theory of economic policy. I assume that the political parties negotiate taxes by playing a bargaining game whose structure is similar to that of Rubinstein (1982). However, my model differs from Rubinstein's by allowing tax negotiations and private investment to occur simultaneously. This feature of the model implies a complex but realistic interplay between the behavior of the private sector and the tax negotiations. I show how to identify the bargaining outcomes with the concept of sustainable bargaining

equilibrium (SBE) ¹.

The bargaining approach of this paper contrasts with recent voting models of the determination of public policy. In particular, Persson and Tabellini (1991) and Alesina and Rodrik (1991) have attempted to explain the negative correlation between income inequality and growth as the politico-economic equilibrium of an economy in which people vote for taxes. My analysis complements theirs in several dimensions. One is realism: it is often the case that taxes are not the direct result of a popular vote but of a negotiation between representatives of different groups ². The second dimension is the scope of the theory. Although for concreteness I will talk about negotiations between political parties, it should become clear that the model in this paper is applicable to any economy in which government decisions emerge from the consensus between two players that represent different constituencies. Thus the model yields lessons for some kinds of dictatorships, as well as bipartisan democracies.

Thirdly, while the data shows a connection between income distribution and growth, the voting model of Alesina and Rodrik implies a relation between wealth distribution and growth, and the Persson and Tabellini model implies a connection between the distribution of "basic skills" and growth. In contrast, I establish a connection between growth and an income distribution variable that is the natural counterpart of the variables employed in empirical studies. Finally, while they argue that their empirical evidence implies that

¹The SBE concept is the natural extension, to bargaining problems, of the concept of "sustainable plans" developed by Barro and Gordon (1983), Chari and Kehoe (1990), and Stokey (1991). In Chang (1991a,b) I used the SBE concept to study negotiations about a monetary union and sovereign debt, respectively.

²It may be argued that, although people do not vote for taxes, they vote for the representatives who decide on taxes. Often, however, the elected representatives must respond to different constituencies and do not share a common view about taxes. This conflict is resolved through bargaining.

"inequality hurts growth", my analysis lends no support to such conclusion.

My model is driven by the assumption that the political parties have some power to appropriate resources for their respective constituencies. In adopting this view I follow Lancaster (1973), Benhabib and Rustichini (1991) and Tornell and Velasco (1991). But while they model each social class as a single, strategic player, I assume that the private sector is atomistic and behaves competitively. In addition, the mechanisms by which a social class can appropriate resources from the other are different. In my model, each group's appropriating power emanates from the assumptions that a tax agreement requires approval by both parties and that delaying an agreement implies an inefficient status quo situation. In theirs, one of the social groups has the right to directly expropriate resources from others, a power that is limited only to the extent that other groups can choose outside options.

The paper proceeds as follows. Section 2 presents the world in which tax negotiations take place. Section 3 describes the negotiation process and defines the SBE concept. Section 4 provides sufficient conditions for the existence of a stationary SBE and characterizes it. Section 5 examines the implications of the model for empirical issues and policy analysis. Section 6 concludes. Some technical proofs are delayed to an Appendix.

2. The Model

This section describes the economy under consideration. In the model below, based on Barro (1990), the government provides public services which affect production and the rate of growth of the economy. Public services must be financed with income taxes, which deter investment. Our point of departure from Barro's model will be to assume that some or all of the tax revenues can

be transferred to a class of agents (called "workers" below), who use these transfers to increase their own consumption. It turns out that workers and capitalists have partly conflicting interests about taxes and the allocation of tax revenues. How such conflict is resolved depends on the political structure, and is the subject of later sections.

Time is discrete and indexed by $t = 0, 1, 2, \dots$. We shall consider a closed economy populated by two types of agents: "capitalists" and "workers". Each class has a large number of identical agents. The ratio of the number of capitalists to the number of workers will be denoted by π .

The representative capitalist owns, at the start of each period t , an amount k_t of a durable good called "capital". In period t she can produce more capital according to the Cobb Douglas production function:

$$y_t = A g_t^\alpha k_t^{1-\alpha} \quad (2.1)$$

where g_t denotes (per capitalist) government provision of public services at t and $A > 0$, $\alpha \in (0, 1)$ are technological parameters. The production function (2.1) incorporates the fact that some government expenditures are important for production. One may think of g_t as infrastructure, police, or fire prevention provided by the government. Under some interpretations, one may want to assume that α is fairly small; we will indeed assume a small α in our numerical exercises later.

I assume that the typical capitalist takes as given the ratio of public services to output, $\theta_t = g_t/y_t$. This formulation is plausible for some public services that are enjoyed by different users in proportion to their respective activities. Alternatively, I could have assumed that the capitalist takes the level of public services, g_t , as given, but this assumption

introduces some complications that are peripheral to our discussion³.

In period t , capitalist must pay a proportional tax τ_t on their current income, and decide how much capital to consume and to leave for the next period. There is no depreciation, and therefore the evolution of capital is given by:

$$k_{t+1} = k_t + (1-\tau_t) y_t - c_t = R_t k_t - c_t \quad (2.2)$$

where c_t denotes the capitalist's consumption and, as implied by (2.1) and the definition of θ_t :

$$R_t = 1 + (1-\tau_t) A^{1/(1-\alpha)} \theta_t^{\alpha/(1-\alpha)} \quad (2.3)$$

R_t is the (gross) after tax rate of return on investment. As shown by (2.3), the rate of return in period t is determined by the parameters θ_t and τ_t . A sequence $\{(\theta_t, \tau_t)\}_{t=0}^{\infty}$ will be called a fiscal policy.

I assume that the capitalist's preferences are described by:

$$\sum_{t=0}^{\infty} \beta^t u(c_t) = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \quad (2.4)$$

where $0 < \beta < 1$ and $\sigma > 0$. The representative capitalist's problem is to maximize (2.4) subject to (2.2) and (2.3), given a fiscal policy and the initial quantity of capital.

The rest of the economy is specified to make our analysis as simple as

³ Assuming that the capitalist takes the level of public services as given introduces an externality effect. See Barro (1990).

possible. In each period, the government transfers the difference between tax revenues and government expenditures to the workers⁴. Workers do not have another source of income and do not save; both assumptions can be relaxed at the expense of heavier calculations. Hence each worker's transfer and consumption is given by:

$$c_t^w = \pi (r_t y_t - g_t) = (1 + A^{1/(1-\alpha)} (\theta_t^{\alpha/(1-\alpha)} - \theta_t^{1/(1-\alpha)}) - R_t) \pi k_t \quad (2.5)$$

where the last equality is easily deduced from (2.1)-(2.3).

Given a fiscal policy, workers' lifetime consumption is defined by (2.3), (2.5), and the evolution of k_t determined by the behavior of capitalists. We will assume, for simplicity, that workers' preferences are linear and given by $\sum_{t=0}^{\infty} \delta^t c_t^w$. Note that, for this sum to converge, δ has to be small enough relative to the rate of growth of consumption. Such restriction will always be satisfied below.

The evolution of this economy will, from the preceding description, depend on fiscal policy. But nothing so far tells us what fiscal policy will prevail. What is clear is that workers and capitalists have partly conflicting interests about fiscal policies. Capitalists benefit from government services, but are hurt by taxes. It is intuitively obvious that their most preferred fiscal policy involves some level of government services and taxes but no transfers to workers. Workers benefit from transfers and therefore would like taxes to be strictly larger than the amount needed to finance government services, but small enough not to cause too large a fall in production and investment.

⁴Hence the budget is balanced in each period.

Later I will assume that this conflict is resolved by a negotiation between two political parties, each representing a social class. The political parties will bargain over tax agreements. If implemented, an agreement specifies a fiscal policy for the rest of time. For simplicity, I will restrict attention to constant agreements, that is, agreements that specify a constant θ and a constant τ . In the rest of this section I will state and discuss some results that we will need for our analysis.

Suppose that a constant fiscal policy (θ, τ) is implemented without delay, starting in period zero. Associated with this policy there is a perfect foresight equilibrium of this economy, whose main features are described by ⁵:

Fact One: Let $\theta_t = \theta$ and $\tau_t = \tau$ be such that $\beta R^{1-\sigma} < 1$ and $\delta(\beta R)^{1/\sigma} < 1$, where $R = 1 + (1-\tau)A^{1/(1-\alpha)}\theta^{\alpha/(1-\alpha)}$ is the equilibrium rate of interest. Also, let $k_0 = k$. Then the discounted utility of the representative capitalist is given by:

$$v(k, \theta, \tau) = (1 - (\beta R^{1-\sigma})^{1/\sigma})^{-\sigma} \frac{(Rk)^{1-\sigma}}{1-\sigma} \quad (2.6)$$

and that of workers is:

$$w(k, \theta, \tau) = \frac{(1 + A^{1/(1-\alpha)}(\theta^{\alpha/(1-\alpha)} - \theta^{1/(1-\alpha)})) - R}{1 - \delta(\beta R)^{1/\sigma}} \pi k \quad (2.7)$$

Moreover, the rate of growth of the economy is given by $k_{t+1}/k_t =$

⁵The proofs of Facts One and Two follow easily from Barro (1990) and are left to the reader.

$(\beta R)^{1/\sigma}$. ■

Some remarks are in order. First, Fact One will be useful in describing the discounted payoffs to workers and capitalists of an immediate tax agreement (θ, τ) when the stock of capital is k . In particular, because this economy is recursive, $v(k, \theta, \tau)$ and $w(k, \theta, \tau)$ are the payoffs to workers and capitalists, from any period t on and discounted to period t , of an agreement to set $\theta_s = \theta$ and $\tau_s = \tau$ for $s \geq t$, if the stock of capital at t is $k_t = k$.

Second, this economy displays unbounded growth provided θ and τ are such that $\beta R > 1$. Thus unbounded growth is possible, although not necessary in this economy.

Third, the existence of a perfect foresight equilibrium requires v and w to be finite. This is the reason of requiring $\beta R^{1-\sigma} < 1$ and $\delta(\beta R)^{1/\sigma} < 1$ in Fact One. Note that the first requirement is satisfied if $R > 1$ and $\sigma > 1$ and the second is satisfied if $\delta(\beta R^*)^{1/\sigma} < 1$, where R^* is defined below.

Fourth, what would the median voter theorem tell us about this economy? Suppose that at the beginning of time there is a vote to pick a constant fiscal policy. Then, clearly, the resulting policy would maximize the utility of the representative capitalist if there are more capitalists than workers, that is, if $\pi > 1$. As in Barro (1990), it is easy to show that the capitalist's most preferred policy is given by $\theta = \tau = \alpha$. The condition $\theta = \alpha$ is a condition of productive efficiency, while $\tau = \alpha$ just says that taxes are just enough to finance government expenditure, and that no resources are transferred to workers. It is also easily checked that such policy maximizes the rate of growth in this economy, and that the associated interest rate is given by $R^* \equiv 1 + A^{1/(1-\alpha)} (\alpha^{1/(1-\alpha)} - \alpha^{1/(1-\alpha)})$.

On the other hand, if $\pi < 1$, the winning policy would maximize the utility of workers, $w(k, \theta, \tau)$. Using Fact One, it can be shown that the

workers' most preferred policy requires the productive efficiency condition $\theta = \alpha$. Then, by (2.7), the workers' most preferred policy requires picking τ such that the interest rate is $R_* = 1 + (1-\tau) A^{1/(1-\alpha)} \alpha^{1/(1-\alpha)}$, where R_* is the solution of ⁶:

$$\text{Max}_{R \in [1, R_*]} \frac{R^* - R}{1 - \delta(\beta R)^{1/\sigma}} \quad (2.8)$$

Since (2.8) implies that $R_* < R^*$, it follows that the policy most preferred by workers would not maximize growth. In fact, $R_* = 1$ for many parameterizations of this model, which implies that workers would be willing to sacrifice economic growth for bigger transfers.

Finally, the preceding discussion implies that the set of constrained⁷ Pareto efficient (constant) fiscal policies is given by all pairs (θ, τ) that satisfy the efficiency condition $\theta = \alpha$ and the condition $R = 1 + (1-\tau) A^{1/(1-\alpha)} \alpha^{1/(1-\alpha)} \in [R_*, R^*]$. The last condition ensures that one cannot make both workers and capitalists better off by altering the rate of growth of the economy.

Fact One summarizes the essential facts of the economy under study, assuming that a tax agreement (θ, τ) is implemented at the beginning of time. What happens if there is no immediate agreement in this economy? For all agents to have an incentive to reach agreement quickly, I will assume that absence of agreement implies a loss of potential output : as long as there is no agreement about taxes, τ and θ are both zero. Therefore, in the absence of a tax agreement, workers do not consume and the economy does not grow

⁶If (2.8) has many solutions, we take the largest one.

⁷These policies are constrained because Lump sum taxes are ruled out.

(because production possibilities are given by $k_{t+1} = k_t - c_t$, where c_t denotes capitalists' consumption).

There is one final aspect to take into account. In the absence of a tax agreement, capitalists must decide how much to consume and invest based, presumably, on their expectations about current and future rates of return. But the future rate of return, and therefore private investment, may depend on when a tax agreement will be reached and what the agreement will be. For instance, suppose that there is no agreement at $t = 0$, but that a constant fiscal policy (θ', τ') will be in place from $t = 1$ on, and that all agents know this with perfect foresight. Then, the consumption-saving plans of capitalists in period zero will depend on (θ', τ') . The relevant aspects of the solution of the capitalists' problem are given by :

Fact Two: Suppose that $\theta_0 = \tau_0 = 0$, and $\theta_t = \theta'$, $\tau_t = \tau$, $t \geq 1$. Define $R' = 1 + (1-\tau')A^{1/(1-\alpha)}(\theta')^{\alpha/(1-\alpha)}$. Then, if $k_0 = k$ and capitalists act optimally with perfect foresight, their optimal consumption and investment in period zero are:

$$c_0 = c(k, \theta', \tau') = (1 - (\beta(R')^{1-\sigma})^{1/\sigma}) k \quad (2.9)$$

$$k_1 = k_{+1}(k, \theta', \tau') = (\beta(R')^{1-\sigma})^{1/\sigma} k \quad (2.10)$$

Fact Two describes how investment in period zero depends, if an agreement has not been reached, on capitalists' expectations about the future rate of return, and hence about the policy that will be implemented from period one on. Fact Two assumes that capitalists have perfect foresight, which will be true in equilibrium.

Summarizing, in this economy different tax agreements imply different rates of growth and different degrees of income redistribution. Capitalists would like to choose taxes so as to maximize growth and zero redistribution. Workers would prefer slower growth but some redistribution. Both sides may benefit from a tax agreement, because there is no production in its absence.

The fiscal policy and, therefore, the growth rate and the degree of income inequality that would be observed in this economy depend on political institutions. We have mentioned how fiscal policy would be determined at the beginning of time by the results of voting. One could also examine what would happen if this economy was ruled by a dictator. Our objective in the next sections will be to analyze the determination of fiscal policy in a bipartisan government.

3. The Bargaining Problem and a Definition of Equilibrium

In the remaining sections I will assume that the government is controlled by two parties called L (representing workers) and C (representing capitalists). These parties will be assumed to exchange offers and counter offers over time about fiscal policy. Implementation of a particular proposal requires the consent of both parties. The main objective of this section is to describe the bargaining mechanism and to discuss how to characterize its outcomes.

The bargaining situation is as follows. Let party L represent workers and party C capitalists. At $t = 0$, with k_0 given, L makes an offer $a_0 = (\theta^0, r^0)$ to C. C may then accept the offer (Y) or reject it (N). If a_0 is accepted, bargaining ends, and a (constant) fiscal policy $\theta_t = \theta^0, r_t = r^0, t \geq 0$, is

immediately implemented.

If C rejects L's offer, bargaining continues at $t = 1$. During the remainder of period zero, however, capitalists decide how much to consume (c_0) or to invest (k_1). This decision determines the amount of capital k_1 at $t = 1$. Then, it is C's turn to make an offer $a_1 = (\theta^1, r^1)$ and L's turn to respond. If L accepts C's offer, bargaining ends, and the constant fiscal policy a_1 is implemented. If L rejects the offer, capitalists decide their consumption (c_1) and investment (k_2). Period two then starts with capital k_2 , L making an offer, and C responding. This process continues until an offer is accepted, or forever. Figure 1 depicts the first two periods of the game.

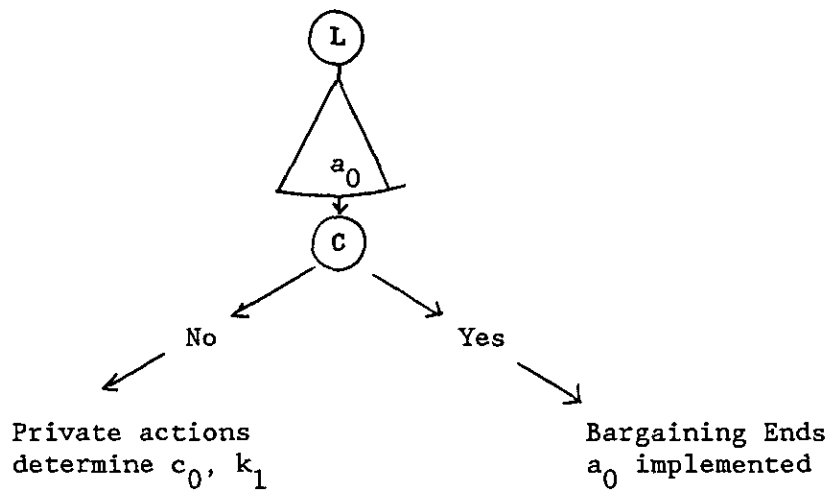
Note that this model makes some concrete assumptions about the political environment. Foremost among them is the need of consensus among the two parties in order to change the tax regime. This assumption is most realistic for countries such as the United States, in which the government is controlled by a small number of parties of similar power. A second assumption is that agreements are final: after an offer is accepted, the corresponding tax agreement is written in stone. However, this aspect of the model is not crucial, as I discuss at the end of Section 4.

The need for consensus grants workers a degree of power that may not be apparent from the economic aspects of the model: the workers' party can veto tax proposals, and cause damage to both sides (because there is no production in the absence of an agreement). Using this veto power, workers may try to appropriate some of the benefits of economic growth. But their power is limited by the fact that their vetoing tax proposals hurts not only capitalists but also themselves.

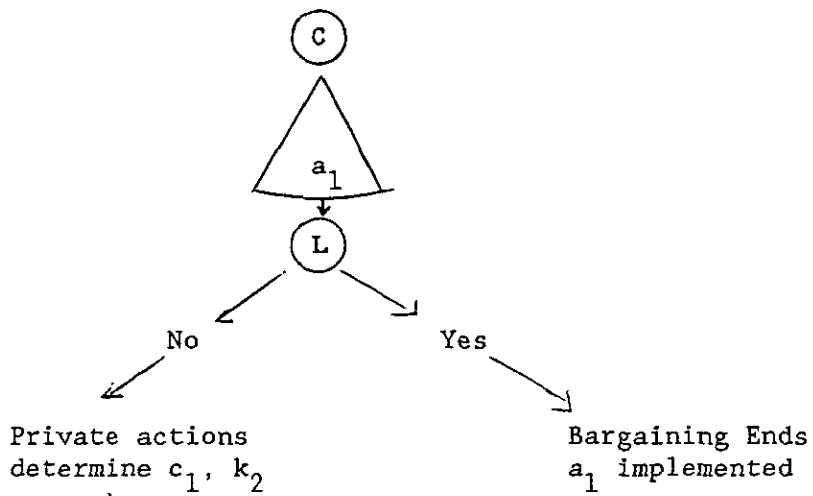
This bargaining model is similar to that postulated by Rubinstein (1982). There is an important difference, however. Rubinstein analyzed a game between two players. In our model, there are two strategic players (the political

FIGURE 1

$t = 0, k_0$ given



$t = 1, k_1$ given



parties) and a large number of competitive agents (workers and capitalists). Capitalists, in particular, have to decide how much capital to save in each period as long as there is no tax agreement. Such decision is based on capitalists' expectations about the future of the tax negotiations, which determine the return on capital. Conversely, private investment changes the bargaining stakes every period from the viewpoint of the political parties. Thus there is an interplay between private investment and tax negotiations that is absent from models of the Rubinstein type. This interplay adds realism to our model, but adds complexity also. In particular, how to characterize the outcomes of our model is not obvious.

In Chang (1991 a,b) I argued that an appropriate equilibrium concept to characterize the solution of this class of bargaining problems is that of sustainable bargaining equilibrium (SBE). In fact, our bargaining problem has the same structure as the problems studied in those two papers, and therefore we can use the tools developed there to solve our present problem. In the remainder of this section I will briefly describe the SBE concept. Readers familiar with the argument of my (1991 a,b) papers may want to go directly to Section 4.

A history at t is a $(t+1)$ -vector of (rejected) offers $h^t = (a_0, \dots, a_t)$. An allocation rule F is a description of capitalists' consumption (c_t) and investment (k_{t+1}) in each period t as a function of history h^t . Intuitively, an allocation rule tells us the behavior of the private sector if no agreement has been reached up to and including period t .

For $j = L, C$, a strategy for player j , denoted σ_j , is a description of what offer to make (if it is j 's turn to offer) and which offers to accept (if j has to respond to an offer) in every period after any history.

A strategy pair $\sigma = (\sigma_C, \sigma_L)$ induces an agreement $\alpha(\sigma)$ at time $\tau(\sigma)$ and, for each $t = 0, 1, \dots, \tau(\sigma)$, a history h^t of rejected offers. Together with the

allocation rule F , a strategy pair induces capitalists' consumptions $(c_t(h^t(\sigma)))$ and investments $(k_{t+1}(h^t(\sigma)))$ in each period $t = 0, 1, \dots, \tau(\sigma)$. Thus, given an allocation rule F , a strategy pair σ implies that the discounted utility of capitalists is given by:

$$\begin{aligned} \lambda_C(k, \sigma, F) &= v(k, \alpha) && \text{if } \tau = 0 \\ &= \sum_{t=0}^{\tau-1} \beta^t u(c_t) + \beta^\tau v(k_\tau, \alpha) && \text{if } \tau > 0 \end{aligned} \quad (3.1)$$

where α , τ , $\{c_t\}_{t=0}^{\tau-1}$, and k_τ are induced by σ and F as described above, and v is described by Fact One. That is, if the strategy pair σ prescribes an immediate agreement α , the payoff to capitalists is given by $v(k, \alpha)$. If not, capitalists' payoff is the discounted utility of their consumption between time zero and the time of agreement, plus the utility of the agreement $v(k_\tau, \alpha)$ discounted by β^τ .

Likewise, given an allocation rule F , a strategy pair σ implies that the discounted utility of workers is:

$$\lambda_L(k, \sigma) = \beta^\tau w(k_\tau, \alpha) \quad (3.2)$$

where α , τ , and k_τ are induced by σ and F , and w is given by Fact One.

Given an allocation rule F , a strategy pair σ is a Nash equilibrium if σ_L maximizes λ_L given σ_C and viceversa. A Nash equilibrium is subgame perfect if its continuation is a Nash equilibrium after any history.

So far we have not restricted the possible allocation rules. To see what restrictions are natural, consider any period t after the offers $(a_0, \dots, a_t) = h^t$ have been made and rejected. The representative capitalist has capital $k_t(h^{t-1})$, and has to decide how much to consume and invest in period t . Now, suppose that a strategy pair σ is given. Then the continuation of σ after h^t ,

call it $\sigma|h^t$, determines that an agreement $\alpha(\sigma|h^t)$ will be reached in period $(\tau(\sigma|h^t) + t + 1)$. Then it is natural to require that c_t and k_{t+1} be part of an optimal plan for a capitalist that starts period t with capital $k_t(h^{t-1})$ and foresees that an agreement $\alpha(\sigma|h^t)$ at time $(\tau(\sigma|h^t) + t + 1)$. If the allocation rule F satisfies this requirement, I will say that F is competitive given σ .

A sustainable bargaining equilibrium is an allocation rule F and a strategy pair σ such that F is competitive given σ and, given F , σ is a subgame perfect Nash equilibrium. Note that, by construction, a SBE implies that, given the behavior of the private sector, the two parties' strategies are optimal against each other after any history. Conversely, given the parties' strategies, the behavior of the private sector is consistent with a perfect foresight equilibrium, also after every history. Thus a SBE is a natural concept based on the assumption that each agent behaves optimally and rationally after any contingency.

The rest of the paper characterizes the outcomes of our bargaining model by its SBEs, and studies empirical and policy implications.

4. Existence and Characterization of a Sustainable Bargaining Equilibrium

This section provides sufficient conditions for the existence of constrained Pareto efficient, stationary SBEs. The main result is that these SBEs solve a pair of normal equations. Although the normal equations are highly nonlinear, they can be analyzed numerically for different values of the underlying parameters of the economy. Such calculation is performed in Section 5.

The SBEs studied in this section will be stationary in the sense that the

players' strategies and the savings rate implied by the allocation rule will be independent of previous history. The SBEs of this section will also be constrained Pareto efficient in implying that agreement is reached without delay after any history and in prescribing that the outcomes be on the Pareto frontier of the set of (constant, constrained) fiscal policies ⁸.

Focusing on constrained Pareto efficient, stationary SBEs is justified on several grounds. First, they are relatively easy to compute, which not only allows us to study them but also lends them plausibility as an equilibrium concept. Second, I conjecture that constrained Pareto efficient, stationary SBEs are the only SBEs in this model, at least for the parameters studied in Section 5. Although I do not have a proof that the normal equations below have a unique solution, in the numerical analysis of Section 5 I was unable to find multiple solutions. This, plus our assumption of complete information, indicates that uniqueness of SBEs is a likely possibility.

Suppose then that there is a constrained Pareto efficient, stationary SBE. Stationarity of the SBE equilibrium strategies implies that L offers $a_0 = (\theta_0, \tau_0)$ in all even periods, and that C offers $a_1 = (\theta_1, \tau_1)$ in all even periods. Constrained Pareto efficiency implies that $\theta_0 = \theta_1 = \alpha$, and that agreement is immediate. Therefore, after any history, there must be an agreement with an interest rate given by:

$$x = 1 + (1-\tau_0) A^{1/(1-\alpha)} \alpha^{\alpha/(1-\alpha)} \quad (4.1)$$

$$y = 1 + (1-\tau_1) A^{1/(1-\alpha)} \alpha^{\alpha/(1-\alpha)} \quad (4.2)$$

depending on whether we consider an even or odd period.

⁸Note that below I do not restrict strategies to be Pareto optimal. Pareto optimality is postulated to be a property of the outcome of the bargaining.

What allocation rules are competitive in this SBE? The postulated strategies imply that, after any history h^t , an agreement will be reached in period $(t+1)$ with an implied interest rate equal to x if t is odd, and y if t is even. Therefore, Fact Two of section 2 implies that the only competitive allocation rule given the postulated strategies is:

$$\begin{aligned} k_{t+1} &= (\beta x^{1-\sigma})^{1/\sigma} k_t && \text{if } t \text{ is odd} \\ &= (\beta y^{1-\sigma})^{1/\sigma} k_t && \text{if } t \text{ is even} \end{aligned} \quad (4.3)$$

and $c_t = k_t - k_{t+1}$.

It remains to show that, given the allocation rule (4.3), the postulated strategies are a subgame perfect Nash equilibrium. As in Rubinstein (1982), this will be shown to be the case if in each period the proposer makes an offer that leaves the responder indifferent between accepting or rejecting the offer. If t is even, this condition is equivalent with $v(k_t, a_0) = u(c_t) + \beta v(k_{t+1}, a_1)$, with c_t and k_{t+1} determined by the allocation rule (4.3), and v given by Fact One. After some simplification and using the definition of x and y , it turns out that C is indifferent between taking a_0 in even periods and rejecting it to offer a_1 in odd periods if and only if:

$$(1 - (\beta x^{1-\sigma})^{1/\sigma})^{-\sigma} x^{1-\sigma} = (1 - (\beta y^{1-\sigma})^{1/\sigma})^{-\sigma} \quad (4.4)$$

Similarly, L is indifferent between taking a_1 in odd periods and rejecting it and agreeing on a_0 in even periods if and only if $w(k_t, a_1) = \delta w(k_{t+1}, a_0)$, or:

$$\frac{R^* - y}{1 - \delta(\beta y)^{1/\sigma}} = \delta (\beta x^{1-\sigma})^{1/\sigma} \frac{R^* - x}{1 - \delta(\beta x)^{1/\sigma}} \quad (4.5)$$

The two equations (4.4) and (4.5) are crucial to characterize the SBE postulated here. Following Rubinstein (1982), I will call (4.4) and (4.5) the normal equations.

We are almost done. The following theorem shows that if x and y satisfy the normal equations, are Pareto optimal, and are not too large, there is a SBE with the anticipated properties:

Proposition 1: Suppose x and y belong to $[R_*, R^*]$ and satisfy the normal equations. Also, assume that $\beta x^{1-\sigma}$, $\beta y^{1-\sigma}$, $\delta(\beta x)^{1/\sigma}$, and $\delta(\beta y)^{1/\sigma}$ are all less than one. Then the following is a sustainable bargaining equilibrium:

Allocation rule F: defined by (4.3) above.

Strategy for L: Offer $a_0 = (\alpha, r_0)$ in even periods, where r_0 satisfies (4.2); in odd periods $t = 1, 3, 5, \dots$, accept any offer a such that $w(k_t, a) \geq \delta w(k_{t+1}, a_0)$.

Strategy for C: Offer $a_1 = (\alpha, r_1)$ in odd periods, where r_1 satisfies (4.3); in even periods $t = 0, 2, 4, \dots$ accept any offer a such that $v(k_t, a) \geq u(c_t) + \beta v(k_{t+1}, a_1)$. ■

The proof is in the Appendix. The usefulness of Proposition 1 is that, under slightly milder restrictions, one can show that a solution to the normal equations exists and identifies a SBE:

Proposition 2: Assume that $R_* = 1$ and that $\sigma > 1$. Then the normal equations (4.4) and (4.5) have a solution (x, y) in $[1, R^*]^2$. ■

The proof is also in the Appendix.

Joining Proposition 1 and Proposition 2 we obtain the following:

Corollary: If $\delta(\beta R^*)^{1/\sigma} < 1$, $R_* = 1$, and $\sigma > 1$, then there is a SBE characterized by the solutions of the normal equations.

In the SBE of the Corollary, bargaining stops in the first period, and the interest rate agreed upon is x if L moves first, and y if C moves first. The outcome is constrained Pareto efficient, as anticipated.

The Corollary suggests how one may study the set of SBEs discussed in this section: if one chooses parameters such that $\sigma > 1$, $R_* = 1$ and $\delta(\beta R^*)^{1/\sigma} < 1$, then finding an SBE corresponding to these parameters reduces to finding a solution of the normal equations. This is the procedure used in the next section.

Before leaving this section, note that the SBEs characterized in this section are independent of the initial quantity of capital k_0 . This fact is a result of our assumption about functional forms and has many consequences. One of them is that we can allow for renegotiation after an initial agreement without changing the results. Suppose, for example, that the original model has a unique SBE. Consider a modified model in which any agreement can be broken after one period. Assume that, if an agreement is broken, taxes are set to zero again until there is a new agreement. Then it should be intuitively clear that, in the modified model, the parties agree on the SBE of the original model and there is never an incentive for either party of breaking such agreement.

5. Empirical Implications and Policy Issues

This section describes some of the implications of the model for empirical analysis and policy formulation. A main result is that the model is capable of generating a negative observable relation between economic growth and income inequality. However, redistributing wealth has no effect on the rate of growth.

We start by analyzing the dependence of the set of SBEs discussed in the previous section to changes in the underlying parameters of the economy. I proceed as follows: I choose an empirically plausible benchmark set of parameters for which the Corollary of the last section applies. Then I find an associated SBE by solving the normal equations, and analyze how it is affected by changes in each of the parameters of the model. The objective of this procedure is to examine what one would observe in a cross section sample of countries which may differ on their fundamental parameters.

In this discussion, I proceed by solving the normal equations numerically. An alternative procedure would have been to find a SBE and then to use the Implicit Function Theorem to study its properties analytically. However, since the normal equations are highly nonlinear, the analytic procedure becomes messy very quickly and yields little insight.

The parameters of the model are the elasticity of intertemporal substitution σ , the discount factors of workers and capitalists δ and β , and the parameters of the production function A and α . Based on studies of consumer behavior, an initial choice of σ equal to 2 seems plausible. As for the parameter α of the production function (2.1), note that in a SBE the efficiency condition $\alpha = \theta = g/y$ holds, which suggests that α can be chosen from National Income Accounts. For the United States, government purchases of goods and services are in the order to twenty percent of GNP; however, one may

argue that such figure includes a lot of "unproductive" government spending. Given this measurement problem and our discussion in Section 2, I conservatively choose $\alpha = 0.1$ for the benchmark. Finally, I choose $\delta = 0.9$, $\beta = 0.95$, and $A = 0.25$. In choosing these values I assume that the relevant period is a year; accommodating other period length is easy, by varying δ , β , and A .

With these benchmark values, one can solve the normal equations⁹ and find that the corresponding solutions of x and y are 1.106 and 1.115. That is, these parameters imply that, in equilibrium, the average rate of return on capital is around eleven percent. The implied growth rate of output can be calculated as $(\beta x)^{1/\sigma}$ or $(\beta y)^{1/\sigma}$ depending on who starts the negotiation; for our discussion, I simply take the mean of the two, which in this case is 1.027. That is, these parameters predict that GNP would grow at a rate of around 2.7% per year¹⁰. Note that tax negotiations may have a sizable effect on the rate of growth of the economy. Our benchmark parameters imply that the economy's maximum possible rate of growth is about 4.5%, but in equilibrium the growth rate is less than three percent.

One can repeat the calculations described in the preceding paragraph for each possible constellation of parameters, and study how the SBE depend on different assumptions. Table 1¹¹ shows the SBEs associated with different values of the capitalists' discount rate β .

⁹Here I solve the normal equations numerically with the help of GAUSS's nonlinear equation procedure. The GAUSS program that perform the calculations is available on request.

¹⁰For these benchmark parameters, one can calculate that $R^* = 1.183$ and $R_* = 1$; the conditions of the Corollary are then satisfied, and the values of x and y correspond in fact to a SBE.

¹¹Tables are collected at the end of the paper.

INSERT TABLE 1 HERE

In Table 1, the rows labeled x and y show the solution of the normal equations. The row labelled "Growth" shows the implied growth rate of the economy. The row labeled "L share" shows the ratio of workers' income to total income in this economy :

$$\text{L share} = (\pi(\tau y - g)) / \pi y = (R^* - R) / (\alpha^{\alpha} / (1 - \alpha))$$

L share corresponds, in the economy under examination, to the measure of income inequality used by Alesina and Rodrik (1991) and Persson and Tabellini (1991) in their empirical work.

We see that as β decreases growth slows down and the workers' share of income increases. An increase in β has two effects in this model. First, it increases the amount of investment given interest rates. Thus a larger β implies a larger "growth cake" to be divided between the two social classes. By itself, however, this effect would not imply a larger L-share. The second effect is familiar from the bargaining literature. A larger β implies that the C party becomes more patient in the tax negotiation. The bargaining situation then becomes relatively more favorable to C, as in other bargaining models. Hence the workers' share decreases.

Table 2 shows the results for different values of workers' discount factor δ . Intuitively, one would expect a smaller δ , which makes workers more impatient, to favor the capitalists' party in the tax negotiations. Therefore a smaller δ should imply a smaller transfer to workers, faster growth, and a smaller workers' share. This is in fact the outcome of the model, as Table 2 shows.

INSERT TABLE 2 HERE

Table 3 shows the effects of changing the elasticity of intertemporal substitution σ . The effects of a larger σ are similar to those of assuming a smaller β : As σ increases, growth slows down, and workers get a larger share of the pie. The intuition is as follows. As σ increases, capitalists save less given any interest rate. Therefore, should there be a disagreement at $t = 0$, the stock of capital from which the economy starts growing at $t = 1$, if agreement is reached, becomes smaller. This hurts capitalists more than workers because workers are not as concerned about growth as with redistribution: So the C party has to offer a more generous tax package for the L party to reach agreement quickly.

At this point, note that L share and growth move in the same direction as we vary parameters in Tables 1-3. Thus, if countries has different β s, δ s, and σ 's, one would observe that "inequality helps growth". This would be, of course, inconsistent with the recent empirical findings of Alesina and Rodrik (1991) and Persson and Tabellini (1991).

INSERT TABLE 3 HERE

Finally, Tables 4 and 5 show the effect of varying the technological parameters α and A. Note that these are the only two parameters that determine R^* in this economy. As noted by the rows labeled " R^* " in the Tables, increases in α decrease R^* , and increases in A increase R^* . Since the "growth pie" depends on R^* , one would expect parameter changes that increase R^* to result in faster growth. Tables 4 and 5 show that this is in, in fact, the case.

Tables 4 and 5 also show that technological changes associated with a larger R^* imply a larger income share for workers. Again, the intuition is

that workers care less about growth than capitalists, so that if the "growth pie" becomes larger the C party has offer a more than proportionally generous offer to the L party for the right to pass a tax reform.

INSERT TABLES 4 AND 5 HERE

In equilibrium, $g/y = \theta - \alpha$. Hence Table 4 implies that observing a negative relation between growth and government expenditure (as a share of income) is consistent with our model. This would be the case if one had observations of growth and g/y for countries with different α 's. If countries had the same α 's but differed in other parameters, Tables 1,2,3, and 5 imply that one would observe no correlation between growth and government expenditure. This aspect of the model is roughly consistent with the empirical literature ¹².

Tables 4 and 5 also imply that our model is consistent with the negative empirical correlation between income inequality and economic growth emphasized by Alesina and Rodrik and Persson and Tabellini. Such correlation would be observed in a world in which technological parameters (the A s and the α s) differ across countries.

Note, however, that growth and income inequality do not depend on the initial amount of capital per capitalist, k_0 , or on the number of capitalists relative to that of workers, π . Since these are the parameters that determine the initial distribution of wealth by any measure, we can conclude that growth is unrelated to the initial distribution of wealth.

The last observation implies the main policy message of this model: the

¹²See Barro (1990) for a discussion. Barro observes that his model would generate a negative relation between growth and g/y if θ is chosen to satisfy productive efficiency.

existence of a negative empirical relation between income inequality and economic growth does not imply that redistribution of initial wealth from capitalists to workers would enhance growth. To see this, imagine that at the beginning of time, before bargaining starts, an omnipotent dictator were to take some amount of capital $0 < \epsilon < k_0$ from each capitalist and divide it equally between the workers. Then the bargaining outcome is the same as if the economy had started with $k_0 - \epsilon$ units of capital per capitalist instead of k_0 . The normal equation for capitalists, (4.4), does not depend on capital in any period. As for workers, since they cannot save, they would consume the transfer $\pi\epsilon$ regardless of whether there is an agreement at $t = 0$ or not. Therefore the normal equation for workers, (4.5), is also unaffected¹³.

Our discussion highlights the fact that observing an empirical relation between income inequality and growth tells us nothing about causality. In our model, income distribution and economic growth are simultaneous outcomes of the same political process. Saying that "income inequality hurts growth" does not make more sense than saying that "growth hurts income inequality". What is exogenously given is the initial allocation of wealth. But wealth does not affect the bargaining outcome, and hence redistributing wealth does not enhance growth.

One may note, correctly, that the implication that wealth redistribution does not enhance growth depends crucially on the fact that the normal equations do not depend on capital, which in turn hinges on the specific functional forms that I have postulated. It may therefore be possible to destroy such homogeneity by assuming different functional forms, implying that

¹³ Although the above argument depends on the assumptions that workers do not save, it seems to me that it would survive if we allowed workers to save or even to lend their capital to the capitalists: in equilibrium, capital can grow at most at the rate $(\beta R^*)^{1/\sigma}$, and we have assumed that $\delta(\beta R^*)^{1/\sigma} < 1$, which implies that workers consume their resources as fast as they can.

redistributing capital may affect SBE growth. But changing functional forms may work both ways, however: I conjecture that one may obtain an observable negative relation between income inequality and growth in a model in which redistributing capital from capitalists to workers decreases growth. The point is that the empirical correlation between income distribution and growth may be a reduced form with no implications for the effects of redistributive policies and growth.

6. Final Remarks

This paper has studied a model in which taxes determine economic growth and income distribution, and in which taxes are the outcome of negotiations between political parties. The main policy lesson from this exercise is that observing a negative correlation between income inequality and growth does not imply that reducing inequality helps growth.

In what sense do these results challenge the studies by Alesina and Rodrik (1991) and Persson and Tabellini (1991)? Their models and the one in this paper may be all consistent with the empirical correlation about income distribution and growth¹⁴, but they have very different policy implications. This implies that efforts should be made to discriminate between the alternative models on the basis of theoretical or empirical criteria other than the income distribution-growth evidence.

The conclusions of this paper must be qualified by the fact that I have treated stationary SBEs as if they were the only SBEs of the model.

¹⁴Note, however, while the data shows a relation between income distribution and growth, Alesina and Rodrik show a connection between wealth distribution and growth, while Persson and Tabellini show a relation between "basic skills" and growth.

In the absence of a formal proof, uniqueness is an open question, although I guess that for this model it does hold. In the absence of uniqueness, many things can happen, and in particular I conjecture that one can construct SBEs in which reducing wealth inequality may actually increase the rate of growth. Such possibility, if existent, would rely on the fact that income inequality may serve as a "coordinating device" in the selection among different SBEs. This question is the subject of ongoing work.

Appendix

Proof of Proposition 1: As discussed in the text, the allocation rule F is competitive by construction. Hence, it is sufficient to verify the optimality of C 's and L 's strategies after any history.

By the recursive structure of the proposed strategies and allocation rule, it is sufficient to verify that no player can unilaterally gain from a one shot deviation from the proposed strategies. Thus, let t be even, and consider the decision problem of C after receiving an offer a . If C takes the offer, its payoff is $v(k_t, a)$. If C rejects it, the continuation strategies imply that an agreement a_1 will be reached at $(t+1)$, giving C $u(c_t) + \beta v(k_{t+1}, a_1)$. As discussed above, (4.4) implies that C is indifferent between the two options.

Likewise, let t be even and consider L 's decision when it has to make an offer. Suppose L does not offer a_0 . Any offer that gives L strictly more than $w(k_t, a_0)$ will be rejected because a_0 is Pareto optimal. So it must be the case that L wants to delay the agreement. The continuation strategies then imply an agreement a_1 at $(t+1)$, giving L $\delta w(k_{t+1}, a_1)$. By construction, $\delta w(k_{t+1}, a_1) = \delta^2 w(k_{t+2}, a_0) < w(k_t, a_0)$. So offering a_0 is in fact optimal for L in even periods.

Finally, suppose that t is odd. Showing that L 's acceptance rule is optimal is easy and left to the reader. Consider C 's decision about what offer to make. Suppose that C does not offer a_1 . Any offer that gives C more than $v(k_t, a_1)$ will be rejected because a_1 is Pareto optimal. Therefore, if C does not offer a_1 , it must be the case that C plans to delay agreement. The continuation strategies then imply that an agreement a_0 will be reached at $(t+1)$. The payoff to C , discounted to t , of delaying agreement is $u(c_t) +$

$\beta v(k_{t+1}, a_0)$. I claim that this cannot be larger than $v(k_t, a_1)$, which is C's payoff if it offers a_1 at t .

The claim will be proved if $v(k_t, a_1) - u(c_t) + \beta v(k_{t+1}, a_0) \geq 0$. By using the definitions of v , u , and the allocation rule, this inequality amounts to:
 $(1-\sigma)^{-1} \{ (1 - (\beta y^{1-\sigma})^{1/\sigma})^{-\sigma} y^{1-\sigma} - (1 - (\beta x^{1-\sigma})^{1/\sigma})^{-\sigma} \} \geq 0$. Using (4.4), the LHS is equal to $(1-\sigma)^{-1} \{ (1 - (\beta x^{1-\sigma})^{1/\sigma})^{-\sigma} [(xy)^{1-\sigma} - 1] \}$, which is nonnegative since x and y are assumed to be not less than one. ■

Proof of Proposition 2: Let the LHS of (4.4) be denoted by $f(x)$. One can show that $f(x)$ is strictly decreasing on $[1, R^*]$, with $f(1) = (1 - \beta^{1/\sigma})^{-\sigma}$ and $f(R^*) = (1 - (\beta(R^*)^{1-\sigma})^{1/\sigma})^{-\sigma} (R^*)^{1-\sigma}$. Given any $y \in [1, R^*]$, the RHS of (4.4) is a number in the interval $[(1 - \beta^{1/\sigma})^{-\sigma}, (1 - (\beta(R^*)^{1-\sigma})^{1/\sigma})^{-\sigma}]$. Call this number z . Since f is continuous, strictly decreasing, and satisfies $f(1) \leq z < f(R^*)$, there is a unique number $T_1(y)$ in $[1, R^*]$ such that $f(T_1(y)) = z$. Thus T_1 maps $[1, R^*]$ to itself. T_1 is clearly continuous.

Likewise, let the LHS of (4.5) be denoted by $g(y)$. Since $R_* \leq 1$, $g(y)$ is decreasing on $[1, R^*]$ and continuous. Also, $g(1) = (R^* - 1) / (1 - \delta\beta^{1/\sigma})$ and $g(R^*) = 0$. Given any x in $[1, R^*]$, the RHS of (4.5) is a number in the interval $[0, \delta\beta^{1/\sigma} g(1)]$; call this number z' . Now $g(1) > \delta\beta^{1/\sigma} g(1) \geq z' \geq 0 = g(R^*)$, so that there is a unique number $T_2(x)$ in $[1, R^*]$ such that $g(T_2(x)) = z'$. Thus T_2 maps $[1, R^*]$ to itself; T_2 is clearly continuous.

Finally, consider the mapping ξ from $[1, R^*]^2$ to itself defined by $\xi(x, y) = (T_1(y), T_2(x))$. It is easy to check that a solution of the normal equations is a fixed point of this mapping. But all the conditions of Brouwer's Fixed Point Theorem are satisfied, and therefore the Proposition is proved. ■

References

Alesina, Alberto, and Dani Rodrik (1991). "Redistributive Politics and Economic Growth" CEPR Discussion Paper 565, June .

Barro, Robert (1990). "Government Spending in a Simple Model of Endogenous Growth". *Journal of Political Economy* 98 (October), S103-S125

Barro, Robert and David Gordon (1983). "Rules, Discretion and Reputation in a Model of Monetary Policy". *Journal of Monetary Economics* 12 (July), 101-21

Benhabib, Jess, and Aldo Rustichini (1991). "Social Conflict, Growth, and Income Distribution". C.V. Starr Center for Applied Economics RR 91-22, New York University

Chang, Roberto (1991a). "Bargaining a Monetary Union." C.V. Starr Center for Applied Economics R.R. 91-02, New York University

Chang, Roberto (1991b). "Private Investment and Sovereign Debt Negotiations". C.V. Starr Center for Applied Economics R.R. 91-47, New York University.

Chari, V.V. and Patrick Kehoe (1990) "Sustainable Plans" *Journal of Political Economy* 98,4 (Aug), 783-802

Lancaster, Kelvin (1973). "The Dynamic Inefficiency of Capitalism". *Journal of Political Economy* 81, 1098-1199

Persson, Torsten, and Guido Tabellini (1991). "Is Inequality Harmful for Growth? Theory and Evidence". NBER Working Paper 3599

Rubinstein, Ariel (1982). "Perfect Equilibrium in a Bargaining Model" *Econometrica* 50, 97-109

Stokey, Nancy (1991). "Credible Fiscal Policy". *Journal of Economic Dynamics and Control* 15, 627-656

Tornell, Aaron, and Andres Velasco (1990). "Why Does Capital Flow from Poor to Rich Countries?" C.V. Starr Center for Applied Economics RR 90-39 New York University.

Table 1
Effect of Capitalists' Discount Rate β

	β			
	0.97	0.95	0.93	0.90
x	1.109	1.106	1.103	1.099
y	1.117	1.115	1.113	1.110
Growth	1.039	1.027	1.015	1.000
L share	0.036	0.039	0.042	0.045
R*	1.149	1.149	1.149	1.149

Fixed Parameters: $A = 0.25$, $\alpha = 0.1$, $\delta = 0.9$, $\sigma = 2$

Table 2
Effect of Workers Discount Rate δ

	δ			
	0.90	0.87	0.84	0.80
x	1.106	1.107	1.110	1.113
y	1.115	1.117	1.119	1.123
Growth	1.027	1.028	1.029	1.030
L share	0.039	0.037	0.035	0.032
R^*	1.149	1.149	1.149	1.149

Fixed Parameters: $A = 0.25$, $\alpha = 0.1$, $\beta = 0.95$, $\sigma = 2$

Table 3
Effect of Elasticity of Substitution σ

	σ			
	2.0	2.5	3.0	3.5
x	1.106	1.103	1.101	1.100
y	1.115	1.112	1.111	1.110
Growth	1.027	1.021	1.017	1.014
L share	0.039	0.042	0.043	0.044
R^*	1.149	1.149	1.149	1.149

Fixed Parameters: $A = 0.25, \alpha = 0.1, \delta = 0.9, \beta = 0.95$

Table 4
Effects of Marginal Productivity α

	α			
	0.1	0.15	0.2	0.25
x	1.106	1.085	1.067	1.055
y	1.115	1.091	1.073	1.058
Growth	1.027	1.017	1.009	1.002
L share	0.039	0.031	0.024	0.018
R^*	1.149	1.119	1.095	1.074

Fixed Parameters: $A = 0.25$, $\delta = 0.9$, $\beta = 0.95$, $\sigma = 2$

Table 5
Effect of Overall Productivity A

	A			
	0.25	0.30	0.35	0.40
x	1.106	1.129	1.155	1.185
y	1.115	1.141	1.173	1.210
Growth	1.027	1.039	1.051	1.067
L share	0.039	0.047	0.053	0.054
R*	1.149	1.183	1.217	1.252

Fixed Parameters: $\alpha = 0.1$, $\delta = 0.9$, $\sigma = 2$, $\beta = 0.95$