

A QUANTITATIVE ANALYSIS OF THE OPTIMAL TAX STRUCTURE UNDER INCOMPLETE MARKETS

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This paper investigates the optimal tax structure in an overlapping generations model in which individuals face idiosyncratic income risk, borrowing constraints and lifetime uncertainty. The calibrated model economy produces some quantitative results that differ significantly from the findings of the previous research. The main finding in this imperfect insurance setup is that moving away from capital income taxation toward higher labor income taxation yields a (steady-state) welfare benefit of 1% of aggregate consumption compared with the 6% figure Lucas (1990) finds in an infinite-horizon, complete markets model. This is because replacing the tax on capital income with a higher tax on labor income redistributes resources away from the young working years during which borrowing constraints are more likely to bind. Furthermore, when the individuals have access to a private annuity market to insure against uncertain lifetimes, it becomes optimal to tax capital. When a consumption tax is made available, it is optimal to switch to consumption taxation. The welfare benefit from implementing this optimal plan is on the order of 1.5-3.2% of GNP.

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I. INTRODUCTION

The choice of the optimal tax base and its effects on private saving and capital accumulation, economic efficiency and individual welfare have been extensively studied in various model economies.¹ Boskin (1978) and Feldstein (1978) focus on the impact of particular tax structures on the saving function of the individuals and argue that capital income taxation slows down economic growth. Summers (1981) uses a (continuous-time) finite-horizon, representative agent, perfect foresight model with productivity growth and obtains a "conservative" estimate of 10% of GNP for the annual welfare gain from switching to consumption taxation. He concludes that capital income taxes are likely to appear to be very undesirable in a realistic life cycle formulation. Judd (1987) computes the dynamic effects of marginal changes in a tax instrument using an infinite-horizon, representative agent, perfect foresight model. He finds that welfare would be improved substantially at the margin by moving away from capital income taxation and toward higher labor income taxation or more investment subsidies at current tax levels.² Chamley (1986) uses an infinitely-lived general equilibrium model and argues that the Ramsey plan calls for zero taxation of capital in the long-run. Auerbach and Kotlikoff (1987) build an overlapping generations model with representative agents, certain lifetimes and complete markets. They find that switching from a 30% income tax to a consumption tax raises the capital stock by 61% and improves welfare by the equivalent of 2.32% of assets plus the present value of full-time lifetime earnings. Lucas (1990) employs an infinite-horizon, representative agent, endogenous growth model and examines the impact of eliminating the tax on income from capital. His major steady-state finding is that eliminating the tax on capital income and raising the lost revenue through higher labor income taxes leads to a 32% increase in the capital stock, and that the welfare benefit from this tax reform is equivalent to 6% of aggregate consumption.³

¹ The empirical evidence on the optimality of taxes is mixed: Barro (1979), Sahasukul (1986), Kingston (1984), Skinner (1989) and Judd (1991) find some support for the optimality of labor income taxes, whereas Bizer and Durlauf (1990) question both the theoretical and the empirical underpinnings of the models used to test the optimality hypothesis.

² Judd (1987) also studies the anticipation effects, temporary tax changes and phased-in tax reforms. Judd (1991) examines the nature of optimal taxation of labor and capital income in dynamic, stochastic infinite-horizon economies. His finding on the optimal labor income tax rate is in line with the earlier results of Barro (1979), Atkinson and Sandmo (1980), and Judd (1985), in that the optimal labor income taxes are highly persistent. The optimal capital income tax, however, turns out to be very volatile, matching the volatility of government's additional revenue requirements.

³ However, when the transitional costs in the form of lower consumption or lower leisure during the transition path are taken into account, the overall welfare benefit drops to about 1% of consumption.

Aiyagari and Peled (1991) examine a Bewley-type economy in which some individuals face binding liquidity constraints. They show that Ramsey taxation with binding liquidity constraints leads to a positive capital income tax rate in the long run. This theoretical result derives from the need to lower the capital stock and raise the pre-tax return to capital so that it matches with the rate of time preference. Hence, the Aiyagari-Peled finding raises the possibility that a realistically parameterized Bewley-type model may support a positive capital income tax rate in the U.S. economy.

This paper examines the optimal tax structure in a parameterized general equilibrium model with incomplete markets. In particular, two experiments are conducted. First, using a parameterization similar to that in Lucas (1990) but with all contingent claims markets shut down exogenously, the optimal capital income tax is calculated and the quantitative improvement in the aggregate capital stock and individual welfare is reported. Second, a consumption tax is made available and the optimal tax structure and the welfare benefit of implementing the optimal plan are studied. For both experiments, the impact of departures from the benchmark model economy on the numerical findings is also reported.

The economy consists of overlapping generations of 65-period-lived individuals who face mortality risk and idiosyncratic income risk. If an individual gets a lucky draw, then he supplies labor inelastically; otherwise he is unemployed and receives unemployment insurance benefits. Both the employed and the unemployed pay labor income taxes and unemployment insurance taxes. The production technology of the economy is described by a constant returns to scale CES production function.

Individuals face borrowing constraints. Together with the idiosyncratic employment risk, this gives rise to a heterogeneous-agent framework in which individuals differ not only with respect to their age but also with respect to their employment history and therefore their accumulated wealth.⁴ Individuals in this economy hold assets for two reasons: first, for life cycle considerations, and second, for precautionary reasons. The latter motive to save arises from two sources. First, the individuals have to self-insure against unlucky draws in the

⁴ The empirical importance of liquidity constraints is examined by several researchers; see for example Hayashi (1985) and Zeldes (1989). It is clearly not the case that all the households in the U.S. economy are liquidity constrained. But the focus in this paper is to find the impact of market incompleteness on the optimal tax structure, and the assumption of liquidity constraints is a simple device to close a large number of markets. The quantitative implications of models with borrowing constraints have been studied by İmrohoroglu (1989), Deaton (1991), Hansen and İmrohoroglu (1992), Prescott and İmrohoroglu (1992), Hubbard and Judd (1987), İmrohoroglu, İmrohoroglu and Joines (1992a) and Altig and Davis (1992), among others.

future since they cannot borrow against their future earnings. Second, uncertainty about the length of life induces individuals to save so that they have enough consumption in the event that they live longer than expected. Their income from saving, however, is subject to a capital income tax. Taxation of capital income is not desirable in this model because of the distortion on private saving decision and the consequent negative impact on the capital stock, output and aggregate consumption. Taxing labor income is also not desirable despite the inelastic supply of labor, since the increase in the labor income tax rate hinders the individuals' ability to self-insure and also to provide for old age consumption. In other words, since the individuals are liquidity constrained, higher labor income tax rates make it more likely that the constraints are binding. Consequently, moving away from capital income taxation toward labor income or consumption taxation has two separately identified effects on individual welfare. First, there is a **capital stock effect**. The proposed tax reform raises the capital stock toward the Golden Rule steady-state capital stock, and as a result aggregate consumption increases. Second, there is a **consumption profile effect**. Moving away from capital income taxation toward higher labor income or consumption taxation reallocates resources over the life cycle since capital is held mostly during middle-aged and old-aged years. This reallocation of resources away from young working years toward older years of life may induce an equilibrium consumption profile which is farther away from that chosen by the social planner. *A priori*, the overall general equilibrium impact of replacing the tax on capital income with a higher labor income or consumption tax is ambiguous. The direction and the magnitude of the effect on individual welfare depend on how strong these two potentially conflicting forces are in a realistic overlapping generations model.

Numerical methods developed in İmrohoroğlu, İmrohoroğlu and Joines (1992a,b) are used to compute the decision rules of the individuals, the age-dependent distributions of the agent types, and equilibrium values of aggregate variables. For a given fiscal policy regime which consists of government purchases, transfer payments, and labor income, capital income, consumption and unemployment insurance tax rates, steady-state equilibria are calculated. The benchmark economy is one in which there is taxation of labor income (and unemployment insurance benefits) and capital income. The exogenous government purchases, which provide no utility to the individuals, and endogenously determined government transfer payments are held constant in the face of a revenue-neutral tax reform. Different tax reforms are examined. First, the tax on capital income is eliminated and the labor income tax rate is increased and the summary statistics of these steady-state equilibria are tabulated. The welfare results are obtained by computing the non-storable, non-tradable consumption supplement that would make an individual indifferent between the benchmark

steady-state equilibrium without the supplement and an alternative steady-state equilibrium under tax reform with the supplement. Second, the same amount of government purchases and transfer payments are financed by eliminating capital income taxation or labor income taxation and introducing a tax on consumption. In addition to the optimality results, the impact of choosing a particular tax base on the overall distribution of consumption and wealth and the reallocation over the life cycle are also described.

The main finding is that moving away from capital income taxation toward higher labor income taxation yields a welfare benefit of 1% of aggregate consumption compared to the 6% benefit that Lucas (1990) finds. Replacing the capital income tax with a higher tax rate on labor income reallocates resources away from the young working years during which borrowing constraints are more likely to bind. Furthermore, when individuals have access to a private annuity market to insure against lifetime uncertainty, it becomes optimal to tax capital income at 10%. Although eliminating capital income taxation brings the economy closer to the golden rule steady-state capital stock which maximizes aggregate consumption, the resulting equilibrium consumption profile is farther away from that chosen by the social planner. A lower elasticity of intertemporal substitution in consumption increases the optimal capital income tax rate to 36%. The numerical findings appear to be robust to using an alternative earnings profile and a more persistent idiosyncratic employment shock relative to the benchmark case.

Although not explicitly analyzed, I would argue that incorporating an exogenous labor-augmenting technological progress would increase the cost of raising the labor income tax and hence would reinforce the quantitative results. Under technological progress, the age-earning profile would become steeper and individuals would have an increased desire to borrow against their future earnings for a given elasticity of substitution in consumption. Borrowing constraints, however, rule out such transactions. As a result, individuals would be more reluctant to any further reallocation of resources away from the early years of life toward the middle-aged and older years of life which is exactly what moving away from capital income taxation toward higher labor income taxation does.⁵ The presence of exogenous productivity growth would make the labor income tax even a worse tax and consequently reinforce the quantitative results of this paper. Allowing for an endogenous labor-leisure choice would also tend to reinforce the present results since in this case taxing labor income would have

⁵ See İmrohoroglu, İmrohoroglu and Joines (1992a) for an explicit treatment of the factor productivity issue in the context of social security analysis.

more severe allocative consequences.⁶

When a consumption tax is made available, it becomes optimal to switch to consumption taxation. This is very much in line with a wide body of findings in the optimal tax literature. The welfare benefits of implementing this optimal tax plan, however, are on the order of 2–4% of aggregate consumption, which are lower than most other findings in the literature. In this case, a positive capital stock effect of moving away from labor and capital income taxation to consumption taxation appears to be partially offset by a negative consumption profile effect. The equilibrium consumption profile under a tax on consumption is farther away from that of the planner than the equilibrium consumption profile under a tax base of labor income and consumption.

Replacing capital income taxation with consumption taxation has little impact on the overall distribution of consumption or wealth. Switching from income taxation to consumption taxation, on the other hand, worsens the inequality of wealth as measured by the coefficient of variation.

The paper is organized as follows. Section II presents the structure of the model economy. Section III discusses calibration. Section IV describes the solution method used in the paper. Section V presents the measures of utility and welfare effects. Quantitative findings are reported in Section VI. Concluding remarks are given in Section VII.

⁶ The presence of borrowing constraints prevents labor income from becoming a pure rent. As a result, there are allocative consequences of labor income taxation under borrowing constraints even though labor is supplied inelastically.

II. THE MODEL ECONOMY

The economy is populated with a large number of individuals with total measure one. Individuals are born at age $j = 1$ with probability $\psi_1 = 1$. After birth, they survive from age $j - 1$ to age j with probability ψ_j . After age $j = J$, death is certain, i.e. $\psi_{J+k} = 0$ for $k = 1, 2, \dots$. Some individuals in the economy will be lucky enough to live for all of J periods with an unconditional probability of $\prod_{j=1}^J \psi_j$. However, a significant fraction of the individuals will experience early death and leave unintended bequests. I assume that these bequests are taxed 100% by the government and then rebated back to the survivors (of all ages) in a lump-sum fashion.⁷

Endowments

Individuals in this economy supply labor inelastically whenever they are given an opportunity to work. At the beginning of each period, an individual draws the realization of the stochastic employment opportunity $s \in S = \{e, u\}$, which follows a two-state, first-order Markov process; $s = e$ indicates that the individual is employed, and $s = u$ indicates that the individual is unemployed. The transition function for s is $\Pi(s', s) = [\pi_{ij}]$, $i, j = e, u$, where $\pi_{ij} = \text{Prob}\{s_{t+1} = j \mid s_t = i\}$. If $s = e$, then $n_j = \hat{h}$; if $s = u$, then $n_j = 0$, where n_j is hours worked by an age- j individual.⁸ When employed, they receive the real wage $w_j^e = w\epsilon_j\hat{h}$ where w is the wage rate (in terms of the consumption good), ϵ_j is the efficiency index (the number of units of work effort into which one unit of time can be turned) of an age- j individual, and $\hat{h}$ is the number of hours of work supplied by an age- j individual.⁹ If an individual is unemployed, he receives unemployment insurance benefits in the amount $w_j^u = \xi w\epsilon_j\hat{h}$, where ξ is the replacement ratio. The allocation of work and leisure must satisfy the time endowment constraint

$$(1) \quad n_j + l_j = 1 \quad .$$

⁷ I will also describe the results from an economy in which individuals enter into private annuity contracts to insure against early death.

⁸ This labor indivisibility follows from Hansen (1985) and Rogerson (1988).

⁹ When there is no aggregate uncertainty or technological progress, aggregate variables in per capita terms will be time-invariant; so will the factor prices. I will restrict the current study to that of steady-states, and hence there will be no time subscript in the model. The analysis of transitional dynamics is left for future research.

The budget constraint of an individual is given by

$$(2) \quad (1 + \tau_c)c_j + a_j = \left[1 + (1 - \tau_k)r\right]a_{j-1} + q_j + \zeta_1 + \zeta_2, \quad j = 1, 2, \dots, J, \quad a_0 \text{ given},$$

$$q_j = \begin{cases} (1 - \tau_n - \tau_u)w_j^e & \text{for } s = e, \\ (1 - \tau_n - \tau_u)w_j^u & \text{for } s = u, \end{cases}$$

where q_j is the disposable income, c_j is consumption, and a_j is the end-of-period asset holdings of an age- j individual. ζ_1 is the lump-sum unintended bequest distribution received by an individual, and ζ_2 is the lump-sum transfer received by an individual from the government. Labor income and unemployment insurance tax rates are denoted by τ_n and τ_u , respectively. The tax rate on income from capital and the tax rate on consumption are denoted by τ_k and τ_c , respectively. Individuals in this economy are liquidity constrained which means their asset holdings cannot be negative:¹⁰

$$(3) \quad a_j \geq 0, \quad \forall j.$$

Technology

The technology of the economy is given by a constant returns to scale CES production function

$$(4) \quad Q = \tilde{A} \left[(1 - \alpha)K^{1-1/\theta} + \alpha N^{1-1/\theta} \right]^{\frac{1}{1-1/\theta}},$$

where $\tilde{A} > 0$, $\alpha \in (0, 1)$ is labor's share of output, and K and N are aggregate capital and labor inputs, respectively. The aggregate capital stock is assumed to depreciate at the rate δ . The intratemporal elasticity of substitution in production is denoted by θ .

Preferences

Let μ_j denote the share of age- j individuals in the population. It is assumed that the population is stable in the sense that the cohort shares are time-invariant. In other words, μ_j evolves according to $\mu_j = \frac{\psi_j}{(1+\rho)}\mu_{j-1}$ for $j = 2, 3, \dots, J$, with $\sum_{j=1}^J \mu_j = 1$. Individuals in

¹⁰ An implication of this and the assumption $\psi_j = 0$ for $j > J$ is that individuals who are alive at period J will choose not to carry over any assets to the next period in the absence of a bequest motive: $a_J = 0$.

the economy rank the sequences of consumption and leisure allocations according to

$$(5) \quad E \sum_{j=1}^J \beta^{j-1} \left[\prod_{k=1}^j \psi_k \right] U(c_j, l_j) ,$$

where β is the subjective discount factor.

Government

The government in this economy makes purchases in the amount g which provides no direct utility to the individuals. It levies taxes on labor income, capital income and consumption. Any surplus of tax revenues over purchases is distributed back to the individuals in a lump-sum fashion. The government budget constraints can be written as:

$$(6) \quad \tau_u \sum_j \sum_a \sum_s w \hat{h} \mu_j \epsilon_j = \xi \sum_j \sum_a \sum_s w \hat{h} \mu_j \epsilon_j ,$$

$$(7) \quad g = \tau_k r K + \tau_n w N + \tau_c C - \zeta_2 ,$$

where K and N are aggregate capital stock and aggregate labor input, respectively, and C is aggregate consumption. Equation (6) indicates that, for a given replacement ratio ξ , the government agency that administers the unemployment insurance benefits program selects the unemployment insurance tax rate so that the program is self-financing. Equation (7) is a standard government budget constraint which determines the equilibrium value of the transfer ζ_2 that corresponds to a given fiscal policy regime $\{g, \xi, \tau_k, \tau_n, \tau_u, \tau_c\}$.

Competitive Equilibrium

The individual's recursive problem is a finite-state, finite-horizon, discounted, dynamic program. Let $V_j(a, s)$ be the value of the objective function of an age- j agent given the state (a, s) , defined as the solution to the dynamic program

$$(8) \quad V_j(a, s) = \max_{(c, a')} \left\{ U(c, l) + \beta \psi_{j+1} E_s V_{j+1}(a', s') \right\} ,$$

subject to

$$(9) \quad \begin{aligned} (1 + \tau_c)c + a' &= \left[1 + (1 - \tau_k)r \right] a + q + \zeta_1 + \zeta_2 , \quad a_0 \text{ given,} \\ q &= \begin{cases} (1 - \tau_n - \tau_u)w\epsilon_j \hat{h} & \text{for } s = e; \\ (1 - \tau_n - \tau_u)\xi w\epsilon_j \hat{h} & \text{for } s = u; \end{cases} \\ n + l &= 1 , \quad c \geq 0, \quad a' \geq 0. \end{aligned}$$

where the notation E_s means that the expectation is over the distribution of s . Let $D = \{d_1, d_2, \dots, d_m\}$ denote the discrete grid of points from which the individuals will be choosing their asset holdings.

Definition: Given a fiscal policy regime $\{g, \xi, \tau_k, \tau_n, \tau_u, \tau_c\}$, an equilibrium for the model economy is a set of value functions $V_j(a, s)$, individual policy rules $C_j : D \times S \rightarrow R_+$, $A_j : D \times S \rightarrow D$, age-dependent (but time-invariant) measures of agent types $\lambda_j(a, s)$ for each age $j = 1, 2, \dots, J$, relative prices of labor and capital $\{w, r\}$, a lump-sum distribution of unintended bequests ζ_1 , and an endogenous lump-sum government transfer ζ_2 such that

i. Aggregate variables are computed from individuals' optimal behavior:

$$(10) \quad K = \sum_j \sum_a \sum_s \mu_j \lambda_j(a, s) A_{j-1} \quad \text{and} \quad N = \sum_j \sum_a \sum_s \mu_j \lambda_j(a, s) \epsilon_j \hat{h} \quad ,$$

where the initial wealth distribution of agents, A_0 , is taken as given,

ii. Factors are paid their marginal products:

$$(11) \quad r = (1 - \alpha) \tilde{A} \left[(1 - \alpha) K^{1-1/\theta} + \alpha N^{1-1/\theta} \right]^{\frac{1}{\theta-1}} K^{-1/\theta} \quad ,$$

$$(12) \quad w = \alpha \tilde{A} \left[(1 - \alpha) K^{1-1/\theta} + \alpha N^{1-1/\theta} \right]^{\frac{1}{\theta-1}} N^{-1/\theta} \quad ,$$

iii. Given relative prices $\{w, r\}$, fiscal policy $\{g, \xi, \tau_k, \tau_n, \tau_u, \tau_c\}$, and a lump-sum transfer ζ_2 , the individual policy rules $C_j(a, s)$, $A_j(a, s)$ solve the individuals' dynamic program (8)-(9),

iv. The commodity market clears,

$$(13) \quad \sum_j \sum_a \sum_s \mu_j \lambda_j(a, s) C_j(a, s) + \rho K + g = Q \quad ,$$

v. The collection of age-dependent, time-invariant measures $\lambda_j(a, s)$ for $j = 1, 2, \dots, J$, satisfies

$$(14) \quad \lambda_j(a', s') = \sum_s \sum_{a: A_j(a, s) = a'} \Pi(s', s) \lambda_{j-1}(a, s) \quad ,$$

where the initial measure of agents at birth, λ_0 , is taken as given,

vi. The government budget equation is satisfied:¹¹

$$(15) \quad g = \tau_k rK + \tau_n wN + \tau_c C - \zeta_2 \quad ,$$

vii. The unemployment insurance benefits program is self-financing:

$$(16) \quad \tau_u = \frac{\sum_{j=1}^J \sum_a \mu_j \lambda_j(a, s = u) \xi w \epsilon_j \hat{h}}{\sum_{j=1}^J \sum_a \mu_j \lambda_j(a, s = e) w \epsilon_j \hat{h}} \quad ,$$

viii. The lump-sum distribution of accidental bequests is determined by

$$(17) \quad \zeta_1 = \sum_j \sum_a \sum_s \mu_j \lambda_j(a, s) (1 - \psi_{j+1}) A_j(a, s) \quad .$$

The Golden Rule Capital Stock and Optimal Allocation of Consumption

Equilibrium allocations of this overlapping generations model will not in general coincide with Pareto optimal allocations. Among the reasons for the inapplicability of the Second Welfare Theorem are the distorting nature of taxes, the presence of borrowing constraints, and possible dynamic inefficiencies in the sense of Diamond (1965). Nevertheless, it will be instructive to characterize the solution to a social planner's problem and compare the optimal consumption allocation to an equilibrium allocation.

Consider the problem of a social planner who wants to maximize the utility function (5) subject to the resource constraint which equates aggregate consumption plus net investment to output:

$$\sum_{j=1}^J \mu_j c_j + \rho K = Q \quad .$$

The objective of the planner is to choose a capital stock and a consumption profile that will maximize aggregate consumption and lifetime utility subject to the resource constraint. Note that the planner can perfectly pool the idiosyncratic employment risk and adjust the consumption profile for lifetime uncertainty. The first order necessary conditions for this

¹¹ This condition is given here for completeness; aggregating the individual's budget and using the market clearing condition guarantees that the government's budget is also satisfied.

problem are:

$$m[f_K(K, N) - \rho] = 0 \quad ,$$

$$\beta^{j-1} \left[\prod_{k=1}^j \psi_k \right] U_c(c_j, l_j) - m\mu_j = 0 \quad , \quad j = 1, 2, \dots, J,$$

where m is the Lagrange multiplier associated with the resource constraint, $f_K(\cdot, \cdot) \equiv \frac{\partial f(\cdot, \cdot)}{\partial K}$ and $U_c(\cdot, \cdot) \equiv \frac{\partial U(\cdot, \cdot)}{\partial c}$. The first equation determines the golden rule steady-state capital stock. The remaining equations determine the optimal consumption profile. By combining the conditions on c_j and c_{j+1} , and using the expression $\mu_{j+1} = \frac{\psi_{j+1}}{(1+\rho)}\mu_j$, the optimal consumption profile can be shown to follow

$$\left(\frac{c_{j+1}}{c_j} \right)^\gamma = \beta(1 + \rho) \quad \text{for } j = 1, 2, \dots, J-1,$$

which is independent of mortality risk and the idiosyncratic employment risk.

The equilibrium allocations of this model can differ from this optimal allocation in two ways. First, equilibrium aggregate consumption can be different from the optimal aggregate consumption. Second, the allocation of equilibrium consumption over the life cycle can be different from the profile chosen by the social planner.

III. CALIBRATION

In order to solve the model and obtain quantitative results, the parameters of preferences, technology, and government fiscal policy have to be calibrated. Since the current model has not been used in estimating these parameters directly from some data set, I will choose values for them based on previous research. However, given that we do not have precise estimates of these parameters, and that most of these parameters have been calibrated with other theoretical constructs in mind, I will repeat the quantitative exercises over a coarse grid of deep parameters. Hopefully, this practice will help provide some insight about the sensitivity of the quantitative results on particular parameters.

A model period is one year. Individuals are born as 21 year olds, which corresponds to model age $j = 1$, and may live until they are 85 years old, which is model age $J = 65$. Death occurs with probability one after model-age 65. Until death, natural or premature, individuals take a draw from nature to determine their employment status. This follows a

first-order, two-state Markov process whose transition matrix is given by

$$\Pi(s, s') = \begin{bmatrix} 0.94 & 0.06 \\ 0.94 & 0.06 \end{bmatrix} ,$$

which implies that the duration of unemployment is $1/(1 - 0.06) = 1.0638$ model periods.

The efficiency index $\{\epsilon_j\}$ is calibrated by using the index constructed by Hansen (1991), interpolated to in-between years, and normalized to average one. I also use the quadratic curve estimated by Welch (1979).

The age-dependent survival probabilities are taken from Faber's (1982) Life Tables of the U.S. Economy. The share of age groups in the population, μ_j , is calculated from $\mu_{j+1} = \frac{\psi_{j+1}}{(1+\rho)}\mu_j$ where ρ is the rate of growth of population, taken as 1.2%. Note that $\sum_{j=1}^J \mu_j = 1$. Raw hours of work, $\hat{h}$, is taken as 0.45, which assumes that individuals devote 45 hours a week (out of a possible 98 hours) to work including commuting. Given an employment rate of 94%, the aggregate labor input is computed as $N = 0.94\hat{h} \sum_{j=1}^J \mu_j \epsilon_j$. Note that μ_j and therefore N depend on the population growth rate ρ and the probability of survival ψ_j .

In choosing the parameters of preferences and technology, I follow Lucas (1990) so that the results are more comparable with his study. Using the Net National Product accounts, Lucas chooses the share of labor in the production function, α , to be 0.76. The elasticity of substitution in production, θ , is selected as 0.6; like Lucas (1990), I also use the Cobb-Douglas case of $\theta = 1$. The parameter $\tilde{A}$ in the production function is fixed at 1.7307026652 so that output is normalized at one for a capital-output ratio of three given an aggregate labor input of 0.443050.

The period sub-utility function is parameterized as

$$U(c, l) = \left[c l^{\alpha_u} \right]^{1-\gamma} / (1-\gamma) ,$$

where $\gamma = 2.0$, and $\alpha_u = 0.5$; as part of the sensitivity analysis, I also use 4.0 for γ , and 5 for α_u .

The econometric evidence on the subjective time discount factor β is somewhat mixed, with a significant number of studies pointing to a value larger than one. In the context of the overlapping generations model used in this paper, this is not problematic. Therefore, I use a value of 1.011 for β , an estimate obtained by Hurd (1989) from the Retirement History

Survey data in a life cycle context.¹² In order to check the sensitivity of the quantitative results to the subjective discount factor, I will also use a value of 0.98.

The discrete set $D = \{d_1, d_2, \dots, d_m\}$ is chosen so that $d_1 = 0$, $d_m = 15$, and $m = 601$. The upper bound $d_m = 15$ is about fifteen times the annual income of an employed individual.¹³ Note that with the choice of $m = 601$ the state space has 601×2 points. The control space is 601×1 .

A government fiscal policy regime in this model is indexed by the choice of the vector $(g, \xi, \tau_k, \tau_n, \tau_c, \tau_u)$ and the endogenously determined government transfer ζ_2 . The unemployment insurance replacement ratio ξ is taken to be 0.25 of the employed wage. Following Lucas (1990), government purchases, g , are taken as 0.21. The value of τ_u is then determined so that the unemployment insurance benefits program is self-financing. This leaves the values of the three tax instruments, τ_k, τ_n and τ_c free. Following Lucas (1990), I will set these equal to 0.40, 0.36 and 0.00, respectively, in the benchmark model economy.

IV. SOLUTION ALGORITHM

The optimization problem faced by an individual in this economy is one of finite horizon dynamic programming.¹⁴ Hence, the decision rules for each cohort j can be found by a single iteration working backwards from the last year of life. The functional form of the value function just after the last year of life is known a priori to be identically zero. That is, $V_{J+1}(\cdot) \equiv 0$. Solutions are found by calculating the single backward recursion starting from $j = J$ until $j = 1$:

$$(8) \quad V_j(a, s) = \max_{(c, a')} \left\{ U(c, l) + \beta \psi_{j+1} E_s V_{j+1}(a', s') \right\} ,$$

subject to (9). In this paper, I use the numerical solution method developed in İmrohoroglu, İmrohoroglu and Joines (1992a,b) to obtain exact numerical solutions for the discrete model economy.

¹² Using the Panel Study of Income Dynamics data, Hotz, Kydland and Sedlacek (1988) estimate β to be between 1.0123 and 1.2041, and argue that these large estimates may be due to a likely increase in the family size over the life cycle and the associated need to increase food expenditures.

¹³ This upper bound is never binding in the numerical experiments reported in this paper.

¹⁴ For details of the numerical solution method used in this paper, see İmrohoroglu, İmrohoroglu and Joines (1992b).

To obtain the distribution of agents, $\lambda_j(a, s)$, into the asset holding levels and employment categories, I start from a given initial wealth distribution λ_0 which influences the equilibrium allocations. It is assumed that newborns come to life with zero asset holdings. So, λ_0 is taken to be an $m \times 2$ matrix with zeros everywhere except the first row which is equal to $(0.94, 0.06)$, the expected employment and unemployment rates, respectively.¹⁵ Starting from this initial wealth distribution λ_0 , some individuals will be employed and some of them will be unemployed at age 1. Depending on the realization of the employment status, individuals will make asset holding decisions which are already calculated and therefore at the end of age 1, they will go to different points in the distribution matrix $\lambda_1(a, s)$. Hence, each entry in the $m \times 2$ matrix $\lambda_1(a, s)$ gives the fraction of 1-year old agents at that particular combination of asset holding-employment status at the end of age-1 optimization problem. Note that, for each j , each element of $\lambda_j(a, s)$ is nonzero, and the sum of all entries equals 1. Hence, given J decision rules A_j and an initial income distribution λ_0 , the age-dependent distributions are computed by a single forward recursion

$$\lambda_j(a', s') = \sum_s \sum_{a: a' \in A_j(a, s)} \Pi(s', s) \lambda_{j-1}(a, s).$$

Given the decision rules of the individuals and the age-dependent measures of agent types, an equilibrium is obtained according to the following algorithm:

Step 1. Start with some initial values for the aggregate capital stock $K^{(0)}$, the lump-sum distribution of unintended bequests $\zeta_1^{(0)}$, and the lump-sum government transfer that balances its budget $\zeta_2^{(0)}$.¹⁶ Compute aggregate employment $N = 0.94 \hat{h} \sum_{j=1}^J \mu_j \epsilon_j$. Use the first-order conditions of the firm's profit maximization problem to obtain the corresponding values for relative factor prices w and r . Substitute these in the individual's budget constraint.

Step 2. Obtain the decision rules and age-dependent measures of agent types following the procedure described in the previous section.

Step 3. Compute the new aggregate capital stock $K^{(1)} = \sum_j \sum_a \sum_s \mu_j \lambda_j(a, s) A_j(a, s)$, the new lump-sum distribution of unintended bequests

$\zeta_1^{(1)} = \sum_j \sum_a \sum_s \mu_j \lambda_j(a, s) (1 - \psi_{j+1}) A_j(a, s)$, and the lump-sum government transfers $\zeta_2^{(1)} = \tau_k r K + \tau_n w N - g$. If $K^{(1)} = K^{(0)}$, $\zeta_1^{(1)} = \zeta_1^{(0)}$ and $\zeta_2^{(1)} = \zeta_2^{(0)}$ up to a convergence criterion of 0.001, then stop. An equilibrium is found. If not, replace the initial guesses with

¹⁵ Note that d_1 is zero.

¹⁶ The last two are monotone functions of the aggregate capital stock. Therefore, the solution algorithm searches for the fixed point of an implicit mapping defined over the aggregate capital stock.

a linear combination of the initial values and those obtained in this last step, and go to Step 1. Iterate until convergence is achieved.

V. UTILITY AND WELFARE MEASURES

As a measure of average utility, the expected discounted utility of a newly-born individual under a given fiscal policy regime is used. This measure is defined as:

$$(18) \quad W(\Omega) = \sum_{j=1}^J \sum_a \sum_s \beta^{j-1} \left[\prod_{k=1}^i \psi_k \right] \lambda_j(a, s) U(C_j(a, s), l_j) \quad .$$

In the previous section, it was mentioned that the benchmark economy is one in which the labor income tax is 40%, the capital income tax rate is 36% and the consumption tax rate is 0%. In order to quantify the welfare effects of departures from the fiscal policy regime in this benchmark case and translate these into familiar measures, I will calculate the non-transferable consumption supplement that would make an individual indifferent between the benchmark model economy without the supplement and a proposed economy under an alternative fiscal policy regime with the supplement.

VI. QUANTITATIVE FINDINGS

The quantitative properties of the model economies are presented in four subsections. First, the results from eliminating the tax on capital income in the absence of a consumption tax are presented. Second, the results from using a consumption tax to replace the tax on capital income or the tax on labor income are discussed. Third, a sensitivity analysis is presented in an attempt to assess the quantitative impact on the numerical findings of departures from the parameter values in the benchmark case. Fourth, the effects of the proposed tax reforms on the overall distribution of consumption and wealth and the reallocation over the life cycle are given.

VI.1 Optimal Tax Structure: Labor and Capital Taxation

The results of this section can be compared to those in Lucas (1990) since the underlying quantitative exercise is the same. Nevertheless, it should be emphasized that the present model differs from Lucas' (1990) setup in two important respects. First, the model of this paper is an overlapping generations model, whereas Lucas' model is an infinite-horizon model. Lucas (1990), however, argues that these two theoretical constructs seem to give similar quantitative answers in the present optimal taxation context. Hence, the bulk of the quantitative differences between the findings of this paper and those in Lucas (1990) presumably arise from the second difference in the two environments. This second difference concerns the degree of market completeness. In the present setup, the ability of the individuals to share idiosyncratic employment risk is completely restricted due to the presence of borrowing constraints. In Lucas' setup, there is a complete set of contingent claims markets.

It will be useful to describe the potential costs of factor taxation in the context of the model used in this paper. Taxing income from capital distorts the saving decision and hence inhibits capital accumulation. As the tax on capital income is reduced the capital stock and therefore aggregate consumption increases. I call this the capital stock effect. Also, there is an impact on the allocation of consumption over the life cycle, which I call the consumption profile effect. Capital is the only asset which the individuals can use to self-insure against bad future draws from nature, and to save for old age consumption. Taxing this asset may therefore impose a big burden on the individuals who may no longer achieve their preferred consumption profile. As the tax on capital income is reduced, however, the tax on labor income is raised in order to keep government purchases and transfer payments unchanged. This will cause a reallocation of resources away from young working years. Hence, borrowing

constraints may be more binding at lower tax rates on capital income (higher tax rates on labor income) than at higher tax rates on capital income (lower tax rates on labor income). The presence of borrowing constraints in this model works to lower the welfare loss associated with taxing capital income.

Starting from a benchmark economy in which the tax rate on labor income is 40% and the tax rate on capital income is 36%, the tax on capital income is reduced in steps of 0.05 while the government spending and transfer payments are held unchanged across these steady-states. As a result, the tax on labor income becomes endogenous and it increases as the tax on capital income is reduced. Table 1 summarizes the results from the benchmark economy. The first two columns give the tax rates on labor and capital incomes, respectively. These indicate that complete elimination of capital taxation (and freezing the amount of government purchases and transfer payments at the benchmark levels) raises the tax on labor income from 40% to 42.42%. The third column gives the aggregate capital stock; elimination of capital income taxation raises the capital stock by 18.13%. This result is of the same order of magnitude relative to those in the literature, although it is smaller than Lucas' (1990) finding of 32%. Table 1 also shows monotonic increases in labor income, due to the increase in the marginal product of labor arising from the higher capital stock. Aggregate output and consumption, and average utility also increase. The optimal tax on income from capital turns out to be zero. This result is consistent with a large body of literature on the Ramsey (1927) plan: it is optimal in the long run to drive the tax on capital income to zero.

Table 1. Eliminating the Tax on Capital Income, $\beta = 1.011$

τ_k	τ_n	Capital Stock	Return to Capital	Relative Wage	Aggregate Output	Aggregate Consumption	Average Utility
0.36	0.4000	2.6215	0.0338	2.0747	1.0079	0.7666	-121.7952
0.30	0.4042	2.7153	0.0321	2.0853	1.0110	0.7685	-121.4632
0.25	0.4077	2.7798	0.0310	2.0923	1.0130	0.7701	-121.1935
0.20	0.4110	2.8565	0.0297	2.1003	1.0154	0.7711	-121.0385
0.15	0.4143	2.9210	0.0287	2.1068	1.0172	0.7723	-120.8376
0.10	0.4178	2.9761	0.0279	2.1122	1.0188	0.7730	-120.7313
0.05	0.4209	3.0454	0.0269	2.1187	1.0207	0.7741	-120.5781
0.00	0.4242	3.0970	0.0262	2.1235	1.0221	0.7750	-120.4489

$G = 0.21, T = 0.1955.$

For the model economy used in Table 1, the golden rule steady-state capital stock is 5.1253 and the corresponding aggregate consumption is 0.7864. Figure 1 shows the consumption profiles from steady-states indexed with $\tau_k = 0.00$ and $\tau_k = 0.36$, and the optimal profile chosen by the planner, after the equilibrium consumption profiles are normalized to give the same aggregate consumption as that of the planner. In addition to the fact that the equilibrium with zero taxation of capital income gets closer to the golden rule steady-state capital stock than the equilibrium with 36% taxation of capital income, this graph indicates that it also generates a consumption profile which is closer to that chosen by the planner. When lifetime utility is evaluated using (5) after correcting for the size of aggregate consumption, lifetime utility in the equilibrium with zero taxation of capital income (-119.8980) is closer to the utility under the social planner's allocation (-119.1681) than that under the equilibrium with 36% capital income taxation (-119.9012). In other words, the capital stock effect and the consumption profile effect reinforce each other.

Table 2. Eliminating the Tax on Capital Income, $\beta = 0.98$

τ_k	τ_n	Capital Stock	Return to Capital	Relative Wage	Aggregate Output	Aggregate Consumption	Average Utility
0.36	0.4000	1.5467	0.0745	1.8940	0.9543	0.7257	-61.7609
0.30	0.4055	1.6132	0.0700	1.9099	0.9591	0.7298	-61.5674
0.25	0.4098	1.6697	0.0665	1.9228	0.9630	0.7329	-61.4319
0.20	0.4141	1.7243	0.0634	1.9346	0.9665	0.7357	-61.3273
0.15	0.4179	1.7880	0.0601	1.9477	0.9704	0.7390	-61.2088
0.10	0.4221	1.8370	0.0578	1.9574	0.9733	0.7413	-61.1397
0.05	0.4263	1.8805	0.0558	1.9657	0.9758	0.7429	-61.1287
0.00	0.4298	1.9422	0.0532	1.9770	0.9792	0.7458	-61.0487

$G = 0.21, T = 0.1725.$

Table 2 presents the findings from a model economy with $\beta = 0.98$. The main result is the same: the optimal tax on capital income is zero. The capital stock is lower, uniformly, relative to the $\beta = 1.011$ case since heavier discounting of the future with $\beta = 0.98$ generates a weaker saving motive. In this case, though, the capital stock effect and the consumption profile effect go in opposite directions. The consumption profile from zero taxation of capital income is farther away from that of the planner's profile than the profile from the equilibrium in which capital is taxed at 36%. This can be seen in Figure 2. Lifetime utility in the equilibrium with 36% taxation of capital income (-57.4826) is closer to the utility under the social planner's allocation (-54.4022) than that under the equilibrium with zero capital income taxation (-58.4087). However, the capital stock effect dominates: the increase in the capital stock from 1.5467 to 1.9422 brings the economy closer to the golden rule steady-state capital stock at which aggregate consumption is maximized. This benefit apparently outweighs the unfavorable reallocation of consumption over the life cycle when the tax on capital income is eliminated.¹⁷

¹⁷ It is worth emphasizing that the main numerical findings in the benchmark case and those that follow are robust to changes in the subjective discount factor.

Given these optimality results which are in line with the findings of a large body of literature in this area, a natural question is "What is the welfare benefit of eliminating the tax on capital income?" Table 3 shows the welfare benefits from the elimination of capital income taxation in the base case, and in some alternative parameter configurations. In the base case, the increase in the capital stock from eliminating capital income taxation is 18.13%. The impact on the capital stock gets larger as we move to smaller subjective discount factors, and higher elasticities of substitution in production. For example, in the Cobb-Douglas case with $\beta = 0.98$, the capital stock increases by 40.66% as a result of the elimination of capital income taxation. However, the welfare benefits from this tax reform appear to be small in all cases, with the largest benefit at 2.82% of consumption. In the case which is more comparable with Lucas' (1990) results, the welfare benefit is only 1.12% of aggregate consumption, much less than the 6% figure calculated by Lucas (1990). This indicates that the potential benefits of eliminating capital income taxation by making the only self-insurance instrument more attractive, labor more productive, and directly increasing output, seem to barely outweigh the potential cost from this tax reform in the form of making the liquidity constraints more likely to bind due to higher labor income tax rates. Put differently, the benefit does not appear to be as great since there is already a strong precautionary saving motive (primarily) to self-insure. When the tax on capital income is eliminated and the after-tax return to capital increased, individuals raise their capital holdings by a "small" amount since the pre-reform capital holdings are already high due to the strong precautionary saving motive. Furthermore, there is a potential cost to eliminating capital income taxation in the form of the increased tax on labor income of individuals who are facing borrowing constraints.

Table 3. Increase in the Capital Stock and Welfare Benefits

	% $\uparrow$ in K	Adjusted τ_n	Welfare Benefit
$\theta = 0.6, \beta = 1.011$	18.13	0.4242	1.12
$\theta = 0.6, \beta = 0.980$	21.10	0.4282	1.18
$\theta = 1.0, \beta = 1.011$	27.12	0.4832	2.82
$\theta = 1.0, \beta = 0.980$	40.66	0.4717	2.46

The Role of Uncertain Lifetimes

When individuals in the economy face uncertain lifetimes, there are two opposing effects on the desired capital stock. First, the presence of lifetime uncertainty provides a precautionary motive, over and above that induced by the self-insurance motive, to increase private saving to provide for future consumption in the event that the individual gets a lucky draw from nature and lives longer than expected. This clearly raises the capital stock. At the same time, however, the presence of mortality risk implies that the individual is discounting the future more heavily relative to the case of certain lifetimes. This works to reduce private saving. Therefore, when there is lifetime uncertainty in the model, its *ex ante* effect on the capital stock is ambiguous. Under borrowing constraints, the additional precautionary saving motive induced by lifetime uncertainty implies a reallocation of consumption away from young working years to old years. Hence, the consumption profile under lifetime uncertainty is likely to be farther away from that of the planner's relative to the case of lifetime certainty.

In order to assess the role of lifetime uncertainty in the present setting, I conducted two experiments. In the first experiment, I computed the steady-state equilibria in a model without lifetime uncertainty. The results are given in Table 4.

Table 4. No Lifetime Uncertainty, $\rho = 1.2\%$, $\beta = 1.011$

τ_k	τ_n	Capital Stock	Return to Capital	Labor Income	Aggregate Output	Aggregate Consumption	Average Utility
0.36	0.4000	3.5276	0.0206	2.1660	1.0070	0.7547	-158.2610
0.30	0.4037	3.6131	0.0199	2.1722	1.0087	0.7554	-158.1439
0.25	0.4070	3.6632	0.0194	2.1757	1.0097	0.7557	-158.1097
0.20	0.4101	3.7165	0.0190	2.1794	1.0107	0.7561	-158.0744
0.15	0.4132	3.7768	0.0185	2.1835	1.0119	0.7565	-158.0516
0.10	0.4160	3.8460	0.0180	2.1880	1.0131	0.7570	-158.0199
0.05	0.4192	3.8843	0.0177	2.1905	1.0138	0.7572	-158.0400
0.00	0.4221	3.9376	0.0174	2.1939	1.0147	0.7575	-158.0487

$G = 0.21, T = 0.1959.$

In this case, it turns out that the tax on capital income is part of the optimal tax package. The average utility attains its maximum at a capital income tax rate of 10%. Note that the young individuals in the economy start out poor; so when the tax on capital income is reduced, the benefit to them is small. But the increase in the labor income tax necessary to keep the government purchases and transfer payments constant imposes a high cost on them due to borrowing constraints. This hinders their ability to smooth their lifetime consumption path for a given elasticity of substitution in consumption.

Note that equilibrium consumption grows more rapidly over the life cycle relative to the profile that the planner chooses. As the tax on capital income is reduced, capital accumulates and the return to capital falls. Everything else constant, this would tend to make the individual's profile flatter than that of the planner's. The revenue-neutral nature of the fiscal policy change from the simultaneous decrease in the tax on capital income and the increase in the tax on labor income redistributes income away from borrowing constrained young working years toward middle-to-old-aged years during which there is high capital ownership. This costs appear to outweigh the benefit they get when they accumulate more capital at a lower tax rate on capital income. Figure 3 shows three equilibrium consumption profiles and the social planner's optimal consumption allocation over the life cycle. Capital income taxation at 36% yields the consumption profile closest to the optimal allocation. However, the capital stock at this high capital income tax rate is almost 10% lower than the capital stock at the equilibrium with $\tau_k = 0.10$. In the absence of lifetime uncertainty, it seems that the profile effect is large enough to make it optimal to tax capital income.

In the second experiment that explores the role of lifetime uncertainty in determining the optimal tax structure, I assume the existence of private annuity markets to insure against uncertain lifetimes. In particular, I follow Rios-Rull (1991) and assume that individuals agree to write contracts that stipulate the distribution of accidental bequests to the survivors of the same cohort in proportion to their asset holdings. This simple annuity contract raises the rate of return to saving for the survivors.¹⁸ Table 5 shows that the capital income tax is part of the optimal tax structure once more. A capital income tax rate of 10% yields maximum

¹⁸ Let y_j denote the beginning of period, post-distribution holdings of capital. Then, $y_j = \frac{a_{j-1}}{\psi_j}$ and the budget constraint (2) becomes $c_j + a_j = [1 + (1 - \tau_k)r]y_j + q_j + \zeta_1 + \zeta_2$.

utility. Just as in the case of lifetime certainty, the consumption profile under a 36% tax on capital income is closer to the optimal profile than any other equilibrium consumption profile. Figure 4 displays these profiles. However, the equilibrium with 36% taxation of capital income suffers from a low capital stock. When the tax rate on capital income is reduced to 10%, the capital stock increases by over 10%. This turns out to outweigh the favorable allocation of consumption over the life cycle in the equilibrium with 36% taxation of capital income. When the tax on capital income is reduced any further, however, the departure from the optimal consumption profile becomes too great to be outweighed by a higher capital stock. Since lowering of the tax on capital income necessarily raises the tax on labor income, the borrowing constraints presumably have a first order effect when the capital income tax is further reduced.¹⁹

Table 5. Uncertain Lifetimes with Annuity Markets, $\rho = 1.2\%$, $\beta = 1.011$

τ_k	τ_n	Capital Stock	Return to Capital	Relative Wage	Aggregate Output	Aggregate Consumption	Average Utility
0.36	0.4000	2.9214	0.0287	2.1068	1.0173	0.7725	-120.5702
0.30	0.4044	2.9827	0.0278	2.1128	1.0190	0.7735	-120.5082
0.25	0.4078	3.0492	0.0269	2.1191	1.0208	0.7746	-120.4507
0.20	0.4114	3.1013	0.0262	2.1239	1.0222	0.7754	-120.4271
0.15	0.4148	3.1526	0.0255	2.1285	1.0235	0.7762	-120.4164
0.10	0.4180	3.2183	0.0247	2.1342	1.0252	0.7770	-120.4081
0.05	0.4215	3.2573	0.0243	2.1375	1.0261	0.7776	-120.4315
0.00	0.4247	3.3155	0.0236	2.1424	1.0275	0.7783	-120.4549

¹⁹ It should be mentioned that the optimality of capital income taxation in the economies with no lifetime uncertainty or in economies with insurance markets for lifetime uncertainty does not arise by way of removing dynamic inefficiencies. All of the above economies are dynamically efficient in the sense of Diamond (1965); there is no overaccumulation of capital since the rate of return to capital exceeds the economy's growth rate in every case. The optimality result is driven by the trade-off between the capital stock effect and the profile effect.

VI.2 Optimal Tax Structure: Consumption, Labor and Capital Taxation

In this section, I report the findings from two experiments designed to evaluate the steady-state effects of introducing a proportional tax on consumption. Given the benchmark case of $\tau_k = 0.36$, $\tau_n = 0.40$, $\tau_c = 0.00$, the first alternative scheme replaces the tax on capital income with a consumption tax. The second scheme replaces both labor and capital income taxes with a consumption tax.

Table 6. Alternative Tax Bases and the Aggregate Economy $\beta = 1.011$

τ_k	τ_n	τ_c	r	K	Q	C	U	λ
0.36	0.40	0.00	0.0338	2.6215	1.0079	0.7666	-121.80	0.00%
0.00	0.40	0.03	0.0258	3.1352	1.0231	0.7755	-120.36	0.80%
0.00	0.00	0.52	0.0191	3.8102	1.0380	0.7835	-119.44	1.98%

λ is the consumption equivalent of welfare benefits.

The main finding is that a pure consumption tax provides the highest utility among the tax bases considered. Capital stock and utility measures are at their highest under this tax base. When a consumption tax replaces the tax on capital income, the capital stock rises by 19.6% in the $\beta = 1.011$ economy and by 26.3% in the $\beta = 0.98$ economy. The tax on consumption that exactly replaces the revenue lost from the elimination of capital income tax turns out to be 3% in the $\beta = 1.011$ case and 4% in the $\beta = 0.98$ case. The welfare benefits, however, are not very large. Table 6 reports 0.80% of consumption as the compensation required by an individual to be indifferent between the benchmark economy and the first alternative in which consumption replaces capital income in the tax base.

When the consumption tax replaces both labor and capital income taxation, the required consumption tax turns out to be about 50%. The capital stock goes up by about 45.3% (42.3% in the $\beta = 0.98$ economy). This switch produces a welfare benefit equal to 1.98% of consumption. The welfare benefits become quite large, however, when the subjective discount factor is taken to be 0.98 instead of 1.011. Table 7 reports a welfare benefit of 4.16% from switching to a consumption tax from the current system.²⁰

²⁰ Although these results are not directly comparable due to many differences in the models and

Table 7. Alternative Tax Bases and the Aggregate Economy $\beta = 0.98$

τ_k	τ_n	τ_c	r	K	Q	C	U	λ
0.36	0.40	0.00	0.0745	1.5467	0.9543	0.7257	-61.76	0.00%
0.00	0.40	0.04	0.0527	1.9529	0.9797	0.7464	-60.85	2.62%
0.00	0.00	0.51	0.0441	2.2003	0.9917	0.7555	-59.31	4.16%

In order to understand the reason why the consumption tax is optimal, I computed the normalized consumption profiles under the three tax schemes in Table 6. Figure 5 shows these equilibrium consumption profiles together with the planner's optimal consumption profile. The equilibrium profile closest to the social planner's optimal profile turns out to be from the economy in which the tax base consists of labor income and consumption. The utility associated with this profile is calculated to be -119.8682. The consumption profile from the economy where the tax base consists of consumption only yields a utility level of -120.0950. This implies that the positive capital stock effect from switching to a consumption tax base is partially offset by a negative consumption profile effect.

VI.3 Sensitivity Analysis

The Role of Intertemporal Elasticity of Substitution in Consumption

Lucas (1990) finds that his numerical findings are fairly robust to changes in the intertemporal elasticity of substitution in consumption, but that they depend critically on the elasticity of leisure.²¹ In this section, I report the findings of a sensitivity analysis which

their calibration, it is still useful to present some of the previous findings. Auerbach and Kotlikoff (1987) estimate that replacing a 30% tax on all income with a consumption tax raises the capital stock by 61%. Summers (1981) estimates that replacing a capital income tax of 50% and a labor income tax of 20% with a consumption tax increases steady-state consumption by 17.2% compared to a 4.1% increase in Table 7.

²¹ In the previous section, I described how Lucas's (1990) results and those of the present paper

indicate that when the elasticity of leisure in the utility function is taken to be 5.0 instead of 0.5 as in the benchmark economy, the results from the benchmark case do not change significantly. This is not surprising since labor is inelastically supplied in this model. On the other hand, when an intertemporal elasticity of substitution in consumption equal to 0.25 is used instead of the benchmark value of 0.50, the capital income tax is part of the optimal tax package. Furthermore, the optimal tax rate on capital income rises when the individuals do not face lifetime uncertainty or when they can insure against lifetime uncertainty.

Table 8. Uncertain Lifetimes, $1/\gamma = 0.25, \beta = 1.011$

τ_k	τ_n	Capital Stock	Return to Capital	Relative Wage	Aggregate Output	Aggregate Consumption	Average Utility
0.36	0.4000	1.9465	0.0530	1.9777	0.9794	0.7460	-134.1458
0.30	0.4057	1.9931	0.0512	1.9859	0.9818	0.7479	-133.5408
0.25	0.4101	2.0376	0.0495	1.9935	0.9841	0.7496	-133.1315
0.20	0.4145	2.0814	0.0479	2.0007	0.9862	0.7512	-132.8002
0.15	0.4189	2.1179	0.0467	2.0065	0.9879	0.7525	-132.5483
0.10	0.4234	2.1493	0.0458	2.0114	0.9894	0.7536	-132.5181
0.05	0.4280	2.1785	0.0448	2.0159	0.9907	0.7545	-132.5645
0.00	0.4325	2.2055	0.0440	2.0200	0.9919	0.7555	-132.6206

Table 8 presents the findings from the benchmark economy with an elasticity of substitution in consumption equal to 0.25. Even though there is uncertainty about the length of life, it is now optimal to tax income from capital. Given a lifetime earnings profile, the precautionary saving motive from self-insurance and uncertain lifetimes induces individuals to shift the bulk of the lifetime consumption toward the end of the life cycle for a given

change when the elasticity of substitution in production is changed.

intertemporal elasticity of substitution in consumption. When this elasticity is lowered from 0.50 to 0.25, the individual is motivated to obtain more consumption smoothing and has a diminished desire to shift the bulk of consumption toward the end of the life cycle. This reduced desire to save is reflected by uniformly smaller capital stocks across the comparable model economies in Tables 1 and 8. For example, at the benchmark case of $\tau_k = 0.36$ and $\tau_n = 0.40$, capital stock decreases from 2.6215 with an intertemporal elasticity of substitution of 0.50 to 1.9465 with an intertemporal elasticity of substitution of 0.25. Given that a revenue-neutral fiscal policy change in the form of eliminating the tax on capital income and raising the tax on labor income redistributes income away from borrowing constrained young working years, the individuals will be more reluctant to a further reallocation of consumption away from the early years in life since their desire to undertake such a shift is reduced with a lower intertemporal elasticity of substitution in consumption. Hence, their response to the revenue-neutral fiscal policy change is to increase their capital stock more sluggishly relative to the economy with a higher intertemporal elasticity of substitution in consumption. In other words, the benefit in terms of a higher capital stock from the elimination of the tax on capital income is now smaller. When this smaller capital stock benefit is combined with the higher profile cost from the elimination of the tax on capital income, the tax on capital income becomes part of the optimal tax package.²²

Table 9. Certain Lifetimes, $1/\gamma = 0.25, \beta = 1.011$

τ_k	τ_n	Capital Stock	Return to Capital	Relative Wage	Aggregate Output	Aggregate Consumption	Average Utility
0.45	0.3925	2.3567	0.0382	2.0500	0.9743	0.7360	-177.4952
0.40	0.3967	2.4060	0.0370	2.0565	0.9761	0.7372	-177.2232
0.36	0.4000	2.4415	0.0362	2.0610	0.9774	0.7381	-177.1306
0.30	0.4052	2.4761	0.0354	2.0654	0.9787	0.7389	-177.3322
0.25	0.4096	2.5010	0.0349	2.0685	0.9795	0.7395	-177.5916

²² Using a subjective discount factor of 0.98 yields very similar results.

Table 9 reports the numerical results from an economy with certain lifetimes and an intertemporal elasticity of substitution in consumption of 0.25. The reduced desire to shift consumption to later years in life is evidenced by a decline in the capital stock from 3.5276 for $\tau_k = 0.36$ in Table 4 to 2.4415 for $\tau_k = 0.36$ in Table 9. Figure 6 shows the equilibrium consumption profiles with zero and 36% taxation of capital income, and the optimal consumption profile chosen by the social planner. In equilibrium, consumption grows even more rapidly (compared to the same economy with $1/\gamma = 0.50$) relative to the social planner's optimal consumption profile. In other words, the profile cost of eliminating the tax on capital income seems to be greater when the intertemporal elasticity of substitution in consumption is reduced. In this case, there is an increase in the optimal tax on capital income from 10% to 36% since the absence of lifetime uncertainty magnifies the profile cost of eliminating the tax on income from capital. The decrease in the intertemporal elasticity of substitution in consumption further increases this profile cost. But now the capital stock benefit is lower relative to the case of a high elasticity of substitution. The overall impact is an increase in the optimal tax rate on capital income.²³

An Alternative Earnings Profile

The shape of the earnings profile is crucial to the age-wealth profile and influences the shape of the consumption profile. So far, all numerical findings rely on the earnings profile calculated by Hansen (1991). An alternative profile is computed by Welch (1979) who obtains estimates of the profile by regressing weekly earnings on a quadratic polynomial in experience. The estimates imply an earnings profile plotted in Figure 7 together with that computed by Hansen (1991). Although there is some difference in the shapes of these profiles, the numerical findings of this paper do not change significantly across the two efficiency indices.

The results from using Welch's index in the benchmark economy are quantitatively very similar to those obtained using Hansen's index.²⁴ The results under consumption taxation are not very different either. The only minor difference in this case is the small reduction in the welfare benefits from switching to a consumption tax. This is probably due to the fact

²³ When lifetime uncertainty is allowed to be present in the model together with a private annuity market of the type mentioned above, the results are almost identical.

²⁴ These results are available from the author on request.

that a typical young worker receives higher earnings for about the first 10 working years and therefore borrowing constraints are less binding under Welch's index. As a result, reducing the tax on labor income and switching to a consumption tax provides less of a relief from the borrowing constraints.

Persistence in the Idiosyncratic Employment Shock

So far, the idiosyncratic employment shock has been assumed to be identically and independently distributed. This assumption implies an expected duration of unemployment of $1/(1 - 0.06) = 1.0638$, given an employment rate of 94%. As an alternative, the expected duration of unemployment is raised by 50% from 1.0638 to 1.5957, and the transition matrix for the Markov process is calibrated based on the new expected duration of unemployment and the same employment rate of 0.94. The new expected duration implies that $\pi_{uu} = 0.3733$ from the definition *duration of unemployment* $= 1/(1 - \pi_{uu})$. π_{ue} is then equal to $1 - 0.3733 = 0.6267$. π_{ee} is found from $0.94\pi_{ee} + 0.06\pi_{ue} = 0.94$ to be 0.96. π_{eu} is 0.04. The resulting Markov transition matrix is

$$\Pi(s', s) = \begin{bmatrix} 0.9600 & 0.0400 \\ 0.6267 & 0.3733 \end{bmatrix}.$$

The findings in this case are very similar to the benchmark i.i.d. case. Hence, a 50% increase in the expected duration of unemployment and the resulting persistence in the idiosyncratic employment shock (given a constant employment rate) leaves the numerical findings largely unchanged.²⁵

VI.4 Reallocation of Consumption and Wealth over the Life Cycle

To reflect the effects on the overall distribution of consumption and wealth, I will use the *coefficient of variation*. In the benchmark model economy, these are 0.0444 and 0.1337, respectively. There is little variation in consumption relative to that in wealth. When a consumption tax is used to replace the capital income tax, there is little change in the distribution of consumption. The distribution of wealth, however, shrinks to 0.1196. Replacing both the capital and labor income taxes with a consumption tax slightly raises the coefficient of variation in consumption to 0.0495 and increases the coefficient of variation in wealth to 0.1586.²⁶ Overall, a consumption tax appears to result in a more widespread distribution

²⁵ These results are available from the author on request.

²⁶ The distributional findings are very similar for the $\beta = 0.98$ case.

than either of the two alternative tax bases.

In order to summarize the implications of different tax bases on the allocation of consumption and wealth over the life cycle, I consider five broad age categories: the very young years when individuals are between 21 and 33, the young years between 34 and 46, the middle-aged years between 47 and 59, the old years between 60 and 72, and the very old years between 73 and 85.

Table 14. Reallocation of Consumption and Wealth, $\beta = 1.011$

Age	Tax Base					
	Capital and Labor Income		Labor Income and Consumption		Consumption	
	<i>C</i>	<i>K</i>	<i>C</i>	<i>K</i>	<i>C</i>	<i>K</i>
21-33	0.5717	0.6254	0.5757	0.6254	0.6145	0.6298
34-46	0.7195	3.0179	0.7214	3.0390	0.7518	3.5802
47-59	0.8874	5.2722	0.8867	5.3486	0.8867	6.6469
60-72	1.0256	5.4933	1.0216	5.5845	0.9803	7.0201
73-85	0.9955	2.7167	0.9890	2.7601	0.9122	3.4081

Table 14 reports the (weighted) average consumption and wealth for each of the five age categories. Replacing capital income taxation with a tax on consumption has very little impact on the reallocation of consumption and wealth. However, when a consumption tax replaces labor and capital income taxes, consumption is shifted toward early years of life from the very old and old years of life. The wealth profile, on the other hand, shifts up almost uniformly, with the exception of the very young years.

VII. CONCLUDING REMARKS

In this paper I investigate the optimal tax structure in an overlapping generations model with lifetime uncertainty, idiosyncratic income risk and borrowing constraints. The model economy consists of overlapping generations of 65-period-lived individuals. Individuals are subject to borrowing constraints; together with the idiosyncratic employment risk, this brings about a strong precautionary saving motive and a heterogeneous-agent framework in which individuals differ not only with respect to age but also with respect to employment status and hence by accumulated wealth. Taxation of capital income is not desirable in this model because of the distortion on private saving and the consequent negative impact on the capital stock, output and aggregate consumption. Taxing labor income is also not desirable despite the inelastic supply of labor, since the increase in the labor income tax hinders the individuals' ability to self-insure and to provide for old age consumption. Since the individuals are liquidity constrained, higher labor income taxes make it more likely that the constraints are binding.

The model economy is calibrated to match certain features of aggregate U.S. data and numerical methods are used to solve the individuals' finite-state, finite-horizon, discounted dynamic programs and to compute steady-state equilibria. The benchmark economy is one in which there is taxation of labor income (and unemployment insurance benefits) and capital income. The exogenous government purchases, which provide no utility to the individuals, and endogenously determined government transfer payments are held constant in the face of tax reform. Different tax reforms are examined. First, the tax on capital income is eliminated, and the labor income tax is increased. Second, the same amount of government purchases and transfer payments are financed by gradually eliminating capital income taxation or labor income taxation and introducing a tax on consumption.

The main finding is that moving away from capital income taxation toward labor income taxation yields a welfare benefit of 1% of aggregate consumption compared to the 6% benefit that Lucas (1990) finds. Replacing the capital income tax with a higher tax rate on labor income redistributes resources away from the young working years during which borrowing constraints are more likely to bind. Furthermore, when the individuals have access to a private annuity market to insure against lifetime uncertainty, the optimal capital income tax is 10%. Although eliminating this tax brings the economy closer to the golden rule steady-state capital stock which maximizes aggregate consumption, the simultaneous increase in

the labor income tax rate produces an equilibrium consumption profile that is farther away from that chosen by the social planner. A lower elasticity of intertemporal substitution in consumption increases the optimal capital income tax rate to 36% since the profile cost increases and capital stock benefit decreases with a decline in the elasticity of substitution in consumption. When a consumption tax is made available, it becomes optimal to switch to consumption taxation. This is very much in line with a wide body of findings in the optimal tax literature. The welfare benefits of implementing this optimal tax plan are on the order of 2-4% of aggregate consumption. At the same time, a consumption tax leads to a worsening of inequality of wealth as measured by the coefficient of variation. Under any tax base, the variability in consumption is small relative to that in wealth.

This paper has focused on a steady-state evaluation of the optimal tax structure in an overlapping generations model with incomplete markets. It has not addressed a number of interesting short run issues. These questions can only be addressed by computing an equilibrium transition path from one steady-state to another, as in Auerbach and Kotlikoff (1987), which would open up new avenues to re-examine the previous results in an incomplete markets setting. This computationally demanding task is left for future research.

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Figure 1. Consumption Profiles. $\beta = 1.011$

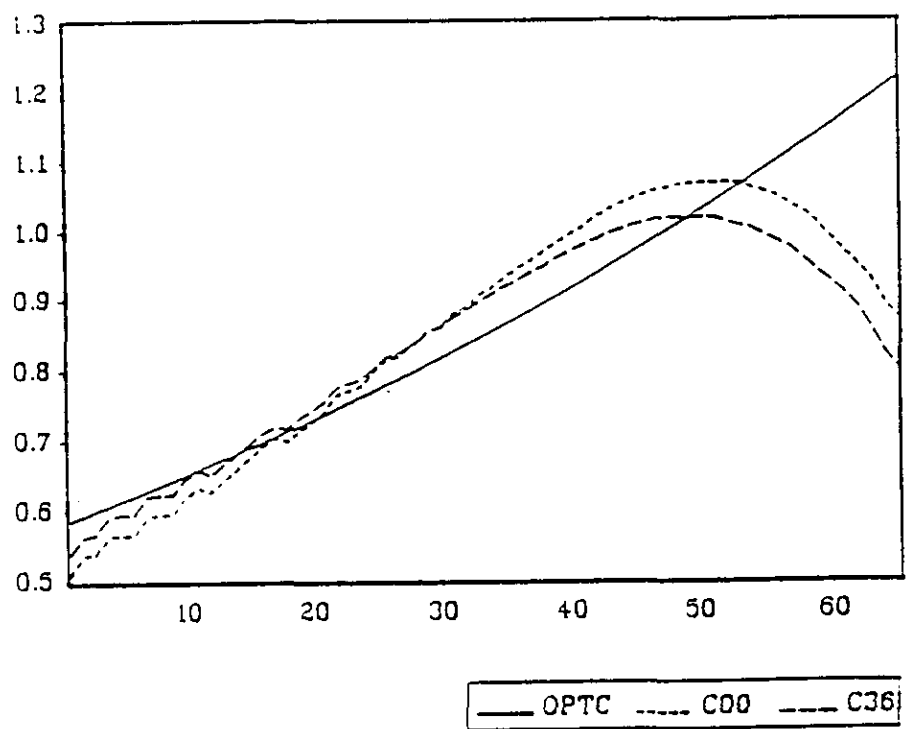


Figure 2. Consumption Profiles, $\beta=0.98$

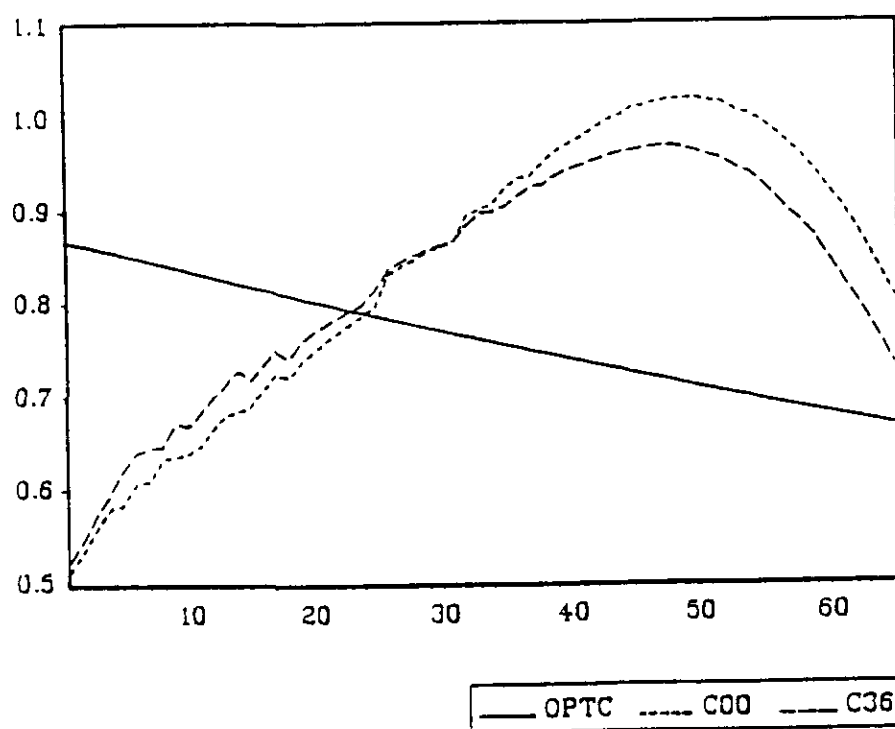


Figure 3. Consumption Profiles, $\beta=1.011$
No Mortality Risk

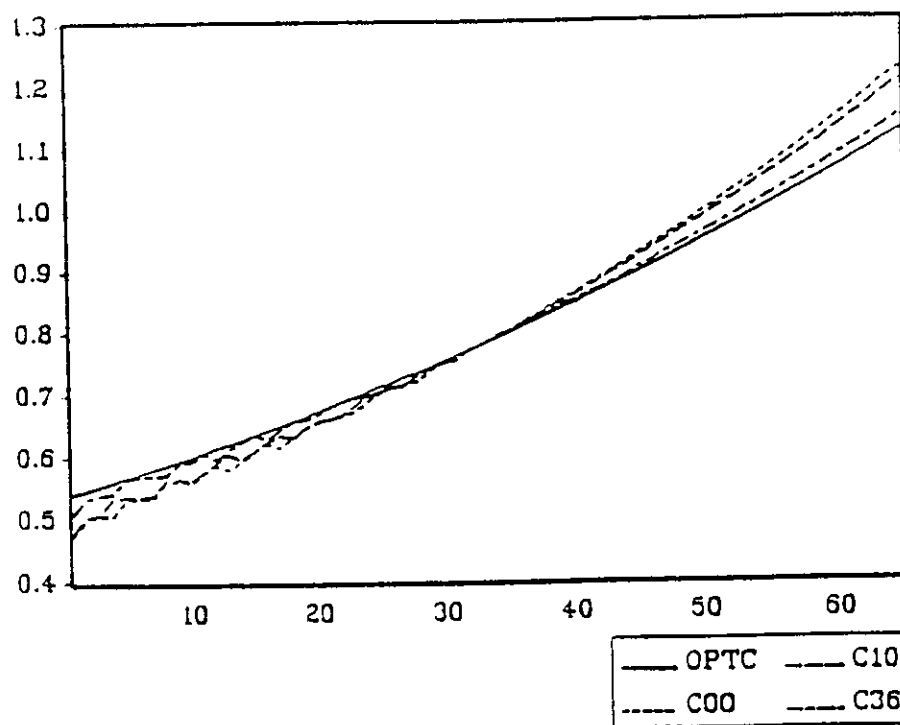


Figure 4. Consumption Profiles, $\beta=1.011$
Private Annuity Markets

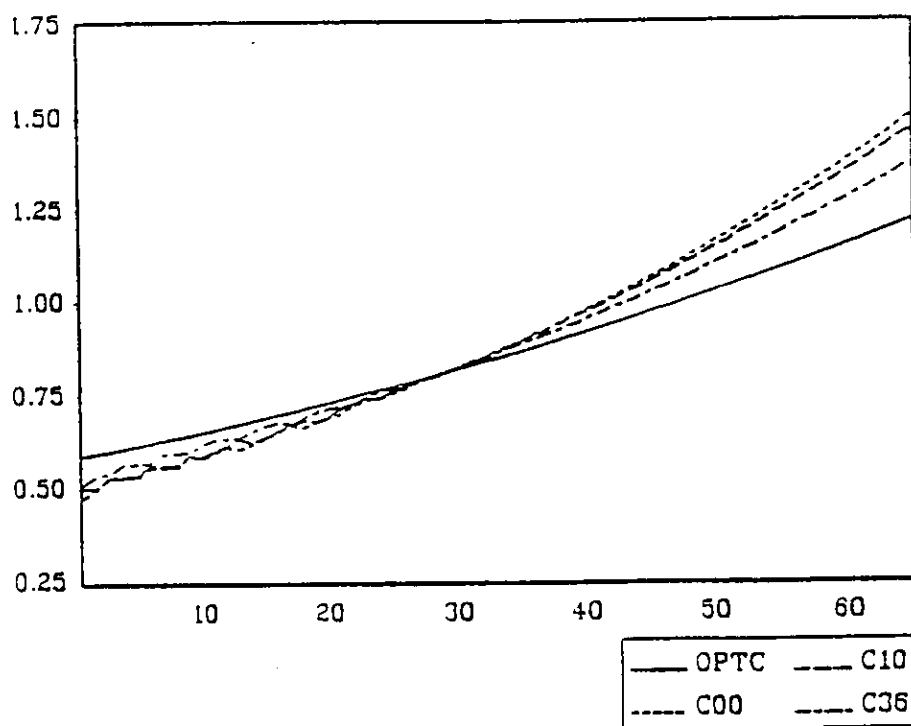


Figure 5. Consumption Profiles Under Alternative
Tax Bases

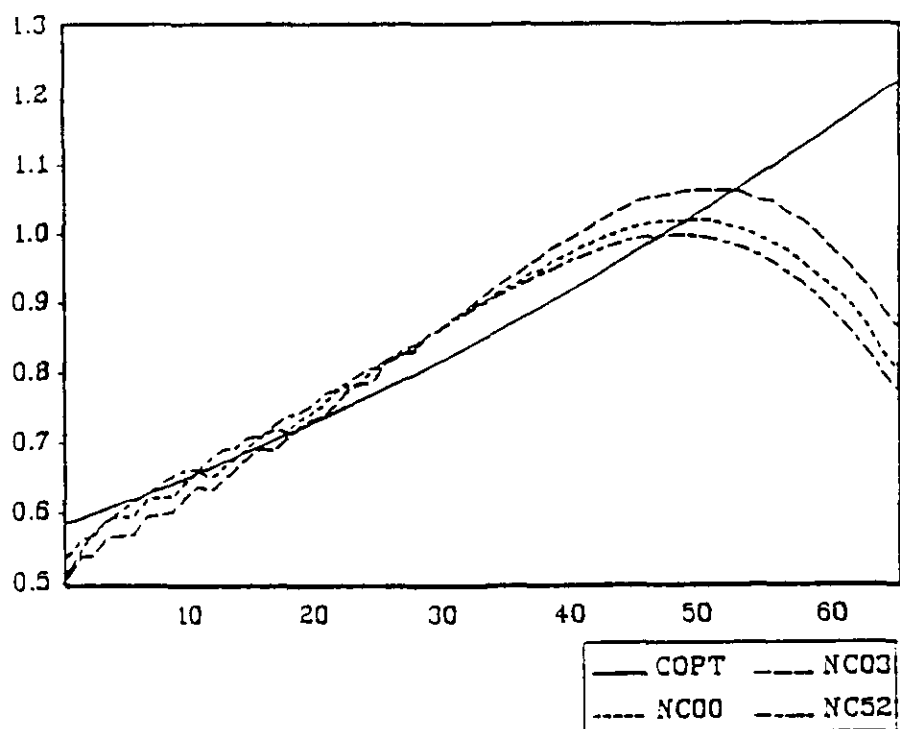


Figure 6. Consumption Profiles with $1/\gamma=0.25$

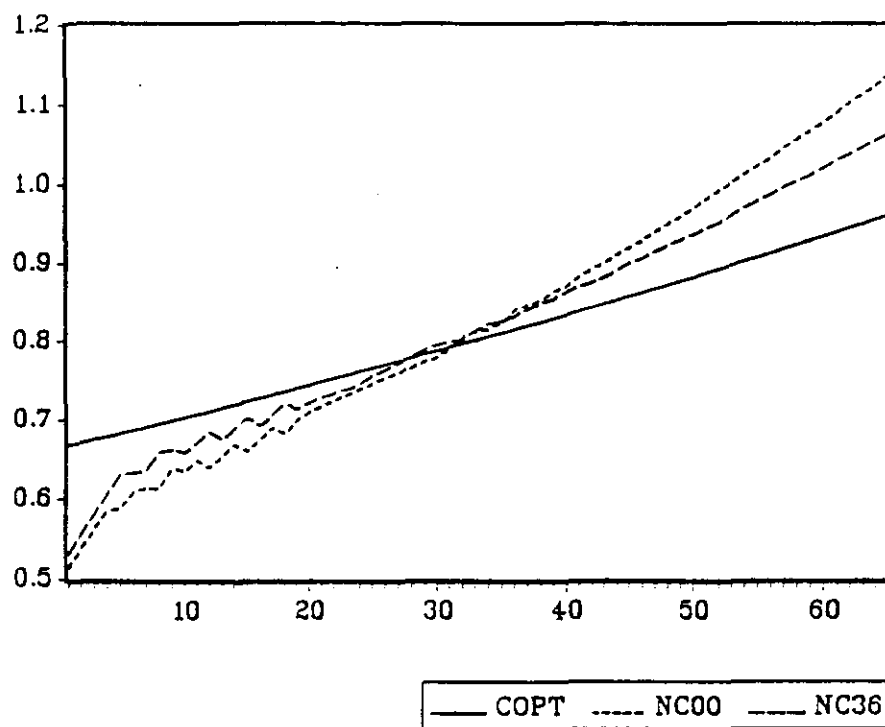


Figure 7. Alternative Efficiency Indices

