Credit Cycles

by

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Not for Quotation

1. Introduction

This paper is a theoretical study into how credit constraints interact with aggregate economic activity over the business cycle. In particular, for an economy where credit limits are endogenously determined, we investigate how relatively small, temporary shocks to technology or income distribution might generate large, persistent fluctuations in output and asset prices. Also we ask whether sector-specific shocks can be contagious, in the sense that they spill over to other sectors and get amplified through time.

For this purpose, we construct a model of a dynamic economy in which credit constraints arise naturally, due to the fact that lenders cannot force borrowers to repay their debts unless the debts are secured.[1] In such an economy, fixed assets such as land, buildings and machinery play a dual role: they are not only factors of production, but they also serve as collateral for loans. Borrowers' credit limits are affected by the price of the collateralized assets. And at the same time, the price of these assets is affected by the size of the credit limits. The dynamic interaction between credit limits and asset prices turns out to be a powerful transmission mechanism by which the effects of shocks persist, amplify, and spill over to other sectors.

The transmission mechanism works as follows. Consider an economy in which land is used to secure loans. Some firms are credit constrained; others are not. Suppose that in some period t, the constrained firms in a particular sector experience a temporary productivity shock which reduces their internal funds. Given that their borrowing is constrained, the firms cut back on investment expenditure, including investment in land. The price of land then falls in response to the reduction in demand. In turn, the fall in price lowers the value of the firms' existing collateral, which tightens their credit limits, further reduces their available funds, and causes them to cut back their investment still more. Through this multiplier process,

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[1] The specific model of debt which we use is a simple version of that in Hart and Moore (1991).

the impact of the initial shock is amplified.[2] The effects do not stop here, though. The reduction in investment hurts the firms in the next period: not only do they earn less revenue because they have cut back their scale of operation, but they also have a smaller asset base against which to borrow. The temporary shock in period t therefore has a knock-on effect in period t+1 -- and, indeed, in subsequent periods. Figure 1 summarises the discussion so far.
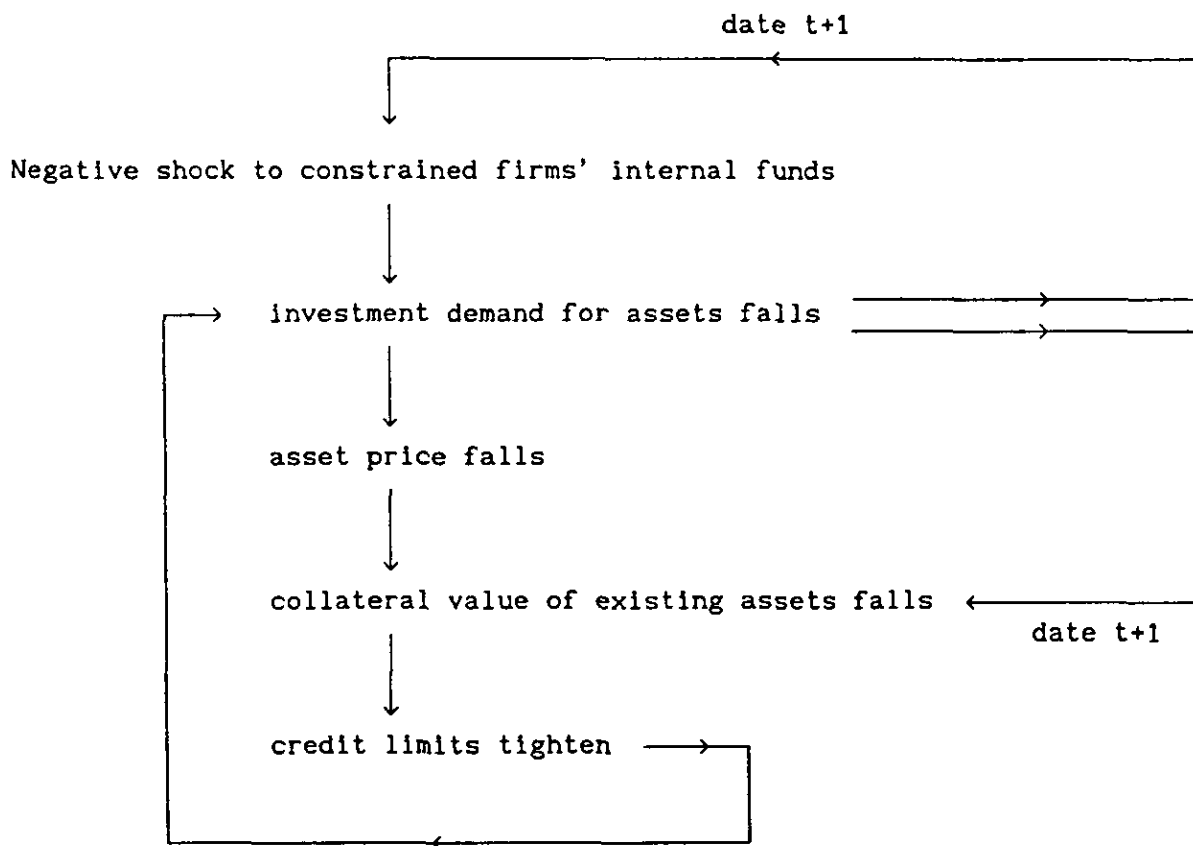
date t+1

Negative shock to constrained firms' internal funds

investment demand for assets falls

asset price falls

collateral value of existing assets falls

date t+1

credit limits tighten

Figure 1

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[2] Of course, the sequence of events we have described is a fiction, because everything occurs instantaneously within period t.

Credit constrained firms in other sectors are affected too, even though they may not have experienced any shock of their own. The reason is that the fall in land price lowers the value of their collateral, and their credit limits tighten. Consequently these firms also cut back on investment, which means that there is an additional force pushing down the land price. In other words, the effects of a shock are contagious: they spread out and build up over time.

The downturn is arrested as the firms which are not credit constrained buy up cheap land. Eventually, the price of land starts to rise, which raises the collateral value and borrowing limits of the constrained firms. Added to this, the internal fund position of the constrained firms begins to ease, because whatever land investments they have made have been at a low price. In time, then, the constrained firms find that they are able to expand their investment levels. The downturn is followed by an upturn, characterised by rising investment and land price.

The first model we construct is of an economy in which the stock of the collateralizable asset, land, is fixed. We find that in such an economy, a cyclical pattern emerges: recessions lead to booms, and booms lead to recessions. A simple way to understand why the economy cycles is to use the analogy of a predator-prey model. Imagine populations of deer and wolves. If the deer population rises, the wolves that feed on them also multiply. However, as the wolves grow in number, they kill off the deer. Eventually, the deer population falls, which means that fewer wolves can survive. But with fewer wolves, the deer population can in time start to grow again; and so on. That is, away from steady state, the two populations cycle, with the deer leading the wolves. Now the deer correspond to the land holdings K_t of the credit constrained firms, and the wolves correspond to their debts B_t. On the one hand, a rise in these firms' land holdings means that they have more collateral against which to borrow $(\partial B_t/\partial K_{t-1} > 0)$: the deer feed the wolves. On the other hand, a high level of debt erodes the firms' available funds and curtails their investment in land $(\partial K_t/\partial B_{t-1} < 0)$: the wolves kill off the deer. In simulations of our model, we find that small, temporary shocks can easily lead to significant cycles that only slowly decay over

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time.[3] Because in equilibrium the credit constrained firms turn out to have a marginal product of land higher than that of the unconstrained firms, aggregate output moves together with amount of land held by the constrained firms. Given the fixed stock of land, average productivity also moves procyclically in the aggregate -- not because there are variations in the underlying technologies (aside from the initial shock), but rather because the change in land use has a compositional effect.[4] The land price leads the fluctuations in output, due to the forward-looking behavior of the unconstrained firms; and, as the deer-wolf analogy suggests, debt lags the cycle.[5]

In the latter part of the paper, we construct a second model in which a durable capital asset is produced, thus allowing the total stock of collateralizable asset to change. Here too cycles occur. While the economic logic is similar to that of the land model, we find that the capital-producing sector moves in tandem with the rest of the economy, which accentuates the overall fluctuations in output and asset price.

There is empirical evidence to support the view that investment decisions are not solely determined by the net present value of new projects, but are also affected by an investing firm's balance sheet position and the value of its collaterized assets. See, for example, Black and de Meza (1992); Black, de Meza and Jeffreys (1992); Evans and Jovanovic (1989); Fazzari, Hubbard and Peterson (1988); Gertler and Gilchrist (1992); Hoshi, Kashap and Scharfstein (1991); Hubbard and Kashap (1992); and Whited (1992). At the aggregate level, a number of studies have highlighted the importance of credit constraints in explaining fluctuations in activity; see in particular Bernanke (1983); Friedman (1982); Eckstein and Sinai (1986); Kashap, Stein and Wilcox

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[3]For some interesting economic applications of the predator-prey model, see Das (1993).

[4]This may shed light on why the aggregate Solow residual fluctuates so much over the business cycle.

[5]We also find evidence in our simulations that there are asymmetries in the speed of adjustment between the upturns and downturns: booms take longer than recessions.

(1993).[6]

The literature on financial structure and aggregate economic activity
is vast, and it would be unwise to attempt to review it here. Gertler
(1989) has written an excellent survey, which not only identifies and
clarifies the issues, but also provides an account of the historical
developments, from Fisher and Keynes onwards. For the most part, the recent
theoretical literature on financial constraints and the aggregate economy has
concentrated on the implications of private information on the part of
borrowers.[7] Although models with private information have proved to be a
fruitful way of thinking about financial market imperfections, the framework
quickly becomes quite complex when extended to a dynamic setting. Aside from
the fact that part of our aim is to build a simple dynamic model, our
perspective differs from the asymmetric information literature in that we are
interested in the role that durable capital plays as a collateralized asset,
whose price movement is an important component of the fluctuations in the net
worth of firms.[8]

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[6]Recently, land values have been experiencing huge fluctuations in Japan,
where a large part business investment is financed by bank loans secured
against real estate. In the Tokyo metropolitan area, land prices more than
doubled in the period 1986 to 1990, and then dropped by around 30% between
1990 and the start of 1993. (Throughout, the consumer price level was
stable.) To give some perspective: the magnitude of each capital gain and
capital loss on the land stock of the whole of Japan during these two periods
is comparable to the U.S. annual GDP (which is roughly twice as large as the
Japanese GDP). Meantime, in the early 1990's Japan has suffered the largest
recession since the first oil shock of 1973.

[7]See, for example, Bernanke and Gertler (1989); Gertler (1992); Green (1987);
Green and Oh (1992); Greenwald and Stiglitz (1988); Phelan (1992); and
Williamson (1987).

[8]Other recent papers that, like ours, consider enforcement problems in debt
contracts include: Bulow and Rogoff (1989); Black and de Meza (1992); Hart
and Moore (1989, 1991); Kehoe and Levine (1991); and Marimon and Marcet
(1991). Shleifer and Vishny (1992) also investigated an interaction
between liquidation values and debt capacity of firms using a three-period
partial equilibrium framework.

An interesting recent paper by Banerjee and Newman (1991a) has initiated a
literature on the question of how financial constraints affect growth and the
long run distribution of income. See, for example, Aghion and Bolton (1991);
Banerjee and Newman (1991b); Newman (1991); and Piketty (1992). This work is
related to ours in that financial constraints due to enforcement problems are

The paper has four more sections. Section 2 examines a "land economy", in which the stock of the collaterizable asset is fixed. A variant of this model is briefly discussed in Section 3. Section 4 concerns a "sea economy", in which the supply of durable capital (boats) is endogenized. Finally, Section 5 discusses directions for further research.

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modelled in a dynamic aggregate setting. However, the focus of these papers is different from ours. We are concerned with explictly analysing the nature of fluctuations around a steady state, and the role that the prices of assets play in amplifying these fluctuations.

2. Land Economy

Consider a discrete time economy with two goods, a durable asset k, and a nondurable commodity x. It is helpful to think of the durable asset as land, which does not depreciate and which has total stock $\bar{K}$. The nondurable commodity may be thought of as fruit, which grows on the land but which cannot be stored. At each date $t \geq 0$ there is a competitive spot market in which land is exchanged for fruit at a price of q_t. (Throughout, fruit is taken as the numeraire.) The only other market is a credit market in which agents write one-period debt contracts, specifying a return of R_t fruit at date t+1 in exchange for one unit of fruit at date t.[1]

There is a continuum of agents. Some are farmers, some are hunters; and both produce and eat fruit. Everyone lives forever and has the same risk neutral preferences

$$E_0 \left(\sum_{t=0}^{\infty} R^{-t} x_t \right) .$$

over consumption $\{x_t | t \geq 0\}$ of fruit -- where R > 1 is a constant, and the operator E_0 denotes expectations formed at t=0. Thanks to the linearity of preferences, the gross interest rate R_t always equals the time preference parameter R. Farmers and hunters differ only in their technologies. Both groups take one period to produce fruit from land. Hunters simply collect wild fruit off the land. Later we shall specify the hunters' (common) production function, but it is best to start with the farmers, who play the central role in the model.

Farmers grow fruit. This entails planting fruit in the land, to grow trees. Consider a particular farmer. We say that his (or her) land is <u>cultivated</u> if he has trees growing on it. The technology exhibits constant returns to scale: for any $t \geq 1$,

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[1] It will become clear that there are no gains from longer term debt contracts.

date t-1	date t

k_{t-1} cultivated land   ——————> λk_{t-1} land still cultivated
(these trees are alive)

$(1-\lambda)k_{t-1}$ land no longer cultivated
(these trees have died)

ak_{t-1} fruit attached to trees
(tradeable)

ck_{t-1} fruit fallen to ground
(nontradeable)

A fraction $(1-\lambda)$ of the trees are assumed to die, and so this part of the land is no longer cultivated. This does not mean that the land cannot be used; it may be used by hunters, or it may be cultivated again (possibly by another farmer). While we assume that the fruit ak_{t-1} attached to the trees can be harvested by anyone, the fruit ck_{t-1} that has fallen to the ground is bruised and cannot be transported to the market. Nevertheless, it can be consumed by the farmer. This is merely a device to finess the farmer's consumption decision: by making consumption technological we simplify the analysis.

To expand the scale of his operation, the farmer has to plant more fruit to grow more trees. Specifically, if at date t he wishes to increase his holding of cultivated land from λk_{t-1} to k_t, he must plant $\phi(k_t - \lambda k_{t-1})$ fruit, as well as acquire $(k_t - k_{t-1})$ more land.[2] However, we assume that a new investment opportunity to plant fruit only arises with probability π. With probability $(1-\pi)$, the farmer is unable to expand, so the scale of his operations is limited to λk_{t-1} and he sells off the $(1-\lambda)k_{t-1}$ uncultivated land. This probabilistic investment assumption simply captures the idea that

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[2]It may be that $\lambda k_{t-1} < k_t < k_{t-1}$, in which case the farmer will sell off $k_{t-1} - k_t$ land. Here, the farm is shrinking, because investment does not cover depreciation.

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In practice, at the level of the individual enterprize, investment in fixed assets is typically occasional and lumpy.

For future use, we make two assumptions on the underlying parameters:

Assumption A1: $\pi > 1 - \frac{1}{R};$ and

Assumption A2: $c > \frac{\phi}{\pi}(R-1)[1 - \lambda(1-\pi)].$

Assumption A1 says that the probability of a new investment opportunity is not too small. It is a weak condition, since R will typically be close to 1. Assumption A2 requires that a farmer's rate of consumption is not too small.

There are two further critical assumptions we make about farming. First, we assume that each farmer's technology is idiosyncratic, in the sense that he grows his own specific trees, and only he has the skill necessary for them to bear fruit. That is, one farmer's trees cannot be successfully tended by other farmers (e.g., they do not know how to prune them). This means that, having (literally) sunk the cost (in terms of fruit) of growing trees, there is wedge between the inside value to a farmer of his cultivated land and the outside value of the land to everyone else. Second, we assume that a farmer cannot precommit to tend his trees. In the language of Hart and Moore (1991), the farmer's specific human capital is inalienable.

The upshot of these two assumptions is that if a farmer has a lot of debt he may find it advantageous to threaten his creditors by withdrawing his labor and repudiating (i.e. tearing up) his debt contract. Creditors protect themselves from the threat of repudiation by securing their loans. A secured debt contract stipulates that in the event of a farmer repudiating, his assets become the property of the creditors, who are then free to dispose of them as they see fit. It is tempting to think that the threat having his assets liquidated would be enough to deter a farmer from ever repudiating in the first place. However, since the liquidation value (the outside value) of the assets is going to be less than what the assets would earn under his control (the inside value), the farmer can bribe creditors into letting him

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continue to use the assets. In effect, the farmer can renegotiate a smaller loan. The division of surplus in this renegotiation process is moot, but Hart and Moore (1991) give an argument to suggest that the farmer may be able to negotiate the debt down to the liquidation value of the assets.[3]

Creditors know of this possibility in advance, and so take care never to allow the size of the debt to exceed the value of the collateral. Specifically, if at date t the farmer has cultivated land k_t, then there are two inequality constraints imposed on the total amount b_t which he can borrow. When the loan is agreed, it must not exceed the value of the collateral:

$$(1) \qquad b_t \leq q_t k_t.$$

And when the loan is due for repayment at date t+1, the (gross) value of the debt, Rb_t, must not exceed the value of the collateral:

$$(2) \qquad Rb_t \leq (q_{t+1} + a)k_t.$$

Note that at date t+1, the value of the farmer's collateral comprises not only the value of the land, $q_{t+1}k_t$, but also the ak_t fruit attached to the trees, which can be harvested by anyone.[4] The collateral does not include

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[3]The case we have in mind is where the liquidation (outside) value is greater than the _share_ of the continuation (inside) value that creditors would get if the liquidation option were not available to them -- albeit that the liquidation value is less than the _total_ continuation value. In this case, the creditors' "outside option" (the option to liquidate) is binding, which pins down the division of surplus in the renegotiation process. (For a discussion of the noncooperative foundations of the so-called Outside Option Principal, see Sutton (1986).) In the interests of saving space, we do not rehearse the details of the Hart-Moore argument here.

[4]In Section 3 we examine a variant of the model where ak_t is not part of the collateral.

the ck_t fruit on the ground, which the farmer consumes.

It is worth noting that the basic reason why a farmer faces credit constaints is that he uses fruit as well as land as an input in production. And, unlike land, the fruit input is not collateralizable, because once the fruit is sunk it has no outside value. As a result, the farmer must be able to acquire the fruit from his own resources -- i.e., from his net wealth.[5] This problem does not face the hunters, since they only use land as an input in production. The hunters are thus not credit-constrained. Indeed, it turns out that hunters are always net creditors: they supply farmers with fruit as an input.

Later we show that assumption A1 implies that

(3) $\qquad q_t \; < \; (q_{t+1} + a)/R.$

Hence the tighter constraint on b_t is the inequality (1), and we can safely ignore (2).[6]

We now examine a farmer's choice of the size of his cultivated land holding k_t at date t when he has the opportunity to plant fruit to grow new trees -- which, recall, only happens with probability π. Suppose at date t-1 he had k_{t-1} cultivated land, and incurred a total debt of b_{t-1}. Then at date t he harvests ak_{t-1} fruit (in addition to the bruised fruit ck_{t-1} which he

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[5]Readers may wonder why farmers cannot find some other way to raise capital -- e.g., by issuing equity. Unfortunately, given the specific nature of a farmer's technology, and the fact that he can withdraw his labor, equity holders could not be assured that they would receive a dividend. Debt contracts secured on the farmer's assets are the only financial instrument which investors can rely on. The same considerations rule out partnerships between farmers, or larger farming cooperatives.

[6]In the variant of the model we look at in Section 3, where ak_t is not part of the collateral, the tighter constraint on b_t is the date t+1 inequality (2).

consumes), which, together with a new loan b_t, is available to cover the costs of planting, buying new land, and repaying the accumulated debt Rb_{t-1} (which includes interest). The farmer's flow of funds constraint is

$$(4) \qquad \phi(k_t - \lambda k_{t-1}) \;+\; q_t(k_t - k_{t-1}) \;+\; Rb_{t-1} \;\leq\; ak_{t-1} + b_t.$$

Notice that much (if not all) of the old debt is paid off by the new loan: in effect, the debt is rolled over. Any difference between the right hand side and the left hand side of (4) is available for (additional) consumption. However, if the present value of future returns (the sum of tradeable and nontradeable output and receipts from sales of uncultivated land) exceeds the unit investment cost:

$$(5) \qquad \sum_{s=1}^{\infty} R^{-s} \lambda^{s-1} [a + c + (1-\lambda)q_{t+s}] \;>\; q_t + \phi,$$

then the farmer will always prefer to expand k_t to the maximum, rather than consume more than the bruised fruit ck_{t-1}. Later we show that assumption A2 implies that (5) holds, and so the farmer's optimal choice of k_t, b_t satisfy (4) as an equality, and the borrowing constraint (1) is binding. That is,

$$(6) \qquad k_t \;=\; \lambda k_{t-1} \;+\; \frac{1}{\phi}\,[(a + q_t)k_{t-1} - Rb_{t-1}]; \quad \text{and}$$

$$(7) \qquad b_t \;=\; q_t k_t \;=\; \lambda q_t k_{t-1} \;+\; \frac{1}{\phi}\, q_t\,[(a + q_t)k_{t-1} - Rb_{t-1}].$$

The term is square brackets, $[(a + q_t)k_{t-1} - Rb_{t-1}]$ is the farmer's net wealth at date t, were his assets to be liquidated. This is the value that the farmer's creditors' are interested in, given that the farmer can repudiate. Notice that the expression for k_t in (6) equals the cultivated land λk_{t-1} inhe.ited from date t-1, plus the amount of land that he can plant

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with new fruit given that he has to supply the fruit out of his net wealth. (He does not have to buy the new land $k_t - k_{t-1}$ out of his net wealth, since he can borrow against land.) The expression for b_t in (7) is simply q_t times k_t, reflecting the binding borrowing constraint (1).

Next, we must consider the position of a farmer at date t who does not have the opportunity to plant fruit to grow new trees -- which happens with probability $1 - \pi$. Assume that he had k_{t-1} cultivated land at date t-1, and incurred debt $b_{t-1} \leq q_{t-1} k_{t-1}$. (This need not necessarily be an equality, since he may not have had the opportunity to expand at date t-1 either.) He inherits λk_{t-1} cultivated land, which is therefore the upper limit on his scale of operations at date t. Given that (5) holds, he will prefer not to consume more than the bruised fruit $c k_{t-1}$, but rather to leave all the cultivated land intact:

$$(6') \qquad k_t = \lambda k_{t-1}.$$

This entails borrowing enough to pay off his accumulated debt Rb_{t-1}, net of his harvested fruit $a k_{t-1}$ plus the receipts from the sale of $(1-\lambda) k_{t-1}$ newly uncultivated land:

$$(7') \qquad b_t = Rb_{t-1} - a k_{t-1} - (1-\lambda) q_t k_{t-1}.$$

But will he be able to borrow this much? The creditors require that borrowing does not exceed the value his collateral, $\lambda q_t k_{t-1}$. Therefore, to take the worst case where the farmer's debt b_{t-1} at date t-1 was at the upper limit of $q_{t-1} k_{t-1}$, the creditors require that his net wealth at date t is nonnegative:

$$(8) \qquad (a + q_t) k_{t-1} - Rb_{t-1} \geq 0.$$

Later we show that assumption A1 ensures that (8) is always <u>strictly</u> satisfied. (Note that we have now promised to check for a total of three inequality conditions: (3), (5) and (8).)

Expressions (6), (7), (6′) and (7′) have the great virtue that they are linear in k_{t-1} and b_{t-1}. Hence we can aggregate across farmers, and appeal to the law of large numbers (the farmers' opportunities to expand are assumed to be independently distributed across individuals and through time), to derive expressions for the aggregate size of the farmers' land holdings and borrowing -- K_t and B_t, say -- without having to keep track of the distribution of the individual farmers' k_t's and b_t's. Normalising the measure of farmers to unity, it follows that

(10) $$K_t = \lambda K_{t-1} + \frac{\pi}{\phi}[(a + q_t)K_{t-1} - RB_{t-1}]; \quad \text{and}$$

(11) $$B_t = \lambda q_t K_{t-1} + [\frac{\pi}{\phi}q_t - (1-\pi)][(a + q_t)K_{t-1} - RB_{t-1}].$$

There are certain points to note about the expressions for K_t and B_t in (10) and (11). First, it is not the case that $K_t = q_t B_t$. Rather, $K_t < q_t B_t$, because the farmers who are not expanding stay strictly within their borrowing limits (since (8) is strictly satisfied). In fact, if by chance an individual farmer has a long history of no opportunity to expand, he may eventually become a net creditor (i.e. his b_t may become negative) -- while his stock of cultivated land is always positive and declining geometrically at the rate λ. Second, although land is a durable asset, at each date t the farmers' demand for land K_t is entirely <u>backward-looking</u>: K_t does not depend on any of the future prices $\{q_{t+s}|s\geq1\}$. This reflects the fact that a farmer either does not have the opportunity to expand; or, even if he can expand, the credit constraint (1) is binding so that he is not at the conventional margin where future land prices impinge on his choice of k_t. Third, the farmers' demand for land is <u>upward-sloping</u> in price q_t. The usual notion that a higher land price reduces demand is offset by the fact that the collateral value of the farmers' existing land holdings is higher, which enables them to borrow more and demand more land.

We now return to the hunters. They are not credit constrained, and so their demand for land is quite familiar, and negatively related to the user cost of holding land:

$$u_t \equiv Rq_t - q_{t+1}.$$

To keep matters simple, we posit that at each date t the aggregate demand for land by the hunters is a simple (negative) linear function of u_t:

(12) $$\bar{K} - \frac{1}{h}(u_t - v),$$

where h and v are constants.[7] In contrast to the farmers, the hunters demand for land is entirely forward-looking. Both their land holding at date t-1 and their debt/credit position are irrelevant: the decision to buy land depends only on whether the investment has positive value.

Thus the total demand for land by farmers and hunters is given by the sum of (10) and (12). In equilibrium, this equals the fixed supply, $\bar{K}$. Simplifying,

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[7]The specific production function which yields (12) is as follows. If a hunter has k land at date t, then he produces

$$(v + h\bar{K})k - (\frac{hm}{2})k^2$$

fruit at date t+1 (as well as k land, which is undepreciated) -- where m is the measure of hunters. This is a conventional decreasing returns to scale technology. Note that it makes no difference whether or not k is cultivated land (in the sense that we have defined the term), because by assumption a hunter collects fruit from wild trees. Another gloss one might put on the model is that hunters do not produce fruit at all, but instead hunt animals; and, to keep matters simple, fruit and meat are assumed to be perfect food substitutes.

(13) $u_t = v + hK_t.$

This positive relation between u_t and K_t in equilibrium is quite straightforward. As the demand for land by the farmers, K_t, rises, the demand for land by the hunters has to be choked off by a rise in the user cost, u_t.

We shall concentrate on the case where there is no uncertainty about aggregate variables. Hence, from (13) with t replaced by t-1, and using the identity $u_{t-1} = Rq_{t-1} - q_t$, we obtain

(14) $q_t = Rq_{t-1} - v - hK_{t-1}$

in a rational expectations equilibrium (or, more precisely, in a perfect foresight equilibrium).

The three equations (10), (11) and (14) summarise the equilibrium of the basic land model. (Given the linearity of preferences, we need only consider the land market. The credit market automatically clears since the hunters offer a perfectly elastic supply of loanable funds at the gross interest rate R.) The three equations together amount to a first order nonlinear difference system with state variables q_t, K_t, and B_t. It easily shown that there is a unique steady state (q^*, K^*, B^*), with $u^* \equiv (R-1)q^*$, given by

$$u^* = (R-1)q^* = a - \frac{\phi}{\pi}(1-\lambda)[1 - R(1-\pi)];$$

(15) $K^* = (u^* - v)/h;$ and

$$B^* = K^*[q^* - \frac{\phi}{\pi}(1-\pi)(1-\lambda)].$$

As promised earlier, it is readily confirmed that, in the neighbourhood of the steady state, assumption A1 ensures that inequalities (3) and (8) are always satisfied (the latter as a strict inequality), and assumption A2 ensures that inequality (5) is always satisfied.

It is instructive to compare the credit constrained equilibrium in (15) with the first-best allocation, where there are no credit constraints. In the first-best, farmers break even on their investment; i.e. (5) holds as an equality. Denoting the first-best by (q^O, K^O, B^O), with $u^O \equiv (R-1)q^O$, we therefore have

$$(16) \qquad a + c - (R-\lambda)\phi = (R-1)q^O = u^O = v + hK^O.$$

The left hand side of (16) is a farmer's one-period return on a unit investment in land, net of the user cost of trees. The right hand side is a hunter's marginal return on land. In a first-best allocation, these returns are the same, and equal the user cost of land. It follows from assumption A2 that $K^* < K^O$ and $q^* < q^O$ (or equivalently, $u^* < u^O$). That is, relative to first-best, in the credit constrained equilibrium too little land is used by the farmers, even though the user cost u^* is lower than their return on investment u^O.

Figure 2 provides a useful summary of the economy. On the vertical axis is the user cost of capital u. The farmers' demand for land is measured conventionally along the horizontal; the hunters' demand is measured leftwards from the right hand axis. The sum of the two equals the fixed supply $\bar{K}$. The area under the solid line, Y^* say, is the steady state net aggregate output of fruit per period (i.e., net of the user cost of trees) in the credit constrained equilibrium. The triangular shaded area is the loss relative to the first-best.
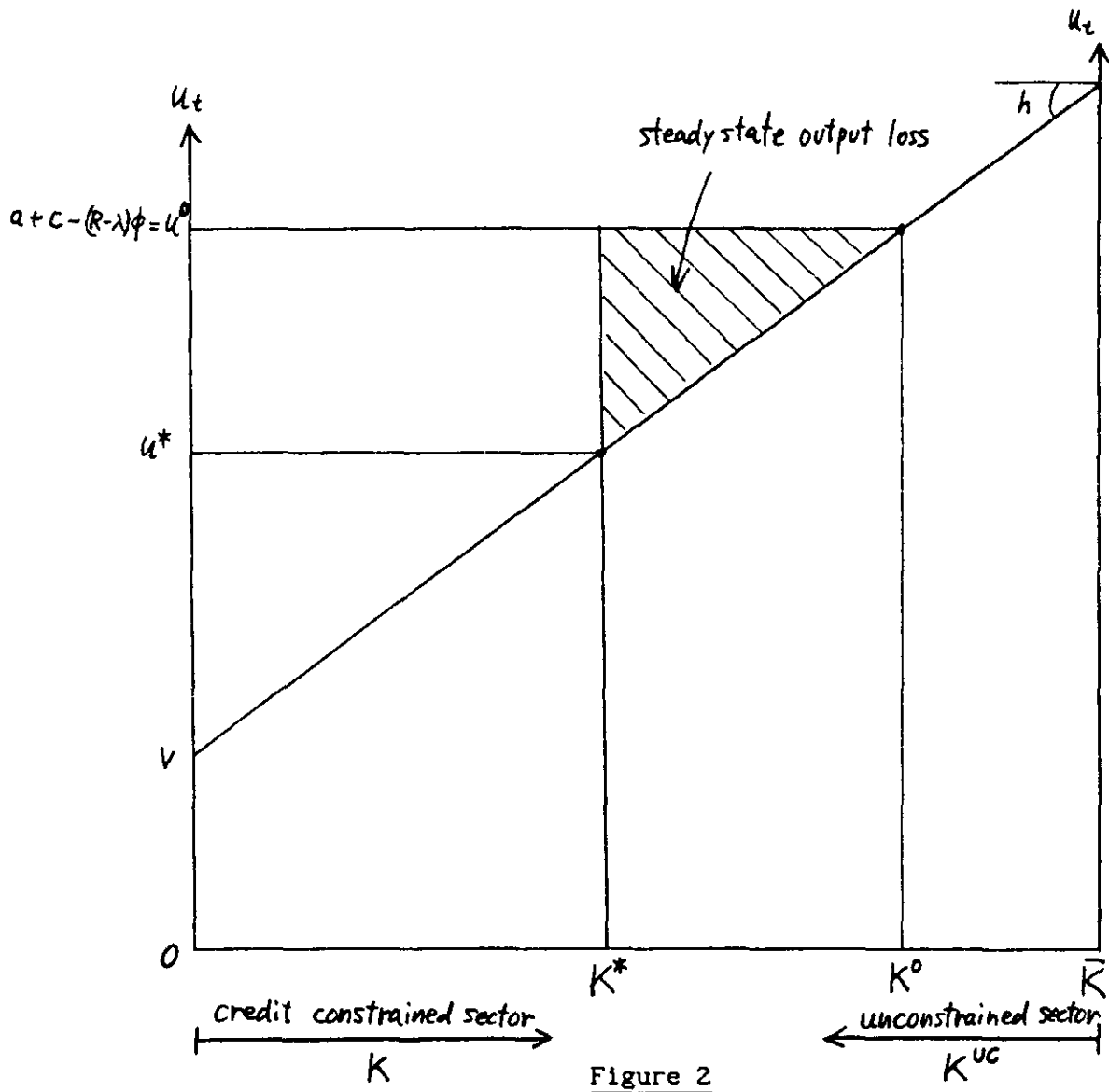
Figure 2

It is important to observe that in any period t, the economy's net aggregate output of fruit, Y_t say, is perfectly correlated with the farmers' land holding K_t. To confirm why, note that the hunters' marginal product is given by the inverse of (12), and the farmers' marginal product (net of the user cost of trees) is $u^o = v + hK^o$. In sum,

$$(17) \qquad Y_t = K_t(v + hK^o) + \int_{K_t}^{\bar{K}} (v + hK)dK.$$

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For K_t near to the steady state of the credit constrained equilibrium, $K_t < K^O$; and so from (17) we have that Y_t increases with K_t. This is because the farmers' marginal product higher than the hunters'. In fact, in the neighborhood of K^*, a rise in K_t causes a non-negligable increase in Y_t: the area under the continuous line in Figure 2 changes by a trapezoid. This would not be true in the first-best: near K^O the change in area would only a small triangle.

For future reference, it is worth asking what would happen in the first-best if, at some date t, there were a single, unanticipated, temporary technology shock. Suppose, for example, that $a \rightarrow (a+\Delta a)$ because of weather conditions affecting the farm sector. This would have no effect on the land price q_t or the land usage K_t (or on any future variable): q_t and K_t would remain at q^O and K^O respectively. This is in marked contrast to happens in the credit constrained equilibrium, as we now see.

To understand the dynamics of the model, we take a linear approximation of (10), (11) and (14) around the steady state:

$$
(18) \qquad \begin{pmatrix} q_t - q^* \\ K_t - K^* \\ B_t - B^* \end{pmatrix} = J \begin{pmatrix} q_{t-1} - q^* \\ K_{t-1} - K^* \\ B_{t-1} - B^* \end{pmatrix},
$$

where the J is the Jacobian.[8] It is straightforward but tedious to show that the characteristic equation for the eigenvalues χ of J can be reduced to

(19) $(\chi - R)[\chi^2 - e_1\chi + e_0] = 0,$

where $e_1 = (2-\lambda) + (2\lambda-1)[1-R(1-\pi)] - (a-v)(\pi/\phi)$

and $e_0 = \lambda R(1-\pi).$

Thus one the eigenvalues of J equals R > 1, which corresponds to an explosive path for the forward-looking variable q_t. The other two eigenvalues will be stable and complex if

$(2-\lambda) + R(2\lambda-1)(1-\pi) - \sqrt{\lambda R(1-\pi)}$

(20) $< (a-v)(\pi/\phi)$

$< (2-\lambda) + R(2\lambda-1)(1-\pi) + \sqrt{\lambda R(1-\pi)}.$

Note that a sufficient condition for (20) is that π lies in the neighbourhood

---

[8]For the record, J is given by

$$
\begin{pmatrix}
R & -h & 0 \\[2em]
(\pi/\phi)RK^* & \lambda + (\pi/\phi)[a+v+(2-R)q^*] & -(\pi/\phi)R \\[2em]
(q^*+\phi)(\pi/\phi)RK^* & \begin{aligned}&-(a+q^*+\lambda\phi) + (q^*+\phi)\lambda + \\ &(q^*+\phi)(\pi/\phi)[a+v+(2-R)q^*]\end{aligned} & R - (q^*+\phi)(\pi/\phi)R
\end{pmatrix}
$$

20

of $1 - \frac{\lambda}{R}$ (which is consistent with assumption A1), and

$$(21) \qquad (1-\lambda)^2 < (a-v)(\pi/2\phi) < 1 + \lambda^2.$$

There is little difficulty in meeting condition (21), insofar as λ is typically close to 1. For the rest of this section, we suppose that (20) is satisfied.

We can take the price q to be a jump variable and assume that (q_t, K_t, B_t) always lie on the two-dimensional stable manifold. Thus the system exhibits damped oscillations. The intuition for why the system cycles can be best understood by orthogonalizing out the explosive q-component. The crucial interaction is between the land holding and debt of the farming sector, K and B:

$$(22) \qquad \begin{pmatrix} K_t - K^* \\ \\ B_t - B^* \end{pmatrix} = \begin{pmatrix} + & - \\ \\ + & + \end{pmatrix} \begin{pmatrix} K_{t-1} - K^* \\ \\ B_{t-1} - B^* \end{pmatrix}.$$

(22) is a classic predator-prey model, which we mentioned in the Introduction. Farmers' debt B_t plays the role of predator, and their land holding K_t acts as prey. A rise in K_{t-1} means that farmers inherit more land at date t, which enables them to borrow more ($\partial B_t/\partial K_{t-1} > 0$). However, a rise in B_{t-1} implies that farmers have a greater debt overhang at date t, which restricts their ability to expand ($\partial K_t/\partial B_{t-1} < 0$). Note K leads B through the cycle. Since net aggregate output Y_t is perfectly correlated the farmers' land holding K_t, the model thus predicts that total debt should be a lag indicator.

Suppose that at date t-1 the economy is at the steady state (q^*, K^*, B^*). Consider the impluse response of a single, unanticipated, temporary technology shock at date t -- say a $\rightarrow$ (a+Δa). The perfect foresight path continues to be described by (10), (11) and (14), except that q_t does not

satisfy (14) due to the surprize. Instead, q_t jumps so that the new equilibrium path $\{(q_{t+s}, K_{t+s}, B_{t+s}) | s \geq 0\}$ does not explode as $s \to \infty$.[9]

From (10), we see that the farmers' land demand K_t rises as $a \to (a+\Delta a)$, which raises the price q_t. This in turn raises K_t still further, since the farmers' demand is upward sloping. There is thus a multiplier effect within period t. But this is not all. Since K_t rises, farmers will have more collateral at date t+1, which means that they will be in a position to demand more land; i.e., there will be arise in K_{t+1}. Similarly, K_{t+2}, K_{t+3}, $\cdots$ will be higher too. Recall, from (13), we have

$$(23) \qquad q_t = \sum_{s=1}^{\infty} R^{-s} u_{t+s} = \sum_{s=1}^{\infty} R^{-s} (v + hK_{t+s}).$$

Thus the rise in K_{t+1}, K_{t+2}, K_{t+3}, $\cdots$ induces a further rise in q_t, which raises the farmers' demand K_t for land at date t still further, which raises K_{t+1}, K_{t+2}, K_{t+3}, $\cdots$ still further, etc. Figure 3 attempts to show the interplay between the market clearing condition and the borrowing constraints:

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[9] We should remark that there is a danger that one or both of the inequalities (3) and (8) may not hold when the land price jumps. However, it is easily confirmed that if the technology shock Δa is positive, this danger does not arise. (In fact it does not even arise if Δa is small and negative.)

If one were to investigate a stochastic version of the model -- with anticipated shocks (negative and positive) -- then not only would the agents' behavior entail more intricate analysis, but also one would need to allow for the possibility that inequalities (3) and (8) may not hold. Thus we gain a lot by restricting attention to a deterministic model (at least, deterministic in aggregate) -- albeit that we introduce a single, surprize technology shock to kick the economy away from steady state.
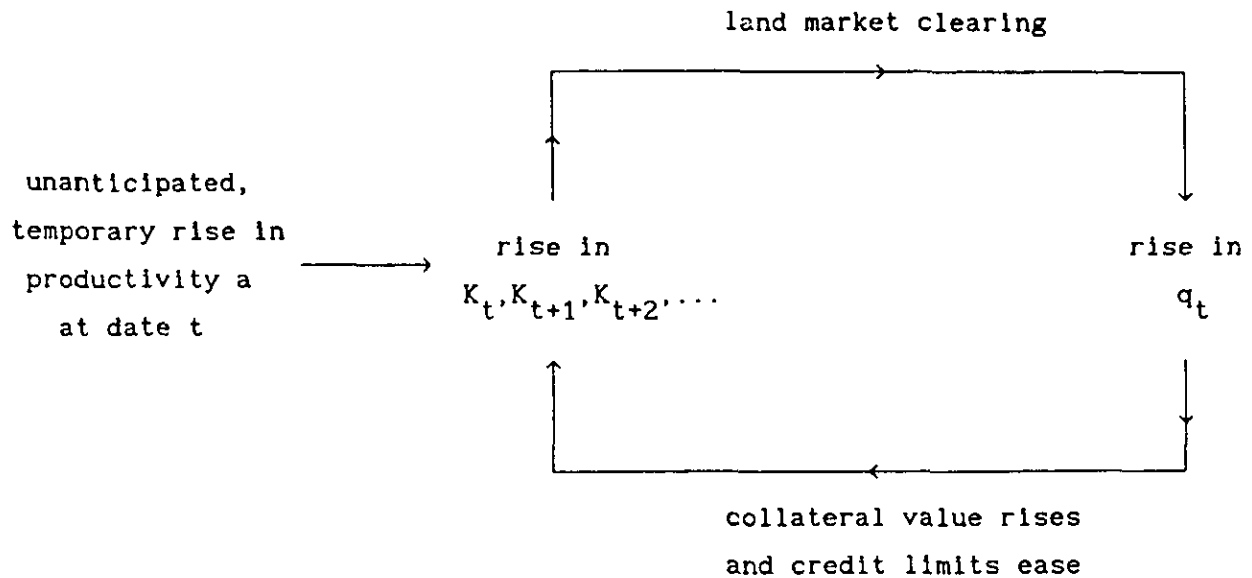
land market clearing

```
              ┌─────────────────────────────►──────────────────────────┐
              │                                                         │
              ▲                                                         ▼
unanticipated,
temporary rise in          rise in                          rise in
productivity a    ──────►   Kₜ,Kₜ₊₁,Kₜ₊₂,···                    qₜ
at date t
              ▲                                                         │
              │                                                         ▼
              └──────────────────────◄──────────────────────────────────┘
```

collateral value rises
and credit limits ease

Figure 3

In short, both price q and quantity K (and, concomitantly, net aggregate output Y) move significantly as a result of the shock Δa; and these effects persist through time. Contrast this with the first best, where Δa had no effect on either q or K!

Figure 4 shows the path of a calculated example in which a is subject to an unanticipated, temporary increase of 10% at date t. In the example, we have used parameters that might be reasonable for a quarterly model: $a = 100$; $\phi = 100$; $\lambda = 0.975$ ($\cong$ 10% per annum death rate of trees); $\pi = 0.05$ ($\cong$ a new investment opportunity every 5 years); $R = 1.01$ ($\cong$ 4% annual rate of interest); $h = 1$, and $v = 0$. The paths of (q,K,B) are drawn relative the their steady state values (q^*,K^*,B^*). Also drawn is the path of net aggregate output Y relative to Y^*.
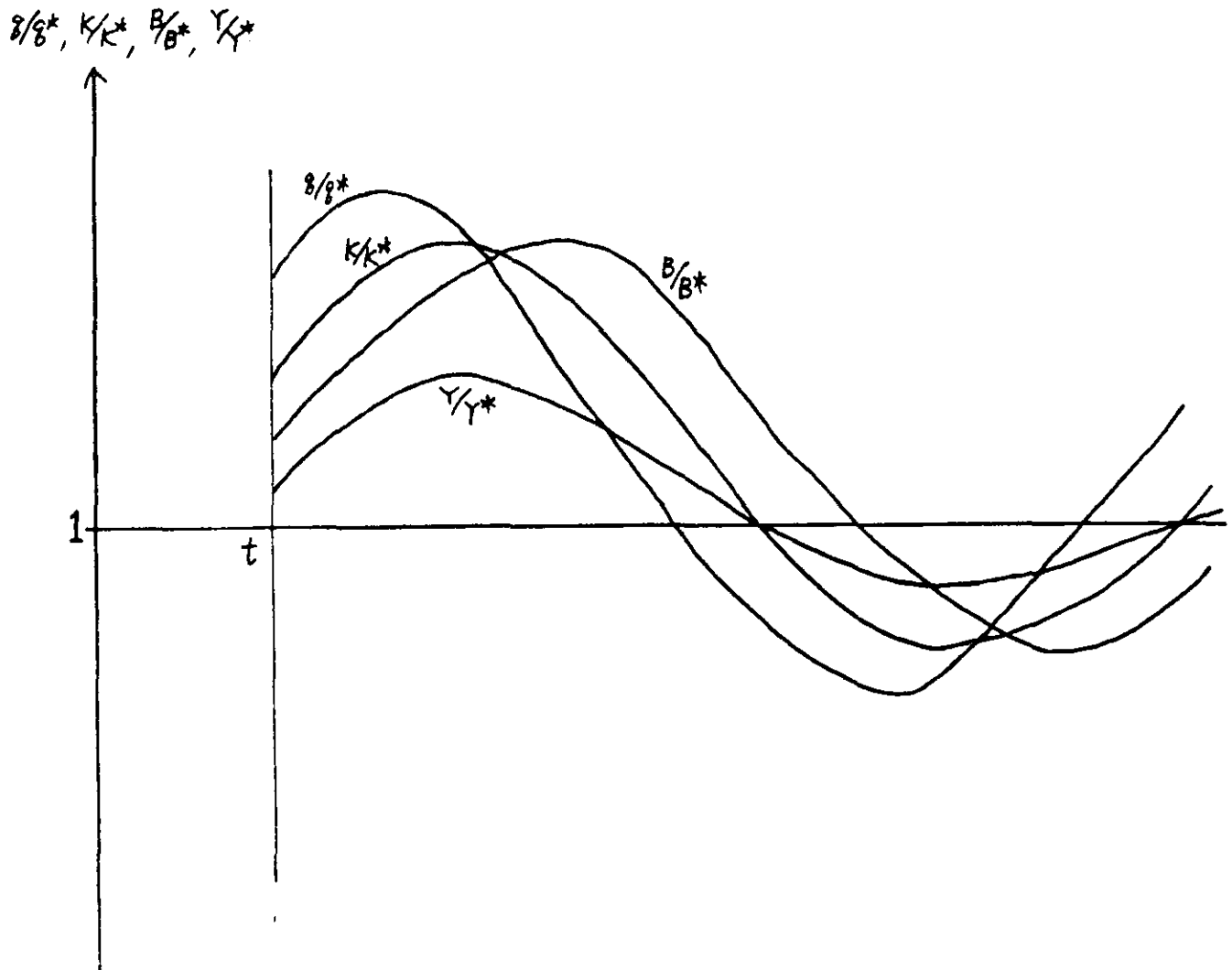
Figure 4

The cycle is defined by the movement in net aggregate output Y (which moves with the farmers' land holding K).[17] The land price q leads the cycle; the farmers' debt B lags the cycle. Notice that there is a further rise in q, Y, K, q and B _after_ the temporary shock has hit the economy at date t: that is,

[17]The length of the cycle in Figure 4 is roughly 20 periods (though this can be varied by, among other things, vaying ϕ) -- which seems sensible given that the parameters were chosen to correspond to a quarterly model.

there is amplification through time.

One (overly simple) way to appreciate why the economy experiences so much movement in both price and quantities is as follows. We know that the farmers' demand schedule for land, K_t, is upward sloping in price, q_t; see (10). Moreover, the residual supply schedule of land -- the total stock minus the hunters' demand -- is also upward sloping in q_t, ceteris paribus (i.e., holding the hunters' expectations over q_{t+1} constant); see (12). Hence both the price q_t and quantity K_t are very susceptible to small shifts in either schedule. The argument is of course more complicated than this, because the hunters' expectations over q_{t+1} are not fixed; but endogenizing these expectations only serves to exacerbate the movement in q_t and K_t.

It is worth tracing out the sequence of events following the shock. Initially, q_t jumps, relecting the anticipated rise in K_{t+1}, K_{t+2}, K_{t+3}, $\cdots$ (see (23)). At date t, since the farmers demand more land K_t, the user cost

$$(24) \qquad u_t = (R-1)q_t - (q_{t+1} - q_t)$$

must rise in order to choke off the hunters' demand (see (12)). The first term on the right hand side of (24) -- the interest component of user cost -- is already high because q_t is high; and so there is scope for the second term -- the capital gains component of user cost -- to be positive. That is, q_{t+1} can be even higher than q_t. As time progresses, however, the farmers demand yet more land, since their collateral is rising in value (not only is the price of land going up, but they have more of it). The hunters' demand therefore has to be further choked off, entailing a further rise in the user cost of land, and, eventually, a fall in the price (i.e., the capital gains component of user cost becomes negative). Thus q is the first variable to peak. At this point, the farmers begin to suffer. They are having to pay large interest payments on their recent land acquisitions (which were bought at a high price); and at the same time the price of their collateral is dropping. In time, they find themselves unable to maintain the necessary replacement investment in new trees, and they have to cut back the scale of

their operations K.[11] The boom has peaked. Debt B continues to accumulate, but is subsequently brought under control by the falling rate of investment. The economy slides into a trough, with falling output and prices. The bottom of trough is heralded by a rise in land price, brought on by the need to attract hunters into the market. This in turn assists the farmers by raising the collateral value of their land holdings. The economy starts to pick up again, with rising ouput and prices, and we are back (near) to where we started.

The behaviour of measured aggregate productivity, $Y/\bar{K}$, is procyclical, even though there are no productivity shocks (beyond the temporary shock at date t). The explanation lies in the composition effect: it is the usage of land which changes through the cycle.

Although we are interested in the fact that the model cycles so easily, we think that the important finding is that credit constraints cause shocks to be amplified and to persist. Of equal importance, perhaps, is the idea that one sector can spill over to another. In the land model, by definition, the farming and hunting sectors move against each other. One simple way to investigate co-movement in the land model is to introduce a second farm sector, with its own independent technology. It is simple to see that if one farm sector experiences a (positive) shock, the price of land will rise, which in turn will cause the other farm sector to expand, through the effect on collateral value. So co-movement is quite natural. In Section 4, we look at a different class of model -- a "sea economy" -- where both the credit-constrained and the unconstrained sectors move together. Beforehand, in Section 3 we briefly consider a variant of the land model.

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[11]Footnote 9 applies.

3. Land Economy with Tighter Credit Limit

In this section, we modify the basic land economy model so that the tradeable output of fruit, ak_t, is not part of the collateral which secures debt. In this case, creditors will not lend more than the discounted value of the land in the next period, and the credit constraint (2) becomes

(25) $$b_t \leq (q_{t+1}/R)\, k_t = \theta_t q_t k_t, \text{ where}$$

(26) $$\theta_t \equiv q_{t+1}/(q_t R).$$

Since $\theta_t < 1$ in the neighborhood of the steady state, the creditor does not lend the full value of land and (25) becomes a tighter constraint than the credit constraint (1): $b_t \leq q_t k_t$. An interesting aspect of this is that the ratio of the credit limit to the land value (θ_t) is an increasing function of the expected rate of increase in the land price, which we would expect to strengthen the amplification mechanism of the earlier land model.

Consider the farmer with an investment opportunity. Now the farmer has to finance internally not only the investment cost of the fruit input but also the difference between the land value and the credit limit. The farmer will invest up to the maximum subject to the credit constraint (25) and the flow of funds condition (4), if the present value of future returns exceeds the unit investment cost as in (5). Thus we have

(26) $$k_t = \lambda k_{t-1} + [\phi + (1-\theta_t)q_t]^{-1}\{[a + q_t - \lambda(1-\theta_t)q_t]k_{t-1} - Rb_{t-1}\},$$

(27) $$b_t = (q_{t+1}/R)k_t = \theta_t q_t k_t.$$

The term in curly brackets, $\{[a + q_t - \lambda(1-\theta_t)q_t]k_{t-1} - Rb_{t-1}\}$, is the

farmer's net wealth minus the fund required to finance internally the old cultivated land; $(1-\theta_t)q_t \lambda k_{t-1}$.

The aggregate land and debt of the farmers (equations (10) and (11)) will be modified to:

$$(28) \quad K_t = \lambda K_{t-1} + \frac{\pi}{\phi + (1-\theta_t)q_t} \{[a + q_t - \lambda(1-\theta_t)q_t]K_{t-1} - RB_{t-1}\},$$

$$(29) \quad B_t = \theta_t q_t K_t + [\frac{\pi \theta_t q_t}{\phi + (1-\theta_t)q_t} - (1-\pi)] \cdot$$
$$\{[a + q_t - \lambda(1-\theta_t)q_t]K_{t-1} - RB_{t-1}\},$$

These equations together with the land market clearing condition (14) summarize the equilibrium of the economy.

We can analyze the dynamic property of this economy in a similar way as the previous economy. Although we are not going to present the details of the analysis, a few remarks are worth noting. First, output and land price tend to oscillate in response to a temporary productivity shock for a fairly large set of parameters.[12] Second, we find that the reaction of output and land price to a shock on farmers' productivity is somewhat larger than the previous land model, because the difference between the land value and the credit limit $((q_t-(q_{t+1}/R))k_t)$ increases when the land price is expected to fall during downturn, which further depresses the farmer's investment. Third, the we find in simulation that the fluctuations of output and land price are asymmetric in the sense that the downturn is faster than the upturn.

---

[12]The conditions for oscillation are that (a/ϕ) is larger than a certain constant which is a function of π, λ and R, and

$\pi \in (\pi_1,\pi_2)$, where $\pi_1,\pi_2 = \{R+(2\lambda-1)^2 \pm 2\lambda[R^{-1}-(1-\lambda)^2(2-\lambda)/\lambda]^{1/2}\}/(R+4\lambda^2)$

4. Sea Economy

In this section, we consider an economy in which capital is produced and total stock of collateralizable capital fluctuates over the cycles. There are two sectors in this economy. One is consumption goods sector (fisherman sector), and the other is investment goods sector (ship building sector). The population size are 1 and m respectively, and the preference is the same as before.

The *fisherman sector* is the same as the *farmer sector* in the previous model, except the capital depreciates at the rate $1-\lambda$ every period. We assume the every fisherman has an investment opportunity to expand every period (i.e. $\pi = 1$) in order to simplify the analysis. We also modify Assumption A1 and A2 to:

Assumption A3: $a > (1-\lambda)\phi$, and

Assumption A4: $c > (R-1)\phi$.

Assumption A3 implies that tradable output from one unit of capital exceeds the replacement cost of consumption goods input, which will guarantee that the capital is productive so that the price of capital is positive in equilibrium. Assumption A4 requires that the fisherman's rate of consumption is not too small. Denoting q_t and k_t as the price and stock of capital (or ships) of an individual fisherman, his credit limit is the same as in the basic land model as:

$$(30) \qquad b_t \leq q_t k_t$$

Modifying equations (10,11), the aggregate capital stock (K_t) and debt (B_t) of the credit constrained fishermen will be:

$$(31) \qquad K_t = \lambda K_{t-1} + (1/\phi)\,[(a + \lambda q_t)K_{t-1} - RB_{t-1}]; \text{ and}$$

$$(32) \qquad B_t = q_t K_t.$$

The term in the square brackets, $[(a + \lambda q_t)K_{t-1} - RB_{t-1}]$ is average

fishermen's net wealth before investing at date t, were he liquidated.

The ship builder has an identical, decreasing returns to scale technology to produce ships from old ships with her own labor input:

period t   period t+1

k^1 ships   ——————————→ $1 = [2(\gamma/m)k^1]^{1/2}$ ships

Since the ship builder only purchase ships as input and ships are fully collateralizable, she is not constrained in the credit market. Thus the ship builder simply chooses input k^1 to maximize the expected profit: $q_{t+1}1_{t+1} - Rq_t k_t^1$. From the first order conditions, the aggregate supply of new ships and the demand for ship input by m number of ship builders are given by:

(33) $\qquad I_{t+1} = \gamma\,\theta_t,$

(34) $\qquad K_t^1 = (\gamma/2)\,\theta_t^2,$ where

(35) $\qquad \theta_t = q_{t+1}/(Rq_t).$

The value of θ_t is the same as the section 3, and here, one may think of θ_t as the relative price of output to input for the ship builder.

The market equilibrium of capital (ships) implies that the demand for ships by the fishermen and the ship builders is equal to the total stock of the ships, which is the sum of old ships in fishing sector and the new supply of ships:

(36) $\qquad K_t + K_t^1 = \lambda\,K_{t-1} + I_t.$

Putting together with equations (31-35), we have the equilibrium condition:

(37) $\quad 0 = (1/\phi)(a+\lambda q_t-Rq_{t-1})K_{t-1} + (\gamma/2)\theta_t^2 - \gamma\theta_{t-1}.$

From (31) and (32), we have the aggregate capital stock of the fishermen

(38) $K_t = \lambda K_{t-1} + (1/\phi)(a + \lambda q_t - R q_{t-1}) K_{t-1}.$

Along the rational expectations equilibrium path, we also have from (35):

(39) $q_t = R\, q_{t-1}\, \theta_{t-1}.$

The three equations (37), (38) and (39) summarizes the equilibrium of the sea economy. The three equations consists a first-order difference equation system with state variables K_t, q_t, and θ_t. The steady state condition implies:

(40) $\theta^* = 1/R, \quad q^* = [a-(1-\lambda)\phi]/(R-\lambda), \quad K^* = \gamma(2R-1)/[2R^2(1-\lambda)].$

Assumption A4 implies that, in the neighborhood of the steady state, the present value of the returns on one unit of investment exceeds the unit investment cost:

(41) $\sum_{s=1}^{\infty} R^{-s}\lambda^{s-1} (a + c) > q_t + \phi.$

Assumption A3 guarantees that the steady state capital price is positive.

In order to study the dynamic property of the equilibrium, we linearize dynamical system (37,38,39) around the steady state:

(42) $\begin{pmatrix} K_t - K^* \\ q_t - q^* \\ \theta_t - \theta^* \end{pmatrix} = J \begin{pmatrix} K_{t-1} - K^* \\ q_{t-1} - q^* \\ \theta_{t-1} - \theta^* \end{pmatrix}$, where J is Jacobian and

$J = \begin{pmatrix} 1 & -(R-\lambda)K/\phi & \lambda RqK/\phi \\ 0 & 1 & Rq \\ -R(1-\lambda)/\gamma & R(R-\lambda)K/(\phi\gamma) & R - \lambda R^2 qK/(\phi\gamma) \end{pmatrix}$

The eigenvalue x of Jacobian solves the characteristic equation

$$(43) \quad \varphi(x) \equiv \det(J-xI) = (1-x)^2(R-x) + M(\lambda-x)[x-(R/\lambda)],$$

where $\quad M \equiv \lambda R^2 qK/(\phi\gamma) = \lambda(2R-1)[a-(1-\lambda)\phi]/[2\phi(1-\lambda)(R-\lambda)].$
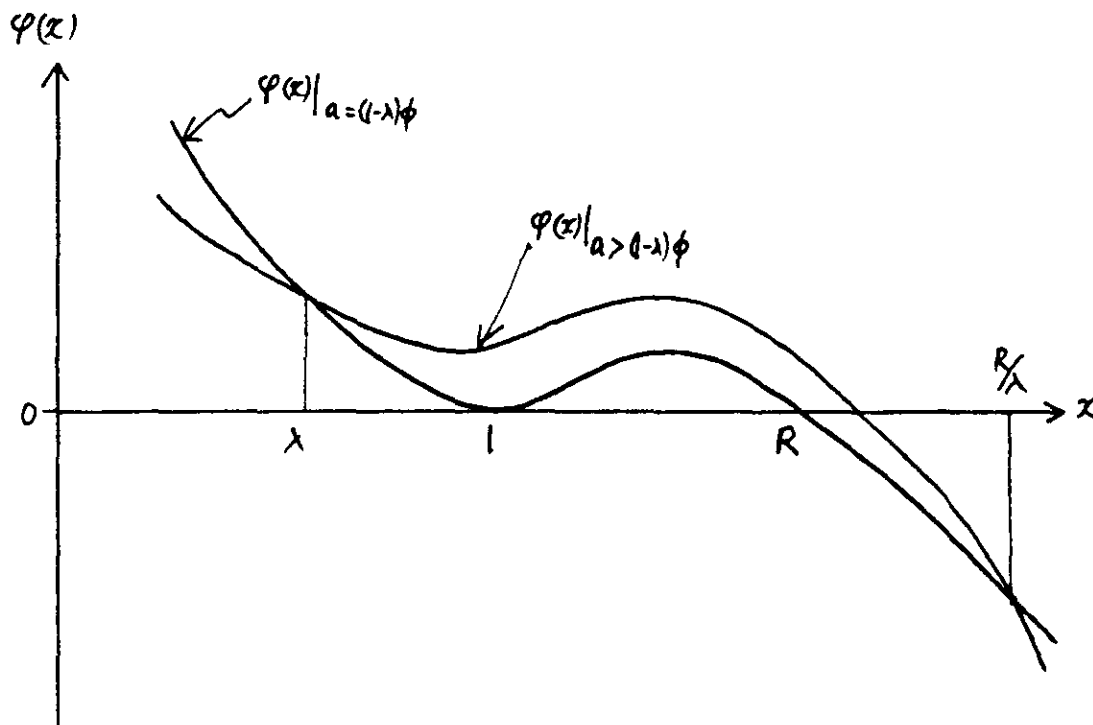
Figure 5

When net productivity of capital in the consumption goods sector is zero (i.e. $a = (1-\lambda)\phi$), then $M = 0$ and the eigenvalue are R and 1. When the net productivity, $a - (1-\lambda)\phi$, increases from zero, $\varphi(x)$ will shift upward in $x \in (\lambda, R/\lambda)$. Thus we get one real eigenvalue which is larger than R, and two stable complex eigenvalue for a small enough net productivity, which we are going to assume in the following. Here, we takes the relative price $\theta_{t-1} = q_t/(Rq_{t-1})$ as a jump variable and assume that (K_t, q_t, θ_t) always lie on the two dimensional stable manifold. Thus the system exhibits dumped oscillations.

We can also consider the impulse response of a temporary, unanticipated technology shock on productivity of the farmer a_t. In our

simulation, we can get a large, persistent fluctuations of output of consumption goods and investment goods as well as the price of capital. Investment tends to leads the cycles. Intuitively, the capital of the consumption goods sector plays a role of prey, while the capital price in the previous period plays a role of predator which causes debt over-hang. Major difference from the previous two models are that the output of the credit-constrained sector tends to move together with the output of the unconstrained sector, because the total stock of capital is no longer constant. The fluctuation of capital stock also appears to make the output fluctuation larger than the constant aggregate land case. We also find some asymmetry that the decrease of output and capital price is faster than their increase over the cycles.