

Repeated Moral Hazard and One-Sided Commitment

by

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Current version: September 1, 1993

Abstract. This paper considers the unobserved endowment economy of Green (1987) with a restriction that agents can walk away from insurance contracts at the beginning of any period and contract with another insurer (one-sided commitment). An equilibrium is derived characterized by a unique, market determined insurance contract with the property that agents never want to walk away from it. I show that trade (or insurance) still occurs and that a non-degenerate long-run distribution of consumption exists.

The author would like to thank Ned Prescott, Larry Samuelson, Robert Townsend and participants at seminars at the University of Wisconsin, the University of Illinois, the University of Iowa, and the Northwestern-MEDS Summer Macroeconomics Workshop for helpful comments. The author received financial support from National Science Foundation grant SES-9111-926.

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1. Introduction

Many long-term economic relationships are characterized by differing abilities of parties to commit to long term contracts. In labor markets, while an employer could conceivably sign a contract that offered a worker a job for life (and face legal sanctions upon renegeing), workers cannot sign contracts promising to never quit or work for another firm. Likewise, while an insurance company can promise not to ever drop coverage for a customer or raise premiums beyond a set schedule, a customer cannot promise to never switch to another insurance company. In credit markets, banks have a much greater ability to commit to long term credit arrangements with borrowers than borrowers have with banks. This paper will consider markets where players on one side of the market, "firms," have an unlimited ability to commit to long-term contracts, while players on the other side of the market, "agents," have no ability to commit to long term contracts.

Limiting commitment possibilities is important to economic theorists because attempts to model long term agency relationships (such as employer-employee, banker-borrower, or insurer-insuree) under full commitment assumptions, while successful in many respects, have wildly missed other important characteristics of the real world. Understanding the issues involved requires or short literature review.

Given an assumption of unlimited commitment, long-term contracts have been shown to optimal in a variety of repeated moral hazard environments. Green (1987) and Thomas and Worrall (1990) consider economies where agents receive repeated unobservable endowment shocks. Spear and Srivastava (1988), Phelan and Townsend (1991) and Phelan (1992) consider a mutli-period

economy where workers take unobservable actions which affect a stochastic output or productivity variable. Atkeson and Lucas (1992) consider repeated unobservable taste shocks. In each of these environments a similar result occurs — the assumption of unlimited commitment is used without limit and drives the long run results.

In Green (1987), a continuum of agents each has a random unobserved endowment which is i.i.d. over both agents and time. He shows that if agents have exponential utility, and society can borrow and lend at the rate of discount, the socially optimal insurance contract has each agent's consumption being the sum of an i.i.d. term and a term which follows a random walk with a negative drift. Thus we have the eventual "immiserization" of almost all of the population. Eventually, the expected discounted utility of an arbitrarily high fraction of the population is below any arbitrarily low utility.

Thomas and Worrall (1990) consider the same problem but with a single risk averse borrower with a stochastic endowment and a single lender or insurer. Under more general utility specifications they show that the utility of the borrower eventually becomes arbitrarily low with probability one. In all of the above papers, at least half the population would eventually be willing to pay any amount to get out of the ex-ante optimal insurance contract. All of these long-run results depend crucially on the assumption that any incentive-compatible contract is enforceable.

The logic is as follows. Because of the moral hazard problem, contracts must punish or reward based on observable or announced variables. When considering a private action or announcing the value of a private variable, an agent will consider only the expected lifetime utility of taking one action or another or announcing one value for a private variable or another. Firms must design contracts to provide these expected discounted utilities so that agents act correctly but wish to do so at the lowest expected cost. This cost minimization implies that firms will spread punishments and rewards over time. (For an agent with a concave utility function, it is cheaper to provide a given expected utility with a smooth profile.) While the exact implications of this depend on the

particular model and utility specification, the general implication is that consumption changes tend to have a strong permanent component, and this permanent component drives the extreme long run results.

Nevertheless, it is not trivial to further restrict the contract set to achieve reasonable long-run distributions. Assuming that agents can unobservably borrow and lend as in Fudenberg, Holmstrom, and Milgrom (1991) only causes less insurance but leaves long run properties unaffected. What is needed to eliminate the result that at least half of all agents in the long run will pay any amount to get out of contracts is a restriction on how low people can be pushed, or allow themselves to be pushed.

A popular avenue is to allow a reversion to autarky. That is, agents can quit out of a contract at any time and receive their autarkic utility from that point on. This approach is taken by Thomas and Worrall and Atkeson and Lucas (II).¹ Thomas and Worrall take an extreme approach and allow both agents and firms to renege and receive their autarkic continuation utility. They show that this eliminates all trade for finite period problems but allows some trade for infinite period problems with low enough discounting. I take a different approach here. Allowing agents to quit out of contracts and receive the utility associated with autarky assumes that there is enough commitment between firms to keep the agent from quitting out of a contract with one firm and contracting with another. Somehow agents can be banished from contract markets. This does not seem realistic. Here, agents can only be held to one-period contracts, but can quit a contract at any time and contract with another firm.

This assumption results in an equilibrium presented where all firms offer a common, market-determined contract characterized by a unique endogenous "market utility" that performs two roles.

¹ Another avenue is to assume an overlapping generations context and limit the amount of debt that parents can pass to children. This approach is taken by Banerjee and Newman (1991).

First, the market utility is the expected discounted utility of an agent at the beginning of signing the market contract. Second, the market utility acts as a lower bound on the expected discounted utility of a worker during the actual execution of the market contract. The lower bound role comes from the ability of a worker to quit out of an insurance contract with one firm and sign with another. If an agent's expected discounted utility ever fell below the starting utility of contracts offered by other firms, he would quit out of the current contract. That is, the market contract reflects the fact that agents will be “stolen” by other firms when it is profitable for them to do so. The market contract also reflects the fact that the conditions under which it becomes worthwhile for a firm to steal and agent are endogenous and dependent on the fact that the agent can be stolen again.

The paper has three main results. First is that there is a sensible market equilibrium manner to discuss contracting with one-sided commitment (Lemma 1). Second is the result that this is not a trivial equilibrium — intertemporal trade still occurs (Lemma 2). Assuming that long-term contracts cannot bind one side of the market does not wipe them out. After these results are established, I consider relaxing assumptions regarding the form of the agent's utility function and the assumption that firms and agents discount the future at the same rate. The third main result is that the long-run distribution of continuation utilities exists and is not degenerate (Lemma 3).

2. The Model

Consider a world with many infinitely lived agents and financial intermediaries (or firms). The physical aspects of each period are identical. In each period, the agent receives an unobservable perishable endowment e_i . For ease of exposition, I simply assume that $i \in \{1, 2\}$, where $e_2 > e_1 > 0$. The probabilities of these endowments are i.i.d. over both time and agents and are denoted π_i . Agents care about expected discounted utility where their instantaneous utility function over consumption is denoted $U(c_t)$. I assume here, as in Green (1987), that $U(c_t) = -\exp(-\gamma c_t)$, or

constant absolute risk aversion. (I discuss an assumption of constant relative risk aversion utility in a later section.) Agents discount future utils by the parameter $\beta < 1$.

Firms are assumed to have access to credit markets which discount the future at the same rate as agents, or $\beta = \frac{1}{1+r}$ where r is the interest rate faced by firms. Firms are assumed to contract with large numbers of agents and thus are risk neutral regarding the transfers (positive or negative) to any particular agent. All of the results in the paper hold when firms discount at least as much as agents. Allowing for firms to discount at rates different from agents is discussed in more detail in a later section.

The assumed commitment possibilities between agents and firms are crucial and thus will be spelled out carefully. Agents can sign contracts with firms but cannot be prevented from walking away from a contract with one firm and signing another contract with another firm. It is simply assumed here that courts, (or whoever enforces contracts) will not force an agent to stay with a contract he doesn't like. (An agent can also not commit to paying damages to the firm for walking away from a contract, since by setting these high enough, all quitting could be ruled out.) However, it is assumed that such quitting can only occur at the beginning of a period. Courts will enforce one-period contracts. This is equivalent to assuming that one-period insurance contracts are feasible if endowments are observable. The issue is preventing those receiving high endowments from immediately walking away from contracts. If agents cannot commit to one-period contracts, no trade is possible even if endowments are observable. Firms are assumed not to be able to walk away from contracts at any cost.

The only equilibrium considered here is a stationary, symmetric equilibrium as follows. At each date, agents have an opportunity to contract with any of many competing firms. Each agent receives an identical market-determined infinite-period contract which gives him an expected discounted utility w^* from the perspective of the contracting date. This market utility w^* is assumed the same at every date. Further, agents are not differentiated by firms by the agent's announced

endowment history, or the agent's history of walking away from contracts. Each agent is offered the same contract by the market. (Note that since all agents are identical, no history could reveal an agent to be "the type to walk away from a contract," or a general "bad risk," no matter how bad the agent's endowment history.)²

3. Firm's Problem

Consider the decision problem of the typical firm when that firm takes as given the market utility, w^* . This decision problem is posed in terms of a contracting problem: What are the efficient or profit maximizing contracts to offer agents? In this context, a "contract" specifies a possibly negative transfer from the firm to the agent at each date in the agent's life as a function of his history of endowment announcements.

The firm takes as given the contracts offered to agents by the other firms. The only relevant characteristic of these contracts is the expected discounted utility w^* that the agent associates with the other contracts. If a particular firm offers a contract with an initial expected discounted utility greater than or equal to w^* , it can have as many customers (agents) as it wishes, and if it offers a contract with a lower expected discounted utility, it will have zero customers.

This typical firm also takes as given that any agent-customer it contracts with will be able to quit out of the insurance contract at the beginning of any future period and receive a new contract from another firm delivering this same market utility, w^* . This is the assumed one-sided

² It may be possible to have equilibria where the market utility w^* is not the same at all dates, or where agents are differentiated by whether they ever walked away from a contract. A general definition of an equilibrium in this context is a sequence of market utilities $w_t^*(h^t)$ where t is the date and h^t represents the announced endowment history of the agent and a history of when the agent walked away from contracts. The utility $w_t^*(h^t)$ is then the expected discounted utility associated with the market contract offered to a worker at date t with history h^t . This utility would also determine the conditions under which an agent would quit. In the equilibrium considered in this paper, $w_t^*(h^t)$ equals the same constant w^* for all dates t and individual specific histories h^t .

commitment. While it is assumed that agents cannot formally commit to contracts, agents *can* believably commit to contracts where under no contingency will they wish to walk away. A firm can without loss restrict itself to offering only such contracts. Such contracts will have the property that under all histories of endowments for the agent, the agent's beginning of period utility, w , is greater than or equal to the market utility, w^* .

Given that w^* is the same for all periods, efficient contracts can be shown to be recursive (see Spear and Srivastava (1987), Phelan and Townsend (1991) or Thomas and Worrall (1991)). This allows the firm to consider the expected discounted utility of the agent as a state variable. Given this, the choice variables of the firm are as follows. For each w , the firm chooses a transfer from the firm to the agent $t_i \in [-e_i, \infty)$, and a utility promise $w_i \in [w^*, 0)$ specifying the agent's expected discounted utility from the beginning of the next period, for each given announced endowment realization e_i . The functions $t_i(w)$ and $w'_i(w)$ completely specify a lifetime contract to an agent with an initial utility of w^* .

Firms are not free to choose just any contract. For an agent who was promised a specific expected discounted utility of w , a firm must pick policies which actually deliver this expected utility. This can be expressed in the form of the promise-keeping constraint (for each $w \in [w^*, 0)$)

$$w = \sum_i \pi_i \{-\exp(-\gamma(e_i + t_i(w))) + \beta w'_i(w)\}. \quad (1)$$

The firm must also respect the unobservability of endowments, again for each $w \in [w^*, 0)$. This is expressed as incentive constraint

$$-\exp(-\gamma(e_2 + t_2(w))) + \beta w'_2(w) \geq -\exp(-\gamma(e_2 + t_1(w))) + \beta w'_1(w). \quad (2)$$

This states that agents with high endowments must be willing to reveal them.

Let $C_{w^*}(w) : [w^*, 0) \rightarrow R$ denote the cost (or expected loss) in terms of current consumption of providing an expected discounted utility w to an agent. (This cost will depend on w^* since w^*

determines the feasible range of the promised utility function $w'_i(w)$. Efficient contracts can be found by noting that $C_{w^*}(w)$ is a fixed point of the operator T defined by the following programming problem. The operator T takes as given a cost function governing utility promises made today for tomorrow on, and generates the cost function governing utility promises from the perspective of today. The objective of the program is to minimize the value of net consumption transfers in terms of current consumption, or,

$$(TC_{w^*})(w) \equiv \min_{t_i(w), w'_i(w) \in [w^*, 0)} \sum_i \pi_i \{t_i(w) + \beta C_{w^*}(w'_i(w))\} \quad (3)$$

subject to (1) and (2).

One can show that $C_{w^*}(w)$ is an increasing, convex function from the fact that the T is a contraction mapping which maps the space of increasing, convex functions to itself.³

4. Existence and Uniqueness of Stationary Equilibrium

This section discusses the existence of a unique market utility w^* , that clears the insurance market at every date. In this model, market clearing essentially boils down to a zero profit condition on firms. If the market utility w^* is such that contracting with an agent causes positive profits ($C_{w^*}(w^*) < 0$), every firm will wish to contract with an infinite number of agents. Every firm will also wish to contract with an infinite number of agents if there exists a $w > w^*$ such that $C_{w^*}(w) < 0$ since it can offer a contract offering utility $w > w^*$ and thus contract with as many agents as it wishes. On the other side, if contracting with an agent at a utility w^* causes a negative profit ($C_{w^*}(w^*) > 0$), and all higher utilities $w > w^*$ also cause a negative profit, no firm will wish to contract with an agent.

³ To be more careful, $C_{w^*}(w)$ can be shown to be bounded on $[w^*, 0)$ by the cost function associated with repeating the one-period problem and the cost function associated with full-information. The set of continuous functions between these bounds is a normed vector space with the sup norm. The operator T is a contraction mapping on this space which preserves convexity and monotonicity.

The agent side is equally simple. Every agent will contract with a firm as long as the market utility w^* is greater than or equal to the utility associated with autarky.

Lemma 1: There exists a unique expected discounted utility w^* such that $C_{w^*}(w^*) = 0$, and for all $w > w^*$, $C_{w^*}(w) > 0$.

Proof. Let $\underline{w}$ be the utility associated with autarky and $\bar{w}$ be the utility associated with the full insurance contract. We know $C_{\underline{w}}(\underline{w}) \leq 0$ since autarky ($t_i = 0$ for all i and all dates) is feasible and has a cost of zero to firms. Further, we know $C_{\bar{w}}(\bar{w}) > 0$ since full insurance is not feasible. Let $f(w) = C_w(w)$. The Theorem of the Maximum delivers f as a continuous function on $[\underline{w}, \bar{w}]$, thus there exists a w^* such that $C_{w^*}(w^*) = 0$. The fact that $C_{w^*}(w)$ is increasing in w delivers for all $w > w^*$, $C_{w^*}(w) > 0$.

This leaves uniqueness. Let $g(w) : [\underline{w}, \bar{w}] \rightarrow [\underline{w}, \bar{w}]$ be defined as follows. If C_w is always greater than zero on $[w, \bar{w}]$, then $g(w) = \underline{w}$. Otherwise, $g(w)$ equals the point where C_w equals zero. This is a weakly decreasing function and thus has at most one fixed point. (It is decreasing because raising w only tightens the constraint set defining C_w .) It is easy to show that if $C_{w^*}(w^*) = 0$, then w^* is a fixed point of g . ■

5. Existence of Trade

I now consider whether the equilibrium is the trivial equilibrium associated with autarky. Note that since the equilibrium in the static context is autarky, this question is equivalent to asking whether the inability of agents to commit to intertemporal contracts wipes out intertemporal trade. I show that this is not the case. To do this, it is sufficient to show that the continuation utilities of the market contract are not trivial, or it is not the case that $w'_1(w^*) = w'_2(w^*) = w^*$.

Lemma 2: Let w^* denote the market utility and $t_i(w) : [w^*, 0) \rightarrow [-e_i, \infty)$ and $w'_i(w) : [w^*, 0) \rightarrow [w^*, 0)$, ($i \in \{1, 2\}$) denote a cost-minimizing, incentive-compatible contract delivering utility w^* . Then it is not the case that $w'_1(w^*) = w'_2(w^*) = w^*$.

Proof. Suppose $w'_1(w^*) = w^*$. Let $u_i(w^*) = -\exp(-\gamma(e_i + t_i(w^*)))$, or the current period utility of an agent who receives endowment $i \in \{1, 2\}$, and let $d_e = e_2 - e_1$. Assume the incentive constraint binds since at autarky the incentive constraint binds, and thus if not, the result is proved. Given $w'_1(w^*) = w^*$ the incentive constraint (2) and the promise keeping constraint (1) can be used to solve for two of the remaining three choice variables $u_1(w^*)$, $u_2(w^*)$, and $w'_2(w^*)$ as a function of the third. Here I solve for $u_1(w^*)$ and $u_2(w^*)$ as functions of $w'_2(w^*)$. These conditions deliver

$$u_1(w^*) = \frac{(1 - \beta)w^*}{\pi_1 + \pi_2 \exp(-\gamma d_e)} \quad (4)$$

which does not happen to depend on $w'_2(w^*)$, and

$$u_2(w^*) = u_1(w^*) \exp(-\gamma d_e) + \beta w^* - \beta w'_2(w^*). \quad (5)$$

With $u_1(w^*)$ and $w'_1(w^*)$ set and independent of $u_2(w^*)$ and $w'_2(w^*)$, the cost minimization problem for a firm can be written as

$$\min_{u_2(w^*), w'_2(w^*) \in [w^*, 0)} -\left(\frac{1}{\gamma}\right) \log(-u_2(w^*)) - e_2 + \beta C_{w^*}(w'_2(w^*)). \quad (6)$$

subject to (5) and (4). Let $C'_{w^*}(w^*)$ denote the right hand side derivative of C_{w^*} at w^* . The first order condition of (6) (an inequality because of the condition that $w'_2(w^*) \geq w^*$) can be written

$$\frac{-1}{\gamma(u_1(w^*) \exp(-\gamma d_e) + \beta w^* - \beta w'_2(w^*))} \leq C'_{w^*}(w'_2(w^*)). \quad (7)$$

If $w'_2(w^*) = w^*$, then this implies

$$\frac{-1}{\gamma u_1(w^*) \exp(-\gamma d_e)} \leq C'_{w^*}(w^*). \quad (8)$$

Now let $\bar{C}(w)$ denote the cost function associated with an explicit restriction to one-period contracts, or $w'_1(w) = w'_2(w) = w$. One can show that $\bar{C}(w)$ takes the form

$$\bar{C}(w) = \frac{-1}{\gamma(1-\beta)} \log(-w) + X, \quad (9)$$

where X is a constant. If $w'_1(w^*) = w'_2(w^*) = w^*$ then there is no trade at w^* so $C_{w^*}(w^*) = \bar{C}(w^*)$. Further, the fact that $C_{w^*}(w^*) \leq \bar{C}(w^*)$ for $w > w^*$ implies $C'_{w^*}(w^*) \leq \bar{C}'(w^*)$. Substituting for the derivative of $\bar{C}$ and the equation for $u_1(w^*)$ (4) into (8) and simplifying delivers the inequality

$$\frac{\exp(-\gamma d_e)}{\pi_1 + \pi_2 \exp(-\gamma d_e)} \geq 1, \quad (10)$$

which is a contradiction since $0 < \exp(-d_e) < 1$. If $w_1(w^*) = w^*$, then firms will strictly want to set $w'_2(w^*) > w^*$. ■

The fact that given two-sided commitment $w'_1(w^*) < w^*$ implies that given one-sided commitment, $w'_1(w^*) = w^*$. Once it is shown that $w'_2(w^*) > w^*$, one can use the incentive constraint (2), the promise keeping constraint (1), the fact that $C_{w^*}(w^*) = 0$, and the fact that the market utility w^* is higher than that associated with autarky to more fully characterize the market contract at w^* . Specifically, one can show that $t_1(w^*) > 0$, and $t_2(w^*) < 0$. Since agents with the low endowment essentially start over ($w'_1(w^*) = w^*$), since $t_1(w^*) > 0$, real insurance is provided.

The intuition behind the proof is that even if one-sided commitment implies $w'_1(w^*) = w^*$, cost minimization will still imply $w'_2(w^*) > w^*$ since it is still possible to delay rewards for agents who announce a high realization. Agents facing a delayed reward will not walk away from the contract. Insurance is still possible for essentially the same reason as in the two-sided commitment model. An agent with a high endowment has a lower marginal utility of consumption (at his endowment point) than an agent with a low endowment, but each has the same expected marginal utility of consumption for the next period under autarky, which is between the two. This difference in preferences allows firms to offer those who announce low endowments a positive transfer, but at

the cost of starting the contract over tomorrow ($w'_1(w^*) = w^*$). Agents with high endowments must give up a give up current consumption $t_1 - t_2$, but receive a higher utility $w'_2(w^*)$ from tomorrow on. The difference in preferences regarding tradeoffs between current period consumption and future utility promises are sufficient to allow truthful revelation where $t_1 - t_2 > 0$. Providing insurance allows the firm to deliver w^* at a lower expected discounted cost.

6. Taste Shocks and Constant Relative Risk Aversion

The previous sections assumed the constant absolute risk aversion utility function $U(c) = -\exp(-\gamma c)$. With this utility function, an unobserved endowment economy is *exactly* the same as an unobserved taste shock economy where the taste shock is multiplicative as in Atkeson and Lucas (1992). That is, let the common observed endowment equal $\bar{e} = \pi_1 e_1 + \pi_2 e_2$, and let the agent's utility function be a function of his consumption c and an i.i.d. taste shock z_i where $U(c, z_i) = -\exp(-\gamma c) z_i$, $i \in \{1, 2\}$. If $z_i = \exp(-\gamma(e_i - \bar{e}))$, then the environments are isomorphic. Receiving a low endowment in the unobserved endowment model is exactly the same as receiving a high taste shock (or high urgency to consume shock) in the taste shock model.

For other utility functions, this correspondence does not occur. In fact, the unobserved taste shock environment tends to be easier to analyze than the unobserved endowment environment for utility functions other than constant absolute risk aversion. For the case of constant relative risk aversion where $U(c, z_i) = (c^\sigma / \sigma) z_i$, ($\sigma \in [-\infty, 1)$, and $\sigma = 0$ denoting $U(c, z_i) = \log(c) z_i$), both Lemma (1) and Lemma (2) hold. The proof of Lemma (1) is identical. The steps in the proof of Lemma (2) can simply be repeated for this specification, resulting in a similar contradicting inequality. What is necessary for this proof is that the derivative of the cost function for the one-period problem $\bar{C}'(w)$ be analytically derived. Given this, the results of the next section apply to the taste shock model as well.

7. Long Run Properties

Obviously, if w^* is the minimum beginning-of-period expected discounted utility an agent can enjoy under one-sided commitment, the results of Green (1987) and Thomas and Worrall (1990) that almost all agents have w go to negative infinity is overturned. However, a stronger result can be proved. Here, we get the result that a non-degenerate long-run distribution of utilities w exists, as well as a corresponding non-degenerate long-run distribution of consumption. Agents do not go to bliss with positive probability nor do they converge to a single point.

Lemma 3: Let w_t be the expected discounted utility of an agent at date t . The markov process defined by $w_{t+1} = w'_1(w_t)$ with probability π_1 and $w_{t+1} = w'_2(w_t)$ with probability π_2 has a non-degenerate limiting distribution from the starting point w^* .

Proof.

Let $W \subset [w^*, 0)$ denote the *countable* set of utilities which can be reached from w^* . This is the state space of the markov chain defined in the statement of the lemma. (It can be ordered by listing those states which can be reached in one period, followed by those which can be reached in two, and so forth.) Let P denote the transition matrix of this markov chain.

It can be shown that all states communicate. With a long enough string of low endowments, it is possible to reach w^* from any point in the state space, and by definition all points can be reached from w^* . This implies that all states are of the same type, either recurrent (where the probability of eventually revisiting the state after visiting it once is one) or transient (where this return probability is less than one).

We will show that all states are recurrent by showing that w^* is recurrent, as follows: Let E denote the set of w such that $w'_1(w) = w^*$. That is, E represents the states near enough to the

lower bound such that if the agent has a bad outcome, he goes to w^* in one move. (The set E is non-empty because $w'_1(w^*) = w^*$.) Let $\tilde{E}$ represent the complement of E . Choose a state $w \in \tilde{E}$. For such a w , the constraint that $w'_1(w) \geq w^*$ does not bind. Since the constraint $w'_i(w) \geq w^*$ is not binding, the first order conditions of the firm's problem (3) with respect to $w'_1(w)$ and $w'_2(w)$ are

$$\begin{aligned}\pi_1 C'_{w^*}(w'_1(w)) - \lambda \pi_1 - \mu &= 0 \\ \pi_2 C'_{w^*}(w'_2(w)) - \lambda \pi_2 + \mu &= 0,\end{aligned}\tag{11}$$

where λ and μ are the multipliers associated with the promise keeping constraint (1) and the incentive constraint (2). (The function C_{w^*} can be shown differentiable using the same arguments as Thomas and Worrall and applying Benveniste and Scheinkman (1979)). Adding these equations together and using the envelope condition for λ delivers

$$C'_{w^*}(w_t) = \pi_1 C'_{w^*}(w'_1(w_t)) + \pi_2 C'_{w^*}(w'_2(w_t)).\tag{12}$$

The range of C'_{w^*} is (x, ∞) where $x = C'_{w^*}(w^*) > 0$. Let $x_t = C'_{w^*}(w_t)$. The process $(x_0, x_1, \dots)$ is itself a Markov chain on the state space $C'_{w^*}(W)$. Define a new markov chain $(y_0, y_1, \dots)$ by altering the process $(x_0, x_1, \dots)$ so that state x_i is absorbing if the utility w_i corresponding to x_i is an element of E . This new markov chain is a non-negative martingale by equation (12), and thus almost all paths converge by Doob's martingale convergence result. (see Billingsley). However, Thomas and Worrall (1990) show, using arguments that apply here as well, that almost all paths $(y_1, y_2, y_3, \dots)$ cannot converge to an element of $C_{w^*}(\tilde{E})$. Simply put, if a path converges, then future utilities are not being used to punish and reward, which causes a contradiction. Thus almost all paths $(y_1, y_2, y_3, \dots)$ converge to an element of $C_{w^*}(E)$. This implies that when the process $(w^*, w_1, w_2, \dots)$ escapes E , it eventually returns with probability one. That within E there is a positive probability of reaching w^* proves that w^* is recurrent.

The process $(w^*, w_1, w_2, \dots)$ can be shown to be non-cyclical because $P(w^*, w^*) > 0$. If P represents the transition probabilities of a recurrent non-cyclic markov chain, then $\lim_{t \rightarrow \infty} P^t$ exists and defines the long-run distribution. (Kemeny, Snell, and Knapp).

It only remains to be shown that the limiting distribution does not put all mass on one point. It is straightforward to show that no points are absorbing by generalizing the arguments in the proof of Lemma 2, which showed that w^* is not absorbing. ■

An immediate corollary of Lemma 3 is that a non-degenerate long-run distribution of consumption exists.

8. Differing Interest Rates

In the previous analysis, I have set the discount parameter for firms implied by the rate at which they can borrow and lend (denote δ) equal to the discount parameter for agents, β . This is a useful benchmark case because in the full information case, equal discount rates implies no trend in consumption over time. On the other hand, if firms discount the future more than agents (or $\delta < \beta$), then the consumption of agents increases over time. With unobserved endowments, an assumption that $\delta < \beta$ gives firms added incentive to postpone consumption payments to the agent and thus one-sided commitment is less likely to bind. The result that trade still occurs given one-sided commitment still holds given $\delta < \beta$. On still another hand, if $\delta > \beta$, firms are given an incentive to move consumption forward in time. This makes one-sided commitment more binding since agents can walk away from contracts with decreasing consumption paths. One can show that if δ is sufficiently greater than β , then in the two-sided commitment environment, even those who are being rewarded start the second period with a lower continuation utility than the utility they started the first period with. That is, $w^* > w'_2(w^*) > w'_1(w^*)$. Given this, it is possible that the inequality derived in the proof of Lemma (2), (equation (10) with extra terms reflecting the

differing discount rates), is no longer a contradiction. If the two-sided commitment case delivers a sufficiently downward trend in consumption, all agents will walk away from the contract if given the chance, and thus the ability to do this eliminates all trade.

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