Sectoral Economics

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Draft January 1994

Chapter Intended for *Handbook of Computational Economics*
Hans Amman, David Kendrick and John Rust, editors
North-Holland Publishing Company, Amsterdam

1. Introduction

Much of economic analysis falls into the two large domains of the microeconomics of individual firms and the macroeconomics of national economies. In between these two domains lies the study of individual sectors of the economy such as the automobile industry or the oil industry. Perhaps this should be called mesoeconomics.* This has traditionally been the area covered by industrial organization. However, the focus of industrial organization has been narrowly on the regulation and behavior of firms in monopolistic or competitive environments rather than on the broad set of economic issues which arise in sectors such as energy, agriculture, environment, education, or transportation.

Therefore, substantial areas of economics analysis have developed separately as fields such as energy economics, agricultural economics or the economics of education. An important part of the work in these studies of separate sectors of the economy has employed numerical economic models which are solved on computers. This has occurred because when one studies a sector of the economy, analytical methods with their restriction to models with only a few equations, one or two commodities, one or two locations, a focus on steady state results and only convex functions have proved to be inadequate. Rather sectoral models frequently address issues involving many commodities, many locations, substitution and economies of scale and this has meant that computational methods have a decided advantage over analytical methods in sectoral economics studies.

There are three purposes which underlie the discussion of sectoral economics here. First is to discuss a common set of methods which are not currently in wide use in either microeconomics or macroeconomics but which constitute a core of the methodology for models in many different sectors of the economy. Some of these methods will be discussed here and pointers will be given to the location in the literature for discussion of others.

The second purpose is to provide something of a tour of the varieties of sectoral models. This is a growing and vibrant area of economic research but one that is relatively unknown to many economists. The aim here is not to provide a comprehensive discussion of the models for any given sector but to provide illustrations of the types of models in use. Neither will a comprehensive bibliography be provided but rather only citations to illustrative works.

The third purpose is to provide discussion of how some of the methods of sectoral economics can be used to great advantage in microeconomics or macroeconomics. One can think of a progression from the microeconomics of a single firm to sectoral economics with many firms in a single industry to macroeconomics covering the entire economy. The theme here is that the interfaces between these three fields are not such much lines as permeable cell walls in which the methods which have been developed in sectoral economics offer substantial opportunities for improving some of the ways we do microeconomics and macroeconomics.

* The name mesoeconomics was suggested to me by Bruce Smith.

For microeconomics in particular it is the view of this author that we economists missed an early opportunity to incorporate these tools fully into our tool kit. Tjalling Koopmans, Robert Dorfman, Paul Samuelson and Robert Solow and many other economists were contributors to the beginning of mathematical programming. However, these tools were not widely adopted in standard microeconomics but rather were taken up by the fields of Operations Research and Management Science and fashioned into some of the basic methods which are now used in industry and commerce. This occurred because of too much emphasis in economics on analytical results and not enough attention to the rapidly growing power of computational methods. So this chapter will contain discussion at various points as to how some of the tools of sectoral economics might be adopted again in standard microeconomics with substantial gain to that field.

Progress in sectoral economics depends not only upon the development of new methods of specifying the economic and mathematical relationships in the models, rather it also depends crucially on improvements in the software which is used for creating and solving the models. Therefore, this chapter includes a section on modeling languages, expert systems and graphical user interfaces and the role they are playing in sectoral economics.

The chapter begins with a discussion of a core of methods used in sectoral economics. This is followed by a sections on software and then by a section which cover some of the major areas of sectoral modeling, i.e. process industries, computer industry, energy, environment and agriculture.

2. Methods

An introductory discussion of three of the basic methods of sectoral economics follows in this section. The first of these is the modeling of production with activity analysis. This draws on the work of Koopmans (1951) and grew naturally out of the use of linear programming methods in economics, viz Dorfman, Samuelson and Solow (1958). The methods have proven to be so useful for modeling diverse production activities and for analyzing substitution possibilities in the face of changing relative prices that they have remained in substantial use even as nonlinear programming methods have been widely adopted.

The second set of methods concern the modeling of location. This springs in part from the early work of Dantzig (1951) and of Koopmans and Reiter (1951) on the linear programming transportation problem. Raw materials, plants and markets are located at fixed points in space and the shipment of materials between these locations is an important part of the economics of a number of sectors. Therefore, sectoral models have made wide use of the linear programming transportation problem and its generalizations.

The third methods is the modeling of economies of scale. Much of microeconomic theory is built on the assumption that production functions exhibit diminishing returns to scale and are therefore convex. However, this assumption is not valid in many parts of production in modern industrial economies. Rather economies of scale are widespread in the process industries and in transportation and communication. So numerical methods were devised for solving models in which production was characterized by economies of scale using non-convex production functions.

As these methods are discussed both the mathematics and the computational development will be shown. The computational development for sectoral economic models is done with a wide variety of types of software systems. One of the most widely used of these systems is GAMS, cf. Brooke, et.al. (1988) so this is the system which will be used here for illustrative purposes. GAMS is useful for solving algebraic models of all kind including linear, non-linear and mix-integer mathematical programming problems. Examples of similar software systems are AMPL by Fourer et al (1987), AIMMS by Bischopp (19xx) and Structured Modeling by Geoffrion (1987).

Also, as the mathematical and computational representations of various components are developed there will be some discussion of "style". This follows the approach of the famous little book *The Elements of Style* by William Strunk, Jr. and E. B. White (1959). This book has been the Bible for generations of American college students as they labor to learn to express themselves well in writing. It is the authors opinion that style is as important in mathematical and computational communication as in written communication, cf. Kendrick (1984) so that theme is interlaced throughout the discussion below.

The following sections is written in an introductory fashion so the reader who is already familiar with these methods may want to simply scan the material on the way to Section 3 on sectoral models of the process industries.

2.1. Activity Analysis

Activity analysis has been widely used in sectoral studies but is not commonly taught in courses on microeconomics which rather focus on neoclassical production functions like the Cobb-Douglas or constant elasticity of substitution (CES) functions. Therefore a brief introduction to the subject of activity analysis is provided here.

Consider an environmental study which is concerned with the production of carbon dioxide in electric power production. *Commodities* are both inputs and outputs. For example, coal and labor are used to produce electric power. In addition carbon dioxide is produced as a by-product. Such a *production process* can be represented by a vector in the tradition of Koopmans (1951) as

	electric power production
coal (tons)	$a_{coal,elec\ power}$
labor (man-hours)	$a_{labor,elec\ power}$
electricity (kwh)	1
carbon dioxide	$a_{carbon-dioxide,elec\ power}$

This is a production process with two inputs and two outputs. The convention is used that inputs are negative so the sign of the coal and labor inputs in the vector above will be negative while the sign of the electricity and carbon dioxide coefficients will be positive. Thus $a_{coal,elec\ power}$ tons of coal and $a_{labor,elec\ power}$ man-hours of labor are required to produce one kilowatt hour of electricity and $a_{carbon-dioxide,elec\ power}$ of carbon dioxide. While most neoclassical production functions have a single output, it is natural in process analysis to have multiple outputs as is the case above. This is one of the aspects of activity analysis which accounts for its widespread use in sectoral models.

Substitution is important in most sectoral models. For example one might want to use an environmental model to analyze the effect of carbon taxes on electricity production. As the carbon taxes are increased one would expect power producers to shift from fuels which yield larger quantities of carbon dioxide like coal to those which yield smaller quantities like fuel oil or natural gas. A model to study this phenomena could have three activities as is shown below

Table 2.1.1 Three Processes for Producing Electric Power

	electric power with coal	electric power with fuel oil	electric power with natural gas
coal (tons)	$a_{coal,elec\ with\ coal}$		
fuel oil (barrels)		$a_{fuel\ oil,elec\ with\ fuel\ oil}$	
natural gas (mcf)			$a_{gas,elec\ with\ gas}$
labor (hours)	$a_{labor,elec\ with\ coal}$	$a_{labor,elec\ with\ fuel\ oil}$	$a_{labor,elec\ with\ gas}$
electricity (kwh)	1	1	1
carbon dioxide	$a_{carbon-dioxide,elec\ with\ coal}$	$a_{carbon-dioxide,elec\ with\ fuel\ oil}$	$a_{carbon-dioxide,elec\ with\ gas}$

In Table 2.1.1 the coefficients in the first four rows will be negative and those in the last two rows will be positive since coal, fuel oil, natural gas and labor are inputs and electricity and carbon dioxide are outputs. The three vectors in Table 2.1.1 represent the three production processes. In the first vector coal is used to produce a kilowatt hour of electricity as well as $a_{carbon-dioxide,elec\ with\ coal}$ of carbon dioxide. In the second vector $a_{fuel\ oil,elec\ with\ fuel\ oil}$ barrels of oil are used to produce the same amount of electricity but the carbon dioxide output is the smaller amount

$a_{carbon-dioxide, elec\ with\ fuel\ oil}$. In the last vector natural gas is used and the still smaller amount $a_{carbon-dioxide, elec\ with\ gas}$ of carbon dioxide is produced.

Mathematically the relationships above can be stated

$$y_c = \sum_{p \in P} a_{cp} z_p \qquad c \in C \qquad (1)$$

where

y_c = output (+) or input (-) of commodity c
a_{cp} = input-output coefficient for commodity c in process p
z_p = level of process p
P = set of processes
= $\{elec\ with\ coal,\quad elec\ with\ fuel\ oil,\quad elec\ with\ gas\}$
C = set of commodities
= $\{coal, fuel\ oil, gas, labor, electricity, carbon\ dioxide\}$

For computational purposes the constraint (1) can be written in GAMS. First the sets are defined with the statements

```
SETS
     C     Commodities
           / coal,fuel-oil,gas,labor,elec,car-diox /
     P     Processes
           /   elec-coal, elec-fo, elec-gas /
```

Many of the computer languages which are used for sectoral modeling are set-driven in the sense that sets are defined as here and then parameters, variables and equations are declared over these sets. This contrast with most econometric modeling packages where set operators are not used and each parameter, variable and equation is separately defined.

Then the parameters, variables and equations are declared.

```
PARAMETERS
     A(C,P)       Input-Output Coefficients
VARIABLES
     Y(C)         Output or Input Levels
     Z(P)         Process Levels
EQUATIONS
     ONE(C)       Input-Output Equation;
```

In each case the mathematical symbol is defined over the appropriate set or sets, i.e. the input-output coefficients are defined over the commodity set C and the process set P.

Recall that the mathematical statement of equation (1) is

$$y_c = \sum_{p \in P} a_{cp} z_p \qquad\qquad c \in C \qquad\qquad (1)$$

With the declarations above Eq. (1) can be written in the GAMS language as

```
ONE(C)..   Y(C)  =E= SUM(P, A(C,P)  *  Z(P))  ;
```

There is almost a one-to-one mapping from the mathematical statement to the computational statement of the equation. This is an important property of upper-level computational languages like GAMS. Sectoral models are frequently notationally complex and this one-to-one mapping means that the mathematics of the model can be translated with ease into the computational language thereby minimizing the number of errors.

The set-driven nature of the GAMS language makes it necessary to write only one equation even though there may be five to five hundred commodities in the model. Thus set driven languages are extremely useful in sectoral modeling because there are usually numerous commodities, processes, production units and locations so it would be a tedious and error prone activity to define each parameter, variable and equation individually.

Consider next an example of the use of Eq. (1) to calculate the output of carbon dioxide from a power plant using three alternative fuels. If coal was used to produce 20 kwh's, fuel oil was used to produce 30 kwh and natural gas was used to produce 10 kwh's the output of carbon dioxide would be

$$\begin{aligned} y_{carbon\ dioxide} = \; & a_{carbon\ dioxide, elec\ with\ coal}\,(20) + a_{carbon\ dioxide, elec\ with\ fuel\ oil}\,(30) + \\ & a_{carbon\ dioxide, elec\ with\ gas}\,(10) \end{aligned}$$

An environmental model might include a constraint that carbon emissions should be less than or equal to a certain level, $\bar{e}$. This constraint would be written in the model as

$$y_{carbon\ dioxide} \le \bar{e} \qquad\qquad (2)$$

or using (1) as

$$\sum_{p \in P} a_{carbon-dioxide, p} z_p \le \bar{e} \qquad\qquad (3)$$

This shows how a production function in activity analysis form might be incorporated into a mathematical programming model of the economy and the environment. In GAMS this would be represented by defining an additional parameter and inequality as follows:

```
PARAMETERS
     EBAR        Upper Limit on Carbon Emissions;

EQUATIONS
     THREE(C)    Carbon dioxide limit constraint;

THREE("car-diox").. SUM(P,A("car-diox",P)*Z(P))=L=EBAR;
```

Eq. (3) applies to only carbon dioxide; however, it is first declared over all commodities and then defined over a single element of the set.

Models with activity analysis also frequently include cascades of production functions in which the output of one activity becomes the input of another activity. Consider for example an oil refinery model with processes for distillation and catalytic reforming. The vectors for these two processes could be

Table 2.1.2 Two Processes in an Oil Refinery

	distillation	catalytic reforming
crude oil (barrels)	-1.0	
naphtha (barrels)	.4	-1.2
gasoline (barrels)	.6	1.0

For each barrel of crude oil that is input to the distillation process there is production of .4 of a barrel of naphtha and .6 of a barrel of gasoline. The naphtha in turn serves as an input to the catalytic reforming process where 1.2 barrels of naphtha is required to produce a barrel of gasoline. The constraint for naphtha takes the form

$$\sum_{p \in P} a_{cp} z_p \geq 0 \qquad c = naptha \qquad (4)$$

i.e.

$$a_{naptha,distillation} z_{distillation} + a_{naptha,reforming} z_{reforming} \geq 0$$

In the summation when $p = distillation$ the a_{cp} coefficient will be positive to indicate that naphtha is produced as is shown in Table 2.1.2. When $p = reforming$ the a_{cp} coefficient will be negative to indicate that naphtha is consumed as is also shown in Table 2.1.2. Thus the inequality (4) requires that as much naphtha be produced by the first process as is used by the second process.

For an introductory discussion of the use of activity analysis in sectoral models see Kendrick and Stoutjesdijk (1978) Ch. 3. Sectoral models frequently contains dozens of commodities and processes in which commodities are produced in some processes and used in others. Some examples are the fertilizer industry in Choksi, Meeraus and Stoutjesdijk (1980), the personal computer industry in Ang (1992), the copper industry in Dammert and Palaniappan (1985) or the steel industry in Westphal (1971).

The discussion above introduces the concept of commodities and processes as they are used in activity analysis models. A third concept that is important is that of a *productive unit*. Consider a boiler that is used to produce steam which turns a turbine to produce electric power. Three processes for these production activities are shown in Table 2.1.3.

Table 2.1.3 Three Processes for Producing Steam

	electric power with coal	electric power with fuel oil	electric power with natural gas
coal (tons)	$a_{coal,elec\ with\ coal}$		
fuel oil (barrels)		$a_{fuel\ oil,elec\ with\ fuel\ oil}$	
natural gas (mcf)			$a_{gas,elec\ with\ gas}$
steam (cubic feet)	1	1	1

Either coal, fuel oil or natural gas can be used to produce steam. In this case the boiler is designed to burn coal, fuel oil or natural gas, so that as the relative price of these three fuels change the power company can switch among the fuels.

The boiler is called a productive unit and has a fixed capacity of so many cubic feet of steam per year. The model then includes an inequality to represent the fact that the three processes can be used in any mix during the year so long as the total usage does not exceed the capacity of the productive unit. This is written mathematically as

$$\sum_{p \in P} b_{mp} z_p \leq k_m \qquad m \in M \tag{5}$$

where

k_m = the capacity of productive unit (machine) m

b_{mp} = a coefficient which is one if process p uses productive unit m and is zero otherwise.

M = the set of productive units.

Then the constraint (5) for the boiler would be written as

$$z_{elec\ with\ coal} + z_{elec\ with\ fuel\ oil} + z_{elec\ with\ gas} \leq \quad k_{boiler} \tag{6}$$

Thus the three processes compete for the use of the capacity of the boiler and there will be substitution between these processes in response to changes in the prices of fuels including the level of any carbon taxes.

In summary activity analysis offers a natural way to model the production side in sectoral models. In these models *commodities* are transformed by *processes* from raw materials, to intermediates and then to final products and the processes compete for the use of the capacity of the *productive units.*

2.2 Location

One of the most important aspects of sectoral economics is space. Traditional economics which concerns space is divided into regional economics which focuses on regions in a country and on international trade which focuses on nations in a worldwide economy. Sectoral economics deals with a wide variety of spatial settings which include both of these traditional areas of study. Sectoral studies may range from a city or province to a country or even to the globe. For example, transportation sector models are frequently developed for a single city while global warming environmental models are most commonly national or international. Electric power models usually cover states or provinces while petrochemical models may cover all the countries in a region like the European Common Market or may be worldwide in scope.

In sectoral models raw materials, processing plants and markets are usually spatially separated and transportation costs play a substantial role in the economics of the sector. Consider the petroleum industry. A petroleum model might begin with crude oil in the Mid-East which is shipped to Rotterdam or Houston to be transformed to gasoline and marketed in Paris or Dallas. In the automobile industry the various parts for an automobile might be manufactured in several countries, assembled in another country and marketed in yet another.

For this reason sectoral models frequent consider location. This type of modeling originated with the use of the linear programming transportation problem which can be stated simply as minimize

$$\xi = \sum_{i \in I} \sum_{j \in J} \mu_{ij} x_{ij} \qquad (7)$$

where

ξ = total transportation cost
μ_{ij} = unit transportation cost for shipments from plant i to market j
x_{ij} = shipments from plant i to market j
I = set of plants
J = set of markets

In this model there is a set of plants and a set of markets and one seeks to find the shipments which minimize total transportation cost while providing the requirements of each markets and without violating the capacity constraints of the plants.

The computational representation of the criterion function (7) in GAMS is developed by first defining the mathematical elements as

```
SETS
     I          Plants
     J          Markets
PARAMETERS
     MU(I,J)    Transportation Cost Per Unit Shipped
VARIABLES
     XI         Total Transportation Cost (Greek xi)
     X(I,J)     Number of Units Shipped
EQUATIONS
     CRITERION  Objective Function;
```

Then the objective function can be written as

```
CRITERION..    XI =E=  SUM((I,J), MU(I,J) * X(I,J));
```

The constraints for the model are

$$\sum_{j \in J} x_{ij} \le k_i \qquad i \in I \qquad (8)$$

$$\begin{bmatrix} \textit{sum of shipments} \\ \textit{from plant } i \textit{ to} \\ \textit{all markets} \end{bmatrix} \le \begin{bmatrix} \textit{capacity of} \\ \textit{plant } i \end{bmatrix}$$

i.e. the shipments to all markets from each plant must be less than or equal to the capacity k_i of the plant.

In GAMS this is represented as

```
PARAMETERS
     K(I)              Capacity
EQUATIONS
     CAPACITY(I)       Capacity Constraint;

CAPACITY(I)..   SUM((J), X(I,J))   =LE=   K(I) ;
```

The second set of constraints for the model are

$$\sum_{i \in I} x_{ij} \geq d_j \qquad\qquad j \in J \qquad\qquad (9)$$

$$\begin{bmatrix} \textit{sum of shipments} \\ \textit{from all plants} \\ \textit{to market } j \end{bmatrix} \geq \begin{bmatrix} \textit{demand at} \\ \textit{market } j \end{bmatrix}$$

i.e. the shipments from all plants to market j must exceed the demand in that market. The style of using words in brackets like those under Eqs. 8 and 9 is called the "Alan Manne Notation" after its originator. This style of notation is one of the methods which are used to make complex models easier to understand.

In GAMS Eq. 9 is represented as

```
PARAMETERS
     D(J)              Demand
EQUATIONS
     DEMAND(J)         Demand Constraint;

DEMAND(J)..   SUM((I), X(I,J))   =GE=   D(J) ;
```

The complete representation of the simple linear programming transportation model in the GAMS language is shown below. This model has two plants and three markets.

```
SETS
     I     Plants
           / CHI       Chicago
             DAL       Dallas   /
     J     Markets
           / CLEV      Cleveland
             PITTS     Pittsburgh
             ATL       Atlanta   /
PARAMETERS
     K(I)       Capacity
           / CHI                42
             DAL                20 /
     D(J)       Demand
           /  CLEV              22
              PITTS             25
              ATL               15 /
     MU(I,J)   Transportation Cost Per Unit Shipped
           CLEV       PITTS          ATL
CHI         6.6        7.9           10.3
DAL        16.3       16.9           12.2

VARIABLES
     XI         Total Transportation Cost (Greek xi)
     X(I,J)     Number of Units Shipped
EQUATIONS
     CRITERION       Objective Function
     CAPACITY(I)     Capacity Constraint
     DEMAND(J)       Demand Constraint;

CRITERION.. XI =E=  SUM( (I,J),  MU(I,J) * X(I,J));

CAPACITY(I)..  SUM( J,  X(I,J) )  =LE=  K(I) ;

DEMAND(J)..    SUM( I,  X(I,J) )  =GE=  D(J) ;

MODEL   TRANSPORT  /ALL/ ;

SOLVE TRANSPORT USING LP MINIMIZING XI ;

DISPLAY X.L ;
```

After the last of the three equations are specified there is a "MODEL" statement which indicates that all of the three equations are included in the model. The "SOLVE" statement then determines that the model will be solved with linear programming (LP) methods and that the criterion value XI will be minimized. Finally, the results of the optimization are displayed with the "DISPLAY" statement.

This model would look somewhat different in one of the other modeling languages but the basic elements of sets, parameters, variables, equations, model, solve and display statements would be present though called by different names and have different syntax and capabilities.

This simple linear programming transportation model is the basis for much more elaborate specifications of location. A full model may include mines where raw materials are extracted, plants where the raw materials are shaped into intermediate commodities, other plants where the intermediate commodities are fashioned into final products and markets where the final products are consumed. In all these locations the simple principle remains that you cannot ship more than you have capacity to produce and that you must receive as much as is required at each stage of processing or final use.

The linear programming transportation problem above focuses on operations and not on investment. In dynamic variants of the transportation problem there is capacity for each productive unit in each plant in each time period. Also, there are investment activities which can be used to add to capacity. This is the problem which is the focus of much of sectoral economics. Given an existing set of plants, which productive units should be expanded in which plants in which years and by how much? Also, where should new plants be built and which existing plants should be shut down?

The economics of investment location highlights another important attribute of many sectoral models: namely, economies of scale.

2.3 Economies of Scale

Consider a country with markets in a number of large cities. If economies of scale in investment are strong and transportation cost are small then a single large factory could most efficiently serve the country. If, on the other hand, transportation cost were large and/or economies of scale in investment were weak, then many small plants located near the markets could most efficiently serve the country.

This is a scene that is replayed in many sectoral models though the stage may not be a single country, but rather a region within a country, a collection of countries in a common market or even the entire world economy. For the electric power industry one may be interested in a region within a country like the southern part of India. For the steel industry one may be concerned with a collection of countries like the European Common Market. For the microprocessor industry the relevant economic playing field might be the whole world.

When there are economies of scale the investment cost function looks like Fig. 2.3.1.

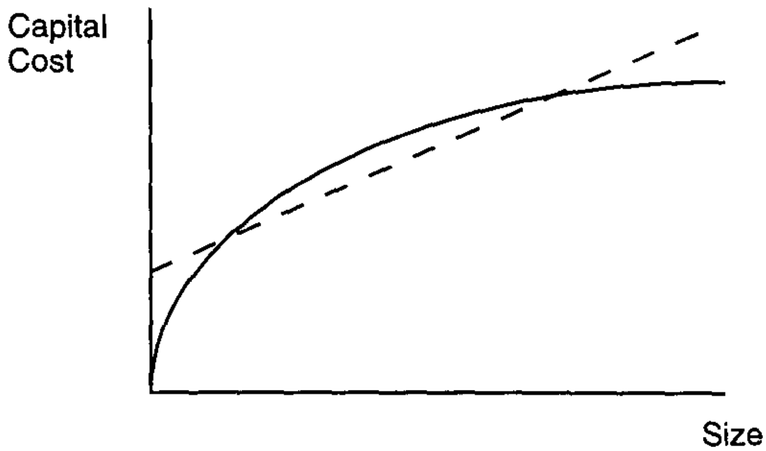

Figure 2.3.1 The Investment Cost Function

The curved line in the figure above is an investment cost function which is characterized by declining marginal cost, i.e.

$$\phi = f(h) \tag{10}$$

where

ϕ = investment cost
h = size of the productive unit

This function can be approximated by a linear function with a positive intercept and a positive slope as in shown by the dashed line in Fig. 2.3.1, i.e.

$$\phi = \omega y + vh \tag{11}$$

$$\begin{bmatrix} investment \\ cost \end{bmatrix} = \begin{bmatrix} fixed \\ cost \end{bmatrix} + \begin{bmatrix} variable \\ cost \end{bmatrix}$$

where

y = a zero-one integer variable
ω = the fixed-charge portion of the capital cost
h = the size of the productive unit
v = the slope of the approximate cost function

In addition to Eq. 11 two other constraints must be added to the models

$$h \le \bar{h} y \tag{12}$$

and

$$y = 0 \ or \ 1 \tag{13}$$

where

$\bar{h}$ = an upper bound on the size of the productive unit

The constraints (12) and (13) require that the full fixed charge ω be incurred if an investment of even the smallest size is made.

In GAMS Equations (12) and (13) are specified as

```
SCALAR
     HBAR Maximum Size of Productive Unit
VARIABLES
     H          Size of Productive Unit
     Y          Zero-One Integer Variable
POSITIVE VARIABLES   H;
BINARY VARIABLES     Y;
EQUATIONS
     FIXCOST    Fixed Cost Constraint

FIXCOST.. H =L= HBAR * Y ;
```

The key to this specification is the use of the "BINARY VARIABLES" statement which requires that the Y variables be restricted to the values of zero or one. This is coupled with "MODEL" and "SOLVE" statements as follows:

```
MODEL  STEEL   /ALL/ ;

SOLVE STEEL MINIMIZING PHI USING MIP ;
```

Here "STEEL" is the model name, "PHI" is the criterion function value and MIP (mixed integer programming) is the solver which is used to obtain the solution to the model.

Strictly speaking GAMS is a modeling system rather than a solution method, but it has associated with it a number of solvers for linear, nonlinear, linear mixed integer and nonlinear mixed integer programming. For example, both MINOS and CONOPT are nonlinear programming solvers which are associated with GAMS. MINOS was developed by Murtagh and Saunders(1987) and CONOPT was developed by Drud (19xx). Thus the user who has purchased both of these solvers packaged with GAMS can solve a given nonlinear programming model using these two codes by specifying "USING MINOS" or "USING CONOPT" respectively in the solve statement. Since different codes are effective in solving different nonlinear programming problems, this is an important option for the user.

Constraint (13) requires that the model be solved as a mixed integer programming model since the y variable cannot take on any non-negative value but rather only the values zero or one. This approach to the approximation of non-convex investment cost functions was developed by Markowitz and Manne (1957).

Essentially the investment problem in models with economies of scale is combinatorial. Usually there are many different productive units which can be expanded as well as new plant sites which can be opened at various locations. So the problem is one of finding which combination of these expansions is the most efficient. Mixed integer programming codes essentially go through the combinations and search for the best solution. However, the search is conducted in an efficient manner which makes use of the fact that some solutions are dominated. Still the computational cost of solving mixed integer programs increases very rapidly with the number of investment options under consideration. High speed computers have made this work possible and are now permitting the solution of substantial investment problems even on desktop computers. A recent example is the work on electric power planning where there are integer variables not only for productive units but also for transmission lines in the distribution network, cf. Baughman, Siddiqi and Zarnikau (1993).

From this background in some of the core methods of sectoral model we turn to a discussion first of the software which can be used to develop sectoral models and then of the models themselves.

3. Software

The GAMS language has been used in this chapter to illustrate the development in a computer language of the mathematics of sectoral modeling. While GAMS (Meeraus, (1983)) was one of the first upper level software systems which was widely used for sectoral modeling a number of competing systems have gained recognition in recent years.

These include the AMPL system of Fourer, Gay and Kernighan (1987) which was developed at Northwestern University and Bell Laboratories and was originally Unix-based. The MIMI (Manager for Interactive Modeling Interfaces) system has been developed by Baker (1990) to include a stronger database orientation than other modeling languages. Among other uses it has been applied to process industry models including distribution planning, operations planning and production scheduling. The Structured Modeling system of Geoffrion (1987) has a very carefully worked out theoretical background and an emphasis on developing the structure of the model independent of the data. One of the most recently developed system is the AIMMS software of Bisschop(19xx). This software is said to include a effective graphical interface to facilitate model development.

One of the most interesting ways to come up to speed in sectoral modeling is to look through the papers and books which describe the modeling languages mentioned above. There are numerous references there to the use of these systems for modeling a wide variety of sectors and different activities within those sectors.

Two recent development in software development hold interesting possibilities for sectoral models. The first is the use of artificial intelligence or expert systems to develop and maintain the models. An example of this is in Krishnan (1988) which utilizes the Prolog language to create an expert system called PM for process industry model development. A small portion of the representation of a model of the Mexican steel industry is shown in Table 3.1

Table 3.1 PM Representation

```
model_name  (io_for_iron).

index (years, [t]).
index (mills, [i]).
index (input, [c]).
index (iron, [r]).
index (iron_production,[p]).

isa (years, planning_period ).
isa (mills, plant ).
isa (input, raw_material ).
isa (iron, product ).
isa (iron_production, production_process ).

ins_of ( [1989], years).
ins_of ( [hylsa], mills).
ins_of ( [coking_coal], input).
ins_of ( [iron_ore], input).
ins_of ( [pig_iron], iron).
ins_of ( [pig_iron_production], iron_production).
```

The declaration of an "**index**" in PM is similar to set specification in the GAMS language except that the set is represented by the index rather than by the set symbol. The "**isa**" predicate is used to relate the sets (or indices) to internal knowledge variables which are used by PM. Thus the set of *years* is a "**planning-period**" and *mills* are "**plant**". This internal knowledge base gives PM some of its power by permitting logical test on the structure of the model and in helping the user to provide a complete model. The "**ins_of**" predicates provides instances or elements of the sets. Thus coking_coal and iron_ore are elements of the set of inputs.

Logic programming can provide powerful tools to help the user construct a model. They can also provide a very useful query structure like a database to answer questions about the sector. However, at this stage in their development they are tedious to construct. However, this tedium will probably be relieved in the future by the use of graphical interfaces for these systems. In the meantime, graphical systems are beginning to provide some of the functions of logic programming systems but in more intuitive framework.

This leads to the second software development for sectoral modeling, namely graphical interfaces which are specific to certain classes of models but which offer substantial ease of use. An example is the production and transportation modeling system, PTS, which was created by Kendrick (1991) to run under Windows. This system is specific to production and transportation modeling but the graphical interface greatly decreases the entry cost for first time modelers. A similar system of the Macintosh computer has been developed by Jones(1989).

These graphical systems are the wave of the future since they offer the possibility of maintaining parallel representations of a model in mathematical, graphical, database, spreadsheet, logic programming and upper-level language forms. Figure 3.1 shows the graphical representation of a simple linear programming transportation model in PTS running under Windows.

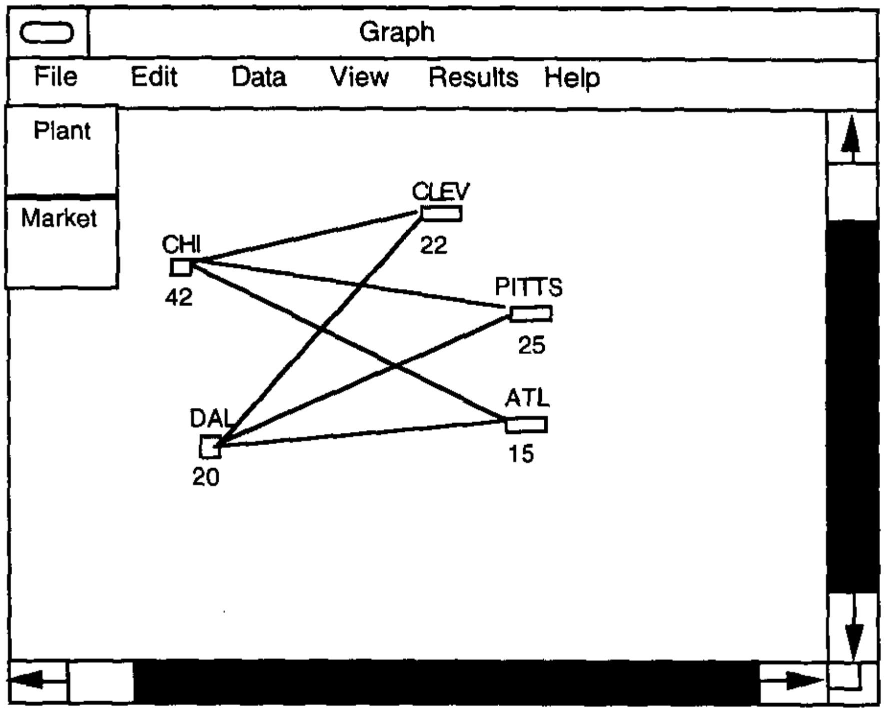

Figure 3.1 The Graph Window

The square symbols represent plants and the rectangles represent markets. A new plants can be added to the model by clicking on the "Plant" box in the upper left hand corner of the window and then moving the mouse to the place in the graph where it should be located and clicking again.

Figure 3.2 provides the GAMS representation of the same model. If the user adds a plant to the graphical view then the GAMS view will immediately be updated to reflect the change.

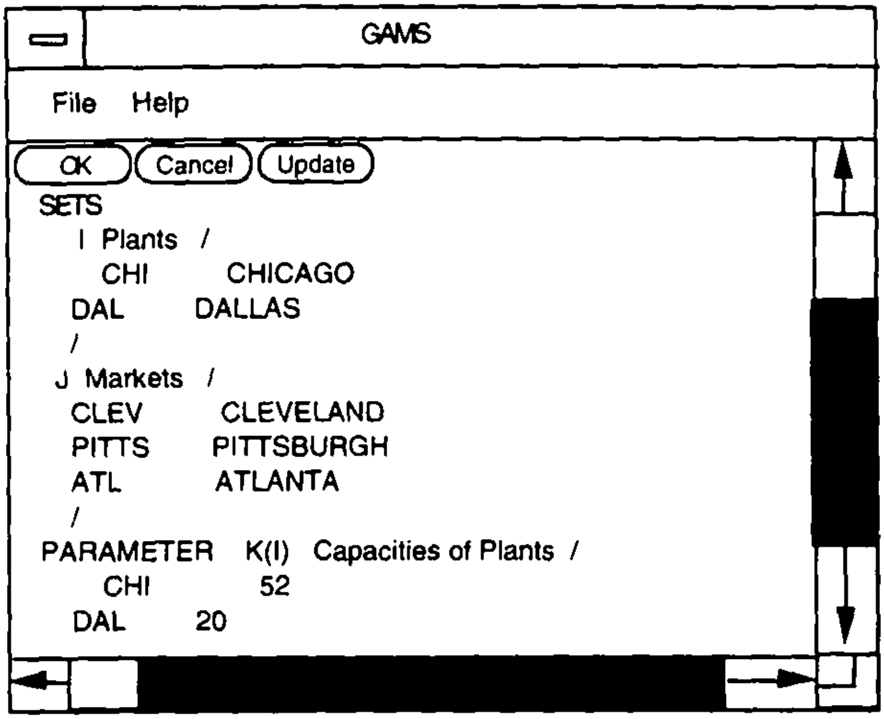

Figure 3.2 The GAMS Window

Some changes to the model are most easily made by changes in the graph window and others by changes in the GAMS window. Other views might also be used. For example PTS maintains a spreadsheet view of the transportation cost between plants and markets. This may be the view in which most users will prefer to enter changes in the transportation cost though one could also make that type of change in the graphical view or the GAMS view.

Each view may represent only a portion of the information about the model. For example the spreadsheet view might be used only for the transportation cost but not include data from the graphical view about where plants and markets are located.

PTS is experimental software which is designed to work with simple linear programming transportation problems and only provides three or four different views. However, it illustrates the principle of parallel model representations which will most likely be provided in the graphical user interface sectoral modeling systems of the future. PTS was implemented in the C language using the Software Development Kit for Windows. At the time PTS was developed OLE (Object Linking and Embedding) was not yet available; however future sectoral modeling system will make use of this capability. Then for example the spreadsheet implementation instead of being hand-crafted as in PTS might make use of the Microsoft Excel spreadsheet.

From the preceding discussion of the core methods of sectoral modeling and the software we now turn to a description of some of the sectoral models which have been developed in recent years. We begin with models from the process industries.

A comprehensive survey of the literature in each area is beyond the scope of this chapter. Rather highlights of the core methodology in each area are given along with illustrative references which will enable the interested reader to get a start in the literature.

4. Process Industries

In order to highlight the attributes of various sectoral models this section will contain a discussion of an illustrative set of models followed by an annotated listing of a group of models organized by geographical coverage and sector.

When one first approaches a sectoral economic analysis there is a tendency to build a single model which encompasses all the matters of interest. However, this strategy frequently runs afoul of the fact that even as fast a modern computers are they are not up to the task of solving quickly and easily fully specified models of sectors. Moreover, the analyst usually confront the fact that large models are difficult to understand and to verify that they are indeed correctly specified and entered into the computer in an error-free way.

Therefore, it usually valuable to plan on developing a number of different models in a given project. For example, the first model or models may focus on the operations problem of the firm and abstract from investment. Then at a later stage in the project models which include investment can be developed. However, when investment is added it is frequently necessary to use an aggregated version of the operations model which contains fewer commodities, productive units and processes.

It is important as one progresses from one model to another not to throw away the first models developed but rather to preserve them. For example, the first highly-aggregated, static, operations-only model is usually the model which is easiest for readers to fully comprehend. This model can therefore can be put to important use as an introduction to the project in a form that can be widely understood and therefore serve as a useful basis for communications between the modelers and the supervisors.

As an example of this kind of a system of models consider the models which were developed in a World Bank project to analyze the steel industry in Mexico, cf. Kendrick, Meeraus and Alatorre (1984). The project included small static, large static and small dynamic models which are used below to illustrate the strengths and weaknesses of such models.

4.1 Small Static

The work began with a small static model of the industry. It included five plants, three markets, five raw materials, two intermediate products, a single final product and a single time period. A list of the commodities gives an idea of the degree of disaggregation in the model.

raw materials
- pellets
- coke
- natural gas
- electricity
- scrap iron

intermediate products
- sponge iron
- pig iron

final product
- steel

A more disaggregated model would begin with iron ore and coal as raw materials rather than pellets and coke. Also, it would include shapes, reinforcing rods, hot rolled sheets, cold rolled sheets and tin as final products. However, even in its highly aggregated form, the small static model is sufficient to address many of the important operational issues of the industry.

This model includes cascades of production functions of the sort that were discussed in the Activity Analysis section above. In the first stage (i) pellets and coke are transformed into pig iron or (ii) pellets and natural gas are transformed into sponge iron. In the second stage (i) pig iron and scrap iron or (ii) sponge iron and electricity are transformed into steel. Also, since there is more than one process for making steel there is substitution between inputs as is described in the first part of the section above on Activity Analysis.

The criterion was to minimize the cost to transform the raw materials first into intermediate and then into final products in order to satisfy the demand requirements at the markets. So the basic structure of the original linear programming transportation model is maintained. This is a strength and a weakness. It is a strength in that cost minimization enables one to focus clearly on the relative efficiency of the different steel mills and the productive processes and productive units within those plants. It is also a strength in that three of the five steel mills in the model were owned by the government at the time of the study and minimizing the cost to serve the steel needs of the country was one of the key goals of the industry. Cost minimization in the linear programming transportation model framework is a weakness in that such models usually focus on meeting fixed requirements and thereby ignore the use of demand functions. Also, the goal of cost minimization is not appropriate for the two private steel mills among the five in the model.

While there are ways to modify the linear programming transportation problem to include demand functions and to convert the model to profit maximization, cf. Kendrick and Stoutjesdijk (1978) Ch. 7, these methods have so far not been widely adopted. However, there is a tendency in that direction which is being reinforced by the increased efficiency of nonlinear programming solvers and by the existence of nonlinear mixed integer programming codes. An example of this can be seen in the electric power study of Baughman, Siddiqi and Zarnikau (1993).

While there are a number of ways in which this small static Mexican steel model could be improved it none-the-less permitted an analysis of some of the key issues of that industry at that time. One of those issues was the substantial subsidies which were effectively passed from the government to the private sector via the medium of artificially low natural gas prices. The effects on the industry of modifications of these subsidies could be studied with even such a small model. And indeed because the model was small and relatively easy to understand the message of the effects of these subsidies was strengthened.

4.2 Large Static

The large static model worked backward to include more details of raw material production and preparation and forward to include more intermediate and final products. Seven iron ore mines and a coal mine were added to the model as well as three pellet plants and a coke oven facility. One more small steel mill was added to raise the total from five to six. The number of market areas was increased from three to eight. All of these changes added to the importance of transportation cost in the model. Also the model was expanded to include about fifty commodities including twelve types of final products.

Since the production of coking coal was explicitly included in this larger model, the model could be used to study the effects of loosening the restriction on imported coal which were in place in Mexico at that time. Also, the industry was focused on the domestic market and did not give much importance to potential exports; therefore a study was made of the effects on the industry of greater export promotion.

There was a tendency to run each mill separately and to ignore the possibility of substantial interplant shipments of intermediate materials to ease bottlenecks in some of the plants. An experiment was conducted with the model to analyze these possibilities. Also, an experimental run was done to study where strikes would be most effective from the point of view of the unions or of management.

This large static model was sufficiently disaggregated that it included the kind of detail which the operating officials in the industry would recognize as the level at which they thought about the economics of the industry. It contained capacities for many of the individual productive units in the steel mills and thus was able to analyze the effects of various bottlenecks on production.

While the model was a large one for its day it could now probably be solved with ease on a microcomputer. However, while speed increases help in understanding such a model, the complexity is still such that one should not jump into such a model development project without first constructing a small static model of the type described in the previous section. As one measure of the complexity the GAMS statement of the model included about 1200 lines, i.e. twenty to thirty pages of set specification, tables, variable list and equations.

Complexity of this degree is one of the reasons that graphical methods like those discussed in the software section above will be important in the evolution of sectoral models. It is easier to understand an iconical-graphical representation of an industry than it is to sort out the details from a GAMS statement. For example, software like the PTS system discussed above will eventually include icons of plants on a map. One will be able to click on these plants and effectively "go inside" to find an array of icons which represent the main productive units linked by lines of product flows. These productive units could in turn be clicked on to permit the user to "go inside" and see the alternative production processes which can be used in the productive unit.

The results of solving the model will not appear as tables in a GAMS output but rather as graphs showing product flows between productive units as well as from plants to markets. This will enable one to quickly digest the results of even large models and to develop additional experiments in order to learn first about the functioning of the model and then later about the functioning of the economics of the sector.

4.3 Small Dynamic

The small dynamic model focuses on investment. The model not only include multiple time periods but also specification of investment cost with economies of scale. As was discussed earlier in the paper mixed integer programming methods are required to solve investment problems where there are economies of scale. Such problems are inherently combinatorial. Thus if the problem has four productive units to be considered in each of two time periods the computational problem is of the order of 2^8 or 256 and if one other productive unit is added the order is 2^{10} or 1024. Thus the computational complexity increased exponential with the number of productive units and the number of time periods.

Because of the substantial computational cost associated with mixed integer programming problems it is necessary to aggregate again when one moves from static to dynamic models. In the case of the small dynamic model of the Mexican steel industry there are seven plants, three mines, three markets, eight commodities, five productive units and five time periods. Thus in order to gain the investment analysis it is necessary to give up much of the disaggregation of mines, plants, markets and commodities which were used in the large static model. While the algorithms for solving mixed integer programming problems are continuing to improve and the speed of computers is advancing even more rapidly, the fundamental tradeoff between investment consideration and model size is likely to remain.

At the time of the study natural gas prices in Mexico were controlled at a level about one-tenth of the world price. Since this was a key input for the sponge iron production of the privately owned steel mills it amounted to a substantial subsidy from the government to the competition to its state owned steel mills. At the same time the quality of domestic coal and iron ore was declining. Also, there was increasing concern about congestion and air pollution in Mexico's larger cities and a willingness to provide subsidies to industries which would locate elsewhere. These ingredient provided grist for the mill of the small dynamic model which was used to study the sensitivity of the minimum cost plan for the sector.

The study found that the policies affecting the regulation of the natural gas price were very important to the investment plan of the coming decades. It also found that the expansion of the industry should be near ports because of the declining quality of the iron ore reserves and therefore the expectation that iron ore pellets would be imported in the future. Finally it found that substantial investment in plants on the Gulf Coast were not economical unless the natural gas prices were held at the low regulated levels.

In summary, a dynamic model can prove to be an effective tool for studying investment in productive units in a set of plants located near mines or ports and serving both domestic and foreign markets. The strength of this type of models lies in it ability to represent the transformation of commodities in a cascade of production processes running in productive units. The strength also lies in the ability to represent economies of scale in investment cost and to solve the resulting model using mixed integer programming methods. The weakness lies in the lack of demand functions and in the specification of cost minimization as the criterion function. Also, the weakness lies in the fact that insufficient attention is given to the game nature of investment in heavy industry. While the basic model described above could be used as a part of a game analysis of investment plans in the various steel mills this was not attempted in the World Bank studies, but would seem to be an important part of future sectoral investment models.

4.4 Examples of Sectoral Models

One of the most useful ways to review sectoral models in the process industries is by the geographical scope of the models. As Table 4.4.1 shows there is a fair sampling of models which are single country, multi-country and worldwide in scope. For a more extensive discussion of models covering different geographical areas see Kendrick (1990).

Table 4.4.1 Linear Mixed-Integer Programming Sectoral Models of Processes Industries by Geographic Area

single country
- fertilizer
  - Choksi, Meeraus and Stoutjesdijk (1980)
- petrochemicals
  - Westphal (1971a) (1971b)
  - Suh (1981)
- steel
  - Westphal (1971a) (1971b)

multi-country
- fertilizer
  - Manne and Vietorisz (1963)
  - Mennes and Stoutjesdijk (1985)

worldwide
- aluminum
  - Brown, Dammert, Meeraus and Stoutjesdijk (1983)
- copper
  - Dammert and Palaniappan (1985)
- petrochemicals
  - Adib (1985)
- steel
  - Wei (1984)

As the discussion of the Mexican steel models above shows, one logical application of this class of sectoral models is to an industry located in a single country. Since national governments regulate or control processes industries in most countries, this is the appropriate geographical scope for many studies. These studies may cover a single country such as the fertilizer industry in Egypt in Choksi, Meeraus and Stoutjesdijk (1980) or the petrochemical industry in South Korea in Suh (1981). Or they may cover large projects in more than one industry as is the case for petrochemicals and steel in South Korea in Westphal (1971a) (1971b). Westphal's model demonstrates how large projects for two sectors can be embodied in an economy-wide model.

The multi-country models have typically been motivated by prospective or existing free trade areas or common markets. For example, one of the first sectoral models was the Manne and Vietorisz (1963) study of the fertilizer industry in Latin America. Similarly, the Mennes and Stoutjesdijk (1985) study of the fertilizer sector was done for the Andean Common Market.

A different methodology for sectoral economic modeling was put forth by Manne (1967). In these studies Manne used dynamic programming methods to calculate the optimal period between capacity expansions and to determine the amount of each expansion. The methodology was applied to the aluminum, caustic soda, cement and fertilizer industries. While this methodology has not been widely used yet, it seems likely that it could provide a basis for the use of stochastic control methods in analyzing the expansion of an industry over time.

5. Computers

Two different methodologies have been applied in developing sectoral models of various portions of the computer industry. The first of these is the use of the linear programming transportation model framework to the personal computer industry by Ang (1992).

The commodities in this model are shown in Table 5.1. The focus here is on memory chips and microprocessors which are assembled into motherboards and then into personal computers.

Table 5.1 *Commodities in Personal Computer Model*

raw material	*intermediate*	*final*
printed circuit boards		IBM PC
memory chips	68xxx motherboard	Macintosh
68xxx processors	80xxx motherboard	Compaq
80xxx processors		AST
hard disk		DELL

The model included sixteen raw material suppliers such as Toshiba, NEC, Mitsubishi, Quantum, Seagate, Conner, Intel, AMD and Motorola along with nine assembly plants located in Texas, Florida, California, New York and Massachusetts as well as thirteen market area in the U.S., Europe, Japan and the Far East.

Models of this type are potentially very useful because there are occasionally shortages of memory chips or other components and the capacity of the productive units would quickly reflect this kind of problem. Also, from time to time there are tariffs and quotas which effect this industry substantially and this class of model are ideal for reflecting the economics of these constraints on international trade.

However, there are at least three major problems with the development of models of this type. First, there is a tendency in linear programming models for the low cost supplier to gain the entire market, while the reality is that product differentiation is very important in this industry and market shares do not shift rapidly between suppliers. Therefore, adaptations of the models to permit slow shifts between suppliers will be important. Second, demand functions need to be built into the models. Third it is difficult to obtain the capacity and cost data which such a model requires. Private firms in this highly competitive industry are reluctant to reveal these statistics. There are some firms that collect and sell information on this industry, but academic projects cannot easily afford to pay the price for such data.

All three of the problems can be solved with creativity and resources and this industry will continue to be important for a number of countries. Therefore, it seems likely that a number of sectoral models of the personal computer industry will be developed in the years to come.

A different class of models of the computer industry are simulation models which are driven by the decreasing feature size of elements on microprocessor and random access memory chips. A schematic for a model of this type is in Figure 5.1.

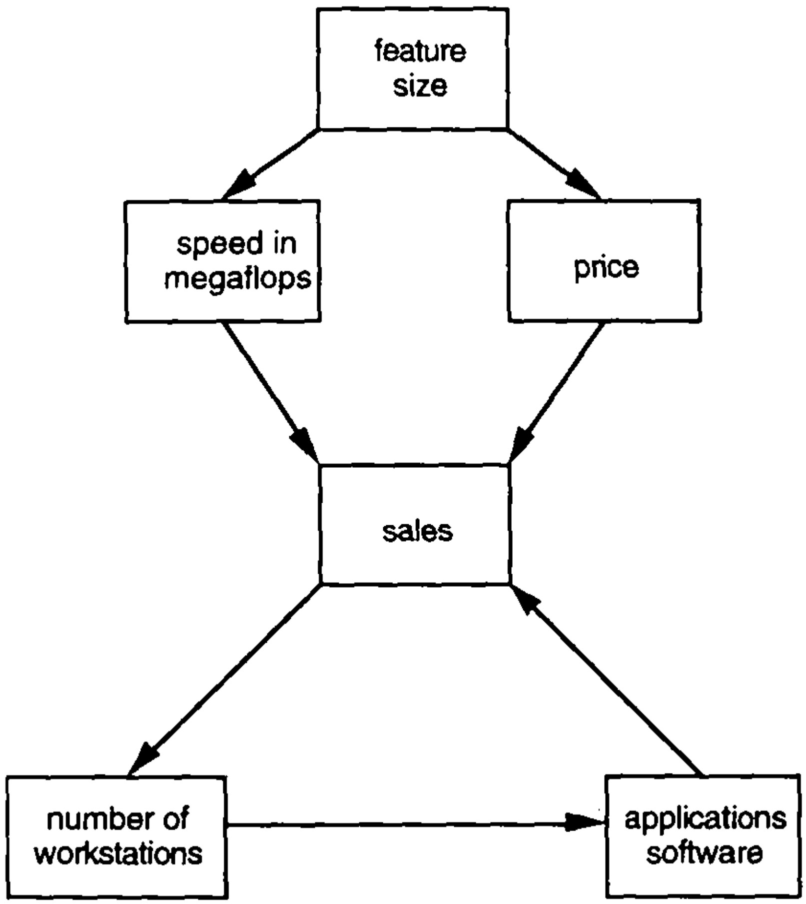

Figure 5.1 A Computer Workstation Model

The decrease in the feature size of the elements on computer chips is the driving force of the model. As this size decreased from around twenty microns in the early 1960's to less than one in the early 1990's the speed of the chips increased and their cost decreased. This in turn resulted in increases in sales so that there were more workstations in existence. This in turn augmented the development of applications software which in turn added to the increase in sales.

Touma (1993) has developed a model of this type for the computer workstation industry using exponential functions. For example in his model the feature size decreases by 5.5 percent per year. This model was developed in the C language.

Touma's training is electrical engineering and the model reflects this in the rich detail of the technical information not only about microprocessors but also disk drives, displays, etc.

The model uses not only feature size but also die size and the number of critical mask as driving forces. Dies are circular disk of several inches in diameter which look a little like a compact disk. The chips are etched onto these dies. The number of critical masks has to do with the number of levels of etching on the die. All three of these factors have to do with the number of flawed chips on a die and therefore on the cost of chips.

Touma's model was designed for parametric use so that one can alter the expected rate of change of feature size, the size of the dies for the chips or the number of critical mask and see the effect of these changes on the number of flawed chips and the effect of this on microprocessor cost and eventually on workstation cost. As a simulation model it runs very fast and is a useful tool for analyzing the effects of changes in feature size, die size and number of masking levels on the future of the computer workstation industry.

6. Energy

While most of the models discussed in this chapter are activity analysis models, some of the most influential are not. Consider for example Pindyck's classic 1978 paper on the OPEC cartel. This is a multiperiod mathematical programming model which can also be thought of as a deterministic nonlinear control theory model.

The model is used to analyze oil supply and price in the world economy. Oil suppliers are divided into two groups, OPEC and the "fringe" of oil producers not in OPEC. The objective function is to maximize OPEC's discounted profit stream.

OPEC faces a dynamic tradeoff. If it raises prices, its immediate revenues are higher but this also has the effect of decreasing demand with a lagged response and of slowly but surely increasing the oil supply from the fringe. Since OPEC is modeled as the residual supplier this increase in supply from the non-OPEC fringe erodes OPEC profits over time.

The model has fifteen time periods and roughly one hundred constraints. This is large enough to capture the essence of the problem and small enough that it is a manageable nonlinear programming problem. The model is in the GAMS Library, cf. Brooke, et al (1988) and has been used by many students over the years.

In contrast to Pindyck's control theory model of oil supply and demand, the activity analysis models have been used to great advantage for many years in modeling oil refineries. One of the first mathematical programming models in the energy sector was the refining model developed by Charnes, Cooper and Mellon (1954). This was followed by Alan Manne's 1956 book on the scheduling of refinery operations and his 1958 article on a U.S. model of the petroleum refining industry. A latter model by Langston (1983) extended this work to the world-wide oil refining industry. Among other thing, Langston analyzed the effects of increases in oil import taxes on the use of U.S. versus European oil refineries to supply gasoline to U.S. markets.

Much of the world oil supply is delivered to refineries in tankers. However, natural gas is difficult to ship economically in tankers and therefore most of it is transported in pipelines. These pipeline networks are naturally modeled with mathematical programming models. For example, Waverman (1973) developed a model of natural gas supply in Canada and the U.S. In this region most natural gas is produced in Texas and Louisiana as well as in the western Canadian province of Alberta. In contrast, the largest markets for the gas are in the eastern U.S. and Canada as well as in California.

Since the natural gas fields in Texas and Louisiana are closer to the markets of eastern Canada than are the Alberta fields, the solution to Waverman's model suggest shipping the Alberta gas to California and the Texas and Louisiana gas to eastern Canada rather than building a trans-Canadian natural gas pipeline to attain autarky in the Canadian natural gas industry. The model provides a very effective vehicle for analyzing this economic policy issue.

A more recent use of a mathematical programming models to analyze the natural gas industry is in Beltramo, Manne and Weyant (1986). Also, Rowse (1992) provides a discussion of U.S.-Canadian natural gas trade using a group of models analyzed by the Stanford University Energy Modeling Forum.

A rather different approach to natural gas pipeline modeling is in McCabe, Rassenti and Smith (1991). This paper discusses the uses of linear programming models of gas pipeline systems to assist market solutions.

Another important application of mathematical programming models to sectoral models is in electric power. The pioneering work in this area was done by Masse and Gibrat (1957). Some of the studies done in the 1970's were Gately (1971), Fernandez, Manne and Valencia (1973) and Anderson and Turvey (1977) and in the 1980's were Kang and Kendrick (1985) and Kwun (1986).

The basic structure of most of these models is power plants linked by transmission lines to markets for electric power. The plants may use coal, oil or natural gas. Also, some of the plants may be nuclear and in most cases there are also hydroelectric facilities. Each of these types of plants have different economic characteristics. For example, hydroelectric facilities have high capital cost, low operating cost and a comparative advantage in certain seasons of the year while coal-fired power plants have relative low capital cost and high operating cost. There are substantial economies of scale in investment particularly in nuclear and hydroelectric facilities so mixed integer programming methods are frequently used to solve these models. Also, some models such as Baughman, Siddiqi and Zarnikau (1993) consider investment not only in power plants but also in transmission lines and model these investment options with integer variables.

The earlier models were linear programming or mixed integer linear programs and the more recent models have moved toward nonlinear programming and even nonlinear mixed integer programming. Similarly, the earlier models were mostly deterministic and there is a tendency now to begin using stochastic methods to model uncertain in outage for power plants and transmission lines.

While most models in this area have confined themselves to a single industry such as oil and gas or electricity, some models have considered the entire energy sector, viz Fernandez and Manne (1973). In a recent model of this class Linden (1992) used a mixed integer programming model to represent coal mining, oil production, refining, natural gas production, and electric power production and distribution for Colombia. This model demonstrates that we are moving into an era in which one can build substantially disaggregated models of a number of related industries and solve them as a single large sectoral model. At the time the model was being developed it was solved on a Sun workstation but Linden has recently reported that improvements in solvers has enabled him to solve the model even on a 486 microcomputer.

7. Environment

Water pollution was one of the first concerns of the environmental movement in the post World War II period in the U.S. In these models point sources of pollution such as urban sewage are processed in part before being dumped into rivers and streams. The pollutants lower the dissolved oxygen level in the rivers down stream from the point source. So the river plays the important role of processing the pollutant but at the cost of degrading the quality of the stream.

The models of water pollution have therefore included activities which provide various degrees of processing of wastewater at increasing cost before the affluent is passed into the stream. Then flow equations are used to model the reduction in the dissolved oxygen level downstream from the sewage plant as the remaining waste is processed. In most models there are a number of sewage plants, sometimes owned by different municipalities, which effect the water quality in any given leg of the stream.

Some models in this class have sought to find the minimum operating and capital cost to meet quality standards for dissolved oxygen levels in the steams. Other models have consider the use of markets for dischargeable permits. A recent model of this type is Letson (1992a) for the Colorado River in Texas. He used thirteen wastewater treatment plants which were planned for the year 2000 to study the effects on two segments of the river. In another aspect of the same studied Letson (1992b) analyzed investment issues.

Recently, the global warming problem has become a matter of increasing concern. Fuels such as coal, oil and natural gas release carbon dioxide as they are burned. As the carbon dioxide concentration increases in the atmosphere it effects the release of heat from the earth and therefore tends to bring about an increase in the earth's temperature. This increase in temperature in turn effects the output of the agricultural and other sectors. There are different degrees of uncertainty associated with the various links in this chain of causation, the most uncertain of which is the link between increases in the carbon dioxide concentration and changes in the earth's temperature. Also, the dynamics are a matter of substantial concern since the full effect of burning fossil fuels today will not be felt until some years in the future.

A small and elegant model for analyzing global warming is Nordhaus (1992). This models is solved as a nonlinear programming model using the GAMS software. It can be used to analyze the effects of various control policies including carbon taxes on fossil fuels.

One aspect of the global warming problem is the potential future conflict between developing and developed nations. While most agree that it may be necessary to reduce carbon emissions in order to stabilize or reduce the carbon dioxide concentrations in the atmosphere; there is less agreement about how this responsibility should be shared around the globe. Consider for example India and China. Both of these countries have substantial coal reserves and strong ambitions for economic development. In the past the industrialized countries of Japan, the U.S. and the Europe have been the largest contributors to the increase in the carbon dioxide concentration in the atmosphere. China, India and the other developing countries feel that it is their turn now and that the developed countries should curtain their emissions even more sharply in order to permit the developing countries to develop their fossil fuel capability.

This is one of the policy questions analyzed in Duraiappah (1993). This model which is also in GAMS includes not only equations to represent the effects of the economy on carbon dioxide emissions and thus the temperature, but also the effect of temperature changes back on the economy. Also the model includes alternative technologies with different degrees of capital intensity which can be used to ameliorate the emissions. Thus one policy examined in the study is transfers from the developed to the developing countries to aid the developing countries to adopt capital intensive technologies which are less polluting.

8. Agriculture

Two of the main classes of agricultural models are those for annual crops and those for tree crops and livestock. The central element in the annual crop models is the allocation of land to crops in multiple growing seasons within a year. In contrast the tree crop and livestock models are usually dynamic models with state equations for the trees or for the various types of livestock.

The annual crop models usually have two or three growing seasons and a set of crops which can occupy the land during certain months of the year. Frequently the models also include irrigation activities as well as careful attention to the use of both permanent and temporary labor. The problem is to find that mix of crops which maximize profits within the land, labor and water constraints.

One of the most well known models in this class is the CHAC model which was developed at the World Bank for Mexican agriculture, cf. Duloy and Norton (1973) and Norton and Solis (1983). Following on the methodology of Kutcher, Meeraus and O'Mara (1986) a recent model in this class is Lofgren's (1990) study of Egyptian agriculture. The model includes eight crops and seven sets of one or two month long time periods within the year. Each crop would occupy the land in a subset of these time intervals. There are constraints not only for land and irrigation water but also for fertilizers and for four types of labor.

During the time of Lofgren's study Egyptian agriculture was closed regulated by the government with many prices for both inputs and outputs fixed. So the model is used to analyze the possible effects of freeing various combinations of regulations and prices.

Tree crop and livestock models use dynamic equations to model the number of trees or animals. For example a cattle model will include equations for the numbers of steers, bulls, cows and calves. Births add to the stocks and slaughter of cows and steers produces meat for sale while reducing the stocks. When prices rise the desired herd size rises so fewer cows are slaughtered in order to produce more calves. This can produce short term price rises even greater than the initial uptick in prices.

Tree crop models may carry vintage information about the age of trees since there is sometimes a long lag between planting of new trees and their first yield. In cocoa this lag is about seven years and affects the price and production dyanmics of the sector substantially. For an example of the application of control theory methods to stabilization of cocoa market prices see, Kim et. al.(1975). The book edited by Labys (1975) contains a number of sectoral model studies for agricultural and industrial commodities.

9. Conclusions

Studies of various sectors of the economy have been done for many years; however, the increased power of computers is bringing new possibilities to this area of economics. In the past it was possible to run a few regressions and to make policy prescriptions on this basis. Similarly, rates of return could be computed on individual projects to aid in investment decisions.

In contrast, it is now possible to estimate entire models and to solve these models using nonlinear programming or stochastic control theory methods. These methods permit the consideration of more accurate models and the inclusion in the analysis of the effects of uncertainty. Also, it is no longer necessary to consider rate of return on a single project. Rather, models can be developed which include many investment projects in a sector and computer codes can be used to find the best combinations of these investments in the various productive units at dispersed mines and plants.

This progress in sectoral modeling is based on the solid foundation of earlier work in activity analysis, linear programming and the solution of models with economies of scale which permits the development and solution of realistic models with many commodities, processes, productive units, plants and markets. The progress is being accelerated by the availability of modeling languages which permit the specification and solution of substantially disaggregated models even on personal computers and by the development of graphical interfaces for sectoral models which reduce the complexity associated with large models. As the speed of microprocessors continue to increase there is strong reason to believe these trends will continue as we develop substantially better ways to understand sectors in economies.

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