Unemployment and the Dynamic Effects of Factor Taxes and Subsidies*

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Abstract

This paper characterizes a dynamic search equilibrium with capital accumulation and examines the dynamic effects of taxes on factor income and subsidies on job search. The paper shows the following results: (i) job vacancies and unemployment are the main variables which respond in the short-run to changes in taxes and subsidies, and their responses generate different dynamics in variables from the standard model without unemployment; (ii) the presence of unemployment reduces the welfare cost of capital income taxation and the marginal gain of switching from capital income taxes to labor income taxes; (iii) reducing the replacement ratio increases welfare; (iv) the welfare costs of these policies increase with the bargaining power of labor in wage determination and decrease with the efficiency of job search.

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1. Introduction

Unemployment is a very important fact. Any discussion on the welfare cost of factor taxation should explicitly take unemployment into consideration. Past examinations have just done the opposite. Although theoretical developments in the past 15 years have promoted dynamic equilibrium models as the useful framework to evaluate the welfare cost of factor taxation, they have typically ignored unemployment. The omission is only justified by the difficulty of incorporating unemployment into a general equilibrium model with capital accumulation.[1] Because of the omission, those existing models cannot answer some of the basic questions in public finance and fiscal policy making. First, existing models usually predict a large welfare gain from switching a marginal tax from capital income to labor income. Is this result still valid when there is unemployment? Second, what is the welfare cost of subsidies to job search? Third, how do the welfare costs respond to changes in labor market frictions? Incorporating unemployment into a dynamic model, we wish to answer these questions in this paper.

Our analysis has two building blocks, and the references to each can be found in the representative works cited below. The first building block is a framework to evaluate the efficiency cost. For this, we follow Chamley (1981), Judd (1985, 1987), and Auerbach and Kotlikoff (1987) to focus on the change in intertemporal utility caused by tax changes. In particular, a measure of marginal deadweight loss proposed by Judd (1987) is used to measure the welfare cost of taxes. The second is a theory of unemployment. To choose among competing theories on unemployment for the present investigation, we require that the theory allow capital accumulation and be easy to have a long horizon. Based on these criteria, we choose to introduce unemployment through the search model described by Mortensen (1992), Pissarides (1990) and Merz (1993).

The theoretical contribution of this paper is to characterize the decentralized equilibrium in an economy with unemployment and capital accumulation. The characterization contributes to the search theory of unemployment in two respects. First, it provides an in-

[1]For the dynamic framework, see for example Judd (1985, 1987) and Auerbach and Kotlikoff (1987). Later discussions in this section briefly outline the analytical difficulty of incorporating unemployment.

tegrated framework for welfare analysis. In a typical search model, it is difficult to evaluate welfare because it is rather arbitrary to impose a set of welfare weights on the employed, the unemployed and the firms. Here we use a representative household utility function to integrate naturally the different types of agents. Second, by embedding agents' search decisions into the household's intertemporal utility maximization problem, we tie reservation wages to the household's marginal utility of leisure and of wealth.

However, for the following reason, the presence of capital accumulation can make the characterization of equilibrium very difficult. In a search model of unemployment, jobs are created by random matching, which creates uncertain streams of income and leisure for each individual. Agents' optimization decisions have to be characterized by intractable dynamic programming problems with uncertainty. To avoid such complexity, Merz (1993) analyzes the optimization of a planner who can smooth individuals' risks, and then seeks the corresponding prices which support the planner's allocation. Merz's approach does not directly model how wages are determined. We directly characterize the decentralized equilibrium, although the equilibrium allocation turns out to be the same as in Merz (1993). To make the characterization tractable, we replace the usual representative agent by a representative household which consists of a continuum of agents, and base our efficiency cost calculation on the household's preferences. Individual risks are completely smoothed out within each household, and wages are determined by some bargaining framework.

The paper establishes the following results. First, job vacancies and unemployment are the main variables which respond to tax changes in the short run. Labor employment and output respond only slowly. As a result, job vacancies and unemployment exhibit overshooting. In contrast, standard models without unemployment imply immediately responses of labor employment and output to changes in taxes and subsidies. This contrast enables the current model to generate different dynamics in variables from the standard models. One of this differences is that tax changes can induce positive comovement between labor employment and consumption. This stylized positive comovement is impossible in standard models with time separable preferences (Barro and King 1984).

Second, the welfare cost of capital income taxation is smaller than in the previous models. It is so because firms can adjust job vacancies in addition to the capital stock to absorb shocks. With a set of parameter values used by Judd (1987), we find that the marginal deadweight loss of capital income taxation is below 50 cents for a dollar increase in tax revenue. In contrast, Judd (1987) found that the corresponding figure exceeds 50 cents and easily exceeds a dollar. He also found that "when any proposed estimates are used for taste and technology parameters, welfare would be improved substantially at the margin by moving away from capital income taxation toward higher labor income taxation." The figures in Judd (1987) indicate that the welfare cost of capital income taxation is on average 3 to 4 times as large as that of labor income taxation. We find much smaller welfare gains from such switch. For some reasonable labor market conditions, labor income taxation can even be more costly than capital income taxation.

Third, reducing subsidies to job search increases governmental revenue *and* welfare. The marginal welfare gain of reducing subsidy easily exceeds 50 cents. Cutting subsidies to search to finance a cut in either labor income tax or capital income tax can easily raise welfare by 50 cents in real income. This suggests that if replacement benefits function only as search subsidies, then they are very inefficient. Even when some of the benefits, such as unemployment insurance, also function as an insurance across states of employment, the large welfare cost of search subsidy (in absolute values) may render these benefits socially inefficient.

Fourth, the welfare costs of taxation depend on labor market conditions. We examine two of such conditions. One is the bargaining power of labor in wage determination; the other is the marginal productivity of search in producing job matches. A high bargaining power of labor in wage determination increases the welfare costs of factor income taxes and subsidies to search. A high marginal productivity of search effort in producing job matches reduces these welfare costs.

This paper is organized as follows. Section 2 builds the dynamic model. Section 3 examines how taxes and subsidies affect the steady state. Section 4 discusses how to measure the welfare costs of taxation and parameterizes the model. Section 5 analyzes the effects of taxes on the transitional path. Section 6 reports the welfare cost of factor taxation. Section 7 concludes this paper.

2. Decentralized Economy

2.1. Households

Consider an economy with many identical households. The number of households is normalized to one. Each household consists of many agents. An agent is infinitely-lived, endowed with a flow of one unit of time and chooses to work for wages, search for jobs or enjoy leisure.[2] Search generates matches between job vacancies and agents. For a specific agent who is searching for job, the timing of a match is random. Random matching results in uncertain streams of income and leisure for any specific agent.

Randomness of such type causes analytical complexity. Directly tackling this problem would require complicated dynamic programming with uncertainty and the model would be too complicated to be parameterized as easily as standard models such as Judd (1987). A similar difficulty exists in the literature of indivisible labor. There, various authors have shown that employment lotteries can be used to smooth agent's consumption across states of employment (see Rogerson 1988, Hansen 1985, and Rogerson and Wright 1988). This technique has its limitation, particularly in a model with capital accumulation. That is, lotteries cannot smooth leisure. This implies that optimal capital accumulation will depend on the state of employment, and dynamic programming with uncertainty is still required. To circumvent this analytical problem, we assume that each household consists of a continuum of agents with measure one and all members care only about the household's utility. Thus, individual risks in consumption and leisure are completely smoothed out within each household.[3]

The utility function of a household is

$$U = \int_0^\infty u(C, 1 - n - s)e^{-\rho t}dt \tag{2.1}$$

[2]For tractability, we choose to adopt an infinite-horizon model instead of an overlapping generations model. For examinations on welfare costs of taxation in overlapping generations models, see Auerbach and Kotlikoff (1987).

[3]Merz (1993) implicitly used this assumption for the social planner. A similar but not so extreme assumption is used in a monetary model by Lucas (1990). He assumes that a household consists of several members who go to different markets to conduct different activities; at the end of each period, receipts are pooled in the household.

where $u(\cdot,\cdot)$ has the standard properties, with an additional simplifying assumption $u_{12} = 0$. C is the household consumption, n the proportion of hours in work and s the proportion of hours in search. Since the size of a household is one, n and s can be equivalently interpreted as the proportion of household members in work and in search respectively. With this interpretation, the notation s conforms with the standard notion of unemployment; and $n+s$ the labor force participation. Denote the rate of unemployment by $UN = s/(n+s)$.

Different from the standard representative agent model but in common with search models of unemployment, the variable n is predetermined at each given time. It can change only gradually as workers quit or searching agents find jobs. The law of motion of n is

$$\dot{n} = ms - \theta n. \tag{2.2}$$

The parameter θ is the (constant) rate of natural separation from jobs. The notation m denotes the rate at which searching agents find job matches. As discussed later, m depends on the aggregate vacancies and aggregate number of search agents. For an individual agent, however, m is taken as given.

A representative household's maximization problem is

$$(PH) \qquad \max_{C,n,s,K} U \qquad s.t.$$

(2.2) *holds*;

$$\dot{K} = (1-\tau_K)rK + (1-\tau_W)wn + \tau_u ws + (1-\tau_K)\Pi - C + L \tag{2.3}$$

$K(0) = K_0$, $n(0) = n_0$ *given*.

In the constraint (2.3), r is the rental rate of capital, w the wage rate and K the household's capital stock. Π is a pure profit defined later. τ_K and τ_W are the tax rates on capital and labor incomes, τ_u is the subsidy to search or the replacement ratio. Finally, L is the lump-sum transfer from the government to the household.

For all the following examinations, we will focus on permanent tax changes, so $\dot{\tau} = 0$. With this additional assumption, applying the standard optimization technique to the household's maximization problem generates the following equations:

$$\dot{C} = \frac{u_1}{u_{11}}(\rho - r(1-\tau_K)) \tag{2.4}$$

$$\dot{s} = -\dot{n} - \frac{1}{u_{22}} \left[\begin{array}{c} \left(\theta + \rho + \frac{\dot{m}}{m}\right)(u_2 - u_1\tau_u w) \\ +m(u_2 - u_1(1-\tau_w)w) + \tau_u w u_{11}\dot{C} + u_1\tau_u\dot{w} \end{array} \right]. \tag{2.5}$$

Also, the current-value shadow price of constraint (2.3) is u_1.

2.2. Firms

There are many identical firms in the economy. Each firm has job vacancies. The unit cost of maintaining a vacancy is b (> 0). The rate at which vacancies turn into job matches is μ. As m, μ depends on the numbers of aggregate job vacancies and aggregate search agents. But an individual firm takes μ as given. Let v be the number of vacancies. A firm's labor employment evolves according to

$$\dot{n} = \mu v - \theta n. \tag{2.6}$$

An individual firm takes as given the wage rate w offered by other firms. It also takes as given the wage rates offered by itself to its existing workers.[4] Let the production function $F(K, n)$ be linearly homogenous and exhibit the standard properties: increasing and concave in each argument. The representative firm maximizes its value:

$$(PF) \quad \max_{(I,v,K,n)} H = \int_0^\infty e^{-\int_0^t r(s)ds}\left[F(K,n) - wn - I - bv\right] dt \quad s.t.$$

$$\begin{aligned} &(2.6) \textit{ holds and} \\ &\dot{K} = I - \delta K \\ &K(0) = K_0,\ n(0) = n_0 \textit{ given} \end{aligned} \tag{2.7}$$

where I is gross investment, and δ the rate of depreciation of capital.

Optimization implies that

$$F_1 = r + \delta \tag{2.8}$$

$$\frac{\dot{\mu}}{\mu} = \frac{\mu}{b}(F_2 - w) - (\theta + r). \tag{2.9}$$

[4]See later discussion for a justification.

Finally, the pure profit is defined as $\Pi = r(H-K)-(\dot{H}-\dot{K})$. Since $rH-\dot{H} = F-wn-I-bv$, then

$$\Pi = F(K,n) - (r+\delta)K - wn - bv. \tag{2.10}$$

The pure profit can be positive. It can be so because firms must maintain job vacancies in order to employ labor. In this sense, a positive profit resembles that in the Tobin's q model which has costs to adjust the capital stock. However, the two models differ in their long-run implications. In the adjustment cost model, the adjustment cost disappears when the economy approaches the steady state. In the current model, the cost is positive even in the steady state because job vacancies are positive in the steady state.

2.3. Matching and Wage Determination

Job vacancies and searching agents create job matches. The number of matches is assumed to be a function of v and s:

$$M(v,s) = Av^{\alpha}s^{1-\alpha}, \qquad \alpha \in (0,1). \tag{2.11}$$

The matching technology exhibits constant-returns to scale. Besides its apparent similarity to the usual production technology, constant-returns to scale matching technology has also be supported empirically (see Pissarides 1986, and Blanchard and Diamond 1989). The Cobb-Douglas form is adopted for analytical simplicity. We call $(1-\alpha)$ the efficiency of search effort in creating job matches. Also for simplicity, we abstract from the choice of search intensities (see Pissarides 1984). With constant-returns to scale, the number of matches per vacancy or searching agents depends only on the vacancy-unemployment ratio. Let $x = v/s$ be such a ratio. Then

$$m = m(x) = Ax^{\alpha}, \qquad \mu = \mu(x) = m(x)/x. \tag{2.12}$$

Once a searching agent is matched with a vacancy, the agent and the firm decide current *and* future wage rates for this agent[5]. These wage rates are assumed to be determined by

[5] That a firm has power to determine a new worker's wage is not inconsistent with our earlier assumption that a firm takes as given the wage rates offered to existing workers. The same approach has been taken by Pissarides (1990, pp. 11-12).

a Nash bargaining solution which maximizes a product of weighted surpluses of household and the firm. To be precise, let T be the time when the match is created. Denote by $\{w^*(t)\}_{t\geq T}$ the path of wage rates for the new worker conditional on the continuation of employment of the worker. Then the firm's current-valued surplus at time $t \geq T$ increases by $[F_2(t) - w^*(t)]dn$ from hiring an additional worker, dn, with the wage schedule. Similarly, having an additional member working at the wage schedule increases the household's utility at $t \geq T$ by $[(1 - \tau_W(t))w^*(t)u_1(t) - u_2(t)]dn$. The Nash bargaining solution solves[6]

$$\max_{w^*(t)}[F_2(t) - w^*(t)]^{1-\lambda}[(1 - \tau_W(t))w^*(t)u_1(t) - u_2(t)]^{\lambda}, \quad for\ t \geq T.$$

The parameter $\lambda \in (0, 1)$ can be interpreted as the bargaining power of labor.

Two remarks follow. First, in the steady state, it is well known that the Nash bargaining framework proposed above generates the same outcome as some noncooperative sequential bargaining games (Binmore 1987). In a non-steady state environment, the equivalence no longer holds. But it is still possible to find some other Nash bargaining formulation which delivers the same outcome as the sequential bargaining game for any time t and the same steady state outcome as the Nash formulation proposed above. Thus the proposed form is a useful approximation around the steady state of the underlying noncooperative bargaining framework. Second, a firm and a new worker bargain to decide the entire wage path $\{w^*(t)\}_{t\geq T}$, not just the wage rate $w^*(T)$. Since the above formulation stipulates that $w^*(t)$ solves the period-t problem, the wage path is time-consistent. That is, workers and firms do not have incentive to reopen the negotiation in the future. This justifies our earlier assumption that firms take as given the wages rates offered to existing workers.

Solving the Nash bargaining problem yields

$$w^*(t) = \lambda F_2(t) + (1 - \lambda)\frac{u_2(t)}{(1 - \tau_W(t))u_1(t)}, \quad for\ t \geq T. \tag{2.13}$$

Since all firms are identical, the wage rates offered by different firms must be the same in any symmetric equilibrium. Also, since the wage path is time consistent, two workers who are hired by the same firm at different times must be paid the same wage at any given time. That

[6]The bargaining solution proposed here implicitly assumes that the agents involved in bargaining cannot search. If agents can search during baragining, the formulas for agents' surpluses are different from the one formulated here (See Wolinsky 1987).

is, the wage rate w^* also equals the one offered to the existing workers. So $w^* = w$. By (2.9), $F_2 \neq w$ around the steady state. Thus the wage rate equals neither the marginal product of labor nor the after-tax marginal rate of substitution between leisure and consumption. In contrast, all three equal in the standard representative agent model.

2.4. The Government

The government faces the following budget constraint:

$$L \leq \tau_K(rK + \Pi) + \tau_W wn - \tau_u ws - g. \tag{2.14}$$

Any changes in revenue caused by taxes and subsidies are rebated to households through the lump-sum transfer L. Government bonds are abstracted from the model for simplicity. As stated earlier, all changes in taxes are permanent in the following discussion.[7]

3. Competitive Equilibrium and Steady State

A *search equilibrium* is a collection $\{C(t), s(t), x(t), n(t), K(t), r(t), w(t), \Pi(t)\}_{t=0}^{\infty}$ such that
(i) given $\{r(t), w(t), \Pi(t)\}$, $\{C(t), s(t), n(t), K(t)\}$ solve the problem (PH);
(ii) given $\{w(t), r(t)\}$, $\{v(t), n(t), K(t)\}$ solve the problem (PF) with $v(t) = s(t)x(t)$;
(iii) $w(t)$ solves the Nash bargaining problem;
(iv) $r(t)$ and $\Pi(t)$ satisfy (2.8) and (2.10) respectively;
(v) the government budget constraint (2.14) is satisfied.

The equilibrium conditions are (2.2), (2.3), (2.4), (2.5), (2.8), (2.9), (2.10), (2.12), (2.13), and (2.14). After suitable substitutions, they give rise to a dynamic system of $Y \equiv (C, s, x, n, K)^T$. In particular,

$$\dot{x} = \frac{1}{1-\alpha}\left[(\theta + F_1 - \delta)x - \frac{(1-\lambda)m}{b}\left(F_2 - \frac{u_2}{(1-\tau_W)u_1}\right)\right] \tag{3.1}$$

$$\dot{K} = F - \delta K - bsx - C - g. \tag{3.2}$$

[7] When tax changes are compensated by lump-sum transfers, as in the current model, the existence of government bonds adds little to the discussion of welfare cost of taxation. Also, for an analysis of anticipated tax changes, see Judd (1987).

(3.1) is derived from (2.9), and (3.2) from (2.3), (2.10) and (2.14). Other equations of the dynamic system are given by (2.2), (2.4) and (2.5). To emphasize its dependence on $\tau = (\tau_K, \tau_W, \tau_u)$, denote this dynamic system by

$$\dot{Y} = h(Y, \tau). \tag{3.3}$$

Before studying the dynamic equilibrium, we remark on its efficiency. Besides distortionary taxes, there are externalities in the labor market. The number of job matches per search, m, depends on search effort s, and the number of job matches per job vacancy, μ, depends on job vacancies v. However, Individual agents and firms do not take the dependence into account when they make their decisions. Thus even when there are no distortionary taxes, the search equilibrium will not deliver efficient allocations. However, when there are no distortionary taxes, these externalities can be internalized with special matching functions and wage bargaining rules. Hosios (1990) specifies the conditions for the internalization in an environment without capital accumulation. With capital accumulation, the corresponding planner's problem for the current model is given by Merz (1993).[8] Using our notation, the conditions found in Merz for the internalization can be written as

$$vM_1(v,s)/M = 1-\lambda, \quad sM_2(v,s)/M = \lambda.$$

where λ is the bargaining power of labor in wage determination. With the Cobb-Douglas matching function used in our model, these two conditions are equivalent to a single condition $\lambda = 1-\alpha$. To examine the dependence of welfare cost of taxation on labor market conditions, we do not force this condition to hold, but will explore its implications in Section 6.

We now examine the steady state of the equilibrium dynamic system. The steady state $Y^* = (C^*, s^*, x^*, n^*, K^*)^T$ is given by the solution to $h(Y^*, \tau) = 0$:

$$F_1 = \delta + \frac{\rho}{1-\tau_K} \tag{3.4}$$

$$\begin{aligned} &\tfrac{x^*}{m(x^*)}\left[\lambda m(x^*) + (\theta+\rho)\left(1 - \tfrac{(1-\lambda)\tau_u}{1-\tau_W}\right)\right] \\ &\quad = (\theta+\rho)\tfrac{1-\lambda}{b}\left(1 - \tfrac{\tau_u}{1-\tau_W}\right)\tfrac{F_2}{F_1+\theta-\delta} \end{aligned} \tag{3.5}$$

[8]The only difference is that Merz's model has discrete-time.

$$s^* = \frac{\theta n^*}{m(x^*)} \tag{3.6}$$

$$C^* = \left(\frac{F - \delta K^*}{n^*} - \frac{b\theta x^*}{m(x^*)}\right) n^* - g \tag{3.7}$$

$$\frac{u_2}{u_1} = \frac{(\theta+\rho)\tau_u + (1-\tau_W)m(x^*)}{(\theta+\rho)\left[1 - \frac{(1-\lambda)\tau_u}{1-\tau_W}\right] + \lambda m(x^*)} \cdot \lambda F_2. \tag{3.8}$$

In particular, (3.5) and (3.8) are derived from $\dot{x} = 0$ and $\dot{s} = 0$. Equation (3.6) deserves special attention. In a typical model where an agent either work or search, $n^* = 1 - s^*$. In this special case, (3.6) gives the Beveridge curve, depicting a negative relationship between v^* and s^*.

The equation system (3.4)–(3.8) can be solved sequentially to determine a unique steady state. First, note that F_1 and F_2 only depend on the capital-labor ratio, so (3.4) uniquely solves for K^*/n^*. Second, (3.5) becomes an equation of only x^* after substituting the value for K^*/n^*, and its left-hand-side is an increasing function of x^*. Thus, the equation uniquely solves for x^*. Third, after substituting the values of K^*/n^* and x^* into (3.6) and (3.7), s^* and C^* become linear functions of n^*. With these functions, (3.8) is an equation of only n^*. In particular, u_2/u_1 is an increasing function of n^*. Thus (3.8) uniquely solves for n^*. Finally, s^*, C^* and K^* can be recovered from (3.6), (3.7) and the ratio K^*/n^*.

All changes in τ have unambiguous effects on x^*. It is easy to check from (3.4) and (3.5) that x^* decreases in τ_u, τ_W and τ_K. That x^* decreases in τ_u and τ_W is quite intuitive. An increase in τ_u or τ_W increases the searching agents' reservation wage. This tends to increase search effort in the long-run. Also, a high equilibrium wage rate increases the cost of hiring so the firms will maintain few vacancies. Both lead to a lower vacancy-unemployment ratio. That x^* decreases in τ_K is also intuitive. A high capital income tax induces a low capital-labor ratio and low marginal productivity of each worker. Because the marginal benefit to the firm from hiring is low in this case, the firm maintains few vacancies.

In contrast to the unambiguous effects of τ on x^*, the effects of τ on n^* are all analytically ambiguous. For example, an increase in τ_u raises the left hand side of (3.8) for any given level of n^*, but has ambiguous effect on the right hand side of (3.8), resulting in an ambiguous change in n^*. This ambiguity makes it difficult to assess analytically the effects of tax changes on the steady state. The same problem exists for the dynamic analysis. Although it is possible to characterize analytically the stable dynamic path that the system will follow after tax changes, the analytical results are not revealing. For this reason, we resort to numerical exercises. The next section sets up the framework to do so.

4. Simulation Method

Following Judd (1987), we model marginal changes in taxes by letting $\tau(t) = \tau_0 + \Delta \cdot \tau_1(t)$, where τ_0 is the initial tax and Δ is an arbitrarily small number. Since we only examine permanent tax changes, $\tau_1(t) = \tau_1$ for all t. All tax changes take place at time 0 unexpectedly. We want to calculate the corresponding changes in the intertemporal utility. Apparently, one has to find the corresponding changes of the entire dynamic path $\{Y(t)\}_{t=0}^{\infty}$, not just the changes in the steady state. Since the tax changes are infinitesimal and permanent, the changes of the dynamic path can be calculated in the following way.

First, differentiate the dynamic system (3.3) with respect to Δ and approximate the resulted system by a liner system evaluated at $\Delta = 0$. This procedure creates

$$\dot{Y}_\Delta = J(Y_\Delta - Y_\Delta^*) \tag{4.1}$$

where $Y_\Delta(t)$ is the change of $Y(t)$ with respect to the tax change; Y_Δ^* is the corresponding change of the steady state Y. J is a 5×5 matrix defined by $J = h_Y(Y^*, \tau_0)$.

Second, solve the linear differential system (4.1) under the initial conditions $K_\Delta(0) = n_\Delta(0) = 0$. Since there are two predetermined variables, stability requires that matrix J have two stable eigenvalues and three unstable eigenvalues.[9] Let the stable eigenvalues be ω_1 and ω_2. Let $Z_i = (Z_{i1}, Z_{i2}, Z_{i3}, Z_{i4}, Z_{i5})^T$ be the eigenvector of J corresponding to ω_i. Then the stable dynamic path of (4.1) is

$$Y_\Delta - Y_\Delta^* = (Z_1, Z_2)\begin{pmatrix} a_1 e^{\omega_1 t} \\ a_2 e^{\omega_2 t} \end{pmatrix} \tag{4.2}$$

[9] Under certain regularity conditions specified in Scheinkman (1976) and Epstein (1987), the correct number of stable eigenvalues is also sufficient for local stability. Those regularity conditions are met in the following numerical exercises.

where a_1 and a_2 are uniquely determined by the two initial conditions $K_\Delta(0) = n_\Delta(0) = 0$.

Now, the change in the intertemporal utility is

$$U_\Delta = u_1 \int_0^\infty C_\Delta e^{-\rho t} dt - u_2 \int_0^\infty (n_\Delta + s_\Delta) e^{-\rho t} dt.$$

Utilizing (4.2), U_Δ can be computed analytically. This change in utility is equivalent to a change U_Δ / u_1 in the present value of real income. We measure this change in real income against the governmental revenue raised by the tax changes. Since additional tax revenue is rebated through lump-sum transfers, the present value of additional tax revenue is

$$R \equiv \int_0^\infty L_\Delta e^{-\rho t} dt.$$

The function $L_\Delta(t)$ can be derived from the government budget constraint and (4.2). The marginal deadweight loss (MDL) proposed by Judd (1987) takes the following form:[10]

$$MDL = -\frac{U_\Delta}{u_1 R}. \tag{4.3}$$

For example, the marginal deadweight loss of capital taxation is computed in the above way by setting $\tau_{K1} \neq 0$ and $\tau_{W1} = \tau_{u1} = 0$. Denote the marginal deadweight loss of capital taxation (labor taxation, search subsidy) by MDL_K (MDL_W, MDL_u). In normal cases, MDL is a positive number. If MDL is a negative number for a tax, then cutting that tax increases governmental revenue and welfare.

To calculate MDL numerically, we parameterize the model. Most of the parameter values used below are taken from Judd (1987). On the production side, we let the production function be $F(K, n) = K^\gamma n^{1-\gamma}$ with a capital share $\gamma = 0.25$. The rate of capital depreciation δ is such that capital consumption allowance is 12 percent of the net output. On the preference side, we let the rate of time preference be $\rho = 0.01$. If one interprets the unit of time as a quarter, this value of ρ gives roughly a 4 percent steady state annual interest rate. The instantaneous utility function is

$$u(C, 1-n-s) = \frac{C^\sigma}{\sigma} - \beta \frac{(n+s)^\eta}{\eta}.$$

[10] Judd defines MDL by $\rho U_\Delta/(u_1 R)$. The results reported in Judd (1987) are actually calculated with the definition $MDL = U_\Delta/(u_1 R)$. We correct this typograghical error in our definition. Also, we add a negative sign to Judd's definition in order to report welfare losses as positive numbers.

It is controversial what values σ and η take. We let σ vary from -0.1 to -5.0. This range gives a relative risk aversion from 1.1 to 6, which is consistent with the macro-econometric literature (for example, see Hansen and Singleton 1983). η is chosen such that the elasticity of labor supply $\varepsilon = 1/(\eta - 1)$ covers the range from 0.1 to 0.6, a range consistent with Killingsworth (1983). To determine β through (3.8), we let the steady state employment n^* be 0.28 to match the empirical observation by Christiano (1988). On the policy side, we choose $\tau_{K0} = \tau_{W0} = 0.3$ to be consistent with King and Fullerton (1984). We later allow $\tau_{K0} = 0.5$ and $\tau_{W0} = 0.4$. The rate of search subsidy τ_{u0} is chosen to vary in the range (0.30, 0.55) which covers a reasonable subset of the replacement ratio observed in different states of the U.S.. Government spending is assumed to be 20 percent of the output.

Taking these values to the standard model without unemployment, we generate similar results as in Table 1 in Judd (1987). Later differences between our results and Judd's must then be caused by the presence of unemployment. There are five parameters related to search and unemployment, $(\theta, A, \alpha, \lambda, b)$. We normalize $A = 1$ and let $\theta = 0.05$. The value of θ resembles the quarterly rate of transition from employment to unemployment used by Mortensen and Pissarides (1992).[11] The parameter b is such that the steady state unemployment rate, $s^*/(s^* + n^*)$, is 7 percent. To check the response of MDL to the labor market conditions, we let α and λ vary from 0.2, 0.4, 0.6 to 0.8. This covers the value $\alpha = 0.6$ estimated by Blanchard and Diamond (1989).

5. Dynamic Effects of Factor Taxes and Subsidies

Figures 1 to 3 report the dynamic effects of a marginal increase in τ_W, τ_K and τ_u respectively. The parameter values used in those figures are $\varepsilon = 0.4, \sigma = -2, \gamma = 0.4, \alpha = 0.6, \tau_{K0} = \tau_{W0} = 0.3$ and $\tau_{u0} = 0.45$. We compare two pairs of policies.

Figures 1,2 and 3 here.

[11]Burdett et al indicate a different estimate of $\theta = 0.15$. We have done simulation also with this estimate but found results similar to the ones reported here.

5.1. Compare increases in τ_W and τ_u

To a working agent, an increase in labor income taxes is similar to an increase in search subsidy. Both increase the opportunity cost of working. The similarity is represented by the same sign of $n_\Delta(t)$ in Figures 1 and 2. Both policies lower the entire path of employment. However, the two policies are different for agents who are not working. An increase in labor income tax lowers the perspective earnings of a searching agent from future employment. In so doing, it discourages participation in the labor force. An increase in search subsidy, on the other hand, encourages participation in the labor force. We can see this difference in Figures 1 and 2. An increase in labor income tax decreases search effort and labor force immediately. Along the dynamic path, employment declines as workers who separated from jobs either give up searching or take longer time to search. Unemployment increases after the initial fall. Thus the dynamic pattern of unemployment is non-monotonic, exhibiting over-shooting. So is the dynamic pattern of the unemployment rate UN. The labor force remains lower along the dynamic path than before the increase in τ_W.

An increase in search subsidy increases the labor force, both immediately and along the dynamic path (Figure 2). However, agents who are attracted into the labor force spend longer time to search than before. Fewer agents are employed in equilibrium. To increase employment, a policy maker may find it attractive at the margin to cut search subsidy to finance a cut in labor income tax. Figures 1 and 2 suggest that such a policy increases employment along the entire dynamic path. The policy immediately increases unemployment when it is carried out, but reduces unemployment after a short time. In Section 6, we will see that this policy also increases welfare.

A labor income tax and a search subsidy also differ in their effects on job vacancies. An increase in labor income taxes immediately reduces job vacancies while an increase in search subsidy increases job vacancies. This difference is induced by the different reactions of search effort in the two cases. An increase in τ_W immediately reduces search effort. Because job vacancies and search effort are complementary in the matching technology, firms find it optimal to reduce job vacancies. An increase in τ_u has the opposite immediate effect on search effort and hence opposite effect on job vacancies.

Despite these differences, τ_W and τ_u both reduce consumption and the capital stock. That is, both policies generate negative wealth effect. It is interesting that a search subsidy generates negative wealth effect. An increase in search subsidy increases a household's receipts of unemployment compensation by increasing both the compensation rate and search effort. Presumably, this will increase the household's income and wealth. This presumption is invalid because the household's wage income falls as a result of the fall in employment. This fall in wage income exceeds the increase in search compensation. The negative wealth effect of τ_u, combined with its effect on the labor force examined earlier, will generate interesting welfare result in Section 6.

5.2. Compare increases in τ_K and τ_W

Similar to a labor income tax, an increase in a capital income tax immediately reduces job vacancies and search effort, creating non-monotonic dynamic paths for unemployment and the unemployment rate. Different from a labor income tax, the sequence of effects runs from job vacancies to search effort rather than the reverse. The increase in the capital income tax reduces the return to capital stocks and induces capital decumulation. However, the capital stock and labor employment can only adjust gradually in this model, so firms absorb the tax increase by reducing job vacancies. Because there are fewer jobs than before, some agents give up searching (see Figure 3).

The immediate response of consumption to an increase in capital income taxes is opposite to its response to labor income taxes. In Figure 3, consumption jumps up immediately rather than drops down as in Figures 1 and 2. This immediate response is easy to understand with the help of (3.2). Because both vacancies and net investment drop, consumption rises to absorb the extra income. It should be noted that this upward reaction in consumption is not typical in standard dynamic models. In those models, a capital income tax reduces labor employment instantaneously and consumption falls as a result of the low income.

The key difference between a capital income tax and a labor income tax is their effects on the transitional path of labor employment. An increase in capital income taxes induces labor employment first to fall and then to rise. When the transition approaches the new steady state, employment rises above the level before the tax increase. Note that a capital income tax can increase labor employment is novel with respect to standard models without unemployment. Since a capital income tax reduces the capital stock and the marginal productivity of labor, it is intuitive that the tax reduces labor employment. Why does a capital income tax increase labor employment?

Two explanations are possible. First, because the capital stock cannot have discrete changes, job vacancies might have dropped below the new steady state level at the beginning of the transition. When the capital stock falls during the transition, job vacancies *rises* to approach the new steady state. Since search effort and job vacancies are complementary in the matching technology, more agents participate in the labor force. More job matches are created and labor employment increases. The second explanation can be found through the wage equation (2.13). Equilibrium wage rate is a linear combination of the marginal productivity of labor and the marginal rate of substitution between leisure and consumption u_2/u_1. Although the falling capital stock reduces the marginal productivity of labor along the transitional path, the jump in consumption at the beginning and the falling employment during the transition raise the marginal rate of substitution between leisure and consumption u_2/u_1. It is possible that the marginal rate of substitution is raised so high that the wage rate increases. A high wage rate induces agents to search and to work.

5.3. Key features of the dynamic responses

We summarize some key differences between the transitional path in this model and that in the standard model without unemployment. In this model, the economic system responds immediately to changes in factor taxes and subsidies through job vacancies and unemployment; in the standard model, none of these is available and the system responds immediately through labor employment. Although unemployment and job vacancies both immediately respond to tax changes and both overshoot, job vacancies overshoot in a much larger degree. In Figures 1 – 3, the vacancy-unemployment ratio x falls right after the tax change, indicating a larger proportional change (overshooting) in job vacancies than in unemployment. This relatively large response in job vacancies seems consistent with stylized facts.

The slow response of labor employment in the current model implies three novel features of the transitional path. First, responding to changes in taxes and subsidies, output is more closely related to lagged unemployment and job vacancies than to contemporaneous ones. Second, as discussed above, consumption may immediately increase rather than decrease as a result of a tax increase. Finally, taxes and subsidies can generate positive comovement between consumption and labor employment (see Figures 1 and 2). In contrast, tax changes generate negative comovement between the two variables in the standard model when preferences are (weakly) separable over time (Barro and King 1984). This negative comovement has led to the perception that disturbances in taxes cannot generate the stylized positive comovement between consumption and labor employment during business cycles. The formal argument for the negative comovement is as follows. The capital stock is the only state variable, so the saddle path can be written as

$$C_\Delta - C_\Delta^* = A_1 (K_\Delta - K_\Delta^*), \qquad n_\Delta - n_\Delta^* = -A_2 (K_\Delta - K_\Delta^*).$$

Normality of consumption and leisure requires $A_1 > 0$ and $A_2 > 0$. Therefore, $\dot{C}_\Delta = -(A_1/A_2)\dot{n}_\Delta$.

Two elements of our model invalidate the above argument. The first is the substitution between search effort and employment. The substitution creates the possibility that labor employment moves in the same direction as leisure. So even when leisure and consumption always move in the same direction, it is possible that labor employment also moves in the same direction as consumption. The second element is the presence of labor employment as a state variable. The saddle path (4.2) gives

$$\dot{C}_\Delta = (Z_{11}, Z_{21}) \begin{pmatrix} Z_{14} & Z_{24} \\ Z_{15} & Z_{25} \end{pmatrix}^{-1} \begin{pmatrix} \dot{K}_\Delta \\ \dot{n}_\Delta \end{pmatrix}.$$

Thus consumption does not necessarily move in the opposite direction to labor employment.

6. Welfare Costs of Factor Taxes and Subsidies

6.1. Comparison between different τ's

The welfare results are presented in Tables 1 and 2. Table 1 reports marginal deadweight losses for different values of ε, σ and τ_{u0} when $\tau_{K0} = \tau_{W0} = 0.3$. The numbers confirm the intuition that marginal deadweight losses increase with labor supply elasticity ε and consumption demand elasticity $1/(1-\sigma)$. Two conclusions emerge from Table 1. First, the marginal deadweight loss of capital income taxation is much lower than in previous research. In Table 1, no figure of MDL_K exceeds 50 cents. In contrast, Judd (1987) found that MDL_K could easily exceed 50 cents and very often exceed one dollar. The smaller figures in our finding are easy to explain using the argument in subsection 5.2. Facing an increase in capital income taxation, a firm can reduce the number of vacancies in addition to reducing the capital stock. That is, the tax increase reduces the capital stock by a smaller amount and raises revenue by a larger amount than in standard models without unemployment. Consequently, the welfare cost of capital taxation is smaller.

A low marginal deadweight loss of capital income taxation narrows the gap between the marginal welfare costs of capital income taxation and labor income taxation. In the standard model, Judd's (1987) calculation indicates that MDL_K is roughly three to four times as large as MDL_W, suggesting a substantial welfare gain at the margin from such switch. While the gain can still be substantial in our findings for some parameter values, it can be small for other parameter values. For example, when $\tau_{u0}=0.55$, MDL_W is close to MDL_K and the gain of the marginal tax switch is only about 12 cents. When $\alpha=0.8$ and $\tau_{K0}=\tau_{W0}=0.3$, Table 2 reports negligible differences between MDL_W and MDL_K, suggesting negligible marginal gain from the tax switch. As we will see later, the switch may even generate losses.

Tables 1 *and* 2 *here.*

The second conclusion from Table 1 is that MDL_u is negative. The negativity is robust not only with respect to changes in ϵ, σ and τ_{u0} as reported in Table 1 but also with respect to changes in $(\alpha, \lambda, \tau_{K0}, \tau_{W0})$ as reported in Table 2. In Table 1, the absolute value of MDL_u is above 27 cents, increases with τ_{u0} and can easily exceed 50 cents. The corresponding values of R_Δ and U_Δ are negative. That is, an increase in the subsidy to search reduces governmental revenue *and* intertemporal utility. Equivalently, a reduction in the subsidy to search *increases* intertemporal utility and governmental revenue. Therefore search subsidy is very inefficient at the margin. We can look at this inefficiency by computing the welfare gain from switching a marginal tax on labor income to a reduction in search subsidy, $MDL_W - MDL_u$. With the numbers in Table 1, this gain can easily exceed 50 cents. The gain from switching a marginal tax on capital income to a reduction in search subsidy, $MDL_K - MDL_u$, is even larger.

That a search subsidy reduces governmental revenue is a natural result. What is surprising at the first glance is that it also reduces intertemporal utility. Nevertheless, this utility-reducing effect can be explained. As discussed in subsection 5.1, an increase in search subsidy decreases consumption and increases labor force participation. So both consumption and leisure are lower after the subsidy than before the subsidy. Utility falls.

One should be careful about the interpretation of a negative MDL_u. In our model, the rate τ_u functions entirely as a job search subsidy. The negative MDL_u means that if the existing replacement benefits function entirely as a search subsidy, then the replacement ratio is inefficiently high. In reality, however, replacement benefits, such as unemployment insurance, also function as an insurance across states of employment. This insurance function may provide a welfare-improving role for τ_u, but has been abstracted from our model when we assume that each household can smooth the risks. Despite the distance between the modeling assumption and reality, the negative MDL_u obtained in this paper is important for two reasons. First, even if τ_u functions as an insurance, the welfare improving role may be offset or even dominated by its negative role as a search subsidy. The marginal deadweight loss of an increase in τ_u may still be very high. Second, if market institutions exist or develop to insure agents across states of employment, any subsidy to search provided by the government will be inefficient and more inefficient than factor income taxes.

6.2. Importance of labor market frictions

Table 2 enables us to examine how marginal deadweight losses depend on the labor market conditions summarized by (α, λ). Looking across Table 2 vertically, we can find that capital income taxation, labor income taxation and subsidy to search all become more inefficient when λ increases. Basing on the interpretation of λ as the bargaining power of labor in wage determination, we can supply the following explanation. Facing a high bargaining power of labor, firms find it optimal to maintain few vacancies and hire few workers. Because capital and labor are complementary under the assumed technology, equilibrium capital stock is low as well. That is, labor income and capital income are both low when λ is high. An increase in either τ_K or τ_W raises little revenue and generates high marginal welfare loss. In this case, encouraging agents to work by reducing the replacement ratio generates large gains.

Looking across Table 2 horizontally, we can find that capital income taxation, labor income taxation and subsidy to search become more inefficient when α increases. Since α is the efficiency of job vacancies in the matching technology, the result can be explained as follows. An increase in τ_K, τ_W and τ_u all results in a fall in job vacancies (See Section 5). A given magnitude of the fall in job vacancies reduces job matches more significantly when α is larger. In the cases of an increase in τ_W and τ_u, this implies that labor employment and hence capital will decrease more significantly when α is larger, generating a larger welfare loss. In the case of an increase in τ_K, labor employment rises but rises by less when α is larger. But the capital stock falls more significantly, also generating a larger welfare loss.

An interesting alternative explanation exists for the dependence of MDL_u on λ and α. Recall that the condition $\lambda = 1 - \alpha$ is required for the search equilibrium to internalize the labor market externalities when there are no distortionary taxes (see Section 3). If $\tau_u \neq 0$, the labor market is distorted even when $\lambda = 1 - \alpha$ and $\tau_K = \tau_W = 0$. This is because job search is subsidized but maintenance of job vacancies is not. With any pair (λ, α) that satisfies $\lambda = 1 - \alpha$, say $\lambda = 0.6$ and $\alpha = 0.4$, there are too much search subsidy and too little subsidy to job vacancy maintenance. If we fix λ at the level we picked up, 0.6, and reduce α, the resulting increase in search efficiency $1 - \alpha$ reduces the labor market friction and reduces the welfare cost of τ_u. On the other hand, if we fix $\lambda = 0.6$ and increase α, we exacerbate the labor market friction and increase the welfare cost of τ_u. This argument confirms the finding in Table 2 that when $\tau_u \neq 0$, the welfare cost of an increase in τ_u increases with α. Similarly the welfare cost of an increase in τ_u increases with λ.

Table 2 also reports the sensitivity of the results with respect to changes in the base values of taxes (τ_{K0}, τ_{W0}). When τ_{K0} changes from 0.3 to 0.5 and τ_{W0} from 0.3 to 0.4, the welfare cost of capital taxation increases significantly. This sensitivity is common in the dynamic models (see Judd 1987). Different from Judd, the welfare cost of labor income taxation can exceed the welfare cost of capital income taxation with this new set of parameter values. The reversal of the efficiency ranking occurs when α is 0.6 or larger. Despite this sensitivity, the results on MDL_u are robust. MDL_u continues to be negative and exceeds 50 cents in

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Table 1. MDL of Permanent Changes in τ

		$\tau_{u0} = 0.30$			$\tau_{u0} = 0.45$			$\tau_{u0} = 0.55$		
ϵ	σ	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
0.1	-0.1	0.05	0.32	-0.35	0.10	0.35	-0.55	0.22	0.38	-0.71
	-0.5	0.05	0.29	-0.34	0.10	0.33	-0.55	0.21	0.35	-0.71
	-2.0	0.04	0.24	-0.33	0.09	0.27	-0.54	0.20	0.29	-0.70
	-5.0	0.04	0.19	-0.32	0.08	0.21	-0.53	0.18	0.23	-0.69
0.4	-0.1	0.12	0.38	-0.29	0.17	0.42	-0.52	0.29	0.44	-0.69
	-0.5	0.11	0.34	-0.29	0.16	0.37	-0.52	0.27	0.40	-0.69
	-2.0	0.08	0.26	-0.29	0.12	0.29	-0.51	0.23	0.31	-0.68
	-5.0	0.06	0.21	-0.29	0.09	0.23	-0.50	0.19	0.25	-0.67
0.6	-0.1	0.16	0.41	-0.27	0.20	0.45	-0.51	0.33	0.47	-0.69
	-0.5	0.14	0.36	-0.27	0.18	0.39	-0.50	0.30	0.42	-0.68
	-2.0	0.09	0.27	-0.28	0.13	0.30	-0.50	0.23	0.32	-0.67
	-5.0	0.06	0.21	-0.28	0.10	0.24	-0.49	0.19	0.26	-0.67

$\gamma = 0.25$, $\theta = 0.05$, $\alpha = 0.6$, $\lambda = 0.4$, $\tau_{K0} = 0.3, \tau_{W0} = 0.3$. Column (1), (2) and (3) report MDL_W, MDL_K and MDL_u respectively.

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absolute values in most cases.

7. Conclusion

Past examinations on the welfare cost of factor taxes and subsidies have ignored the important fact of unemployment, because of inadequate analytical tools to integrate unemployment into a dynamic, utility-maximization framework. Building on Pissarides (1990), Mortensen (1992) and particularly Merz (1993), we have attempted to create such tools. The resulted model is simple enough to be compared with the standard representative agent model, and at the same time generates quite different results from the standard model. The differences illustrate that unemployment and labor market conditions are important for determining the welfare costs of factor taxes and subsidies.

This paper should be viewed as an attempt to provide a benchmark for future research. Viewed in this way, some of the restrictions and omissions made in this paper are necessary to make the model simple, but may not be strictly required for the issues examined here. Among them, two are worth mentioning. First, there are no quality differences between different job matches, so every match generates employment. Second, the identity of an agent does not matter in this model. For a household, it does not matter whether the same proportion of members (say the first half) are unemployed for two periods or the first half of members are unemployed in the first period and then replaced by the second half in the second period. Therefore this model must be modified in order to be suitable for a study on unemployment duration.

Despite the limitations, the analytical framework in this paper has a wide range of applications. We anticipate it to be useful whenever the relationship between unemployment and capital accumulation is important. For the issue of taxation, the framework can readily be used to examine some normative questions. For example, what are the optimal factor taxes and subsidies? Standard models without unemployment, such as Chamley (1986), show that optimal taxes on capital income converge to zero when the economy converges to the steady state, but optimal labor income tax does not converge to zero. There, the key argument is that capital supply is perfectly inelastic in the short-run but perfectly elastic in the long-run.

In the present model, labor employment has the same feature as capital. This may imply tax smoothing across capital and labor income even in the steady state.

The question of optimal search subsidy is also interesting. The dynamic framework of this paper provides a novel avenue along which this old question can be examined (see Topel and Welch 1980 for a survey of the old arguments). The results in this paper indicate that for given tax rates on capital and labor income, the search subsidy is inefficient. It is interesting to determine the optimal subsidy when factor taxes are chosen optimally.

Table 2. Dependence of MDL on (α, λ)

	τ_{K0}, τ_{W0}		0.3	0.3			0.5	0.4	
	α	0.2	0.4	0.6	0.8	0.2	0.4	0.6	0.8
	(1)	-0.06	0.03	0.15	0.28	-0.06	0.37	0.88	1.89
$\lambda = 0.2$	(2)	0.15	0.20	0.25	0.31	0.44	0.51	0.59	0.69
	(3)	1.14	0.19	-0.36	-0.73	0.18	-0.45	-0.72	-0.88
	(1)	0.05	0.13	0.23	0.34	0.22	0.56	1.18	2.54
$\lambda = 0.4$	(2)	0.22	0.26	0.31	0.37	0.52	0.60	0.69	0.79
	(3)	0.17	-0.38	-0.68	-0.86	-0.39	-0.72	-0.86	-0.93
	(1)	0.09	0.17	0.27	0.38	0.30	0.71	1.50	3.64
$\lambda = 0.6$	(2)	0.25	0.30	0.35	0.40	0.59	0.68	0.79	0.91
	(3)	-0.22	-0.60	-0.79	-0.91	-0.60	-0.82	-0.91	-0.96
	(1)	0.12	0.21	0.31	0.43	0.38	0.88	1.99	6.29
$\lambda = 0.8$	(2)	0.28	0.33	0.38	0.43	0.68	0.78	0.90	1.05
	(3)	-0.43	-0.71	-0.85	-0.94	-0.71	-0.87	-0.93	-0.97

$\sigma = -2$, $\epsilon = 0.4$, $\gamma = 0.25, \theta = 0.05$, $\tau_{u0} = 0.55$. Row (1), (2) and (3) report MDL_W, MDL_K and MDL_u respectively.

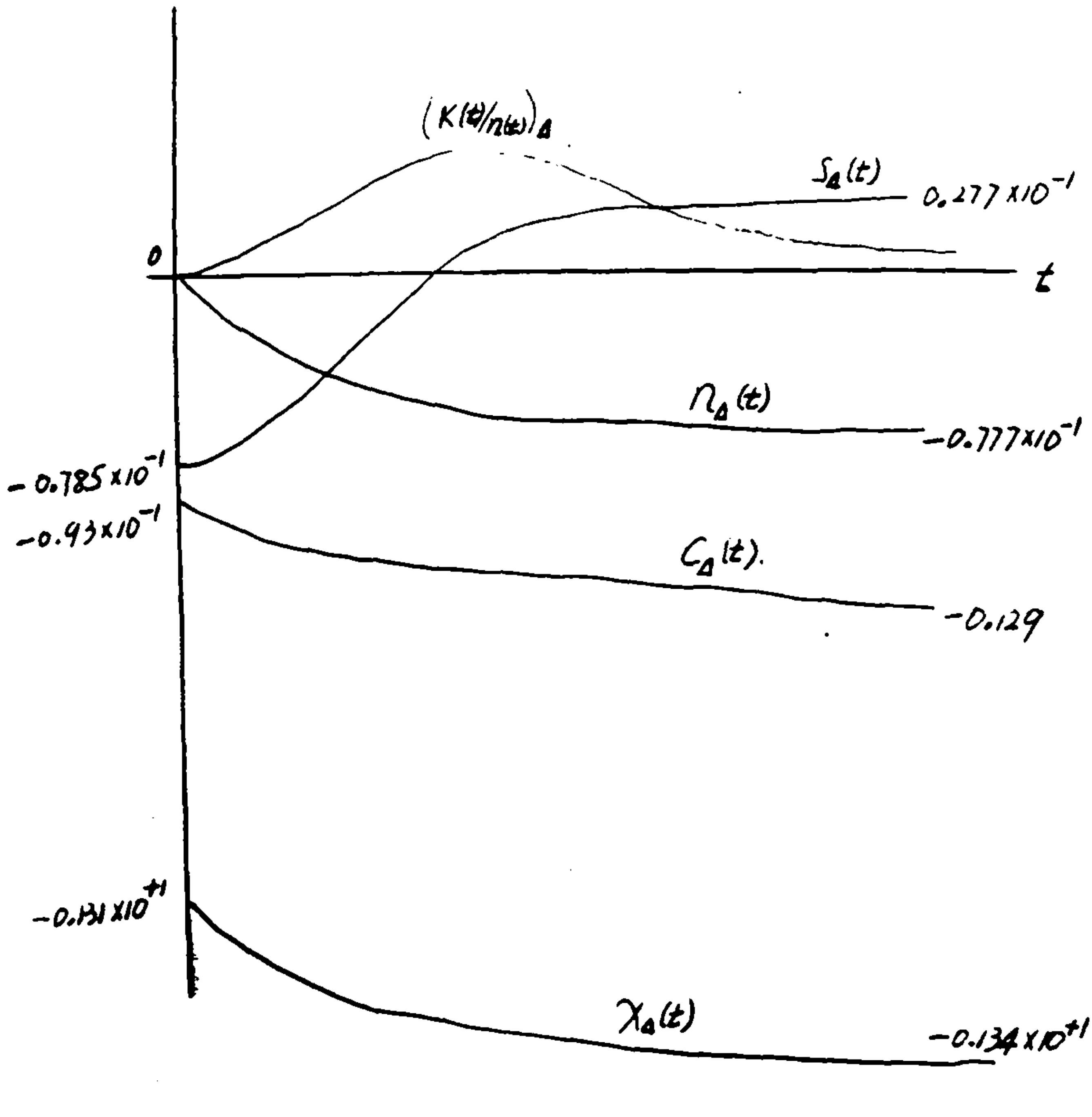

Figure 1 The dynamic effects of an increase in τ_W

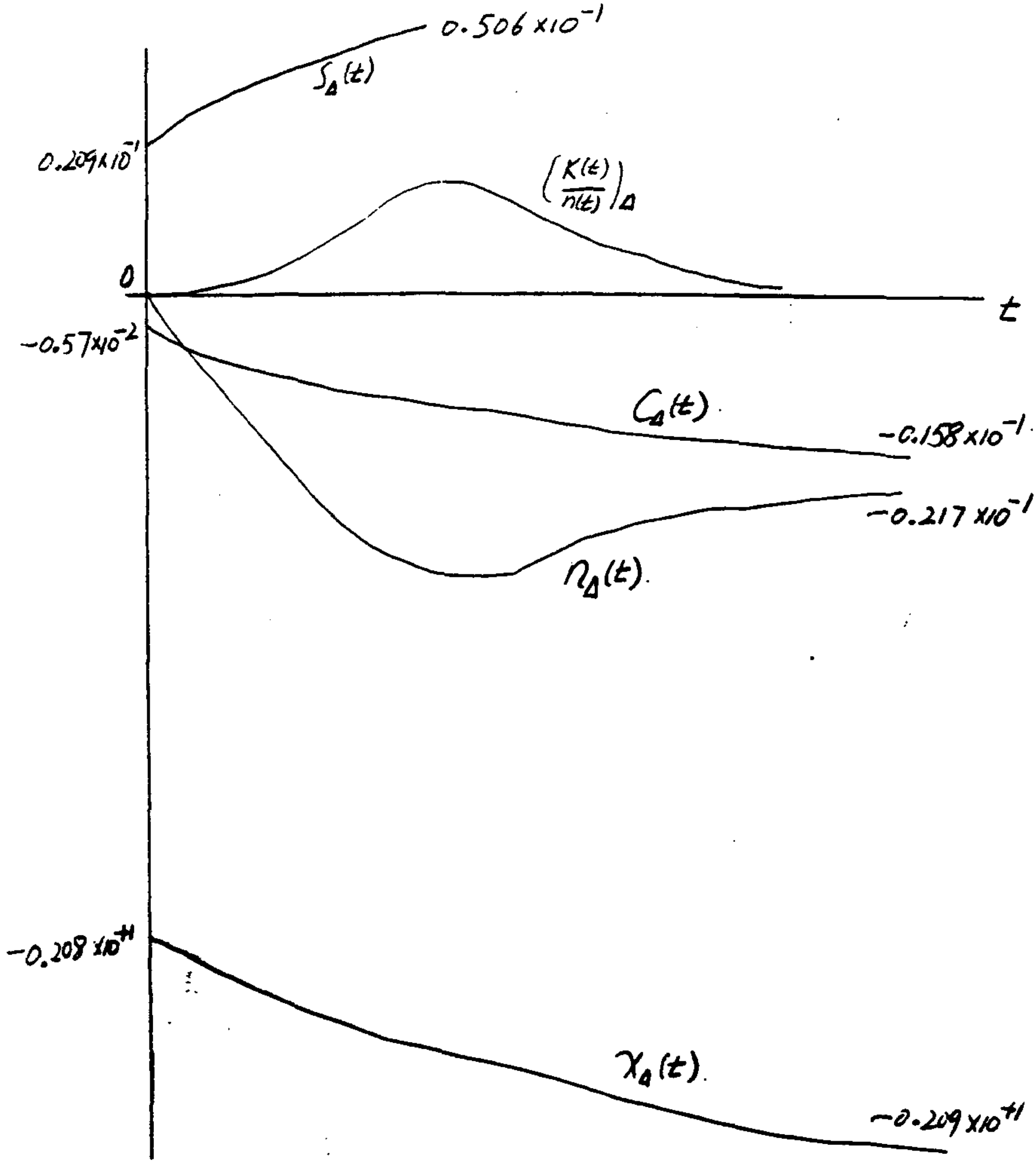

Figure 2 The dynamic effects of an increase in γ_u

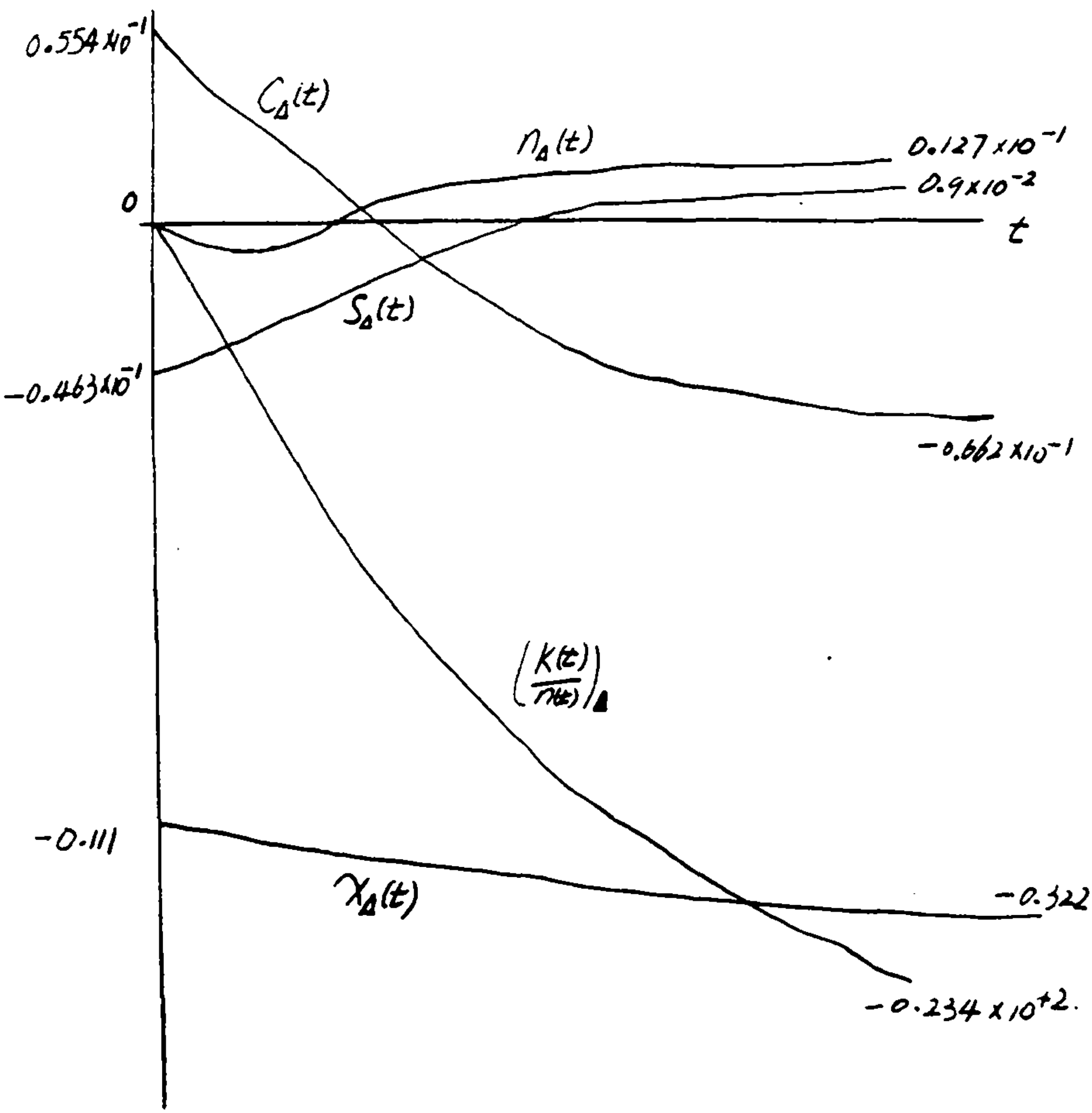

Figure 3 The dynamic effects of an increase in τ_K