FINANCIAL INTERMEDIATION AND REGIME SWITCHING

IN BUSINESS CYCLES[*]

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ABSTRACT

We study a variant of the one-sector neoclassical growth model of Diamond in which capital investment must be credit financed, and an adverse selection problem appears in loan markets. The result is that the unfettered operation of credit markets leads to a one-dimensional indeterminacy of equilibrium. Many equilibria display economic fluctuations which do not vanish asymptotically; such equilibria are characterized by transitions between a Walrasian regime in which the adverse selection problem does not matter, and a regime of credit rationing in which it does. Moreover, for some configurations of parameters, all equilibria display such transitions for two reasons. One, the banking system imposes ceilings on credit when the economy expands and floors when it contracts because the quality of public information about the applicant pool of potential borrowers is negatively correlated with the demand for credit. Two, depositors believe that returns on bank deposits will be low (or high): these beliefs lead them to transfer savings out of (into) the banking system and into less (more) productive uses. The associated disintermediation (or its opposite) causes banks to contract (expand) credit. The result is a set of equilibrium interest rates on loans that validate depositors' original beliefs.

We investigate the existence of perfect foresight equilibria displaying periodic (possibly asymmetric) cycles that consist of m periods of expansion followed by n periods of contraction, and propose an algorithm that detects all such cycles.

1. INTRODUCTION

Monetary economists have frequently expressed the view that the financial system is an important source of -- and propagation mechanism for -- cyclical fluctuations. Indeed, Keynes (1936), Simons (1948), Friedman (1960) and many others have argued that the free and unregulated operation of financial markets can lead to indeterminacy of equilibrium and "excessive economic fluctuations," even in the absence of shocks impinging on the rest of the economy. In modern terms, this argument claims that the financial system itself is a source of endogenously arising economic volatility.

This view has a long empirical foundation. Most of the pre-World War II recessions were associated with substantial transfers of resources out of the banking system and into other assets. For instance, most of the pre-World War II recessions described by Friedman and Schwartz (1963) were associated with increases in the currency-deposit ratio. Particularly severe recessions were associated with particularly sharp increases in this ratio (that is, with bank panics). And even in the last three decades, several recessions have been accompanied by phenomena termed "disintermediation" or "credit crunches." In all of these episodes the volume of bank-extended credit declined, and "credit crunches" have often been associated with the increased incidence of non-price rationing of credit.[1]

Why do we observe sharp fluctuations in the volume of intermediated credit? Can these fluctuations "cause" business cycles, or are they merely a symptom of them? Is credit market activity a potential or (as many have argued) a necessary source of endogenously arising excess fluctuations? Are there economic configurations which dictate that credit market activity must *be associated with endogenous economic volatility?*

This paper considers the relationship between credit and production in a simple model of dynamic general equilibrium, namely the non-monetary overlapping generations economy with production introduced by Diamond (1965).[2] We modify that economy in only two respects: we introduce some intra-generational heterogeneity, and we force some kinds of capital investment to be credit financed. Under complete public information

about the characteristics of potential borrowers, we show that both of these modifications are purely cosmetic, and make no qualitative difference to the properties of competitive equilibria. Specifically, given any positive initial capital stock, the economy monotonically approaches a non-trivial steady state equilibrium, which is unique under our assumptions. Thus there is no scope either for indeterminacy of equilibrium, or for endogenous fluctuations.

Heterogeneity makes a lot of difference when there is private information about borrower characteristics (here ex ante information about loan repayment probabilities). In the presence of an adverse selection problem, lenders will seek to elicit from borrowers information regarding loan repayment probabilities. Lenders will do so by structuring the loan contracts they offer -- which specify both loan quantities and interest rates -- in order to induce potential borrowers to self-select or, in effect, to reveal their type. Thus all loan contracts offered must be incentive compatible. Incentive compatibility can be attained in two distinct ways. In one, the full-information exchange of credit may be consistent with self-selection in the loan market, in which case it can be duplicated under private information by appropriately chosen loan contracts. Alternatively, the exchange of credit that occurs under full information may not be incentive compatible, in which case self-selection constraints will bind on the choice of loan contracts. In this case lenders will use credit rationing as a means of inducing self-selection.

Interestingly, there is a range of values for the current capital stock (equivalently, for factor prices) under which the full information allocation of credit is incentive compatible. Here it is feasible for the equilibrium law of motion of the capital stock to coincide with that which obtains under perfect information. There is also a range of capital stocks for which incentive constraints necessarily bind; here credit must be rationed and the law of motion of the capital stock yields lower values relative to the situation of complete information. In addition, under conditions we describe, there is a non-trivial closed interval of current capital stocks for which either binding or non-binding incentive constraints are

consistent with equilibrium in the loan market. When this happens, the economy can be in either of two regimes: a Walrasian regime in which competitive markets allocate credit in the standard way, and a regime of credit rationing. If credit is rationed, resources leave the banking system and the allocation of investment becomes less efficient in a manner we will make precise. Since each of these regimes is consistent with equilibrium, equilibrium is indeterminate: the economy can follow either the full information or the private information law of motion for the capital stock. Moreover, as we will demonstrate, the economy can switch from one law of motion to the other in either a deterministic or a stochastic manner. The result is that there will be fluctuations in output and the capital stock, and these fluctuations need not dampen over time. Thus both indeterminacy of equilibrium and "excessive" fluctuations can be observed when agents are privately informed about loan repayment characteristics.

Perhaps more interestingly, our model economy generates endogenous reflective barriers, a floor below which incentive constraints cannot bind in equilibrium, and a ceiling above which Walrasian credit allocations cannot be incentive compatible. We then use the presence of these barriers to examine two possible configurations of equilibria.

Since our model economy generates two distinct laws of motion for the capital stock (one corresponding to Walrasian allocations in the credit market, and one corresponding to credit rationing), it may possess two distinct steady state equilibria: one corresponding to Walrasian outcomes and another corresponding to credit rationing. If these two steady states lie inside the reflective barriers they are both equilibria, as are various paths that approach either of them monotonically. In addition, there is a large set of equilibria in which the economy endogenously switches between the two laws of motion. Indeed, we demonstrate the existence of equilibria in which m periods of expansion are followed by n periods of contraction (and so on) for every pair of integer values (m,n). Moreover, all such equilibria are asymptotically stable, so that the economy generates a high dimensional multiplicity of equilibria displaying asymmetric cyclical fluctuations when m ≠ n.

It is also possible that the two steady state equilibria lie <u>outside</u> the reflective barriers, in which case neither one constitutes a legitimate equilibrium. In this case, the economy does not admit any equilibrium with monotonic time paths in the capital stock or output: all equilibrium sequences have turning points, and fluctuations must be observed along every equilibrium path. We derive necessary and sufficient conditions for the existence of deterministic (m,n) cycles (m periods of expansion followed by n periods of contraction). We also propose an algorithm which identifies all such periodic cycles, and isolate the largest pair (m,n), which we associate with the <u>maximally persistent periodic orbit</u>. Asymmetric cycles with m > n (expansions are longer and shallower than contractions) are found to exist if the "floor" of the economy is tighter than its "ceiling".

What happens when these fluctuations occur? When the economy transits from an expansion phase to a contraction phase, resources leave the banking system and are allocated to less efficient uses. This occurs because depositors (correctly, as it turns out) conjecture that rates of return offered by the banking system will be low. As banks lose depositors, less credit is available, and banks are forced to ration it. Hence contractions are associated with disintermediation and "credit crunches." Moreover, when steady state equilibria lie outside the economy's reflecting barriers, these phenomena cannot be avoided. For economies with this property, indeterminacy and excessive fluctuations are not merely possible but actually unavoidable.

To summarize, dynamic equilibria with adverse selection satisfy an unusual, set-valued, discontinuous difference equation which contains two well-defined, partially overlapping domains with distinct structural regimes. At intermediate values of economic activity the two regimes overlap, permitting the economy to switch between them. The resulting indeterminate equilibria may be indexed by a stochastic process that governs an unobserved regime-switching variable in the manner proposed by Hamilton (1989, 1990). The economic interpretation of this state variable is to regard it as an index of savers' expectations about credit market conditions. An expanding body of evidence[3] suggests

that such nonlinear mechanisms accord with the behavior of many aggregate time series.

The remainder of the paper proceeds as follows. Section 2 lays out the environment and the nature of trades, and describes the equilibrium conditions that obtain when credit is and is not rationed. Section 3 shows how the economy can transit between Walrasian regimes and regimes of credit rationing, while Sections 4 and 5 examine the existence of cyclical equilibria which display undamped oscillations. Section 6 contains a variety of numerical examples of such equilibria, and Section 7 concludes.

2. GROWTH WITH ADVERSE SELECTION

A. Environment

We consider a simple variant of Diamond's one-sector neoclassical growth model. In particular, at each date $t = 0,1,...$ a new set of two period lived, overlapping generations is born.[4] Each generation is identical in composition, and consists of a continuum of agents of measure one.

At each date there is a single consumption good, which is produced using a standard constant-returns-to-scale production function with capital and labor as inputs. A capital input of K_t combined with a labor input of L_t permits $F(K_t, L_t)$ units of this good to be produced at t. We let $k_t \equiv K_t/L_t$ denote the capital-labor ratio, and $f(k_t) \equiv F(k_t, 1)$ denote the intensive production function. We assume that $f(0) = 0$, that $f'(k) > 0 > f''(k)\ \forall k \geq 0$, and that f satisfies the usual Inada conditions. We also assume that the consumption good can be stored: one unit of the good stored at t returns a units of the good at t+1. Throughout we think of a as being relatively small, so that storage is a relatively unproductive activity. Finally, the consumption good can be used to produce capital; one unit of consumption placed in a capital investment at t returns one unit of capital at t+1.

Within each generation, agents are divided into two types. A fraction $\gamma > 0.5$ of young agents are of type 1. These agents are endowed with one unit of labor when young, and are retired when old. In addition, they have access to the storage technology just

described, but they have <u>no</u> access to the technology for converting current goods into future capital. A fraction $1-\gamma < 0.5$ of each generation is of type 2. Type 2 agents cannot work when young, but are endowed with one unit of labor when old. In addition, we assume that type 2 agents have no access to the goods storage technology, but that they are endowed with the technology for converting current consumption into future capital. Thus, in many respects, type 1 and 2 agents are mirror images of one another. Finally, we assume that all agents care only about old age consumption, and are risk neutral.[5] In particular, labor generates no disutility.

Notice that type 1 agents are natural lenders at any interest rate because they need to provide for old age consumption. Type 2 agents are natural borrowers who need credit to finance capital investments. Between borrowers and lenders stand financial intermediaries, or banks, which accept deposits and extend loans. Their cost of doing so is zero.

Finally, there is a set of initial old agents who are endowed with a per capita capital stock of $k_o > 0$. We assume that capital depreciates at the rate $\delta \in [0,1]$.

B. Full Information

In this section we describe a competitive equilibrium of this economy under the assumption of full information, and in particular that all allocations and the type of each agent are publicly observable. We also assume throughout, without loss of generality, that each type 2 agent (recall that these agents own capital) runs one firm and works for himself.

At date t each young type 1 agent supplies one unit of labor inelastically, earning the real wage rate w_t. All of this income is saved, to be allocated between storage and bank deposits. We let s_t be per capita storage at t, so $\gamma w_t - s_t$ is per capita deposits. Deposits earn the competitive/gross return R_{t+1} between t and $t+1$, which both banks and depositors treat as parametric.

Each young type 2 agent (firm) borrows b_t, and uses it to produce a per firm capital stock of K_{t+1} at t. Loan market clearing, then, requires that

$$(1-\gamma)b_t + s_t = \gamma w_t \tag{1}$$

In addition, there will be a positive supply of deposits if, and only if, these weakly dominate storage in rate of return, so that

$$a \leq R_t, \quad \forall t. \tag{2}$$

Old type 2 agents at t have the inherited (per firm) capital stock K_t, which they combine with their own labor plus N_t units of young labor in order to produce output. In addition, these agents have an inherited interest obligation of $r_t b_{t-1} = r_t K_t$, where r_t is the gross loan rate of interest, between $t-1$ and t, charged by banks. Then old type 2 agents have an income (consumption) level of

$$F(K_t, 1+N_t) - w_t N_t - r_t K_t + (1-\delta)K_t,$$

which they maximize with respect to N_t. Hence if total labor input for the firm is $L_t = 1 + N_t$, we have

$$w_t = F_2(K_t, L_t) \equiv f(k_t) - k_t f'(k_t) \equiv w(k_t); \quad t \geq 0. \tag{3}$$

Note that $w'(k) > 0$ holds for all k, and in addition we will assume that

w(k) is a concave function. (a.1)

Assumption (a.1) holds if, for instance, F is any CES function with elasticity of substitution no less than one.

We assume that there is free entry into banking, so that

$$r_t = R_t \tag{4}$$

holds for all $t \geq 0$. In addition, competition among banks for borrowers implies that banks must offer type 2 agents at t the loan quantity which maximizes their lifetime utility:

$F(b_t, 1+N_{t+1}) - w_{t+1}N_{t+1} - R_{t+1}b_t + (1-\delta)b_t.$ Hence

$$R_{t+1} = F_1(b_t, L_{t+1}) + 1-\delta = F_1(K_{t+1}, L_{t+1}) + 1-\delta$$

$$= f'(k_{t+1}) + 1-\delta; \quad t \geq 0. \tag{5}$$

Finally, since a fraction $1-\gamma$ of the population is firms and γ is workers at each date, clearly $N_t = \gamma/(1-\gamma)$, and $L_t = 1 + N_t = 1/(1-\gamma)$. Thus

$$k_t = K_t/L_t = (1-\gamma)K_t = (1-\gamma)b_{t-1}; \quad t \geq 1. \tag{6}$$

It is now straightforward to describe the equilibrium law of motion for the capital stock. Equations (1), (3) and (6) imply that

$$k_{t+1} = \gamma w(k_t) \cdot s_t. \tag{7}$$

Suppose that $f'(k_{t+1}) + 1-\delta > a$. Then $s_t = 0$ holds, and (7) becomes

$$k_{t+1} = \gamma w(k_t). \tag{7'}$$

Under our assumptions, (7') describes an increasing, concave locus that passes through the origin, as depicted in Figure 1. If $\gamma w'(0) > 1$ holds, then there is a unique non-trivial steady state equilibrium, denoted k^O in Figure 1a.[6] Given the initial per capita capital stock k_O, there is a unique sequence $\{k_t\}$ that monotonically converges to the steady state. Thus, under full information, this economy can display neither indeterminacy nor economic fluctuations.

C. Credit Rationing

Next we introduce private information and examine what occurs when incentive constraints bind. Then, in Section 3, we examine a full general equilibrium in which incentive constraints may or may not bind in an endogenous way.

We work with a simple structure of information; household type and storage activity are private information, while all markets (both labor and credit markets) transactions are publicly observed. These assumptions imply that young type 2 agents cannot credibly claim

to be type 1 when young, since they are unable to supply labor at that age. Type 1 agents, however, can borrow when young. If they do so, they must borrow the same amount (b_t) as type 2 agents, and they cannot work. In addition, type 1 agents are unable to produce capital, and hence cannot function as producers in old age. Thus any type 1 agent who borrows will be discovered as having misrepresented his type. In order to avoid punishment, we assume that a dissembling type 1 agent will simply store what he borrowers and "go underground" or abscond with his loan. Thus a loan made to a type 1 agent will never be repaid.[7]

Given these circumstances, bank behavior must be modified in two respects. First, loan contracts must earn non-negative profits. Let μ_t denote the fraction of type 1 agents who claim to be of type 2. Since such agents never repay their loans (and all borrowers borrow the same amount), banks earn non-negative profits iff

$$r_{t+1} \geq R_{t+1} \{1 + \mu_t[\lambda/(1-\lambda)]\}; \quad t \geq 0. \tag{8}$$

Equation (8) requires that type 2 agents, who actually repay loans, pay enough to cover the defaults associated with loans to type 1 agents. Second, if $\mu_t < 1$ holds,[8] type 1 agents must do at least as well by working when young and saving as they would by claiming to be of type 2. Working when young and saving generates a lifetime utility level of $R_{t+1}w_t$ at t, while claiming to be of type 2 generates a lifetime utility level of ab_t. Hence loan contracts must satisfy the incentive constraint (or self-selection condition)

$$R_{t+1}w_t \geq ab_t; \quad t \geq 0. \tag{9}$$

Following Rothschild and Stiglitz (1976), we assume that intermediaries are Nash competitors in loan markets; taking the deposit rate R_t and the announced loan contracts of other banks as given. As in Rothschild and Stiglitz, it is easy to show that any Nash equilibrium contract earns zero profits, so that (8) holds with equality. In addition, it is possible to show that any non-trivial equilibrium (that is, any equilibrium with $k_t > 0$ $\forall t$) has $\mu_t = 0$ $\forall t$. In particular, contracts induce self-selection, and pooling is not a

possibility.[9]

Thus, if an equilibrium with credit rationing exists, it continues to satisfy equations (3) and (6). Therefore (9) reduces to

$$R_{t+1}w(k_t) \geq ak_{t+1}/(1-\gamma). \tag{9'}$$

In addition, it is necessary that type 2 agents be willing to borrow; it is easy to show that they are iff the marginal product of capital, inclusive of depreciation, exceeds the loan rate. In short, the inequality

$$f'(k_{t+1}) + 1-\delta \geq R_{t+1} \tag{10}$$

must hold. If (10) holds as a strict inequality borrowers would like to borrow arbitrarily large amounts; in this case the incentive constraint (9) [or (9')] holds as an equality and determines the equilibrium loan quantity. Thus, when credit rationing obtains, (9') holds as an equality.

Finally, in order for announced loan contracts (b_t, r_{t+1}) to constitute a Nash equilibrium, it must be the case that no intermediary can offer an alternative contract $(\tilde{b}_t, \tilde{r}_{t+1})$ which is preferred by type 2 agents, and which satisfies (8) for some μ_t. It is straightforward to show that, if there is such a contract, it must be a pooling contract $(\mu_t = 1)$ with $\tilde{r}_{t+1} \geq R_{t+1}/(1-\gamma)$ in order to satisfy (8). Moreover, a pooling contract will attract type 2 agents iff $R_{t+1}/(1-\gamma) \geq f'(k_{t+1}) + 1-\delta$ holds.[10] Since it is impossible to observe a pooling contract in a non-trivial equilibrium, the existence of an equilibrium with credit rationing requires that there be no pooling contract which type 2 agents prefer to the contract $(b_t, R_{t+1}) = (R_{t+1}w_t/a, R_{t+1})$. This is so if

$$f'(k_{t+1}) + 1-\delta \leq R_{t+1}/(1-\gamma). \tag{11}$$

To summarize: in a regime of credit rationing, equations (3), (4) and (6) hold, as does (9') at equality. Equations (10) and (11) must hold as well; these may (and typically will) be inequalities. It follows that

Proposition 1: If credit is rationed at t, then $R_{t+1} = a$ holds.

Proposition 1 is proved in Appendix A. It has the immediate implication that, if credit is rationed at t, equation (9') becomes

$$k_{t+1} = (1-\gamma)w(k_t). \tag{12}$$

Substituting (12) into (7) yields

$$s_t = (2\gamma-1)w(k_t) > 0. \tag{13}$$

Thus, credit rationing is accompanied by savings leaving the banking system (here to be used for storage). This justifies why $R_{t+1} = a$ must hold if credit is rationed at t.

When credit rationing does exist at t, equation (12) governs the evolution of the capital stock. (12), of course defines an increasing, concave function (as depicted in Figure 2) which has a unique non-trivial intersection with the 45° line iff $(1-\gamma)w'(0) > 1$. We denote this intersection by k^1. Since $\gamma > 0.5$, the law of motion defined by (12) lies everywhere below the law of motion defined by (7) -- so that credit rationing impedes capital formation -- and $k^0 > k^1$ necessarily holds. Finally we note that, when credit is rationed, (11) reduces to

$$f'(k_{t+1}) \le [a/(1-\gamma)] - (1-\delta). \tag{14}$$

Thus (14) must hold at any t for which there is credit rationing.

If credit is rationed at every date, then (12) gives the equilibrium sequence of capital stocks starting from k_0. Clearly this sequence will monotonically approach k^1, so that again equilibria are unique and display monotone dynamics. Multiple equilibria and endogenous fluctuations are associated with transitions between Walrasian regimes and regimes with credit rationing. Such transitions are the focus of the next section.

3A. ENDOGENOUS REGIME SWITCHING MECHANISMS

Suppose that, given the inherited capital-labor ratio k_t at t, the value of k_{t+1} from (7') satisfies $f'(k_{t+1}) + 1-\delta > a$, and in addition that (k_t, k_{t+1}) satisfies (9'). Then the full information allocation is incentive compatible, and constitutes an equilibrium at t. Alternatively, suppose that, given k_t, equation (9') gives a value k_{t+1} satisfying both $f'(k_{t+1}) + 1-\delta \geq a$ and (14). Then credit rationing is consistent with an equilibrium outcome at t.

These two possibilities define two distinct regimes for each time period; in regime 0, the incentive constraint is slack and borrowers are rationed by prices alone; in regime 1, the incentive constraint is tight and borrowers are rationed by a combination of prices and quantities. A brief description of equilibria in each regime follows.

Regime 0: Assume that the parameters of the economy satisfy

$$a\gamma/(1-\gamma) \geq 1-\delta. \tag{15}$$

Equation (15) implies that there are some full information capital stocks that satisfy the incentive compatibility condition, as well as some that do not.[11] Then we define two critical values $(\hat{k}, k_c^o)$ for the capital stock from the equations

$$f'(\hat{k}) = a\gamma/(1-\gamma) - (1-\delta) \tag{16a}$$

$$\hat{k} = \gamma w(k_c^o). \tag{16b}$$

$\hat{k}$ is the largest "full information capital stock" that can satisfy the incentive constraint, and k_c^o is the capital stock that maps into $\hat{k}$ under (7'). Given these values, it is easy to check that any sequence (k_t, s_t, R_t) conforming to the initial condition k_0 is an equilibrium without credit rationing if, for each t,

$$k_{t+1} = \gamma w(k_t) \tag{17a}$$

$$s_t = 0 \tag{17b}$$

$$R_t = 1 - \delta + f'(k_t) > a \tag{17c}$$

$$k_{t+1} < \hat{k} \quad \text{or} \quad k_t < k_c^0 \tag{17d}$$

Equation (17) defines equilibrium sequences for which borrower characteristics are effectively public information. Solution sequences of (17a) converging to k^0 will be competitive equilibria for regime 0 if they are bounded above by the critical value k_0^c defined in equations (16a) and (16b) and shown in Figure 1.

This figure explains why, for any initial capital stock, equilibria without credit rationing are more likely to be viable if the critical value k_c^0 is large relative to the fixed point k^0 as in panel 1a; less likely to exist if, as in panel 1b, k_c^0 is relatively small. The definition of k_c^0 reveals that the critical value is large when either of the parameters a and δ is itself small, that is, if the unobservable storage technology is much less productive than the observable neoclassical production technology at k^0.

We conclude that equilibria in this regime are more likely to exist when two conditions are met:

i.   the level of economic activity is not too high; and

ii.   the investment projects available to low-quality borrowers (type-1 agents) are sufficiently less productive than those open to high-quality borrowers (type-2 agents).

Regime 1: Again we define two critical values $(\bar{k}, k_c^1)$ from the equations:

$$f'(\bar{k}) = a/(1-\gamma) - (1-\delta) \tag{18a}$$

$$\bar{k} = (1-\gamma)w(k_c^1) \tag{18b}$$

Thus $\bar{k}$ is the smallest capital stock consistent with the existence of a separating Nash equilibrium under private information [see equation (14)], and k_c^1 is the capital stock that maps into $\bar{k}$ under (9'). From (16a) and (18a) we obtain

$$\bar{k} < \hat{k} \tag{18c}$$

Any sequence (k_t, s_t, R_t) that starts from a given initial condition k_0 is a competitive equilibrium with credit rationing if, for each t, it satisfies the following conditions:

$$k_{t+1} = (1-\gamma)w(k_t) \tag{19a}$$

$$s_t = (2\gamma-1)w(k_t) \tag{19b}$$

$$R_t = a < 1-\delta + f'(k_t) \tag{19c}$$

$$k_{t+1} > \bar{k} \quad \text{or} \quad k_t > k_c^1 \tag{19d}$$

As previously, the locus defined by (19a) in Figure 2 lies entirely below the locus defined by (17a), and its fixed point, k^1 lies below k^0.

Figure 2 also depicts the domain of definition, $[k_c^1, \infty)$, for equation (19a) and depicts two possible cases: the unique positive fixed point, k^1, of this equation is in that domain when panel 2a obtains, but not when panel 2b does. Given any initial value k_0 of the capital stock, separating competitive equilibria with credit rationing are more likely to exist if the critical value k_c^1 is low relative to the fixed point k^1, i.e., if:

i.   the level of economic activity is not too low; and

ii.   the investment projects available to low-quality borrowers are not much less productive than the ones open to high-quality borrowers.

3B. REGIME TRANSITIONS

This section discusses somewhat informally how and why an economy may shift from a Walrasian regime of slack incentive constraints to one of credit rationing and tight incentive constraints. Sections 4 and 5 treat the same issues more formally and in greater depth. We start by amalgamating Figures 1 and 2 into the top two panels of Figure 3. Figure 3a contains the fixed point of each regime in the domain of its definition, or in other words, it assumes that

$$k_c^1 < k^1 < k^o < k_c^o \tag{20a}$$

Figure 3b, on the other hand, excludes the fixed point of each regime from its domain by assuming that

$$k^1 < k_c^1 < k_c^0 < k^0 \tag{20b}$$

We also draw in panels 3c and 3d two other cases in which one of the fixed points is within the relevant domain of definition and the other is outside. Intuitively, inequality (20a) holds when the storage technology is much less productive than the neoclassical technology at k^0, while at k^1 banks are able to offer contracts separating high-quality borrowers from low-quality ones. Inequality (20b) on the other hand, means that the storage technology is only a little less productive than the neoclassical technology at k^0, and banks are completely unable to offer contracts permitting them to distinguish low from high quality borrowers at k^1.

Dynamic equilibria are solutions to the discontinuous, set-valued difference equation represented by the solid lines in each panel. To ensure that equilibria exist we require that the critical values (k_c^0, k_c^1) satisfy

$$k_c^1 < k_c^0 \tag{21}$$

Otherwise the solid graph in panel 3b will contain a hole, and a deterministic k_{t+1} will be undefined[12] if k_t were to lie in the interval $[k_c^0, k_c^1]$. Appendix B shows that a necessary and sufficient condition for (21) is

$$\gamma \bar{k} < (1-\gamma)\hat{k} \tag{22}$$

where $(\hat{k}, \bar{k})$ are defined in equations (16a) and (18a).

Given this assumption, it is easy to see that each economy in Figure 3 has an invariant set, that is, a subset in its state space which traps all solution sequences that start in it. That set is the interval $[k^1, k^0]$ in Figure 3a, and the interval $(\bar{k}, \hat{k})$ in Figure 3b. Examples of sequences trapped within the former invariant set are regime-0 equilibria converging to k^0 from below, regime-1 equilibria converging to k^1 from above, as well

as the periodic two-cycle depicted in panel 3a.

As we will show, in economies having the configuration of panel 3a, there is in fact a very large set of equilibria displaying deterministic cycles. Hence both indeterminacy of equilibrium and undamped fluctuations are a very real possibility in such economies. In economies having the configuration of panel 3b, there are no monotone equilibrium sequences (k_t): all equilibrium sequences must display transitions between regimes 0 and 1. Hence, all such economies necessarily display fluctuations despite the absence of any variations in economic fundamentals.

The top panels of Figure 3 suggest how regime switches occur. For example, if depositors pessimistically (optimistically) expect that banks will (will not) ration credit to borrowers, then deposit yields are expected to fall below (to equal) the net marginal product of capital. This makes bank liabilities less (more) attractive relative to autarkic storage, leading to a shortage (surplus) of deposits and validating the original belief about credit rationing. Figure 3a shows how the switching of expectations may force a regime change even if "objective" conditions permit the present regime to continue. In Figure 3b the mechanism is the same, but regime transitions necessarily occur because some capital stocks will be inconsistent with the ability of banks to separate low from high quality borrowers.

Key among objective conditions is how well informed intermediaries are about the quality of their loan portfolios. At low levels of economic activity, the price mechanism is successful in separating low-quality borrowers from high-quality ones, and adverse selection does not pose a serious information problem for lenders. At high levels of activity, however, the price system cannot prevent low-quality borrowers from claiming loans meant for high-quality producers[13] unless it is supplemented by quantity rationing of credit.

We can easily define measures of a "temptation to lie" when credit is not rationed at time t by either the yield differential $a - R_{t+1}$ between storage and bank deposits, or by the differential income $b_t - w_t$ between ill-gotten bank loans and honestly performed

labor. From equations (17a), (17c) and (6) we obtain

$$a \cdot R_{t+1} = a + \delta\text{-}1 \cdot f'(k_{t+1}) = a + \delta\text{-}1 \cdot f'[\gamma w(k_t)] \qquad (23a)$$

$$b_t \cdot w_t = k_{t+1}/(1\text{-}\gamma) \cdot w(k_t) = [\gamma/(1\text{-}\gamma)\text{-}1]w(k_t) \qquad (23b)$$

In each case, a high current stock of capital raises the differential and strengthens incentives for borrowers to misrepresent themselves. When these incentives are strong enough, banks respond by rationing credit. This is why the price-rationing equilibria of regime 0 do not exist above a critical value k_c^0 of the capital stock.

As the composition of the pool of potential borrowers shifts, financial intermediaries respond with loan contracts whose collective outcome is the erection of <u>reflective barriers</u> for the entire economy. An upper bound or <u>ceiling</u> is set up when credit is restricted to discourage applications by low-quality borrowers. A lower bound or <u>floor</u> results from the lifting of credit restrictions that occurs when the price system by itself provides sufficient incentives to keep undesirable borrowers from applying for loans.

As this discussion suggests, it is important whether or not the positive steady states (k^1, k^0) of the two regimes fall outside the interval (k_c^1, k_c^0) defined by the reflective barriers. Accordingly, we arrange our investigation around reflective barriers: we study economies with no binding barriers in Section 4, economies with two binding barriers in Section 5.These discussions will also illustrate what would happen in economies with the configurations depicted in Figures 3c and 3d.

4. ECONOMIES WITH STEADY STATE EQUILIBRIA

When equation (20a) is satisfied, equilibrium sequences (k_t) evolve according to the set-valued difference equation

$$k_{t+1} = \gamma w(k_t); \quad k_t < k_c^1 \qquad (24)$$

$$k_{t+1} \in \{\gamma w(k_t),(1\text{-}\gamma)w(k_t)\}; \quad k_c^1 \le k_t \le k_c^0$$

$$k_{t+1} = (1-\gamma)w(k_t); \quad k_t > k_c^0.$$

We are particularly interested in periodic solutions, that is, in fixed points of iterated maps derived by repeated application of (24) to describe how today's state variable is related to its value $n = 1,2,3,...,$ periods hence. Because the map is set valued, iterates of a given order n depend very much on which branch is chosen at each iteration. For instance, always choosing the lower branch $(1-\gamma)w$ of the map will produce a smaller iterate than if we always choose the upper branch γw.

To simplify the mathematical structure, we endow the economy with logarithmic utility and production functions which reduce the map (24) to a piecewise linear difference equation in the logarithm of the capital stock. We assume, in particular, that

$$f(k) = (A/\theta)k^\theta; \quad A > 0, \quad \theta \in (0,1) \tag{25}$$

This implies the following wage, marginal product of capital, and inverse marginal product of capital functions:

$$w(k) = [A(1-\theta)/\theta]k^\theta, \tag{26a}$$

$$f'(k) = A/k^{1-\theta}, \tag{26b}$$

$$h(z) = (A/z)^{1/(1-\theta)} \tag{26c}$$

Defining $x = \log k$, we may rewrite equation (24) in its linear form:

$$x_{t+1} = f_0(x_t) \equiv \theta(x_t-x^0) + x^0; \quad x_t < x_c^1$$

$$x_{t+1} \in \{f_0(x_t), f_1(x_t)\}; \quad x_c^1 \leq x_t \leq x_c^0 \tag{27}$$

$$x_{t+1} = f_1(x_t) \equiv \theta(x_t-x^1) + x^1; \quad x_t > x_c^0.$$

This map is drawn in Fig. 4a. Here fixed points of regimes 0 and 1 are

$$x^0 = [1/(1-\theta)] \log[\gamma(1-\theta)A/\theta] \tag{28a}$$

$$x^1 = [1/(1-\theta)] \log[(1-\gamma)(1-\theta)A/\theta] \tag{28b}$$

Computing critical points for the two regimes requires some algebra. The end results are:

$$x^O_c = (1/\theta) \log\left\{\frac{\theta}{\gamma(1-\theta)A} \left[\frac{A(1-\gamma)}{a\gamma-(1-\delta)(1-\gamma)}\right]^{1/(1-\theta)}\right\} \tag{29a}$$

$$x^1_c = (1/\theta) \log\left\{\frac{\theta}{(1-\gamma)(1-\theta)A} \left[\frac{A(1-\gamma)}{a-(1-\delta)(1-\gamma)}\right]^{1/(1-\theta)}\right\} \tag{29b}$$

To ensure that the map (27) contains no holes and that deterministic equilibria exist, we assume $x^1_c < x^O_c$ or, equivalently,

$$(1-\delta)(1-\gamma) + [a\gamma-(1-\delta)(1-\gamma)] \left(\frac{\gamma}{1-\gamma}\right)^{1-\theta} < a \tag{30}$$

Storage is an inferior asset if it yields less than the net marginal product of capital at all $k \leq k^O$, that is, if $f'(k^O) + 1-\delta > a$. This proves to be equivalent to the restriction

$$\gamma < \frac{\theta/(1-\theta)}{\max\{0,a-(1-\delta)\}} \tag{31}$$

Finally, an economy has no effective reflective barriers if the map (27) contains in its domain of definition the fixed points (x^O,x^1) of each regime. This inclusion means that

$$x^1_c < x^1 < x^O < x^O_c \tag{32}$$

or, equivalently, that

$$[\gamma/(1-\gamma)][a\gamma-(1-\delta)(1-\gamma)] < \theta/(1-\theta) < a - (1-\delta)(1-\gamma) \tag{33}$$

In the remainder of this section we study solutions to equation (27) maintaining inequalities (30)-(33) as restrictions on the economy's parameter space $(a,\gamma,\delta,\theta,A)$. Inequality (32), in particular, means that there are no binding floors or ceilings; the steady state of each regime is an equilibrium and, hence, this economy does not have to switch regimes. Whatever switching is done will be due to market beliefs, that is, to self-confirming expectations by depositors about the behavior of intermediaries and the yields on bank liabilities.

We are now prepared to establish the main result of this section, which indicates that this economy can display very strong indeterminacies and excessive fluctuations in perfect foresight equilibria. It is an almost immediate implication that there is also a wide variety of equilibria where the economy experiences stochastic shifts between regimes 0 and 1 in a Markovian manner, with the probability of regime transitions depending potentially on time, history, or the state of the system.

We define an (m,n) cycle to be a deterministic cycle of m periods spent in regime 0, followed by n periods spent in regime 1 (and so on). We can now demonstrate the following proposition.

Proposition 2: For all positive integers m and n, there exists an asymptotically stable (m,n) cycle.

Proposition 2 specifies the sense in which this economy displays a high dimensional indeterminacy, and in which there is a wide variety of perfect foresight equilibria that display undamped oscillations. Moreover, these equilibria can be attained starting from a variety of initial capital stocks.

In order to prove Proposition 2 we begin by looking at iterates of the maps f_0 and f_1 defined in equation (27). It is easy to compute the iterates f_0^m and f_1^n: these are given by:

$$f_0^m(x) = \theta^m x + (1-\theta^m)x^0 \tag{34a}$$

$$f_1^n(x) = \theta^n x + (1-\theta^n)x^1 \tag{34b}$$

Compound iterates of the form $f_0^m[f_1^n(x)]$ and $(f_1^n[f_0^m(x)])$ describe orbits that spend n periods in regime 1 (m periods in regime 0) followed by m periods in regime 0 (n periods in regime 1). For given (m,n), these iterates are

$$f_0^m[f_1^n(x)] = \theta^{m+n}x + (1-\theta^m)x^0 + \theta^m(1-\theta^n)x^1 \tag{34c}$$

$$f_1^n[f_0^m(x)] = \theta^{m+n}x + \theta^n(1-\theta^m)x^O + (1-\theta^n)x^1 \qquad (34d)$$

Figure 4b graphs these maps. Since $x^O > x^1$, we have

$$f_0^m(f_1^n) > f_1^n(f_0^m) \quad \forall x \qquad (35)$$

Finally, we may compute double compound iterates that change regime twice, starting with m-q periods in regime 0, continuing with n periods in regime 1 and ending with q periods in regime 0 again. Alternatively, there may be (m,n) orbits that start with n-p periods in regime 1, continue with m periods in regime 0, and finish with p periods in regime 1 again.

The following lemma will prove useful. Its proof appears in Appendix C.

Lemma 1: The compound iterates of the maps f_O and f_1 satisfy

$$f_1^n(f_0^m) < f_0^q[f_1^n(f_0^{m-q})] < f_0^m(f_1^n) \qquad (36a)$$

$$\forall(m,n,q,x), \quad q < m$$

$$f_1^n(f_0^m) < f_1^p[f_0^m(f_1^{n-p})] < f_0^m(f_1^n) \qquad (36b)$$

$$\forall(m,n,p,x), \quad p < n$$

It should immediately be apparent that, for given $m \geq 1$ and $n \geq 1$, the sequence $(\bar{x}_1, \bar{x}_2, ..., \bar{x}_m, \bar{x}_{m+1}, ..., \bar{x}_{m+n})$ constitutes an (m,n) cycle if

(i) $\quad \bar{x}_i \in [x^1, x^O] \; \forall i = 1, ..., m+n$, and

(ii) $\quad \bar{x}_i = f_0^{i-1} \circ f_1^n \circ f_0^{m+1-i}(\bar{x}_i)$,

for $i = 1, ..., m,$[14] and $\bar{x}_i = f_1^{i-(m+1)} \circ f_0^m \circ f_1^{n+m+1-i}(\bar{x}_i)$ for $i = m+1, ..., m+n$.

Thus the values $\bar{x}_i$ are fixed points of the appropriate compound maps, and inequalities (37a) and (37b) reveal that $\bar{x}_1$ is the smallest of these fixed points while $\bar{x}_{m+1}$ is the largest. From (34c,d) it is easy to establish that $(\bar{x}_1, \bar{x}_{m+1})$ are both convex combinations of the steady states x^O and x^1. In particular:

$$\bar{x}_1 = [1-\Gamma(m,n)]x^0 + \Gamma(m,n)x^1 \tag{37a}$$

$$\bar{x}_{m+1} = [1-\theta^m\Gamma(m,n)]x^0 + \theta^m \Gamma(m,n)x^1, \tag{37b}$$

where

$$\Gamma(m,n) \equiv (1-\theta^n)/(1-\theta^{m+n}) \tag{37c}$$

These equations imply that, for any (m,n), both of the extreme periodic points $\bar{x}_1$ and $\bar{x}_{m+1}$ lie within the invariant interval $[x^0, x^1]$; hence the remaining fixed points $(\bar{x}_2,...,\bar{x}_{m-1},\bar{x}_m,...,\bar{x}_{m+n})$ will also lie in that interval.

It only remains to show that the (m,n) cycle is asymptotically stable. But this follows immediately from $\theta < 1$.

Proposition 2 demonstrates how private information in the credit market spawns indeterminate equilibria in an overlapping generations economy which otherwise lacks all the features we normally hold responsible for indeterminacy:[15] there is one sector, no paper assets, no large income effects, no nonconvexities in production or preferences, no monopolistic competition or any other significant departures from the key assumptions of Arrow and Debreu -- except private information.

The difference equation (27) has two overlapping branches but history specifies only one initial condition. The resulting indeterminacy is particularly strong and does not vanish asymptotically unless the economy develops rules to choose unequivocally between regimes. Indeed, a countable infinity of periodic attractors exists, each of which corresponds to an equilibrium displaying cycles of different periodicity and amplitude. Amplitude and periodicity move together: a measure of the amplitude of a deterministic (m,n) cycle is the distance, $d = \|\bar{x}_1 - \bar{x}_{m+1}\|$, between the largest and smallest periodic points. From (37a) and (37b) we obtain the following expression for the amplitude of an (m,n) cycle:

$$d(m,n) = (x^0-x^1)(1-\theta^m)(1-\theta^n)/(1-\theta^{m+n}) \tag{38}$$

For any fixed n, this amplitude is an increasing function of m, and d satisfies the

following relations:

$$d(0,n) = 0 \tag{39a}$$

$$d(n,n) = (x^0 - x^1)(1-\theta^n)/(1+\theta^n) \tag{39b}$$

$$d(\infty,n) = x^0 - x^1 \tag{39c}$$

By the same token, d is an increasing function of n for any fixed m with properties exactly analogous to those contained in equations (39a)-(39c). As periodicity goes up, the amplitude of the resulting cycles approaches the distance, $x^0 - x^1$, between the steady states of the two regimes.

It is also straightforward to construct equilibria in which transitions between the two regimes occur stochastically. In particular, let $\Phi_t \in \{0,1\}$ index the regime, and let

$$\pi_{00}^t = \text{prob}(\Phi_{t+1}=0 \mid \Phi_t = 0) \tag{40a}$$

$$\pi_{11}^t = \text{prob}(\Phi_{t+1}=1 \mid \Phi_t = 1) \tag{40b}$$

Then the sequence of matrices

$$\Pi_t = \begin{pmatrix} \pi_{00}^t & 1-\pi_{00}^t \\ \\ 1-\pi_{11}^t & \pi_{11}^t \end{pmatrix} \tag{41}$$

induces a (possibly non-stationary) Markov process on the regime index Φ_t. Moreover, competitive equilibria exist for arbitrary sequences (π_{00}^t, π_{11}^t), and the elements of these sequences can depend in essentially any fashion on the history of the economy.

To summarize, in an economy where the steady state equilibria of each regime lie within the economy's reflective barriers, the expectations of depositors become paramount in determining the type of equilibrium that will be observed. In particular, depositors must form expectations about yields on bank liabilities. Optimistic expectations of high yields lead to Walrasian regimes without credit rationing, and the absence of rationing validates

depositors' expectations. Pessimistic expectations of low yields lead to regimes of disinter-mediation, which forces banks to ration credit. This credit rationing again validates depositors' beliefs. Periodic cycles correspond to alternating, self-confirming waves of optimism and pessimism.

In Section 5, we consider economies where the steady state equilibria of the two regimes lie outside the economy's reflective barriers. When this occurs, the existence of these barriers limits the scope of depositors' expectations by ruling out persistently optimis-tic or persistently pessimistic rational expectations equilibria. The result is that all competitive equilibria must display endogenous volatility, and that this volatility cannot vanish asymptotically.

5. BINDING REFLECTIVE BARRIERS

We next take up the issue of equilibrium when neither steady state lies within the economy's reflective barriers. Therefore, in this section we assume that

$$x^1 < x^1_c < x^0_c < x^0 \tag{42}$$

Evidently, then, neither steady state constitutes a legitimate competitive equilibrium, nor do any monotonic sequences (x_t) that converge to one of the steady states. We now describe what kinds of equilibria can be observed here.

In order to do so, we define two values, $\bar{x}_F$ and $\bar{x}_c$, by the relations

$$\bar{x}_F \equiv \theta x^1_c + (1-\theta)x^1 = f_1(x^1_c)$$

$$\bar{x}_c \equiv \theta x^0_c + (1-\theta)x^0 = f_0(x^0_c).$$

It follows from these definitions that $\bar{x}_F$ is simply the point that succeeds x^1_c in regime 1, while $\bar{x}_c$ is the point that succeeds x^0_c in regime 0. These two points represent a floor or ceiling, respectively, on the value of x_t that can be attained in each regime. From equations (28) and (29) it is possible to obtain the following closed form expressions for $\bar{x}_F$ and $\bar{x}_c$:

$$\bar{x}_F = \frac{1}{1-\theta} \log\left[\frac{A(1-\gamma)}{a-(1-\delta)(1-\gamma)}\right] \qquad (43a)$$

$$\bar{x}_C = \frac{1}{1-\theta} \log\left[\frac{A(1-\gamma)}{a\gamma-(1-\delta)(1-\gamma)}\right] \qquad (43b)$$

An examination of Figure 3b should now convince the reader of the following proposition.

Proposition 3: (i) Every equilibrium sequence of this economy has an upper and a lower turning point, and (ii) both turning points lie inside the invariant set $[\bar{x}_F,\bar{x}_C]$, which attracts all equilibrium sequences in finite time.

In view of the proposition, we now focus our attention on the potential existence of equilibria displaying (m,n) cycles. Such equilibria are defined exactly as in Section 4, except that the invariant interval is now $[\bar{x}_F,\bar{x}_C]$ rather than $[x^1,x^0]$.

As in Section 4, an (m,n) cycle exists if, and only if, the fixed points of the compound maps $f_0^m(f_1^n)$ and $f_1^n(f_0^m)$ both fall within the invariant set or, in other words, if and only if

$$\bar{x}_F \le \hat{x}_1 < \hat{x}_{m+1} \le \bar{x}_C. \qquad (44)$$

Using equations (37a) and (38), it is straightforward to show that (44) holds iff the following two conditions are satisfied:

$$\theta^n \ge A_1/[1-(1-A_1)\theta^m] \qquad (45a)$$

$$\theta^m \ge (1-A_0)/(1-A_0\theta^n) \qquad (45b)$$

where the parameters A_0 and A_1 are defined by the relations

$$A_0 = (\bar{x}_C-x^1)/(x^0-x^1) \qquad (46a)$$

$$A_1 = (\bar{x}_F\text{-}x^1)/(x^0\text{-}x^1). \tag{46b}$$

As is evident from these definitions,[16]

$$0 < A_1 < A_0 < 1. \tag{46c}$$

Periodic cycles with m periods of expansion and n periods of contraction will exist iff, given the parameters (θ, A_0, A_1), we can find positive integers (m,n) that satisfy inequalities (45a) and (45b). In order to investigate the existence of integers satisfying these conditions, we depict them in Figure 5. Here two cases must be distinguished, corresponding to panels (a) and (b): $A_0 + A_1 < 1$ holds, or $A_0 + A_1 > 1$ holds.

In either case, both inequalities (45a) and (45b) hold iff the point (θ^m, θ^n) lies within the shaded region; the frontiers that define this region are upward sloping and inter-sect the 45° line at $\theta^m = A_1/(1\text{-}A_1)$ and $\theta^m = 1$ in panel (a), and at $\theta^m = (1\text{-}A_0)/A_0$ and $\theta^m = 1$ in panel (b).

When (θ^m, θ^n) lies within the appropriate shaded region, an (m,n) cycle exists. Evidently symmetric cycles lead to values (θ^m, θ^m) lying along the 45° line, while points above (below) the 45° line correspond to asymmetric cycles with longer (shorter) periods of expansion than contraction. Longer periodicities are associated with points that lie closer to the origin. Finally, in each panel of Figure 5, point H denotes the smallest intersection of the 45° line with the relevant region. In panel (a) [panel (b)] this intersection occurs at $\theta^m = A_1/(1\text{-}A_1)$ [$\theta^m = (1\text{-}A_0)/A_0$]. It will be useful to define

$$z \equiv \max\{A_1/(1\text{-}A_1),(1\text{-}A_0)/A_0\}. \tag{47}$$

The following proposition should now be apparent.

Proposition 4: A symmetric periodic equilibrium [that is, an (m,m) cycle] exists if, and only if, there is an integer $m \geq 1$ that satisfies the inequality

$$\theta^m \geq z. \tag{48}$$

When (48) holds, there exists a vector (θ^m, θ^m) lying along the 45° line in Figure 5, and lying to the northeast of point H. Hence an (m,m) cycle exists. Moreover, as the steady states (x^1, x^0) draw closer to the corresponding reflective barriers $(\bar{x}_F, \bar{x}_c)$, $A_0(A_1)$ will become closer to one (zero). In the limit, $z \to 0$, and symmetric cycles will necessarily exist. It is straightforward to show that z lies near zero when the yield on the storage technology is neither very close to nor very far from the productivity of the neoclassical technology.

There is also an immediate corollary of Proposition 4: if a symmetric cycle exists, then so does every other symmetric cycle of shorter periodicity. Thus the existence of an (m,m) cycle implies the existence of an (m-k,m-k) cycle for all integers $k = 1, ..., m-1$.

Apparently the values A_0 and A_1 govern the periodicity that can be observed for symmetric cycles. In order to investigate the relationships between these values and the underlying structural parameters, we use equations (28), (44) and (46) to derive

$$A_0 = \frac{\log\{\theta/(1-\theta)[a\gamma-(1-\delta)(1-\gamma)]\}}{\log[\gamma/(1-\gamma)]} \qquad (49a)$$

$$A_1 = \frac{\log\{\theta/(1-\theta)[a-(1-\delta)(1-\gamma)]\}}{\log[\gamma/(1-\gamma)]} \qquad (49b)$$

Proposition 4 states that no symmetric deterministic equilibrium cycles exist if inequality (48) fails for all integers $m \geq 1$. Appendix C shows that the same event rules out _all_ (m,n) cycles, both symmetric and asymmetric. We therefore have:

Proposition 5: A deterministic (m,n) cycle exists if, and only if $\theta \geq z$ holds.

Proposition 4 establishes conditions under which symmetric cycles exist, and Proposition 5 establishes that asymmetric cycles exist only if symmetric cycles do. We now wish to discuss what kinds of asymmetric (m,n) cycles exist when $\theta \geq z$ holds. Panels (c) and (d) of Figure 5 depict what is necessary for the existence of a symmetric cycle to imply the existence of an asymmetric cycle. In each panel (θ^s, θ^s) lies in the appropriate shaded

region while $(\theta^{s+1}, \theta^{s+1})$ does not. In panel (c) (θ^s, θ^{s+1}) lies in the shaded region as well, as does (θ^{s+1}, θ^s) in panel (d). Notice that, in each case, the existence of the asymmetric cycle implies the existence of a symmetric cycle. This result is, in fact, general, and Appendix C proves the following proposition

Proposition 6: (i) Suppose that $A_0 + A_1 \leq 1$. Then the existence of an (m,n) cycle with $m > n$ implies the existence of an (n,n) cycle. (ii) Suppose $A_0 + A_1 \geq 1$. Then the existence of an (m,n) cycle with $n > m$ implies the existence of an (m,m) cycle.

We now turn our attention to the existence of what we term <u>maximally persistent</u> periodic cycles; these are cycles that display a maximal periodicity $m+n$. Such cycles are of interest for two reasons. First, maximally persistent cycles correspond to least volatile rational depositor expectations. Second, if (m,n) is a maximally persistent cycle, and if $m > n$ $(m < n)$, all cycles $(m-k,n)$ $[(m,n-k)]$ exist for $k = 1,...,m-n$ $(k = 1,...,n-m)$; these cycles correspond to more volatile depositor expectations.

Figures 5c and 5d contain a geometric code that identifies maximally persistent (m,n) cycles. First we find the symmetric cycle with the longest periodicity, that is, the largest integer that satisfies inequality (48). This (s,s) cycle corresponds to point S on the diagonal of panels 5a and 5b; the next largest cycle $(s+1,s+1)$ is indexed by point $\hat{S}$ which lies outside the shaded area.

Maximally persistent periodic cycles have longer contractions than expansions when $A_0 + A_1 < 1$ (because x^1 is closer to the floor than x^0 is to the ceiling); shorter contractions than expansions when $A_0 + A_1 > 1$ (for precisely the opposite reason). Given the longest symmetric cycle (s,s), s is the length of the shorter of the two equilibrium regimes in the maximally persistent cycle. In other words, for $A_0 + A_1 < 1$, the maximally persistent periodic equilibrium is (s,n) and the length $n \geq s$ of the contraction is the largest integer solution to (45a) when we set $m = s$. By the same token, if $A_0 + A_1 > 1$, s is the length of the contraction regime; the duration $m \geq s$ of the expansion is the largest integer solution to (45b) when $n = s$.

Figures 5c and 5d describe this procedure: in 5c we seek the smallest θ^n that lies directly south of the symmetric cycle S and within the shaded region; in Figure 5d we seek the smallest allowable θ^m in a direct westerly direction from S. The outcome is point $\tilde{S}$ in each case.

We give below a compact, formal description of the algorithm used to identify maximally persistent cycles:

Proposition 7: (i) Suppose that $A_0 + A_1 \geq 1$ and that $\hat{s} \geq 1$ is the largest integer solution to $\theta^s \geq A_1/(1-A_1)$. Then there exists a unique periodic equilibrium $(\hat{m},\hat{s})$ of maximal periodicity $\hat{m} + \hat{s}$ in which $\hat{m} \geq \hat{s}$ is the largest integer solution to $\theta^m \geq (1-A_0)/(1-A_0\theta^{\hat{s}})$.

(ii) Suppose that $A_0 + A_1 \leq 1$ and that $\hat{s} \geq 1$ is the largest integer solution to $\theta^s \geq (1-A_0)/A_0$. Then there exists a unique periodic equilibrium $(\hat{s},\hat{n})$ of maximal periodicity $\hat{s} + \hat{n}$ in which $\hat{n} \geq \hat{s}$ is the largest integer solution to $\theta^n \geq A_1/[1-(1-A_1)\theta^{\hat{s}}]$.

In addition to (m,n) cycles, this economy may have other deterministic periodic equilibria as well. For example, there may be equilibria with m periods of expansion followed by n periods of contraction, p periods of expansion again, and then q periods of contraction (and so on), with $m \neq p$ and $n \neq q$. Indeed, such equilibria may exist even if $\theta < z$ holds. In addition, stochastic equilibria of the type described in Section 4 will typically exist. What does not exist here are equilibrium sequences that converge to steady states.

To get a better feel for what sorts of deterministic cycles are possible for the economies described in this section, we calculate some numerical examples in Section 6.

6. NUMERICAL EXAMPLES

In this section we apply Proposition 7 to identify maximally persistent periodic equilibria for a number of economies indexed by the parameters (θ, A_0, A_1). We choose

$\theta = 1/3$ throughout to represent capital's share of output, and allow (A_0, A_1) to vary on a coarse grid in Table 1, and on a finer grid in Table 2.[17] Each box in either table has an entry of either NC or of two integers (m,n). NC means that no deterministic periodic equilibrium exists for the corresponding parameter values $(1/3, A_0, A_1)$; other equilibria -- aperiodic or stochastic -- may or may not exist. Finally, a two integer entry (m,n) means that the maximally persistent cycle has m periods of expansion and n periods of contraction.

As we expect from Section 5, periodic cycles exist in economies with reflective barriers when A_0 is near 1 and A_1 is near zero. Maximally persistent cycles tend to be symmetric if $A_0 + A_1$ is near one; expansions tend to be longer than contractions if $A_0 + A_1 >> 1$, while contractions tend to be longer than expansions if $A_0 + A_1 << 1$.

A. The Roles of a and γ

The parameters θ and δ of the model have obvious empirical real world counterparts. This connection is less transparent for a and γ. Therefore, this section explores in more detail how some aspects of equilibrium depend on the latter two parameters. To do so we set $\theta = 1/3$, as before, and set $\delta = 0.1$. In addition, given these parameter values, we choose a and $\gamma > 0.5$ so that $y_1/y_0 = 0.8$, where y_i is steady state per capita output in regime i, inclusive of output from storage. This puts an upper bound on the amplitude of output fluctuations over any cycle: per capita output at a "trough" cannot be less than 80 percent of per capita output at a "peak" in any (m,n) cycle. This bound is consistent with the amplitude of the largest recession observed in the U.S. in this century.

Since $s_t = 0$ in regime 0, we have that

$$y^0 = f(k^0) = (A/\theta)(k^0)^\theta \tag{50a}$$

$$y^1 = f(k^1) + as^1 = f(k^1) + ak^1(2\gamma-1)/(1-\gamma)$$

$$= (A/\theta)(k^1)^\Theta + ak^1(2\gamma-1)/(1-\gamma) \qquad (50b)$$

Equations (28a,b) describe the logarithms of k^O and k^1. We set $\theta = 1/3$ in these equations as well as in (50) to obtain $\mu = y^1/y^O$ as a function of a and γ:

$$\mu(a,\gamma) \equiv [1 + (2a/3)(2\gamma-1)] \, [(1-\gamma)/\gamma]^{1/2} = .80 \qquad (51)$$

In addition, we insist that a and γ be chosen to satisfy the following maintained assumptions of our analysis:

MA1: A deterministic equilibrium exists for all initial values of the capital stock. This is equivalent to the inequality (30) being satisfied

MA2: Storage is dominated in rate of return by claims on capital. This is equivalent to the inequality (31) being satisfied

MA3: The steady states (x^O, x^1) of the two regimes are not competitive equilibria. This is equivalent to the inequality (42) being satisfied.

Table 3 reports results for economies satisfying these assumptions over a fairly coarse grid for a and γ. The first two columns of the table report values of a and γ which satisfy (51). The next three columns indicate whether maintained assumptions MA1, MA2 and MA3 are satisfied (by an entry of $+$) or not (by an entry of $-$) for the given values of (a,γ). The next two columns denote the presence $(+)$ or absence $(-)$ of a binding floor or ceiling; the following two columns compute the values of A_O and A_1 implied by the (a,γ) combination chosen. Finally, the last two columns show the number of contraction and expansion periods in the maximally persistent periodic cycle.

It turns out that binding ceilings cease to exist for $\gamma \leq .6868$, as do floors for $\gamma \geq .6920$. We recall that Proposition 2 asserts that equilibrium expansions and contractions of arbitrary length can be observed for economies with non-binding reflective barriers. Finally, deterministic (m,n) cycles do not exist when $\gamma > 0.703$. It remains to be seen if a finer parameter grid or slight perturbations of the values $(\theta,\delta,y^1/y^O)$ can produce longer-lasting expansions or contractions.

The outcomes seem to conform to expansions and contractions that last from 1 to 4 periods. It remains to be seen if a finer parameter grid and/or slight changes in the assumed values of the parameters $(\delta, \theta, y^1/y^0)$ can produce longer-lasting expansions or contractions. Also, Proposition 1 states that equilibrium expansions and contractions of arbitrary length may be observed in economics without binding reflective barriers.

7. CONCLUSIONS

A long tradition in monetary economics holds that the unrestricted operation of financial markets can lead to indeterminacies and excessive fluctuations relative to fundamentals. Previous models that deliver this result, however, have typically depended on the existence of nominal assets, large income effects, non-convexities, monopolistic competition, or multiple sectors.[18] We have examined a one-sector neoclassical growth model with none of these features, but in which capital investments must be credit financed, and in which credit markets are characterized by the presence of an adverse selection problem. In this model we have shown that the existence of two equilibrium regimes is possible: a Walrasian regime in which incentive constraints are non-binding, and a regime of credit rationing in which this rationing is necessary to induce self-selection in loan markets. For some range of current capital stocks either regime is consistent with the existence of an equilibrium; therefore, it is possible to observe equilibria in which deterministic or stochastic transitions occur between regimes. Many of these equilibria will display oscillations that do not die out. Moreover, for some configurations of parameters (those considered in Section 5), the only equilibria that can be observed display fluctuations that do not vanish asymptotically. In such economies excessive fluctuations are not only possible, but indeed are a necessary feature of any equilibrium.

These results have clearly been obtained under a variety of fairly strong assumptions. Some of them are not conceptually hard to generalize. For example, the utility function $u(c_1, c_2) = c_2$ is easily replaced with the more general Cobb-Douglas utility function $c_1^{1-\beta} c_2^{\beta}$ for any $\beta \in (0,1]$ with the only modifications being that the savings rate will be

$\beta \leq 1$, and the incentive constraint would also require some modification. Generalizing the utility function beyond this further would cause us to lose the linearity of equation (27), and would lead to a more complicated incentive constraint of the form

$$v(k_{t+1}/(1-\gamma),a) \leq v(w_t,R_{t+1})$$

where $v(y,R)$ is the indirect utility of an agent with income profile $(y,0)$, who faces the intertemporal terms of trade R.

Another possible extension is to allow random disturbances to affect the fundamentals of this economy, for example, the neoclassical technology $F(K,L)$. The outcome we would hope for is that the deterministic piecewise linear difference equation (27) would generalize to a similar stochastic difference equation. If this more general dynamical system is tractable, it may yield quantitative insights about the process by which financial markets amplify exogenous disturbances, both close to and far from cyclical turning points. This is a key concern in the nonlinear time-series research of Hamilton, op. cit., Diebold and Rudebusch, op. cit., Durlauf and Johnson (1992), Pesaran and Potter, (1994), and many others.

An easier task is to introduce money into the economy of this paper. Indeed, a monetary version of our economy is considered by Azariadis and Smith (1994), who demonstrate how inflation can tighten incentive constraints and slow down capital accumulation in economies with nominal assets. As a result, secular improvements in the functioning of credit markets may come about from a reduction in distorting inflationary taxation.

Another issue that we intend to explore in future work is the occurrence of financial deepening. Here it is possible that advances in financial intermediation may also spring from technological progress that favors investments that require intermediation over investments that do not. Imagine, for example, that total factor productivity advances faster in the publicly observed neoclassical technology of our economy than in the privately observed storage technology. Over a long time span, returns to storage will fall relative to

bank deposits, reducing the incentives which exist to misrepresent one's type, and reducing in a completely endogenous manner the severity of the adverse selection problem banks face. If this occurs, it will raise the ceiling associated with regime 0, and hence will permit the existence of progressively longer periods of expansion. As a result, economic growth could -- in part -- be traced to a less frequent occurrence of periods of credit rationing and disintermediation.

ENDNOTES

[1]See, for example, Schreft (1990) or Schreft and Owens (1993).

[2]Azariadis and Smith (1994) study the role of fiat money or national debt in a neoclassical growth model with adverse selection.

[3]See, for instance, Hamilton (1989), Boldin (1990), Potter (1994), Beaudry and Koop (1993), and Diebold and Rudebusch (1994).

[4]We do not take the two-period lifecycle assumption literally, but, rather, as a shortcut to multi-period lifecycle growth models of the sort employed by Auerbach and Kotlikoff (1987) and others. Given enough gross substitutability, we know from Kehoe and Levine (1985) that overlapping generations economies with perfect markets have determinate equilibria for an arbitrary deterministic lifecycle. We conjecture that, if markets are complete, the equilibrium dynamics of any OLG growth model along the stable manifold may well be first order even if the lifecycle has arbitrary deterministic length $T \geq 2$.

[5]The assumption that agents are risk neutral precludes lotteries from playing a useful role under private information. The assumption that agents care only about old age consumption allows us to abstract from consumption/savings decisions. Both are inessential simplifications.

[6]This, of course, assumes that $f'(k^0) + 1-\delta > a$.

[7]The hallmark of any model of credit rationing based on adverse selection or moral hazard [for example Bencivenga and Smith (1993) or Stiglitz and Weiss (1981)] is that some borrowers have a higher probability of repayment than others, and hence care more about the interest rate dimension of the loan contract. Our specification is simply the most extreme (and simplest) version of this possibility: type 1 agents default on loans with probability one, and type 2 agents repay with the same probability. There is no conceptual difficulty associated with allowing type 1 and 2 agents to have loan repayment probabilities

strictly between zero and one. However, this merely adds complexity without bringing any additional substantive issues into the analysis.

[8]If $\mu_t = 1$, no young agents work at t, no savings are supplied, and $k_{t+1} = 0$. The economy jumps to the autarkic steady state.

[9]See Azariadis and Smith (1994) for a proof in this particular context. Parenthetically, the argument they give depends on banks treating the deposit rate R_t parametrically. See Kreps (1990), Ch. 17, for the strategic foundations of this formulation.

[10]Again, see Azariadis and Smith (1994) for a formal proof. Since this condition is important in what follows, Appendix B sketches the main elements of the proof.

[11]If (15) is reversed, then all values of k satisfy (9').

[12]A stochastic equilibrium may exist for k_{t+1} even if the deterministic map is undefined in some region. This solution may require mixed loan strategies from banks, e.g., stochastic credit rationing.

[13]A more realistic interpretation of this process, and one that would require a richer menu of borrowers than the one we are using, is to focus on the quality dispersion of loan applicants through the business cycle. In a business expansion, poorly managed firms expand along with better firms, and poor investment projects become marginally profitable. The resulting high dispersion in the quality of loan applications raises the riskiness of intermediary loan portfolios. Banks respond by rationing credit because they lack accurate information about the risk characteristics of individual projects.

[14]Here f^o denotes the identity map.

[15]See Azariadis (1993), Ch. 26, Farmer (1993), and Guesnerie and Woodford (1993) for recent surveys of indeterminate equilibria.

[16]The parameters A_o and A_1 measure, respectively, the relative distance of the ceiling and floor from the nearest steady state and, hence, reveal how tight reflective

barriers are. As $A_O \to 1$ and $A_1 \to 0$, floors and ceilings become less tight, permitting equilibria to approach the steady states x^O and x^1 more closely before reversing direction.

[17]Corresponding to each pair of values (A_O, A_1) is a set of underlying parameters a, $\delta \in [0,1]$, and $\gamma > 0.5$ that produce these values via the formulas in equation (49).

[18]Large income effects are considered by Azariadis (1981) and Azariadis and Guesnerie (1986). Models with nominal assets and production include Bernanke and Gertler (1989), Azariadis and Smith (1994), Boyd and Smith (1994), or Schreft and Smith (1994). Non-convexities and monopolistic competition are examined by Gali (1993) and Benhabib and Farmer (1994). Kiyotaki and Moore (1993) study a multi-sector economy.

BIBLIOGRAPHY

Auerbach, A., and L. Kotlikoff (1987), <u>Dynamic Fiscal Policy</u>, Cambridge: Cambridge University Press.

Azariadis, C. (1993), <u>Intertemporal Macroeconomics</u>, Oxford: Blackwell Publishers.

——— (1981), "Self-Fulfilling Prophecies," <u>Journal of Economic Theory</u>, 25, 380-96.

——— and B. Smith (1994), "Private Information, Money and Growth: Indeterminacy, Fluctuations, and the Mundell-Tobin Effect," UCLA mimeo.

Beaudry, P., and G. Koop (1993), "Do Recessions Permanently Change Output?", <u>Journal of Monetary Economics</u>, 31, 149-63.

Bencivenga, V.R., and B.D. Smith (1993), "Some Consequences of Credit Rationing in an Endogenous Growth Model," <u>Journal of Economic Dynamics and Control</u>, 58, 97-112.

Benhabib, J., and R. Farmer (1994), "Indeterminacy and Increasing Returns," forthcoming in <u>Journal of Economic Theory</u>.

Bernanke, B., and M. Gertler (1989), "Agency Costs, Net Worth and Business Fluctuations," <u>American Economic Review</u>, 79, 14-31.

Boldin, M. (1990), "Characterizing Business Cycles with a Markov Switching Model," FRB of New York Working Paper.

Boyd, J.H., and B.D. Smith (1994), "Credit Market Imperfections in a Monetary Growth Model," Cornell University mimeo.

De Gregorio, J. (1992), "The Effects of Inflation on Economic Growth," <u>Journal of Development Economics</u>.

Diamond, P. (1965), "National Debt in a Neoclassical Growth Model," <u>American Economic Review</u>, 55, 1026-50.

Diebold, F., and G. Rudebusch (1994), "Measuring Business Cycles: A Modern Perspective," University of Pennsylvania mimeo.

Durlauf, S., and P. Johnson (1992), "Local vs. Global Convergence Across National Economies," NBER Working Paper 3996.

Farmer, R. (1993), The Economics of Self-Fulfilling Prophesies, Cambridge, MA: MIT Press.

Friedman, M. (1960) A Program for Monetary Stability, New York: Fordham University Press.

Friedman, M., and A.J. Schwartz (1963), A Monetary History of the United States, 1867-1960, Princeton: Princeton University Press.

Gali, J. (1993), "Multiple Equilibria in a Growth Model With Monopolistic Competition," Columbia University Press.

Guesnerie, R., and M. Woodford (1993), "Endogenous Fluctuations," DELTA mimeo.

Hamilton, J.D. (1989), "A New Approach to the Analysis of Nonstationary Time Series and the Business Cycle," Econometrica, 57, 357-84.

_____ (1990), "Analysis of Time Series Subject to Changes in Regime," Journal of Econometrics, 45, 39-70.

Kehoe, T., and D. Levine (1985), "Comparative Statics and Perfect Foresight in Infinite Horizon Economies," Econometrica, 53, 433-53.

Keynes, J.M. (1936), The General Theory of Employment, Interest and Prices, London: Macmillan.

Kiyotaki, N., and J. Moore (1993), "Credit Cycles," University of Minnesota mimeo.

Kreps, D. (1990), A Course in Microeconomic Theory, Princeton: Princeton University Press.

Neftçi, S. (1984), "Are Economic Time Series Asymmetric Over the Business Cycle?" Journal of Political Economy, 92, 301-28.

Pesaran, H., and S. Potter (1994), "A Floor and Ceiling Model of U.S. Output," University of Cambridge DAE Working Paper #9407.

Potter, S. (1994), "A Nonlinear Approach to U.S. GNP," UCLA mimeo.

Rothschild, M., and J. Stiglitz (1976), "Equilibrium in Competitive Insurance Markets," *Quarterly Journal of Economics*, 80, 629-49.

Schreft, S.L. (1990), "Credit Controls: 1980," *Federal Reserve Bank of Richmond Economic Review*, 77, 29-34.

Schreft, S.L., and R. Owens (1992), "Identifying Credit Crunches," Federal Reserve Bank of Richmond mimeo.

_____ and B. Smith (1994), "Money, Banking and Capital Formation," Cornell University mimeo.

Simons, H. (1948), *Monetary Policy for a Free Society*, Chicago: University of Chicago Press.

Stiglitz, J., and A. Weiss (1981), "Credit Rationing in Models with Asymmetric Information," *American Economic Review*, 71, 393-410.

APPENDIX A

Proof of Proposition 1

In a regime of credit rationing, the full information allocation is not incentive compatible, or in other words

$$[f'(k_{t+1}) + 1-\delta]w(k_t) < ak_{t+1}/(1-\gamma) \tag{A1}$$

Moreover, $s_t = 0$ holds under full information, so that [from (7)] $k_{t+1} = \gamma w(k_t)$. Substituting this into (A1) gives

$$f'(k_{t+1}) + 1-\delta < a\gamma/(1-\gamma) \tag{A2}$$

Now suppose, for the purpose of a proof by contradiction, that credit is rationed at t and that $R_{t+1} > a$. Then $s_t = 0$ holds, as before, and so does $k_{t+1} = \gamma w(k_t)$. Therefore (9'), which must hold with equality, becomes

$$R_{t+1} = a\gamma/(1-\gamma) \tag{A3}$$

It is also the case that

$$f'(k_{t+1}) + 1-\delta \geq R_{t+1} \tag{A4}$$

must hold in any equilibrium. But (A3) and (A4) contradict (A2), delivering the desired result.

Q.E.D.

APPENDIX B

1. A Proof That Separating Equilibrium Loan Contracts Exist iff $f'(k_{t+1}) + 1-\delta \leq R_{t+1}/(1-\gamma)$.

Given K_t units of capital inherited from period t-1, old agents of type 2 will choose the size of their labor force in period t to maximize profit, that is, to equate the wage rate, w_t, to the marginal product of labor, $f(k_t) - k_t f'(k_t)$. Let R_t be the cost of capital to these producers under a separating loan contract, and use Euler's Law to

compute their net income, which is,

$$\pi_t = w_t + K_t[f'(k_t)-R_t] \tag{B1}$$

If type-2 agents were to pool with type-1 agents, the lowest possible cost of capital under a pooling contract would be $R_t/(1-\gamma)$. Accordingly, the net income of type-2 producers who borrow $\hat{K}_t$ from intermediaries under the best possible pooling contract would be

$$\hat{\pi}_t = w_t + \hat{K}_t[f'(k_t)-R_t/(1-\gamma)] \tag{B2}$$

The capital-labor ratio in (B1) and (B2) is identical because the market wage rate is the same in both situations.

If the incentive constraint binds, type 1 agents prefer a pooling contract to a separating contract whenever type 2 agents do. Type 2 agents prefer the best pooling contract to the candidate separating equilibrium iff there exists a value $\hat{K}_t$ such that

$$[f'(k_t) - R_t/(1-\gamma)]\hat{K}_t > [f'(k_t) - R_t]K_t \tag{B3}$$

Thus the existence of a separating contract requires that there exists no $\hat{K}_t$ for which (B3) holds, given K_t. Since $f'(k_t) \geq R_t$, the absence of such a value $\hat{K}_t$ clearly requires that $f'(k_t) \leq R_t/(1-\gamma)$ holds. Moreover, since $R_t = a$ whenever the incentive constraint binds, we have that $f'(k_t) \leq a/(1-\gamma)$, as claimed. Q.E.D.

2. Proof That (22b) Is Equivalent to (22a)

We start with the fact that the wage function $w(k)$ is increasing for any concave production function $f(k)$. Thus (22a) is equivalent to $w(k_c^1) < w(k_c^0)$ and, hence, to $\hat{k}/\gamma < \bar{k}/(1-\gamma)$ once we consult equations (16b) and (18b). Q.E.D.

APPENDIX C

1. Proof of Inequalities (36a) and (36b)

Combining equations (34a) and (34b), we obtain

$$f_0^q[f_1^n(f_0^{m-q}(x))] = f_0^q[f_1^n(\theta^{m-q}x + (1-\theta^{m-q})x^0)] = f_0^q[\theta^n\theta^{m-q}x + \theta^n(1-\theta^{m-q})x^0 + (1-\theta^n)x^1]$$

$$= f_0^q[\theta^{m+n-q}x + \theta^n(1-\theta^{m-q})x^0 + (1-\theta^n)x^1] = \theta^q[\theta^{m+n-q}x + \theta^n(1-\theta^{m-q})x^0$$

$$+ (1-\theta^n)x^1] + (1-\theta^q)x^0 = \theta^{m+n}x + [\theta^{n+q}(1-\theta^{m-q}) + (1-\theta^q)]x^0 + \theta^q(1-\theta^n)x^1$$

From this equation and equations (34c) and (34d), it follows that (36a) is equivalent to

$$\theta^n(1-\theta^m)x^0 + (1-\theta^n)x^1 < [\theta^{n+q}(1-\theta^{m-q}) + 1-\theta^q]x^0 + \theta^q(1-\theta^n)x^1 <$$

$$(1-\theta^m)x^0 + \theta^m(1-\theta^n)x^1 \tag{C1}$$

The first inequality in (C1) is equivalent to

$$(1-\theta^q)(1-\theta^n)x^1 < (\theta^{n+q} - \theta^n + 1 - \theta^q)x^0$$

that is, to

$$(1-\theta^q)(1-\theta^n)x^1 < (1-\theta^q)(1-\theta^n)x^0 \tag{C2}$$

This inequality holds for all (x,m,n) and $q < m$ because $x^1 < x^0$.

The second inequality in (C1) is established similarly, as is the set of inequalities in (36b).

2. Proof That No Asymmetric Deterministic Cycles Exist if Inequality (48) Fails

Failure of (48) implies no symmetric cycle (m,m) exists for any $m \geq 1$. If an asymmetric cycle (m,n) existed for $m \neq n$, then a symmetric cycle would also exist by Proposition 5, which is a contradiction.   Q.E.D.

3. Proof of Proposition 6

We shall prove part (a) only; part (b) is completely analogous. Let $A_0 + A_1 \leq 1$ and suppose an (m,n) cycle exists with $m > n$. This means there are values (m,n) for which inequalities (45a) and (45b) hold, i.e.,

$$\theta^n \geq A_1/[1-(1-A_1)\theta^m] \tag{C3}$$

$$\theta^m \geq (1-A_0)/(1-A_0\theta^n) \tag{C4}$$

We need to prove that a similar pair of inequalities holds for (θ^n, θ^n) as well, namely

$$\theta^n \geq A_1/[1-(1-A_1)\theta^n] \tag{C5}$$

$$\theta^n \geq (1-A_0)/(1-A_0\theta^n) \tag{C6}$$

Clearly, (C4) implies (C6) since $\theta^n > \theta^m$ for any $n < m$. By the same token, (C4) implies that $\theta^n > \theta^m \geq (1-A_0)/(1-A_0\theta^n) > A_1/[1-(1-A_1)\theta^n]$. The last inequality is equivalent to $(1-A_0 \cdot A_1)(1-\theta^n) > 0$ which holds if $1 > A_0 + A_1$. This proves (C5) and Proposition 6(a) as well.   Q.E.D.

TABLE 1: EXISTENCE OF CYCLES

A_0/A_1	.40	.60	.70	.80	.85	.88	.91	.94	.96	.98
.01	NC	NC	NC	1,3	1,3	1,3	2,4	2,4	2,4	3,4
.02	NC	NC	NC	1,3	1,3	1,3	2,3	2,3	2,3	3,3
.05	NC	NC	NC	1,2	1,2	1,2	2,2	2,2	2,2	3,2
.10	NC	NC	NC	1,1	1,1	1,1	2,2	2,2	2,2	3,2
.15	NC	NC	NC	1,1	1,1	1,1	1,1	2,1	2,1	3,1
.20	NC	NC	NC	1,1	1,1	1,1	1,1	2,1	2,1	3,1
.25	NC	NC	NC	1,1	1,1	1,1	1,1	2,1	2,1	3,1
.35	NC	NC	NC	NC	NC	NC	NC	NC	NC	NC
.50	NC	NC	NC	NC	NC	NC	NC	NC	NC	NC

TABLE 2: PERSISTENCE OF CYCLES

A_0/A_1	.880	.900	.920	.940	.960	.980	.985	.990	.995	.997	.998	.999
.001	1,5	2,6	2,6	2,6	2,6	3,6	3,6	4,6	4,6	5,6	5,6	6,6
.002	1,5	2,5	2,5	2,5	2,5	3,5	3,5	4,5	4,5	5,5	5,5	6,5
.003	1,4	2,5	2,5	2,5	2,5	3,5	3,5	4,5	4,5	5,5	5,5	6,5
.004	1,4	2,4	2,4	2,4	2,4	3,4	3,4	4,5	4,5	5,5	5,5	6,5
.006	1,4	2,4	2,4	2,4	2,4	3,4	3,4	4,4	4,4	5,4	5,4	6,4
.010	1,3	2,4	2,4	2,4	2,4	3,4	3,4	4,4	4,4	5,4	5,4	6,4
.015	1,3	2,3	2,3	2,3	2,3	3,3	3,3	4,3	4,3	5,3	5,3	6,3
.020	1,3	2,3	2,3	2,3	2,3	3,3	3,3	4,3	4,3	5,3	5,3	6,3
.030	1,2	2,3	2,3	2,3	2,3	3,3	3,3	4,3	4,3	5,3	5,3	6,3
.050	1,2	2,2	2,2	2,2	2,2	3,2	3,2	4,2	4,2	5,2	5,2	6,2
.080	1,1	2,2	2,2	2,2	2,2	3,2	3,2	4,2	4,2	5,2	5,2	6,2
.120	1,1	1,1	1,1	1,1	2,1	3,1	3,1	3,1	3,1	4,1	5,1	5,1

TABLE 3: EQUILIBRIUM CYCLES

γ	a	MA1	MA2	MA3	Floor	Ceiling	A_0	A_1	m	n
.6870	.7428	+	+	+	+	+	.9954	.1029	4	2
.6874	.7457	+	+	+	+	+	.9787	.0939	3	2
.6880	.7499	+	+	+	+	+	.9541	.0807	2	2
.6890	.7569	+	+	+	+	+	.9142	.0591	2	2
.6900	.7639	+	+	+	+	+	.8757	.0382	1	3
.6910	.7709	+	+	+	+	+	.8386	.0180	1	3
.6918	.7765	+	+	+	+	+	.8098	.0022	1	3

FIGURE 1
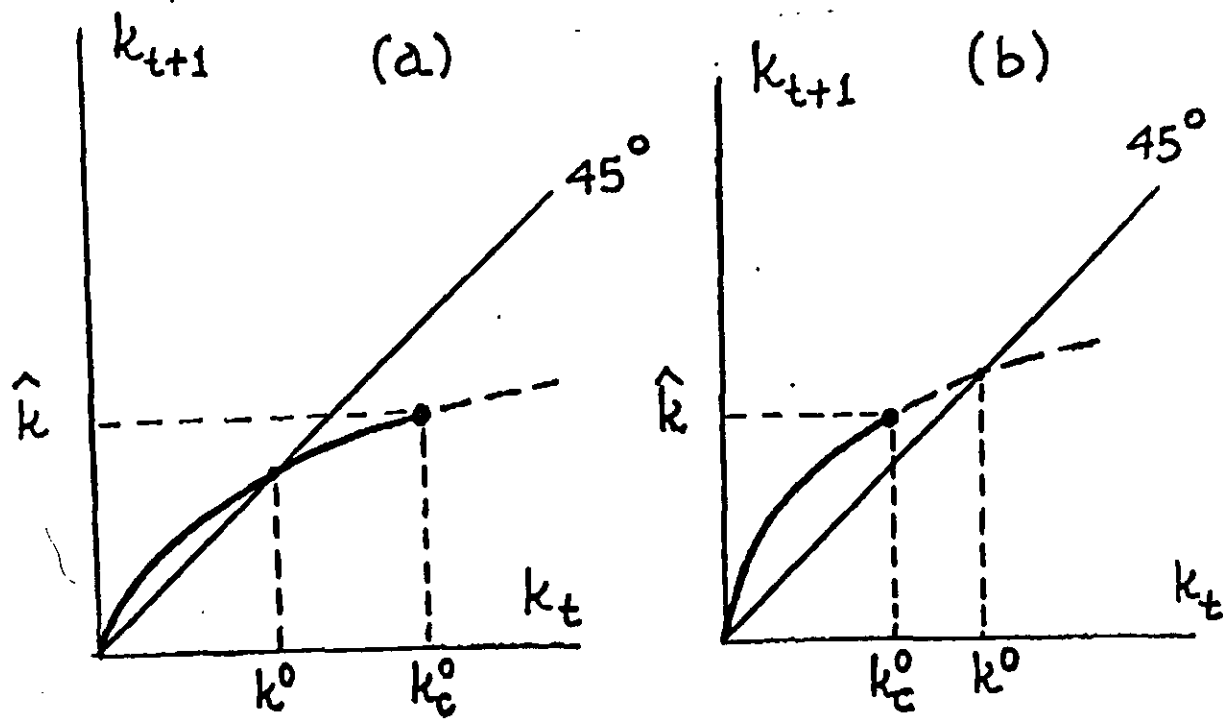
FIGURE 2
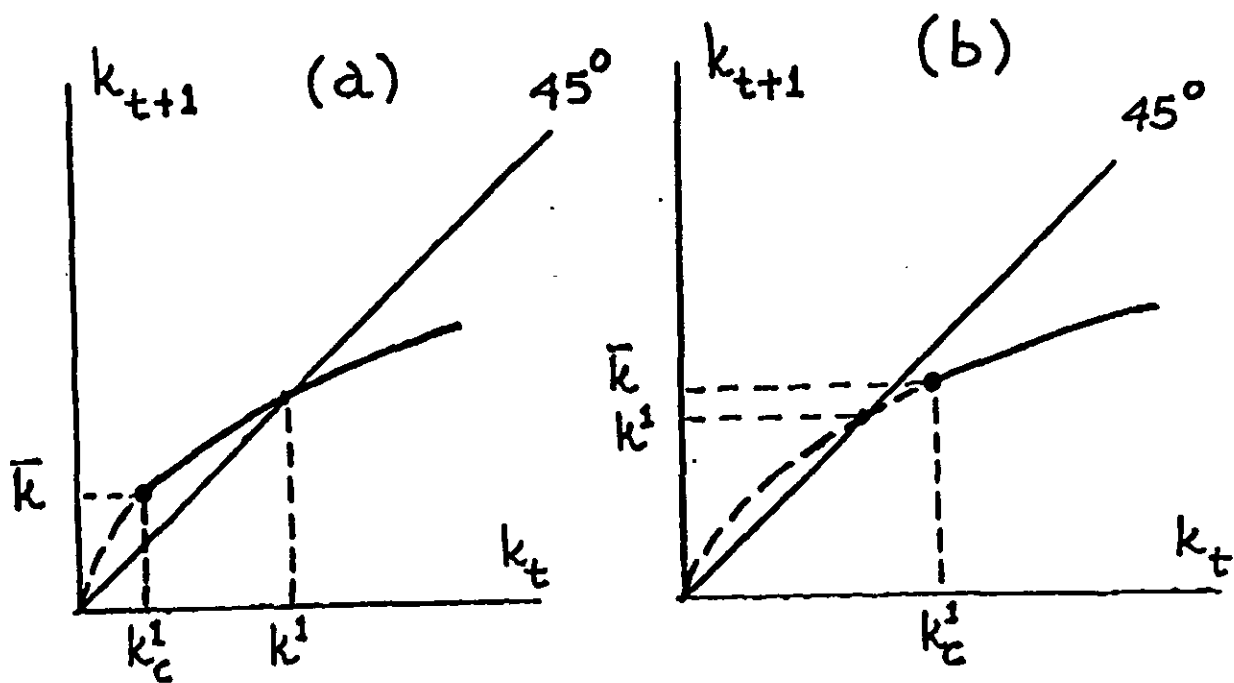
FIGURE 3

FIGURE 4

FIGURE 5