

# Monetary Steady States in a Low Real Interest Rate Economy

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## Abstract

We examine the conditions under which steady states with low real interest rates—real rates substantially below the output growth rate—exist in an overlapping generations model with production, capital accumulation, a labor-leisure trade-off, technological progress, and agents who live for many periods. The number of periods in an agent's life ( $n$ ) is left open for much of the analysis and determines the temporal interpretation of a time period. The qualitative properties of the model are largely invariant to different values of  $n$ . We find that two low real interest rate steady states exist for empirically plausible values of the parameters of the model. Outside liabilities such as fiat currency or unbacked government debt are valued in one of these steady states. *Journal of Economic Literature* Classification D51, E40.

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# 1 Introduction

In recent years macroeconomists have begun to experiment with the use of dynamic general equilibrium models to study questions of policy. This experimentation has been extended, in a limited way, to questions of monetary policy. One problem this effort has encountered is that the most popular class of dynamic general equilibrium models, the infinite horizon representative agent models, have difficulty explaining two stylized “monetary” facts which we regard as fundamental: (1) real rates of return on safe, short-term assets are relatively low—substantially lower, on average, than output growth rates—and (2) there exist large stocks of unbacked liabilities, such as fiat currency and unbacked government debt. In this paper we argue that the overlapping generations model, suitably specified, provides an empirically plausible framework that can explain these two facts, and consequently seems likely to provide a fruitful basis for further research in monetary policy. The specifications we analyze involve multiperiod agent lifetimes, production and capital accumulation, a labor-leisure trade-off, and exogenous technological progress. We make standard assumptions regarding the forms and parameters of preferences, endowment patterns, and technologies.

The specific contributions of our analysis are twofold. First, we show that steady state equilibria in which unbacked liabilities have value—*monetary steady states* in the monetary theory literature—may exist in principle regardless of the number of periods in an agent’s lifetime. This finding suggests that lack of robustness along the time dimension is not, as has sometimes been suggested, a serious conceptual impediment to the use of the overlapping generations model as a model of money.<sup>1</sup> We demonstrate, moreover, that the “folk theorem” that empirically plausible assumptions about production, capital accumulation, and technological progress rule out monetary steady states is not correct, and that the methods used to characterize and analyze monetary steady states in conventional two-period models can be used for the same purpose in our much-augmented multiperiod model.

Second, we show that parameterizations of the model that involve parameter values well within the range of published estimates are capable of generating values of the steady state real interest rate similar to empirical estimates of average real interest rates on safe, short-term assets in the postwar U.S. economy.<sup>2</sup> This result is robust both to variation

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<sup>1</sup>See in particular Tobin (1980) and Aiyagari (1988, 1989).

<sup>2</sup>İmrohoroğlu, İmrohoroğlu, and Joines (1992) obtain relatively low real interest rates in nonmonetary

in the number of periods in agents' lives and to a wide range of alternative parameter specifications. Abel, *et al.*, (1989, p. 15) characterize observed safe short-term real interest rates as "startlingly low"—a characterization that has led us to describe our model as one of a *low real interest rate economy*.

The plan of the paper is as follows. In the next section, we illustrate some of our findings involving the existence and robustness of monetary steady states in pure exchange economies. These results provide useful background for experiments involving a model with production and capital. In the third section we lay out the latter model, and in section four we derive and discuss the conditions for the existence of monetary steady states in the model. In the fifth section we calibrate the model and use numerical methods to compute its steady state equilibria. We compare the features of these equilibria to various features of U.S. postwar data, and investigate the sensitivity of our results to changes in the values of the model's parameters. In section six we discuss some issues that seem important in interpreting our results and suggest some areas for future research. The seventh and final section presents a summary of our findings.

## 2 A pure exchange economy

In this section, we illustrate some features of multiperiod overlapping generations models by examining a simple pure exchange (endowment) economy. This economy endures forever, time  $t$  taking on integer values on the real line. A large, finite number of agents are born at each date  $t$ . The agents are endowed with perfect foresight and live for  $n$  periods. The agents are identical in the sense that, regardless of their birthdate, they have the same preferences and face the same endowment stream over the course of their lives. For the purposes of this section, we denote the endowment stream received by the agents born at time  $t$  as  $\{e_t(t+i)\}_{i=0}^{n-1}$ , where  $e_t(t+i) > 0 \forall i$ . An endowment  $e_t(t+i)$  is the quantity of the single consumption good received at time  $t+i$  by an agent born at time  $t$ .<sup>3</sup> The population of this economy grows at gross rate  $\psi \geq 1$ . Agents born at time  $t$  maximize time-separable logarithmic utility  $U = \sum_{j=0}^{n-1} \beta^j \ln c_t(t+j)$ , which implies that the goods in

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steady states of certain calibrations of their multiperiod overlapping generations model. They do not study the properties of monetary steady states, however.

<sup>3</sup>We adopt the notational convention throughout that birthdates are denoted by subscripts, while real time is denoted inside parentheses.

the various periods of life are gross substitutes.<sup>4</sup> Agents can participate in the consumption loan market, lending or borrowing for one period at gross real interest rate  $R(t)$ , and they can purchase or sell unbacked government liabilities. We assume in this section that the aggregate nominal stock of such liabilities is fixed at  $H > 0$ .

The equilibria of the model can be represented by solutions to a difference equation:<sup>5</sup>

$$S(t) = R(t-1)S(t-1) \quad (1)$$

where  $S(t)$  represents aggregate savings at time  $t$ , and can be written, for  $n \geq 3$ ,

$$S(t) = X - Y$$

where

$$X = \sum_{i=0}^{n-2} \psi^{t-i} e_t(t+i) + \sum_{i=0}^{n-3} \sum_{j=1}^{n-2-i} \psi^{t-i-j} e_t(t+i) \prod_{k=1}^j R(t-k)$$

and

$$Y = \psi^t E_t + \sum_{i=0}^{n-2} \sum_{j=1}^i \psi^{t-i} \beta^j E_{i-1} \prod_{k=1}^i R(t-k)$$

and

$$E_k = \left[ \sum_{i=0}^{n-1} \beta^i \right]^{-1} \left[ e_t(t) + \sum_{i=1}^{n-1} e_t(t+i) \prod_{j=k-1}^{k+i-2} R(t+j) \right]^{-1}.$$

The aggregate savings function at time  $t$  has a steady state or stationary form ( $R(t) = R \forall t$ ) which we will denote  $S(t, R)$ . The function  $S(t, R)$  is strictly increasing in the stationary gross interest rate  $R$ , and has the property that  $\lim_{R \rightarrow 0} S(t, R) = -\infty$ , and  $\lim_{R \rightarrow \infty} S(t, R) > 0$ . Equation (1) consequently has at most two stationary solutions which are the steady states of this model. The first steady state is associated with the value of  $R$  such that  $S(t, R) = 0$ . We denote this interest rate  $R_{nmss}$  and refer to it as the nonmonetary steady state rate of interest. In this steady state unbacked government liabilities are valueless. The second steady state is associated with the gross real rate of interest  $R_{mss} = \psi$  and exists if and only if  $S(t, \psi) > 0$ .

An expanded version of the latter inequality provides the condition for the existence of a monetary steady state in this model:

$$\sum_{i=1}^n e_t(t+i-1) \psi^{1-i} [(n-i) - B] > 0, \quad (2)$$

<sup>4</sup>See Kehoe, *et al.*, (1991) for a comprehensive analysis of endowment economies when all goods are gross substitutes.

<sup>5</sup>See Bullard (1992).

where

$$B = \left[ \sum_{i=0}^{n-1} \beta^i \right]^{-1} \left[ \sum_{i=0}^{n-2} \sum_{j=0}^i \beta^j \right].$$

Consider the case where  $\psi = \beta = 1$ ;  $B$  simplifies to  $\frac{n-1}{2}$ . In this case the endowments received by agents in the first half of their lives contribute to satisfying the condition, while those received in the second halves of their lives work against it. An endowment pattern such as  $e_t(t) = 1$ ,  $e_t(t+i) = 0$ ,  $i = 1, 2, \dots, n-1$ , will always satisfy the condition regardless of  $n$ . This is a simple example of a point espoused throughout this paper that the number of periods  $n$  is in principle not an impediment to the existence of monetary steady states.

Next consider the effects of nonzero rates of time preference, keeping the gross rate of population growth at unity. When  $\beta = 0$ ,  $B = n-1$ , and condition (2) would never be met: all of the terms in the sum would be nonpositive. This result is sensible because if  $\beta = 0$ , agents discount the future completely and will not hold unbacked liabilities or any other assets. When  $\beta > 1$ , the case of negative rates of time preference,  $B < \frac{n-1}{2}$  for large  $n$ ; if  $\beta = 2$ , for example, then  $B \cong 1$  for large  $n$ . In this latter case, agents are extremely concerned about the future and the existence condition is almost certain to be satisfied. Figure 1 displays the value of  $B$ , as a function of  $\beta$ , for large  $n$ . If we set the rate of time preference to zero and consider population growth ( $\psi > 1$ ), we find that positive rates of population growth make the existence condition easier to satisfy: endowments received late in life, which tend to work against the condition, are divided by  $\psi$  raised to progressively higher powers.

Thus we conclude that high rates of population growth facilitate existence of monetary steady states while high rates of time preference work against existence. These effects have limits, however, that arise because we calibrate the temporal interpretation of a time period in the model by the choice of  $n$ . We view  $n = 55$  as constituting an “annual” model; by analogy  $n = 220$  must constitute a “quarterly” model,  $n = 660$  must constitute a “monthly” model, and so on, with  $n \rightarrow \infty$  approaching a continuous time formulation. Thus, parameters such as the population growth rate and the rate of time preference depend on  $n$  and would be better written as  $\psi(n)$  and  $\rho(n)$ , although we will not do so in this paper. When these parameters are properly adjusted for the choice of  $n$  (“annualized”), future discounting continues to work against the existence condition and population growth

continues to work towards it, but the magnitude of each effect is essentially invariant to the value of  $n$ .<sup>6</sup>

With regard to this last point let us return momentarily to Figure 1, which will be used extensively in the analysis below. When we constructed the figure we took  $n$  to be large, but finite, and plotted the values of  $B$  associated with a wide range of possible values of the discount factor  $\beta$ . Empirically plausible values of the *annual* discount factor, however, are relatively close to unity: a range of .9 to 1.1 would capture most published estimates. Since the adjustment procedure raises the annual value of  $\beta$  to the power  $55/n$ , when  $n$  is large the adjusted discount factor will be quite close to unity. Thus for empirically plausible time preference rates the limiting value of  $B$  will be close to  $\frac{n-1}{2}$ , a little higher, or lower, depending on the actual choice of the (annual) rate of time preference. This is why we conclude that while features of a given model specification that affect the value of  $B$  certainly influence the prospects that the specification will satisfy the existence condition, the impact of these influences is limited.

We have shown how the existence of a monetary steady state depends importantly on the nature of the agents' endowment patterns. Since we cannot test all possible endowment patterns we proceed by selecting a class of relatively tractable patterns that has some empirical support. The endowment patterns we employ are based on the pattern used by Auerbach and Kotlikoff (1987), who analyze a 55-period overlapping generations model.<sup>7</sup> The Auerbach-Kotlikoff pattern is

$$e_t(t+j-1) = \exp [4.47 + .033j - .00067j^2]$$

where  $j = 1, 2, \dots, 55$ . We construct endowment patterns for  $n \neq 55$  by dividing the domain into  $n$  intervals of equal length and setting the endowment received in the  $i^{\text{th}}$  period of life equal to the integral of the Auerbach-Kotlikoff endowment function over the  $i^{\text{th}}$  interval. When there is neither future discounting nor population growth (so that  $\beta = \psi = 1$ ) patterns of this type satisfy condition (2) for any  $n \geq 2$ .

Figure 2, which displays results from specifications of the model that use the endowment scheme just described, illustrates the fact that savings *per capita* (which are equal, in this

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<sup>6</sup>See Bullard (1992).

<sup>7</sup>The pattern is based on an empirical investigation conducted by Welch (1979).

pure exchange model, to real balances of unbacked liabilities *per capita*) converges to a constant as  $n$  increases. In these specifications there is no population growth, and the monetary steady state occurs at  $R = 1$ . Aggregate savings *per capita* at this interest rate is evaluated under two different assumptions about the rate of time preference. In the first case,  $\beta = .999$  regardless of the value of  $n$ ; in this *fixed time period interpretation* the length of a time period is taken to be fixed (say, a year) and thus the rate of time preference is not adjusted based on  $n$ .<sup>8</sup> In the second case,  $\beta = .999^{55/n}$ ; in this *variable time period interpretation* the length of a time period is taken to be decreasing as  $n$  increases, and the discount factor is adjusted accordingly. If per capita savings falls below zero, the monetary steady state does not exist in this example. Since per capita savings converges rapidly to a constant in the variable time period interpretation, we conclude that the unbacked liability will be valued in this situation regardless of the value of  $n$ .

The fixed time period length interpretation has been adopted by Aiyagari (1988, 1989), who views this model with  $n \rightarrow \infty$  as one where agents live forever. Aiyagari (1988, 1989) studies pure exchange economies without population growth, and establishes that under the fixed time period interpretation, as  $n$  approaches infinity the real interest rate in the nonmonetary steady state approaches the rate of time preference. It follows that if the rate of time preference is positive monetary steady states are ruled out for sufficiently large values of  $n$ . The fixed time period interpretation plot of per capita savings in Figure 2 confirms Aiyagari's (1988) result for this example, as it falls below zero at about  $n = 250$ , even though the rate of time preference is quite small.

We present Aiyagari's (1988, 1989) fixed time period length interpretation in order to contrast this view with our own and because it is the only example of which we are aware of a systematic treatment of the time period problem in overlapping generations models. Aiyagari's (1988, 1989) results are sometimes used to argue that monetary steady states in overlapping generations models are without empirical relevance: if the rate of time preference is presumed positive, their existence hinges on entirely arbitrary decisions about the number of periods in agents' lifetimes. We think the argument has little force, however. The fundamental assumption of the model is that agents die, which gives the time period length a biological basis for any  $n$ . A great deal of previous literature has viewed the length

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<sup>8</sup>We do, however, adjust the endowment pattern according to the Auerbach-Kotlikoff scheme.

of a time period in the model with  $n = 2$  as being very long, say, 30 years, or half a human productive lifetime, so that by analogy, the length of the time period in the model with large  $n$  should be viewed as very short.<sup>9</sup> In addition, we think that it is desirable to preserve fundamental properties of the model across the (arbitrary) choice of  $n$ , as our interpretation does.

The examples in the endowment economy case therefore suggest, in our view, that the time period problem is not an obstacle to valued outside liabilities in the overlapping generations model. Since the length of a time period, in our interpretation, shrinks as  $n$  increases, criticisms of the type leveled by Tobin (1980) against this model of money, most of which are based on the argument that the “money” in the model is an asset that is held for many years, should probably carry much less force with economists than they commonly do. In the remainder of this paper, we turn to our primary purpose of pushing the question of robustness of monetary steady states further by adding production, capital accumulation, technological progress, and a labor-leisure decision to the model.<sup>10</sup>

### 3 The model with capital and technological change

The model economy we study in this section augments the endowment economy by adding production, capital accumulation, a labor-leisure decision, and technological progress. The economy again endures forever, with time  $t$  taking on integer values on the real line. Many agents are born at each date  $t$ , and agents live for  $n$  periods and are identical except for birthdates. Agents born at date  $t$  are said to be “of generation  $t$ ,” and we do not distinguish between members of the same generation. There is a single good in the model which can be consumed or used as an input into production, in which case it is called “capital.” Capital produced during a period cannot be used in production until the following period, at which point it begins to depreciate at net rate  $\delta \in [0, 1]$  per period. The production process

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<sup>9</sup>We note that in the case of Auerbach and Kotlikoff (1987) and subsequent related studies it is hard to distinguish the interpretation since the fixed time period length view, with the time period being a year, would be equivalent to the variable time period length view at something like  $n = 55$ .

<sup>10</sup>The results presented in this paper help verify the comments of Friedman and Hahn (1990, p. xiv), who write: “Overlapping generations models [of money] are both more robust and more interesting than is sometimes believed.... Of course, the postulate of two period lives is highly unrealistic. On the other hand, it is hard to think of a *qualitative* conclusion of these models ... that is plausibly at risk from more realistic lifetimes.... There may ... be a difference in qualitative conclusions as one passes from finitely to infinitely lived agents.”



displays constant returns to scale:

$$Y(t) = \lambda^{(t-1)(1-\alpha)} L(t) k(t)^\alpha,$$

where  $Y(t)$  is aggregate output at time  $t$ ,  $\lambda \geq 1$  represents the gross rate of technological improvement,  $K(t)$  and  $L(t)$  represent the aggregate employment of capital and effective labor, respectively, at date  $t$ ,  $k(t) \equiv K(t)/L(t)$  is the capital-effective labor ratio at time  $t$ , and  $\alpha \in [0, 1]$  determines the capital share of output. The production technology is available to an arbitrary number of perfectly competitive firms that earn zero profits at each date  $t$ . The firms rent capital and hire effective labor from agents at rates equal to the marginal products of these inputs: the real rental rate on capital is given by  $r(t) = \lambda^{(t-1)(1-\alpha)} \alpha k(t)^{\alpha-1}$ , and the real wage is given by  $w(t) = \lambda^{(t-1)(1-\alpha)} (1 - \alpha) k(t)^\alpha$ .

Each member of generation  $t$  is endowed with  $\bar{\ell}$  units of time per period of life. We adjust the value of this time endowment to the length of an agent's life in periods according to  $\bar{\ell} = 55/n$ . Agents may enjoy leisure by withholding some of their time endowment from the labor supply. We denote  $l_t(t+j)$ ,  $j = 0, \dots, n-1$ , as the amount of time allocated to leisure at time  $t+j$ , so that  $\bar{\ell} - l_t(t+j)$  represents the amount of time allocated to labor supply. Agents possess productivity endowments—proxies for fully internalized human capital—denoted  $e_t(t)$ ,  $e_t(t+1)$ ,  $\dots$ ,  $e_t(t+n-1)$ . The endowment profile is assumed to be identical for all agents regardless of birthdates; because of this we use shorthand notation and denote the productivity endowment stream by  $e_1, e_2, \dots, e_n$ . We determine these amounts according to the Auerbach-Kotlikoff scheme outlined in the previous section. *Effective* labor supply at time  $t$  is the product of the time allocated to labor supply at that date and the agent's productivity endowment at that date. Income at time  $t+j$  for an individual born at time  $t$  is the product of the wage at that date and the effective labor supply,  $w(t+j) e_{j+1} [\bar{\ell} - l_t(t+j)]$ ,  $j = 0, \dots, n-1$ .

The population size of generation  $t$  is denoted  $a_t$ . The total number of agents alive at time  $t$  is  $A(t) = \sum_{j=t-n+1}^t a_j$ . We assume that  $a_t$  grows at gross rate  $\psi \geq 1$  per period, which implies that  $A(t)$  grows at the same rate. The size of generation one is normalized to unity:  $a_1 = 1$ .

There is a government that endures forever. The only role of the government is to collect real revenue by issuing unbacked liabilities and exchanging them for consumption goods.

Government revenue at date  $t$  is

$$G(t) = \frac{H(t) - H(t-1)}{P(t)},$$

where  $H(t)$  represents the nominal stock of unbacked liabilities outstanding at the end of date  $t$  and  $P(t)$  represents the date  $t$  price of a unit of the consumption good in units of these liabilities. The government is assumed to issue unbacked liabilities so that their nominal stock grows at a constant rate  $\theta \geq 1$ ; that is,  $H(t) = \theta H(t-1)$ . When  $\theta = 1$ , the nominal stock of unbacked liabilities is constant and government revenue is zero at each date.

Agents are endowed with perfect foresight and make decisions to supply labor, consume and save. Agents may participate in the consumption loan market, buy or sell the unbacked liability, or rent capital to the firms. The gross real rate of return on consumption loans is  $R(t)$ . The rate of return to holding unbacked liabilities is  $P(t)/P(t+1)$ . Capital rentals earn gross rate  $r(t+1) + 1 - \delta$ . In any perfect foresight equilibrium in which all three types of saving are observed, arbitrage requires  $P(t)/P(t+1) = R(t) = 1 - \delta + r(t+1)$ .

Agents choose consumption and leisure at each date in order to maximize

$$U = \sum_{i=0}^{n-1} (1-\gamma)^{-1} \beta^i \left[ c_t(t+i)^\eta \ell_t(t+i)^{(1-\eta)} \right]^{(1-\gamma)}$$

for  $\gamma > 0$ ,  $\eta \in (0, 1]$ , and  $\beta > 0$ . The parameter  $\gamma$  governs the curvature of the utility function, and the parameter  $\eta$  governs the fraction of total time agents devote to labor supply. The coefficient of relative risk aversion is given by  $1 - \eta(1 - \gamma)$ , although in this nonstochastic model it is perhaps better described as the inverse of the elasticity of intertemporal consumption substitution. The discount factor is given by  $\beta \equiv \frac{1}{1+\rho}$ , and we restrict  $\rho > -1$ . The constraints are

$$c_t(t) + \sum_{i=1}^{n-1} c_t(t+i) \prod_{j=0}^{i-1} R(t+j)^{-1} \leq$$

$$w(t)e_1 [\bar{\ell} - \ell_t(t)] + \sum_{i=1}^{n-1} w(t+i)e_i [\bar{\ell} - \ell_t(t+i)] \prod_{j=0}^{i-1} R(t+j)^{-1},$$

provided  $\ell_t(t+i) \leq \bar{\ell}$  for  $i = 0, \dots, n-1$ . We denote the savings of generation  $i$  at stage of

life cycle  $j$  by  $s_i(i + j)$ . Aggregate savings in the economy at time  $t$  is denoted by

$$S(t) = \sum_{j=0}^{n-2} \psi^{t-j} s_{t-j}(t)$$

and can be expressed as a function of the  $R(t + i)$ ,  $i = 2 - n, \dots, n - 2$ , by solving the maximization problem for interior optima and making use of the assumption that the generations are identical except for birthdates and population size. A derivation of the aggregate savings function is given in Appendix A.

Agents in this model may choose to retire, that is, to work zero hours during a period, and the aggregate savings function given in Appendix A applies only to the case of an interior solution in which retirement does not occur. Generally, we find that retirement matters little for the issues addressed in this paper. Nevertheless, in the next section we handle retirement by computing numerical solutions and checking the conditions for an optimum. In one experiment, we simply force the agents to retire at a fixed date.

Given the aggregate savings function, the economy can be described by two equations:

$$H(t)/P(t) = S(t) - K(t + 1) \quad (3)$$

$$H(t) = \theta H(t - 1). \quad (4)$$

Equilibrium paths can consequently be described as the solutions to

$$S(t) - K(t + 1) = \theta R(t - 1)[S(t - 1) - K(t)]. \quad (5)$$

Since  $K(t) = k(t)L(t)$  and since both  $k(t)$  and  $L(t)$  can be written as functions of interest rates alone (see Appendix A), the equilibrium condition (5) for the economy can be viewed as a complicated difference equation in the gross real interest rate. Stationary solutions to this equation can be found by setting  $R(t) = R \forall t$ . In a stationary equilibrium, both aggregate savings and the aggregate capital stock grow at gross rate  $\lambda\psi$ . Thus equilibria occur at values of  $R$  such that  $S(1) - \lambda\psi K(1) = 0$ , where  $S(t) = (\lambda\psi)^{t-1}S(1)$  and  $K(t) = (\lambda\psi)^{t-1}K(1)$ . There is also a monetary steady state at  $R = \frac{\lambda\psi}{\theta}$  provided  $S(1) - \lambda\psi K(1) > 0$  at this value of  $R$ . The equilibria at values of  $R$  that set  $S(1) - \lambda\psi K(1) = 0$  will be called nonmonetary steady states, with rates of interest denoted  $R_{nmss}$ . Unbacked liabilities are not held at these steady states. The steady state at  $R = \frac{\lambda\psi}{\theta}$ , provided it exists, is the

monetary steady state of the model, with the rate of interest denoted  $R_{mss}$ ; unbacked liabilities are held at this steady state, and the real stock of such liabilities grows at gross rate  $\lambda\psi$  per period.

While it is possible for a specification of the model to have many nonmonetary steady states, in the calibrated specifications we study the aggregate savings function is monotonically increasing in  $R$ , and there is consequently only one such steady state. We will assume such monotonicity in the remainder of this section. We will also focus on monetary steady states in which the nominal stock of unbacked liabilities is constant ( $\theta = 1$ ), so that  $R_{mss} = \lambda\psi$ . Given monotonicity in aggregate savings, the existence of such a monetary steady state is necessary and sufficient for the existence of monetary steady states in which  $\theta > 1$ : if there is a monetary steady state when  $\theta = 1$  there will also be a monetary steady state associated with each  $\theta \in (1, \hat{\theta})$ , where  $\hat{\theta}$  solves  $R_{nmss} = \lambda\psi/\theta$ .

In the next section, we analyze the condition for existence of a monetary steady state:

$$S(1) - \lambda\psi K(1) > 0 \tag{6}$$

at  $R = \lambda\psi$ .<sup>11</sup> We are able to provide a relatively general analysis of the effects of changes in various parameters on the condition for existence of a monetary steady state for the case where there is no labor-leisure trade-off. To analyze the case where there is a labor-leisure trade-off, we turn in the following section to a calibration of the model by choosing parameter values which are standard in the literature or are available from U.S. time series. We compute a baseline case, and check robustness by varying parameters against the baseline case.

## 4 Monetary steady states

### 4.1 The existence condition

In this section we set  $\eta$  equal to unity and study the existence of monetary steady states by analyzing the aggregate savings function. Our analysis is in the tradition of the literature on fiat money in two period pure exchange overlapping generations economies. Most of the intuition for how parameters affect the condition for existence of a monetary steady

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<sup>11</sup>In experiments conducted below we use the test  $R_{nmss} < \lambda\psi$  to determine whether a monetary steady state exists. When the aggregate savings function is upward-sloping, as it is in the calibrated versions of the model that we study, this test is equivalent to (6).

state in the model with retirement can be gleaned from an analysis of the model without a labor-leisure decision. The condition (6) with  $\eta = 1$  can be written compactly from the expanded form given in Appendix A:

$$w(1) \left[ \sum_{i=1}^n e_i \psi^{1-i} [(n-i) - B] \right] - k(1) \lambda \psi \left[ \sum_{i=1}^n e_i \psi^{1-i} \right] > 0 \quad (7)$$

where

$$B = \left[ \sum_{i=1}^n (\lambda \psi)^{1-i} (\lambda \psi \beta)^{(i-1)/\gamma} \right]^{-1} \left[ \sum_{i=1}^n (n-i) (\lambda \psi)^{1-i} (\lambda \psi \beta)^{(i-1)/\gamma} \right]$$

and

$$k(1) = \left[ \frac{\lambda \psi - 1 + \delta}{\alpha} \right]^{\frac{1}{\alpha-1}}$$

and

$$w(1) = (1 - \alpha) \left[ \frac{\lambda \psi - 1 + \delta}{\alpha} \right]^{\frac{\sigma}{\alpha-1}}.$$

The existence condition for the pure exchange economy can be thought of as a special case of this condition. In the pure exchange economy there is no capital, so the capital-labor ratio and the depreciation rate are both equal to zero. There is no technological progress, so  $\lambda = 1$ . The product of the real wage and the productivity index is the analog of the consumption good endowment, so we set  $w(1) = 1$  and let the productivity endowments now represent the endowments of the consumption good. If we then set  $\gamma = 1$ , which is the logarithmic preferences case, we have exactly the existence condition (2). This condition was discussed at length in Bullard (1992). Bullard (1992) concluded that increasing the rate of time preference makes the condition more difficult to satisfy, while increasing the rate of population growth makes the condition easier to satisfy.

We now consider the endowment economy case with  $\gamma > 1$ . The condition is given by

$$\sum_{i=1}^n e_i \psi^{1-i} [(n-i) - B] > 0 \quad (8)$$

with

$$B = \left[ \sum_{i=1}^n \psi^{1-i} (\psi \beta)^{(i-1)/\gamma} \right]^{-1} \left[ \sum_{i=1}^n (n-i) \psi^{1-i} (\psi \beta)^{(i-1)/\gamma} \right].$$

The coefficient of relative risk aversion,  $\gamma$ , appears only in the term  $B$ . To see the effect of  $\gamma \neq 1$ , envision condition (2) from the previous section with  $\beta$  replaced by

$$z = \psi^{\frac{1-\gamma}{\gamma}} \beta^{\frac{1}{\gamma}}.$$

Then it is possible to use Figure 1 to deduce the effects of changes in parameters, replacing  $\beta$  in the figure with  $z$ . It is clear by inspection that  $\lim_{\gamma \rightarrow 0} z = +\infty$ , and thus from the figure,  $B \cong 1$  for  $n$  large; in the other direction,  $\lim_{\gamma \rightarrow \infty} z = \psi^{-1}$ , and thus from Figure 1  $B > \frac{n-1}{2}$  for  $n$  large. We conclude that more risk aversion will damage the prospects for existence of a monetary steady state. An increase in the rate of time preference reduces  $\beta$  which tends to depress  $z$ ; this causes the value of  $B$  to rise and makes the existence condition more difficult to satisfy. The gross rate of population growth,  $\psi$ , presents a more difficult problem, as it enters the existence condition in two different places with opposite effects. The  $\psi$ -terms multiplying the productivity endowments are helpful to satisfying the existence condition because they reduce the weights on endowments received later in life. On the other hand, when  $\gamma > 1$  increases in  $\psi$  cause  $B$  to increase. Consequently the overall effect of increases in the population growth rate on the existence condition is not clear.<sup>12</sup>

These parameters,  $\gamma$ ,  $\beta$ , and  $\psi$ , are the central ones in condition (7). We now turn to the model with capital to illustrate this point.

## 4.2 Effects of parameters on the existence condition

In condition (7), we begin by noting that each productivity endowment is in effect multiplied by a term, the first portion of which involves  $w(1)$  and may be positive, and the second portion of which involves  $k(1)$  and is necessarily negative.<sup>13</sup> The portion involving  $w(1)$  is more likely to be positive for productivity endowments received early in life and less likely to be positive for productivity endowments received late in life; for  $e_n$  this term is definitely negative. Thus, as far as the endowment pattern is concerned, greater productivity early in life relative to late in life is helpful to the satisfaction of condition (7).

The preference parameters  $\gamma$  and  $\rho$  enter condition (7) only through the term  $B$ . If  $B$  is sufficiently large the term multiplying  $w(1)$  will be negative for each of the productivity endowments and the condition will never be satisfied. Changes in these parameters have essentially the same influence on the existence of monetary steady states in the production economy that they had in the endowment economy.

We now turn to other parameters, which must be analyzed in part through the wage

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<sup>12</sup>If  $\gamma < 1$ , more rapid population growth is unambiguously helpful to existence. We regard  $\gamma > 1$  as the more plausible case.

<sup>13</sup>See also the expanded version of the condition in Appendix A.

and the capital-labor ratio. As the wage is strictly increasing in the capital-labor ratio, parameter choices that produce relatively high capital-labor ratios will also produce relatively high wages. This creates some ambiguity in the condition, because higher values of  $k(1)$  work against existence, while higher values of  $w(1)$  may support existence. It is difficult to say *a priori* what the effects of a change in  $w(1)$  will be, but for the purposes of this discussion, we will assume that the positive effects of an increase in the wage (as it multiplies productivity endowments received relatively early in life) are approximately offset by the negative effects (as it multiplies endowments received later in life). Consequently we will ignore the effects of changing parameter values on the wage rate and focus on their effects on the capital-labor ratio.

One case where this assumption plays a role is in the assessment of the effects of a change in the depreciation rate  $\delta$ . The value of  $\delta$  enters condition (7) in two ways. However,  $\delta$  also helps determine  $k(1)$ , and other parameters held constant, higher values of  $\delta$  cause the ratio to decline, improving the prospects for existence. A lower value of  $k(1)$  also means a lower real wage, but we have agreed to ignore the effects of real wage changes. We conclude that higher depreciation rates make it easier to satisfy the condition.

The effects of a change in  $\alpha$ , the capital share parameter, are also clear. An increase in the capital share increases the capital-labor ratio, which makes it more difficult to satisfy the condition; again we ignore real wage effects.

Technological progress also enters the condition in several ways. First,  $\lambda$  enters directly in the term multiplying  $k(1)$ , so that by this channel higher rates of technological progress tend to make it more difficult to satisfy the condition. The value of  $\lambda$  also enters the expression for  $B$ ; if  $\gamma > 1$ , higher values of  $\lambda$  tend to increase  $B$  and again make condition (7) more difficult to satisfy. On the other hand, higher values of  $\lambda$  produce lower capital-labor ratios, an effect which tends to work towards satisfying the condition. Overall, the effects of an increase in  $\lambda$  are ambiguous.

A similar analysis applies to the case of changes in the rate of population growth. The values of the productivity endowments are divided by progressively higher values of  $\psi$  as we move later in an agent's life, which tends to improve the prospects for satisfaction of the condition. Increases in  $\psi$  also reduce the capital-labor ratio, which has the same effect. On the other hand, higher values of  $\psi$  tend to increase the magnitude of the term multiplying

Parameter	Symbol	Effect of a higher value
Rate of time preference	$\rho$	Works against
Curvature in preferences	$\gamma$	Works against
Depreciation rate	$\delta$	Works toward
Capital share	$\alpha$	Works against
Gross rate of technological change	$\lambda$	Ambiguous
Gross rate of population growth	$\psi$	Ambiguous
Share of time devoted to market	$\eta$	Works against

Table 1: Some qualitative conclusions. “Works against” means a higher value of this parameter works against existence of a monetary steady state.

$k(1)$ , and also tend to increase the value of  $B$ ; each of these effects works against existence. Thus the “existence effects” of changes in the population growth rate are also ambiguous.

This concludes our discussion of the qualitative influence of changes in parameter values on the prospects for existence of a monetary steady state in the model without a labor-leisure decision. We comment briefly on the condition when  $\eta < 1$ , noting that our comments will only be strictly valid in the case of an interior solution to the agents’ problem. The parameter  $\eta$  enters the condition in two ways. The first is as part of a term multiplying  $k(1)$  so that lower values clearly work in favor of existence. The second way is through the influence of  $\eta$  on the value of  $B$ . This effect is somewhat ambiguous because it depends in part on the life cycle pattern of productivity endowments. Some intuition about the nature of this effect can be gleaned by considering the special case in which all endowments are equal:  $e_i = \bar{e} \forall i$ . Returning to our previous notation, we can then write

$$z = \lambda^{\frac{(1-\gamma)(\eta-1)}{\gamma}} (\lambda\psi)^{\frac{1-\gamma}{\gamma}} \beta^{\frac{1}{\gamma}}.$$

Using Figure 1 in the same way as before, we can conclude that when  $\gamma > 1$ , lower values of  $\eta$  tend to increase  $z$  and thus lower the value of  $B$ . Thus in this special case a reduction in  $\eta$  unambiguously enhances the prospects for existence of a monetary steady state. The conclusions of the analysis presented in this section are summarized in Table 1. We now turn to numerical methods to analyze the existence of monetary steady states in the presence of a labor-leisure trade-off.



## 5 The model with a labor-leisure trade-off

This section is devoted primarily to reporting the values of nonmonetary steady state gross real interest rates  $R_{nms}$  under a variety of different parameter settings we regard as empirically plausible. We orient these calibration experiments around a *baseline case*: a set of parameter choices that we view as best supported in the empirical literature. In this baseline economy, we allow agents to choose to retire. All parameters are set appropriately according to the choice of  $n$ .

### 5.1 The baseline case

#### 5.1.1 Choice of parameter values

The value of the parameter  $n$ , the number of periods in an agent's lifetime, plays a key role in our analysis. In our baseline case we follow Auerbach and Kotlikoff (1987) and subsequent literature and set  $n = 55$ . We think of the length of a time period for this value of  $n$  as a year.

The value of the preference parameter  $\gamma$  influences the coefficient of relative risk aversion which is  $1 - \eta(1 - \gamma)$ . The coefficient of relative risk aversion is the inverse of the intertemporal elasticity of substitution  $\sigma = 1/(1 - \eta(1 - \gamma))$ . A great deal of empirical analysis has attempted to obtain point estimates for the value of  $\sigma$ . Mehra and Prescott (1985) presented arguments in favor of unity, but empirical estimates of  $\gamma$  obtained in the first half of the 1980s were often substantially lower. We chose  $\gamma = 5$ , which, given our value of  $\eta$ , produces a coefficient of relative risk aversion of 1.8.

We set the preference parameter  $\eta$ , which governs the average share of an agent's time endowment that is devoted to labor supply, at 0.2. We arrive at this choice by interpreting the utility function as applying to a twenty-four hour day where the marginal disutility from working the full day is infinite—agents cannot do without sleep for any extended period. We then attempt to choose  $\eta$  so that the average fraction of the time endowment an agent in the model devotes to labor supply is roughly equal to the average fraction of a 24-hour day a full-time worker spends on the job. We estimate this figure by subtracting weekend days, holidays and vacation days (10) from a calendar year, dividing by three to represent an eight hour day, and dividing the result by the total number of days in a year. This

produces a fraction of time devoted to work of about .22. We choose  $\eta = .2$  because this produces approximately the .22 fraction in our simulations.<sup>14</sup>

The value of the parameter  $\alpha$ , which determines the capital share of output, is set at 0.25. This is the value used by Auerbach and Kotlikoff (1987); the basis for their choice is that it is approximately equal to unity less the average shares of employee compensation and proprietors' income in national income. This value has become standard in the extensive public finance literature that builds on the work of Auerbach and Kotlikoff (1987). A few contributors to this literature use lower values; the lowest value we have encountered is 0.18.<sup>15</sup> Equilibrium business cycle studies based on representative agent models typically use substantially higher values;  $\alpha = 0.36$  is the most common. In the context of our discussion of dynamic efficiency in Appendix C, we present an analysis that helps justify our choice of  $\alpha$  and also helps reconcile it with the higher values used in the equilibrium business cycle literature.

The gross population growth rate  $\psi$  and the gross rate of technological change  $\lambda$  are both set to 1.015. These choices imply an output growth rate of about 3 percent per period, which is consistent with postwar U.S. annual data. The individual values are close to various estimates of the average rates of labor force and productivity growth in the postwar U.S. These parameters are adjusted for various values of  $n$  by raising the annual value to the power  $55/n$ .

The net rate of depreciation  $\delta$  is set to 0.1. This is the standard figure used in equilibrium business cycle studies based on representative agent models. Auerbach and Kotlikoff (1987) abstract from depreciation, as do many of the public finance studies that build on their work. While some life-cycle calibration studies use lower values of  $\delta$ , we have not seen a value less than 0.07. The dynamic efficiency discussion in Appendix C contains some analysis that helps justify our baseline depreciation rate. Values of  $\delta$  must also be adjusted to the value of  $n$ ; if we denote the annual depreciation rate by  $\delta_{annual}$  the adjustment is  $\delta(n) = 1 - (1 - \delta_{annual})^{55/n}$ .

The empirically appropriate value of the rate of time preference  $\rho$  is an unsettled ques-

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<sup>14</sup>We note that the mapping between  $\eta$  and the percent of the time our agents devote to the market is not exact. The standard value for  $\eta$  in the literature is 1/3 which is based on similar reasoning but does not use a 24-hour day and abstracts from weekends, holidays and vacation

<sup>15</sup>See Laitner (1987).

tion. Our choice for  $\rho$  is based on an empirical study by Hurd (1989). Hurd's (1989) model contained a parameter representing the difference between the real interest rate facing agents and their rate of time preference; his favored estimate of this parameter was 0.041.<sup>16</sup> Since our own estimate of the average after-tax real interest rate in the postwar U.S. is approximately zero, we have chosen  $\rho = -0.04$  in our baseline case.<sup>17</sup> We discuss this aspect of our baseline calibration at length in Appendix B. The rate of time preference must be adjusted for the choice of  $n$ , and we make the adjustment by raising the annual value of  $\beta$  to the power  $55/n$ .

The remaining parameter for our baseline calibration is the gross rate of unbacked liability creation  $\theta$ . This value cannot be calibrated to the average growth rate of the monetary base, since the stock of unbacked liabilities presumably also includes a substantial portion of the national debt. Rather than attempting the uncertain process of decomposing a national debt time series into backed and unbacked components, we have simply set the baseline value of the net growth rate of the stock of unbacked debt at 0.03 ( $\theta = 1.03$ ), a value that produces a net real interest rate of zero in the monetary steady state. A net real interest rate of zero is consistent with our estimate of the average after-tax real rate on safe, short-term credit instruments in the U.S. during the postwar period.

### 5.1.2 Features of the baseline calibration

Figure 3 plots the aggregate excess savings function (aggregate savings less aggregate demand for capital) associated with the baseline parameterization of our model. The figure can also be interpreted as a plot of the aggregate demand for outside liabilities as a function of the gross real interest rate  $R$ . Since this function is uniformly upward-sloping and crosses the  $R$ -axis once, it follows that there is a unique nonmonetary steady state with gross interest rate  $R_{nmss}$  given by the point where the aggregate excess savings function

<sup>16</sup>Hurd's (1989) alternative estimate of this difference, which he obtained using a measure of household wealth that was broader but less reliably measured, was approximately 6 percent.

<sup>17</sup>We interpret the real interest rate in our model to be the riskless after-tax return on a safe, short-term security, and we use the three month treasury bill as a real world analog. We use 45 years of U.S. postwar data, 1948 to 1992, and adjust the observed nominal rates by the associated marginal tax rate and subtract the holding period rate of consumer price index inflation. This yields an average real interest rate of about  $-.004$ , which we round to zero, and thus from the estimates of Hurd (1989) we choose  $\rho = -.04$ . We thank Joe Peek at Boston College for his cooperation in providing us with the marginal tax rates used in this calculation.

crosses the  $R$ -axis. Since  $R_{n_{mss}} < \lambda\psi\theta^{-1}$  ( $\cong 1$ ), there is also a unique monetary steady state at  $R_{mss} = \lambda\psi\theta^{-1}$ .

The baseline monetary steady state has a number of interesting features. The most noteworthy is the fact that the net equilibrium real growth rate is approximately three percent and the net equilibrium real interest rate is zero. The former value is not particularly controversial, and the latter is roughly consistent with the findings of a large number of empirical studies that have attempted to estimate the average level of safe, short-term real interest rates. Nevertheless, this combination of equilibrium values has rarely been produced by calibrated versions of dynamic general equilibrium models that make standard neoclassical assumptions regarding the features of preferences, endowments, and technologies. Deterministic, infinite horizon neoclassical models are incapable of generating a steady state real interest rate lower than the output growth rate, and stochastic models of this sort, as conventionally calibrated, yield steady state real interest rates very close to those produced by their nonstochastic counterparts. Multiperiod overlapping generations models have also been calibrated so as to produce relatively high real interest rates; the potential of plausibly calibrated versions to produce low real interest rates and monetary steady states has not been previously recognized.

The capital-output ratio in the baseline monetary steady state is 2.5, which is in line with most empirical estimates of the value of this statistic. Equilibrium business cycle studies tend to produce similar capital-output ratios, but Auerbach and Kotlikoff (1987) obtain substantially higher ratios. The principle explanation for this is the fact that there is no depreciation in their model.

In our baseline case the ratio of net investment to output is approximately 7.5 percent. This value is close to estimates of the U.S. average for the postwar period. Auerbach and Kotlikoff (1987) obtain a figure which is considerably lower. This aspect of their findings stimulated an extensive literature that attempts to generalize the model by, for example, introducing features that induce precautionary savings, in order to generate higher net investment rates. The source of the increased net savings rate in our model is the fact that we have specified preferences and endowments in a way that permits us to obtain steady state equilibria with exogenous technological progress. We find that if we revise our baseline case by setting  $\lambda = 1$  the net savings rate drops to approximately the level reported

Variable	B-R	P	R-R	A-K
Net real interest rate, percent	0.0	4.0	6.1	6.7
Capital-output ratio	2.5	2.6	2.2	3.7
Net savings rate, percent	7.5	–	6.6	3.7
Ratio of unbacked liabilities to output	.55	–	–	–

Table 2: Values of endogenous variables in some baseline calibrations, where B-R refers to this paper, P=Prescott (1986), R-R=Rios-Rull (1993), and A-K=Auerbach and Kotlikoff (1987). Prescott (1986) abstracts from growth of population or productivity, and consequently obtains a net savings rate of zero. Prescott (1986) uses a representative agent model, while the others use overlapping generations models.

by Auerbach and Kotlikoff (1987). Conversely, if we set the parameter values of our model to achieve a specification as close as possible to that of Auerbach and Kotlikoff (1987)—a specification that includes  $\lambda = 1$ —and then increase  $\lambda$  to 1.01, the net savings rate rises from approximately the value reported by Auerbach and Kotlikoff (1987) to approximately the value we obtain in our baseline case.

In our baseline case the ratio of the stock of unbacked liabilities to output is 0.55. There is no easy way to compare this value to the data because, as we have noted, we do not claim to know how to decompose empirical debt time series into backed and unbacked components. The current ratio of total federal debt to output is approximately .75, so that our baseline case may be thought of as reflecting the implicit assumption that about two-thirds of this debt is unbacked.<sup>18</sup> This value produces a steady state deficit of approximately 1.7 percent of output. Comparisons of values of key endogenous variables in our baseline case to those produced by the baseline cases of three prominent calibration studies are presented in Table 2.

## 5.2 Sensitivity analysis

In this section we report the results of some simple experiments in which we vary a single parameter, holding all other parameters (except  $\theta$ ) at their baseline values, and plot the value of the nonmonetary steady state gross real interest rate  $R_{nms}$  associated with each value of the parameter of interest. As long as this rate falls short of the gross output growth rate  $\lambda\psi$  (which is also plotted) a monetary steady state will exist at  $\theta = 1$  and for

<sup>18</sup>Total federal debt is debt held by the public plus base money outstanding (or, almost equivalently, plus federal debt held by the Federal Reserve Banks).

all  $\theta \in (1, \lambda\psi/R_{nmss})$ .

Figure 4 shows how the value of the “annualized” gross interest rate  $R_{nmss}$  (that is, the nonannualized value raised to the power  $n/55$ ) depends on the value of  $n$  as the number of periods in agents’ lives is varied from 2 to 110. Under our variable time period length interpretation, the nonmonetary steady state rate of interest converges rapidly, as it did in the endowment economy case, and remains below the output growth rate  $\lambda\psi$ . We conclude that a monetary steady state exists in the baseline case regardless of the value of  $n$ . In the remainder of this section, we set  $n = 55$ , our baseline value.

Figure 5 displays the values of  $R_{nmss}$  associated with net rates of technological progress between 0 and 10 percent per year. Of course, changing the value of  $\lambda$  also changes the output growth rate. The nonmonetary steady state real interest rate rises somewhat faster than the output growth rate, but remains below that growth rate as long as the net rate of technological progress is less than 6.5 percent. We conclude that changing the pace of technological progress within plausible limits does not alter the ability of the model to deliver steady state real rates of interest less than the output growth rate, at least against our baseline parameterization.

A plot of the relationship between  $\rho$  and  $R_{nmss}$  is displayed in Figure 6. The rate of time preference is varied over a domain of  $-5$  to  $+5$  percent. We find that the rate of interest is virtually linear in the rate of time preference and that some rates of time preference are high enough to preclude existence of a monetary steady state. In the diagram this occurs at about  $\rho = -.006$ ; a monetary steady state does not exist if  $\rho > -.006$ , but does exist if  $\rho < -.006$ . We stress that the approximate relationship between the sign of the time preference rate and the existence of a monetary steady state is not implied by the structure of the model and is simply an artifact of our baseline parameterization. If we reduce our baseline value of  $\gamma$  to unity, for example, a monetary steady state would exist for a range of positive values of  $\rho$ .

In Figure 7 we vary  $\gamma$  over a domain from 1 to 16, implying intertemporal substitution elasticities between unity and 0.25. The relationship between  $\gamma$  and  $R_{nmss}$  is again almost linear, and again relatively high values rule out a monetary steady state. We conclude that the condition for existence of a monetary steady state is somewhat sensitive to the value of this parameter.

In Figure 8 we vary the capital share of output,  $\alpha$ , over a range from .2 to .4, the typical range used in empirical studies. While increases in this parameter tend to increase  $R_{nmss}$ , the gradient is sufficiently small that we conclude that this parameter has little effect on the potential existence of a monetary steady state.

The effects of varying the gross population growth rate  $\psi$  are displayed in Figure 9, which plots the values of  $R_{nmss}$  associated with net population growth rates from 0 to 10 percent. As in the case of the technology growth parameter  $\lambda$ , increases in  $\psi$  increase both the output growth rate and the nonmonetary steady state rate of interest. In this case, however, the latter rises more slowly than the former, so that the range of values of  $\theta$  consistent with the existence of a monetary steady state broadens markedly as  $\psi$  increases.

Figure 10 displays the effects of changes in the depreciation rate  $\delta$  which we vary over a domain from 0 to 10 percent. Reducing the depreciation rate tends to increase  $R_{nmss}$  and narrow the range of  $\theta$ -values consistent with existence of a monetary steady state. Except for values quite close to zero (less than 0.005),  $R_{nmss}$  is less than the output growth rate for all values of  $\delta$  in the range we examine.

In Figure 11 we report on the implications of imposing forced retirement on agents. As we noted above, in the equilibria we study agents choose to “retire” (provide zero labor) during one or more periods of their lives. In our baseline case an agent retires in the very last period of life only. This result is sensitive to the interest rate the agent faces in the monetary steady state, however. If we choose  $\theta = 1$  in our baseline case, so that the net real interest rate in the monetary steady state rises from 0 to 3 percent, the agents will retire during the last 11 periods of life. We can solve agents decision problems if they are constrained to retire for the rest of their lives, beginning in a particular period. In Figure 11 we report the results of experiments in which we impose retirement beginning at a range of ages that varies from 62 to 76 (period 55 = age 76 in this interpretation) so that the agent must retire for 14, 13, ..., 2, 1, 0 periods. We find that reducing the age of forced retirement tends to decrease  $R_{nmss}$  but that the effect is very weak. Part of the reason for this is that even when agents do not retire outright late in life they are often semi-retired, which is to say that they provide relatively small amounts of labor. In our baseline case, for example, during the last ten periods of life an agent devotes an average of 6 percent of the time endowment to labor; this compares to an average of more than 20 percent for an

Parameter	Range	Effect of increase	Critical pt.
Number of periods in life, $n$	2 to 110	Decreases $R_{nmss}$	3
Rate of technological progress, $\lambda$	1.0 to 1.1	Increases $R_{nmss}$	.065
Net rate of time preference, $\rho$	-.05 to .05	Increases $R_{nmss}$	-.006
Preference curvature, $\gamma$	1 to 16	Increases $R_{nmss}$	12
Capital share, $\alpha$	.2 to .4	Increases $R_{nmss}$	None
Rate of population growth, $\psi$	1.0 to 1.1	Decreases $R_{nmss}$	None
Net depreciation rate, $\delta$	0 to .1	Decreases $R_{nmss}$	.005
Age of forced retirement	62 to 76	Increases $R_{nmss}$	None
Labor share of time endowment, $\eta$	.2 to .4	Increases $R_{nmss}$	None

Table 3: Effects of changing particular parameters. The last column shows the point beyond which a monetary steady state ceases to exist (for  $R_{nmss}$  increasing) or begins to exist (for  $R_{nmss}$  decreasing). "None" implies that a monetary steady state exists for all values within the range considered.

entire life.

Figure 12 reports the impact on  $R_{nmss}$  of changes in the value of the parameter  $\eta$  which governs the average share of an agent's time endowment that is devoted to labor. We vary  $\eta$  over a domain from .2 to .4, and we find that this parameter has little effect on the prospects for existence of a monetary steady state. Lower values of  $\eta$  do seem to lower the nonmonetary steady state rate of interest somewhat, as suggested by our earlier discussion. Figure 13 shows that the baseline case value of  $\eta$ , .2, is about right if the goal is to keep the fraction of time agents devote to the market at about .22, which according to our calculations is approximately the correct figure.

We summarize the findings of our sensitivity experiments in Table 3.

We conclude from this discussion that our baseline case is reasonably robust in the sense that many of the individual parameters can be varied within the specified ranges without disturbing the prospects for a monetary steady state. The two parameters that seem to be most important for the existence condition, based on Table 3, are the preference parameters  $\rho$  and  $\gamma$ .<sup>19</sup> We remark that low values of  $\gamma$  and low values of  $\rho$  tend to be substitutes in supporting low values of  $R_{nmss}$ . If we chose  $\gamma = 1$ , for example, the threshold value of  $\rho$  rises to +0.014.

<sup>19</sup>There is, of course, an interrelationship between the parameter values we choose for our baseline case and our conclusions about the sensitivity of our existence results to changes in the values of other parameters.



## 6 Remarks

In this section we comment on three issues that arise in connection with the discussion presented in the previous sections; namely, alternative models of low real interest rate equilibria, monetary neutrality and superneutrality, and rate of return dominance. In two associated appendices, we comment further on two additional issues: in Appendix B we discuss empirical estimates of the rate of time preference, while in Appendix C we provide remarks on the question of dynamic efficiency.

### 6.1 Alternative approaches to low real interest rates

Estimation strategies based on infinite horizon representative agent models have tended to find negative estimates of the rate of time preference (see Appendix B). This fact was not initially reflected in the research of investigators working with calibrated versions of these same models. The main reason for this seems to be that these models were typically specified without growth—that is, so as to produce steady states with constant average consumption—and in this case the agent’s decision problem has a solution only if the rate of time preference is positive. Kocherlakota (1990a) and Benninga and Protopapadakis (1990) have shown that in infinite horizon models with positive steady state growth rates solutions exist even with negative rates of time preference. This point has led to the development of a literature that uses infinite horizon representative agent models calibrated with negative (or positive but very small) rates of time preference in order to explain both the relatively low average level of safe, short-term interest rates (the ‘risk-free rate puzzle’) and the relatively high average level of rates of return on equity (the ‘equity premium puzzle’).<sup>20</sup> Papers in this literature include Benninga and Protopapadakis (1990), Kocherlakota (1990b), and Kandel and Stambaugh (1991). To date, however, these models have typically employed parameter values outside the range estimated in the empirical literature: either quite negative rates of time preference, very high coefficients of relative risk aversion, or both, in order to generate low risk-free real rates.<sup>21</sup>

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<sup>20</sup>See Mehra and Prescott (1985) and Weil (1989).

<sup>21</sup>Weil (1989) considers only positive rates of time preference and is unable to duplicate the observed risk-free interest rate and equity premium in a model with non-expected utility preferences. Kocherlakota (1990b) chooses  $\beta = 1.139$  ( $\rho \cong -0.122$ ) and a coefficient of relative risk aversion of 13.7. Benninga and Protopapadakis (1990) find that values of the coefficient of relative risk aversion from 6 to 19 coupled with values of  $\beta$  from 1.08 to 1.13 can generate risk-free rates and equity premia that match the data; they prefer

In overlapping generations models, unlike infinite-horizon models, there is no theoretical requirement that the rate of time preference be positive, whether or not there is growth. Moreover, as we have argued above (and in Appendix B), the parameter values necessary to generate relatively low real interest rates are not inconsistent with point estimates from the empirical literature. Thus we think of plausibly calibrated multiperiod overlapping generations models as an alternative solution to the risk-free rate puzzle.

## 6.2 Neutrality and superneutrality

An important issue in monetary economics is whether it is possible for plausibly specified dynamic general equilibrium models to generate certain “stylized facts” that involve, or seem to involve, instances of monetary non-neutrality. Examples of such stylized facts include “liquidity effects”—the tendency of nominal interest rates to decline in response to a monetary injection—and “sluggish price adjustment”—the tendency of the price level to respond relatively slowly to money growth rate increases. Infinite horizon representative agent models that generate money demand via special assumptions (typically, through a cash-in-advance constraint) have some difficulty replicating these sorts of facts.<sup>22</sup>

The multiperiod overlapping generations model studied in this paper shares, along with other models specified in the spirit of Diamond (1965), a richer set of potential non-neutralities than are present in typical cash-in-advance models. Permanent increases in the rate of growth of unbacked liabilities are *neutral* in the sense that have no effect on the steady state output growth rate. Such increases are not, however, *superneutral*: they reduce the steady state rate of interest and increase the steady state capital-labor ratio, leading to an increase in the *level* of output and a change in the nature of the intertemporal consumption-leisure allocation. Russell (1993) has shown in the context of a two period

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the case where the coefficient of relative risk aversion is approximately 10 and  $\beta \cong 1.11$  which matches the data along additional dimensions. Kandel and Stambaugh (1991) choose  $\beta$  slightly below unity and a coefficient of relative risk aversion of 29.

Constantinides (1990) inserts preferences characterized by habit persistence into an otherwise standard infinite horizon representative agent and is able, for certain parameter values, to explain both the observed risk-free real interest rate and observe the equity premium. In Constantinides' (1990) analysis, the ‘subsistence’ level of consumption—the level at which the marginal utility of consumption becomes infinite—is about 80 percent of the level of consumption. The resulting willingness of households to take extreme measures to avoid relatively moderate consumption declines is central to Constantinides' success.

<sup>22</sup>On liquidity effects, see Christiano (1991) and Christiano and Eichenbaum (1992); regarding sluggish price response, see Blanchard (1989), Sims (1992), and Matheny (1993).

stochastic pure exchange overlapping generations model that the real interest rate effect of a monetary injection can produce small liquidity effects resulting from permanent monetary growth increases and much magnified effects resulting from temporary ones. We suspect that these results can be extended to stochastic versions of models like the one specified here. In addition, the fact that in overlapping generations models with capital changes in money growth rates can produce significant increases in the level of output suggests the possibility that these models can produce instances of sluggish price adjustment. These possibilities should, we think, encourage future research in monetary multiperiod overlapping generations models.

### 6.3 Rate of return dominance

The model we study in this paper generates steady state equilibria in which an unbacked government liability is valued and held by agents. We have followed the theory literature by describing these equilibria as monetary steady states. While the empirical analog of the category “unbacked government liabilities” certainly includes base money, it also includes, presumably, a large stock of unbacked Treasury debt. There is nothing in our model that allows us to distinguish between these two types of liabilities. Because of this, “money” is not dominated in rate of return by “debt” in the model, in contrast to a well established fact about actual economies.

The questions of the source of rate-of-return dominance and of the proper way to obtain it in dynamic general equilibrium models remain subjects of debate. Wallace’s (1983) argument was that whatever the observed differences in the *uses* of base money and government debt (that is, base money is used as a medium of exchange, and government debt for the most part is not), the *sources* of the differences between them (including differences in their rates of return) must be legal restrictions—in particular, restrictions on private intermediation. Absent such restrictions, private intermediation would produce large stocks of currency backed by government debt, and arbitrage would drive the rates of return on these items to approximate equality, producing nominal interest rates close to zero.

It follows from this argument that one way to produce rate-of-return dominance in dynamic general equilibrium models is to impose legal restrictions on private intermediation. Russell (1993) and Espinosa and Russell (1993) have used two period, pure exchange over-

lapping generations models to show that in specifications in which unbacked liabilities would have value under *laissez faire*, and there are legal restrictions that produce rate of return dominance, both permanent and temporary monetary injections can produce liquidity effects. These papers provide examples of a more general point made by Mourmouras and Russell (1992): the qualitative results of policy analyses may hinge on whether unbacked liabilities have value under *laissez faire* even if the equilibria under study are not *laissez faire* equilibria. Taken together, these findings, combined with the demonstration in this paper that monetary steady states may exist in *laissez faire* versions of empirically plausible multiperiod overlapping generations models, suggest that future research which uses legal restrictions to produce rate-of-return dominance in such models is likely to produce results that are interesting and useful.

## 7 Summary

We have analyzed a multiperiod overlapping generations economy with production, capital, and related features. The model is specified and calibrated so as to be empirically plausible. We find that the model has steady state equilibria in which unbacked liabilities have value—*monetary steady states*—and that both the monetary and nonmonetary steady states are unique. We make three principle points about the model and these equilibria. First, under our *variable time period* interpretation of the model, the features of its equilibria, including the existence and nature of its monetary steady state, are robust to changes in the number of periods in agents lives—a finding that stands in contrast to earlier work that adopts a fixed time period interpretation. Second, the monetary steady state of the baseline calibration of our model can account for a stylized fact that is often regarded as puzzling: real interest rates on short term, essentially riskless securities are “startlingly low.” This steady state is also consistent with a number of other stylized facts that have been difficult to explain in the context of a single equilibrium of a single model. Finally, the existence of monetary steady states is robust to a wide range of changes in the values of the model’s parameters, although large changes in the values of certain key parameters such as the rate of time preference and the coefficient of relative risk aversion can rule out monetary steady states.

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## A Derivation of the condition for existence of a monetary steady state

In this appendix, we present a complete derivation of the condition for existence of a monetary steady state in the model with capital and technological progress. We derive the condition under the assumption that the solution to the agents' maximization problem is interior. We comment on this in the text.

Agents maximize

$$U = \sum_{i=1}^{n-1} (1-\gamma)^{-1} \beta^i \left[ c_t(t+i)^\eta \ell_t(t+i)^{1-\eta} \right]^{1-\gamma}$$

where  $\ell$  is leisure,  $c$  is consumption,  $\gamma$  governs the curvature of the utility function,  $\eta$  controls the share of the agent's time that is devoted to market activities, and  $\beta \equiv \frac{1}{1+\rho}$  is the discount factor where  $\rho$  is the rate of time preference. The agents face the constraint

$$c_t(t) + \sum_{i=1}^{n-1} c_t(t+i) \prod_{j=0}^{i-1} R(t+j)^{-1} \leq w(t)e_1 [\bar{\ell} - \ell_t(t)] + \sum_{i=1}^{n-1} w(t+i)e_i [\bar{\ell} - \ell_t(t+i)] \prod_{j=0}^{i-1} R(t+j)^{-1},$$

provided  $\ell_t(t+i) \leq \bar{\ell}$  for  $i = 0, \dots, n-1$ . Again, for the purposes of this appendix, we simply assume that this latter inequality holds and that the agents' problem has an interior solution. The basic approach is to find the first order conditions pairwise by  $c_t(t+i)$ ,  $\ell_t(t+i)$  for  $i = 0, \dots, n-1$ ; solve for  $\ell_t(t+i)$  in terms of  $c_t(t+i)$ ; and then solve for  $c_t(t+i)$  in terms of  $c_t(t)$ . The first order conditions imply

$$\ell_t(t+i) = \left( \frac{1-\eta}{\eta} \right) \frac{c_t(t+i)}{w(t+i)e_{i+1}}$$

for  $i = 0, \dots, n-1$ , and so

$$c_t(t+i) = c_t(t) \left[ \frac{w(t+i)e_{i+1}}{w(t)e_1} \right]^{\frac{(1-\gamma)(\eta-1)}{\gamma}} \beta^{i/\gamma} \prod_{j=0}^{i-1} R(t+j)^{1/\gamma}$$

for  $i = 1, \dots, n-1$ . Now use these equations in the budget constraint to obtain an expression in terms of wages and interest rates for  $c_t(t)$ . This gives

$$c_t(t) = E_1 \equiv \frac{\eta \left[ w(t)e_1 \bar{\ell} + \sum_{i=1}^{n-1} w(t+i)e_{i+1} \bar{\ell} \prod_{j=0}^{i-1} R(t+j)^{-1} \right]}{1 + \sum_{i=1}^{n-1} \left[ \frac{w(t+i)e_{i+1}}{w(t)e_1} \right]^{\frac{(1-\gamma)(\eta-1)}{\gamma}} \beta^{i/\gamma} \prod_{j=0}^{i-1} R(t+j)^{(1-\gamma)/\gamma}}$$

For later use, we define this quantity displaced in time as

$$E_h \equiv \frac{\eta \left[ w(t+h-1)e_1\bar{\ell} + \sum_{i=1}^{n-1} w(t+i+h-1)e_{i+1}\bar{\ell} \prod_{j=0}^{i-1} R(t+j+h-1)^{-1} \right]}{1 + \sum_{i=1}^{n-1} \left[ \frac{w(t+i+h-1)e_{i+1}}{w(t+h-1)e_1} \right]^{\frac{(1-\gamma)(\eta-1)}{\gamma}} \beta^{i/\gamma} \prod_{j=0}^{i-1} R(t+j+h-1)^{(1-\gamma)/\gamma}}$$

so that  $h = 1 \Rightarrow E_1$ .

We now consider individual savings amounts. These can now be written as

$$s_t(t) = w(t)e_1\bar{\ell} - \frac{1}{\eta}E_1$$

$$s_t(t+1) = w(t+1)e_2\bar{\ell} + R(t)s_t(t) - \frac{1}{\eta}E_1 \left[ \frac{w(t+1)e_2}{w(t)e_1} \right]^{\frac{(1-\gamma)(\eta-1)}{\gamma}} \beta^{1/\gamma} R(t)^{1/\gamma}$$

...

$$s_t(t+n-2) = w(t+n-2)e_{n-1}\bar{\ell} + R(t+n-3)s_t(t+n-3)$$

$$- \frac{1}{\eta}E_1 \left[ \frac{w(t+n-2)e_{n-1}}{w(t)e_1} \right]^{\frac{(1-\gamma)(\eta-1)}{\gamma}} \beta^{(n-2)/\gamma} \prod_{j=0}^{n-3} R(t+j)^{1/\gamma}.$$

Using the assumption that all generations are alike, we can now postdate these amounts to time  $t$ , substitute  $s_t(t) = s_{t-1}(t-1)$ , etc., adjust for population growth (the generation born at time  $t = 1$  has size one), and rearrange to obtain

$$s_t(t) = \psi^{t-1} \left[ w(t)e_1\bar{\ell} - \frac{1}{\eta}E_1 \right]$$

$$s_{t-1}(t) = \psi^{t-2} \left[ w(t)e_2\bar{\ell} + R(t-1)w(t-1)e_1\bar{\ell} \right]$$

$$- \psi^{t-2} \left[ \frac{1}{\eta}E_0 R(t-1) \right]$$

$$- \psi^{t-2} \left[ \frac{1}{\eta}E_0 \left[ \frac{w(t)e_2}{w(t-1)e_1} \right]^{\frac{(1-\gamma)(\eta-1)}{\gamma}} \beta^{1/\gamma} R(t-1)^{1/\gamma} \right]$$

$$s_{t-2}(t) = \psi^{t-3} [w(t)e_3\bar{\ell} + R(t-1)w(t-1)e_2\bar{\ell} + R(t-1)R(t-2)w(t-2)e_1\bar{\ell}]$$

$$-\psi^{t-3} \left[ \frac{1}{\eta} E_{-1} R(t-1)R(t-2) \right]$$

$$-\psi^{t-3} \left[ \frac{1}{\eta} E_{-1} R(t-1) \left[ \frac{w(t-1)e_2}{w(t-2)e_1} \right]^{\frac{(1-\gamma)(\eta-1)}{\gamma}} \beta^{1/\gamma} R(t-2)^{1/\gamma} \right]$$

$$-\psi^{t-3} \left[ \frac{1}{\eta} E_{-1} \left[ \frac{w(t)e_3}{w(t-2)e_1} \right]^{\frac{(1-\gamma)(\eta-1)}{\gamma}} \beta^{2/\gamma} [R(t-2)R(t-1)]^{1/\gamma} \right]$$

...

and so on up to  $s_{t-n+2}(t)$ . We want to write these amounts in terms of a single variable,  $R(t)$ , and to do this we must first find an expression for the wage. For this reason, we now turn to the production side of the economy.

We assume output is produced according to a constant returns to scale production function using labor and capital as inputs. In particular, we assume output is produced according to

$$Y(t) = \lambda^{(t-1)(\alpha-1)} L(t)k(t)^\alpha$$

where  $Y(t)$  is output at time  $t$ ,  $L(t)$  is the aggregate labor input at time  $t$ ,  $k(t)$  is the capital-labor ratio  $K(t)/L(t)$ ,  $K(t)$  is the aggregate capital input,  $\lambda$  is the gross rate of technological progress, and  $\alpha$  is the capital share of output. The wage is given by

$$w(t) = \lambda^{(t-1)(1-\alpha)} (1-\alpha)k(t)^\alpha.$$

The rental rate on capital is given by

$$r(t) = \lambda^{(t-1)(1-\alpha)} \alpha k(t)^{\alpha-1}.$$

Arbitrage requires

$$R(t) = 1 - \delta + r(t+1)$$

so that

$$k(t) = \lambda^{t-1} \left[ \frac{R(t-1) - 1 + \delta}{\alpha} \right]^{\frac{1}{\alpha-1}}.$$

Hence, when  $R(t) = R \forall t$ , the capital-labor ratio grows at a constant gross rate  $\lambda$ . We deduce that

$$w(t) = \lambda^{t-1}(1 - \alpha) \left[ \frac{R(t-1) - 1 + \delta}{\alpha} \right]^{\frac{\alpha}{\alpha-1}},$$

and thus the wage also grows at constant gross rate  $\lambda$  when  $R(t) = R \forall t$ . We can use the facts to substitute appropriately in  $E_h$  and into the individual savings functions given above. Then aggregate savings at time  $t$ ,  $S(t)$ , is given by

$$S(t) = \sum_{i=0}^{n-1} s_{t-i}(t)$$

which is in turn a function of interest rates alone.

We now turn to the stationary case where  $R(t) = R \forall t$ . We make use of the assumption that the integer  $t \in (-\infty, +\infty)$ , that is, that time is doubly infinite, and assume the economy has been in this steady state for all time.

We begin by noting that in the stationary case,

$$w(t) = \lambda^{t-1}w(1)$$

where

$$w(1) = (1 - \alpha) \left[ \frac{R - 1 + \delta}{\alpha} \right]^{\frac{\alpha}{\alpha-1}}.$$

We then define

$$E = \frac{\eta \bar{\ell} w(1) \sum_{i=1}^n \lambda^{i-1} R^{1-i} e_i}{\sum_{i=0}^{n-1} x_i R^{-i}}$$

where

$$x_i = \left[ \frac{\lambda^i e_{i+1}}{e_1} \right]^{\frac{(1-\gamma)(n-1)}{\gamma}} \beta^{i/\gamma} R^{i/\gamma}$$

for  $i = 0, \dots, n-1$ . We then note that, in the stationary case,

$$E_h = \lambda^{h-1} E_1 = \lambda^{h-1} E.$$

We are now ready to write out, in the stationary case, the aggregate savings function at an arbitrary date  $t$ :

$$s_t(t) = \psi^{t-1} \left[ \lambda^{t-1} w(1) e_1 \bar{\ell} - \frac{1}{\eta} \lambda^{t-1} E \right]$$

$$\begin{aligned}
s_{t-1}(t) &= \psi^{t-2} \left[ \lambda^{t-1} w(1) e_2 \bar{\ell} + \lambda^{t-2} w(1) e_1 \bar{\ell} R \right] \\
&\quad - \psi^{t-2} \left[ \frac{1}{\eta} \lambda^{t-2} E R \right] \\
&\quad - \psi^{t-2} \left[ \frac{1}{\eta} \lambda^{t-2} E \left[ \frac{\lambda e_2}{e_1} \right]^{\frac{(1-\gamma)(\eta-1)}{\gamma}} \beta^{1/\gamma} R^{1/\gamma} \right] \\
s_{t-2}(t) &= \psi^{t-3} \left[ \lambda^{t-1} w(1) e_3 \bar{\ell} + \lambda^{t-2} w(1) e_2 \bar{\ell} R + \lambda^{t-3} w(1) e_1 \bar{\ell} R^2 \right] \\
&\quad - \psi^{t-3} \left[ \frac{1}{\eta} \lambda^{t-3} E R^2 \right] \\
&\quad - \psi^{t-3} \left[ \frac{1}{\eta} \lambda^{t-3} E R \left[ \frac{\lambda e_2}{e_1} \right]^{\frac{(1-\gamma)(\eta-1)}{\gamma}} \beta^{1/\gamma} R^{1/\gamma} \right] \\
&\quad - \psi^{t-3} \left[ \frac{1}{\eta} \lambda^{t-3} E \left[ \frac{\lambda^2 e_3}{e_1} \right]^{\frac{(1-\gamma)(\eta-1)}{\gamma}} \beta^{2/\gamma} R^{2/\gamma} \right] \\
&\quad \dots
\end{aligned}$$

and so on up to  $s_{t-n+2}(t)$ . We note that  $\lambda^{t-1}\psi^{t-1}$  enters as a factor in each of these individual savings functions. Thus, in a stationary case,

$$S(t) = (\lambda\psi)^{t-1} S(1)$$

where  $S(1)$  is the sum of

$$s_1(1) = w(1) e_1 \bar{\ell} - \frac{1}{\eta} E$$

$$s_0(1) = \lambda^{-1} \psi^{-2} w(1) e_2 \bar{\ell} + (\lambda\psi)^{-2} w(1) e_1 \bar{\ell} R$$

$$-(R + x_1)(\lambda\psi)^{-2}\frac{1}{\eta}E$$

$$s_{-1}(1) = \lambda^{-1}\psi^{-3}w(1)e_3\bar{\ell} + \lambda^{-2}\psi^{-3}w(1)e_2\bar{\ell}R + (\lambda\psi)^{-3}w(1)e_1\bar{\ell}R^2$$

$$-(R^2 + Rx_1 + x_2)\frac{1}{\eta}(\lambda\psi)^{-3}E$$

...

and so on up to  $s_{2-n}(1)$ . This sum is given by

$$S(1) = w(1)\bar{\ell} \sum_{i=1}^n e_i \sum_{j=1}^{n-i-1} R^{j-1}(\lambda\psi)^{1-j} - \frac{1}{\eta}E - \frac{1}{\eta}E \sum_{i=1}^{n-3} (\lambda\psi)^{-i} \sum_{j=0}^i R^j x_{i-j}.$$

We now turn to finding the entire condition for existence of a monetary steady state. Aggregate savings in this economy consists of consumption loans, capital holdings, and holdings of unbacked government liabilities. Consumption loans net out in equilibrium. If we subtract capital holdings from aggregate savings, we will be left with holdings of unbacked liabilities, which are described by

$$\frac{H(t)}{P(t)} = S(t) - K(t+1).$$

In a steady state, aggregate capital  $K(t)$  grows at rate  $\lambda\psi$ , as does aggregate savings, so that the condition for existence of a monetary steady state is

$$S(1) - \lambda\psi K(1) > 0.$$

Thus, we want an expression for  $K(1)$  in terms of stationary interest rates  $R$ . To obtain this, we note that  $k(1) = K(1)/L(1)$  where

$$k(1) = \left[ \frac{R - 1 + \delta}{\alpha} \right]^{\frac{1}{\alpha-1}}.$$

We can find  $L(1)$  as follows. Labor input at time  $t$  is (in the general case)

$$L(t) = \sum_{i=0}^{n-1} \psi^{t-n+i} e_{n-i} [\bar{\ell} - \ell_{t-n+i+1}(t)]$$

implying that

$$L(1) = \sum_{i=0}^{n-1} \psi^{1-n+i} e_{n-i} [\bar{\ell} - \ell_{i-n+2}(1)].$$

The first order conditions imply that the stationary values for the leisure choices are given by

$$\ell_{1-i}(1) = \frac{1-\eta}{\eta} \frac{\lambda^{-i} E x_i}{w(1) e_{i+1}}$$

for  $i = 0, \dots, n-1$ . Thus we have

$$L(1) = \sum_{i=1}^n \psi^{i-1} e_i \left[ \bar{\ell} - \left( \frac{1-\eta}{\eta} \right) \frac{E x_{i-1}}{w(1) e_i} \right].$$

The entire condition for existence of a monetary steady state is thus

$$S(1) - \lambda \psi k(1) L(1) > 0.$$

This is the condition analyzed extensively in the text. The condition, written in terms of stationary  $R$ , is:

$$\begin{aligned} & e_1 \left\{ w(1) \left[ \left( 1 + \frac{R}{\lambda \psi} + \dots + \frac{R^{n-2}}{(\lambda \psi)^{n-2}} \right) - B \right] - \lambda \psi k(1) [1 - (1-\eta)C] \right\} \\ & + e_2 \left\{ w(1) \left[ \frac{1}{\psi} \left( 1 + \frac{R}{\lambda \psi} + \dots + \frac{R^{n-3}}{(\lambda \psi)^{n-3}} \right) - \frac{\lambda}{R} B \right] - \lambda \psi k(1) \left[ \frac{1}{\psi} - \frac{\lambda}{R} (1-\eta)C \right] \right\} \\ & + e_3 \left\{ w(1) \left[ \frac{1}{\psi^2} \left( 1 + \frac{R}{\lambda \psi} + \dots + \frac{R^{n-4}}{(\lambda \psi)^{n-4}} \right) - \frac{\lambda^2}{R^2} B \right] - \lambda \psi k(1) \left[ \frac{1}{\psi^2} - \frac{\lambda^2}{R^2} (1-\eta)C \right] \right\} \\ & \quad \quad \quad + \dots \\ & + e_{n-2} \left\{ w(1) \left[ \frac{1}{\psi^{n-3}} \left( 1 + \frac{R}{\lambda \psi} \right) - \frac{\lambda^{n-3}}{R^{n-3}} B \right] - \lambda \psi k(1) \left[ \frac{1}{\psi^{n-3}} - \frac{\lambda^{n-3}}{R^{n-3}} (1-\eta)C \right] \right\} \\ & + e_{n-1} \left\{ w(1) \left[ \frac{1}{\psi^{n-2}} - \frac{\lambda^{n-2}}{R^{n-2}} B \right] - \lambda \psi k(1) \left[ \frac{1}{\psi^{n-2}} - \frac{\lambda^{n-2}}{R^{n-2}} (1-\eta)C \right] \right\} \end{aligned}$$



$$+e_n \left\{ w(1) \left[ -\frac{\lambda^{n-1}}{R^{n-1}} B \right] - \lambda \psi k(1) \left[ \frac{1}{\psi^{n-1}} - \frac{\lambda^{n-1}}{R^{n-1}} (1 - \eta) C \right] \right\} > 0,$$

where

$$B = \frac{\left(1 + \frac{R}{\lambda\psi} + \dots + \frac{R^{n-2}}{(\lambda\psi)^{n-2}}\right) + \frac{x_1}{\lambda\psi} \left(1 + \frac{R}{\lambda\psi} + \dots + \frac{R^{n-3}}{(\lambda\psi)^{n-3}}\right) + \dots + \frac{x_{n-2}}{(\lambda\psi)^{n-2}}}{1 + \frac{x_1}{R} + \frac{x_2}{R^2} + \dots + \frac{x_{n-1}}{R^{n-1}}}$$

and

$$C = \frac{1 + \frac{x_1}{\lambda\psi} + \frac{x_2}{(\lambda\psi)^2} + \dots + \frac{x_{n-1}}{(\lambda\psi)^{n-1}}}{1 + \frac{x_1}{R} + \frac{x_2}{R^2} + \dots + \frac{x_{n-1}}{R^{n-1}}}.$$

If  $R = \lambda\psi$ , this condition becomes

$$\begin{aligned} & e_1 \{ w(1) [(n-1) - B] - \lambda \psi k(1) \eta \} \\ & + e_2 \left\{ w(1) \left[ \frac{1}{\psi} (n-2) - \frac{1}{\psi} B \right] - \lambda \psi k(1) \frac{\eta}{\psi} \right\} \\ & + e_3 \left\{ w(1) \left[ \frac{1}{\psi^2} (n-3) - \frac{1}{\psi^2} B \right] - \lambda \psi k(1) \frac{\eta}{\psi^2} \right\} \\ & \quad \quad \quad + \dots \\ & + e_{n-2} \left\{ w(1) \left[ \frac{2}{\psi^{n-3}} - \frac{1}{\psi^{n-3}} B \right] - \lambda \psi k(1) \frac{\eta}{\psi^{n-3}} \right\} \\ & + e_{n-1} \left\{ w(1) \left[ \frac{1}{\psi^{n-2}} - \frac{1}{\psi^{n-2}} B \right] - \lambda \psi k(1) \frac{\eta}{\psi^{n-2}} \right\} \\ & + e_n \left\{ w(1) \left[ -\frac{1}{\psi^{n-1}} B \right] - \lambda \psi k(1) \frac{\eta}{\psi^{n-1}} \right\} > 0, \end{aligned}$$

where

$$B = \frac{(n-1) + \frac{x_1}{\lambda\psi} (n-2) + \dots + \frac{x_{n-2}}{(\lambda\psi)^{n-2}}}{1 + \frac{x_1}{\lambda\psi} + \frac{x_2}{(\lambda\psi)^2} + \dots + \frac{x_{n-1}}{(\lambda\psi)^{n-1}}},$$

$$k(1) = \left[ \frac{\lambda\psi - 1 + \delta}{\alpha} \right]^{\frac{1}{\alpha-1}},$$

$$w(1) = (1 - \alpha) k(1)^\alpha,$$

and

$$x_i = \left[ \frac{\lambda^i e_{i+1}}{e_1} \right]^{\frac{(1-\gamma)(n-1)}{\gamma}} (\lambda\psi\beta)^{i/\gamma}$$

for  $i = 0, \dots, n - 1$ .

## B Estimates of the rate of time preference

There is a long tradition of assuming that the rate of time preference must be positive. Both the notion of time preference and the assumption of positive time preference grew out of the attempts of classical economists like Böhm-Bawerk (1891) to provide a psychological basis for the apparent prevalence of positive real interest rates. Although Fisher (1930) pointed out that positive time preference was neither necessary nor sufficient for positive real rates, the assumption had intuitive appeal and remained largely unchallenged until relatively recently. Becker and Stigler (1977) made a brief but sharp attack on the assumption that the rate of time preference must be positive. Baily and Olson (1980) devoted an entire paper to the subject. They concluded that the classical case for a presumption of positive time preference was unconvincing, and repeated Fisher's conclusion that this presumption was not necessary for positive real interest rates.

Research conducted during the 1980s produced some empirical evidence that the rate of time preference might be negative. This evidence can be summarized succinctly: given the average growth rates of household consumption, the average real interest rates on safe, short-term assets are too low to be consistent with positive rates of time preference. This phenomenon was noted by investigators attempting to estimate infinite horizon representative agent models using aggregate time series data, and also by investigators attempting to estimate infinite or finite horizon models using pooled data. Examples of papers that report negative estimates of time preference rates obtained using infinite horizon representative agent models and aggregate time series data are Hayashi (1982), Hansen and Singleton (1983), Mankiw, Rotemberg, and Summers (1985), Eichenbaum, Hansen, and Singleton (1988), and Singleton (1990). Examples of papers that report negative time preference rate estimates obtained using pooled data are MaCurdy (1981), Courant, Gramlich, and Laitner (1986), Hotz, Kydland, and Sedlacek (1988), and Hurd (1989).

During the early 1990s there have been several additional studies that attempt to es-

timate rates of time preference and other utility function parameters using Euler equation methods. Three of these studies, Lawrance (1991), Engen (1991), and Dynan (1993), use pooled data (from the Panel Study of Income Dynamics). Each study, however, reports different estimates of the average rate of time preference for households in their sample. In the case of Lawrance and Dynan, several of these estimates are negative. Engen, however, reports estimates that range between +4 and +8 percent.<sup>23</sup>

Runkle (1991) has noted a problem that can lead to overestimates of the rate of time preference in studies that use the estimation methods employed by these authors. The estimates of the rate of time preference they report are increasing functions of the estimate of the variance of the forecast errors agents commit as they attempt to satisfy their Euler conditions. Runkle (1991) argues that measurement error in the variables contained in the empirical versions of the Euler equations can lead to overestimates of the forecast error variance; these overestimates will produce, in turn, overestimates of the rate of time preference.<sup>24</sup>

Another recent study, Epstein and Zin (1991), estimates the parameters of a representative agent infinite horizon model and reports mostly negative estimates of the rate of time preference. This study illustrates another possible source of overestimation of the time preference rate. The estimates of the rate obtained by Epstein and Zin (1991) when they use instruments robust to Hall's (1988) critique are substantially lower (more negative) than their other estimates. This suggests that the same intercorrelation problems that lead, according to Hall (1988), to overestimates of the intertemporal elasticity of substitution may also lead to overestimates of the rate of time preference. The studies of Lawrance (1991), Engen (1991), and Dynan (1993) are also vulnerable to Hall's (1988) critique. In the case of these studies it is particularly easy to see how overestimation might arise, since their time

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<sup>23</sup>Lawrance (1991) reports negative estimates of  $\rho$  when she uses the passbook interest rate (as opposed to the Treasury bill rate) as the interest rate variable and does not use time dummies to pick up aggregate shocks; she obtains positive estimates in the other three cases. Lawrance (1991) also reports the results of two experiments in which the passbook rate is used as the interest rate variable for below median income households, and the Treasury bill rate is used for above median income households. She finds negative time preference rate estimates when time dummies are not used, and positive estimates otherwise. Dynan (1993) studies cases similar to the first four cases studied by Lawrance (1991). She obtains a negative estimate of  $\rho$  with the Treasury bill rate and the time dummies, and an estimate of zero when she uses the passbook rate and time dummies. Her other two estimates are positive.

<sup>24</sup>Runkle (1991) does not attempt to estimate the rate of time preference in his study, and does not comment on other attempts to estimate this parameter; instead, he simply notes a potential source of bias in estimates of the Euler equation forecast error.

preference rate estimates are also increasing functions of their estimates of  $\sigma$  (which are, in turn, considerably higher than Hall's 'maximum estimate'). This aspect of Hall's (1988) critique may, of course, also imply that earlier estimates of the time preference rate are also biased upward.

A different approach to the question of the sign of the rate of time preference is provided by recent research that attempts to account for a number of observations about consumer behavior that seem inconsistent with the predictions of the standard discounted additively separable utility model of intertemporal choice.<sup>25</sup> A recurrent finding of this literature is that individuals tend to behave differently when they perceive their actions as influencing the nature of a sequence of rewards or penalties rather than as influencing the value and timing of a single reward or penalty. In particular, while most individuals will prefer to receive a given (nondurable) reward earlier rather than later, most will also prefer, when confronted with a choice regarding the timing of a pair of rewards that are to be received in sequence, to receive the most attractive reward later rather than earlier. The first observation, it is argued, accounts for the widespread belief in positive time preference, while the second accounts for a variety of empirical observations that seem consistent with negative time preference. These latter observations include evidence from a variety of industries that the wage rate of an employee tends to rise more rapidly than productivity, survey evidence that workers seem to prefer increasing wage profiles to flat or decreasing profiles of equal or higher present value, and evidence that household consumption tends to increase over the life cycle until (and even beyond) retirement.<sup>26</sup>

A variety of theories have been proposed to explain this apparent tendency of individuals to prefer increasing sequences. These include the theories of "habit persistence" (Duesenberry, 1949), "loss aversion" (Kahneman and Tversky (1979), and "anticipatory savoring and dread" (Loewenstein, 1987).<sup>27</sup> Each of these theories posits departures from the conventional assumption that the utility derived from consumption is additively separable

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<sup>25</sup>A concise survey of a portion of this literature is provided by Loewenstein and Prelec (1991). See also Loewenstein and Thaler (1989).

<sup>26</sup>For descriptions of some of this evidence see Loewenstein and Thaler (1989), Loewenstein and Prelec (1991), and Loewenstein and Sicherman (1991). In our baseline case in the main text an agent's consumption grows at an average rate of 1.5 percent per year. This value is roughly consistent with the observed range of values for different age and income groups reported by Courant, Gramlich and Laitner (1986).

<sup>27</sup>Constantinides (1990) uses preferences that display habit persistence in an attempt to resolve the equity premium puzzle.

over time in an attempt, in part, to reconcile utility theory with the empirical evidence on increasing reward sequences.

For our purposes, it makes little difference whether observed consumption and savings behavior is driven by negative rates of time preference or by departures from standard separability assumptions for which the assumption that the time preference rate is negative can act as a proxy. What matters is the behavior and its implications for interest rates and asset demand.

## C Dynamic efficiency

The nonmonetary steady states that arise in our model are dynamically inefficient in that they involve steady state capital stocks in excess of those consistent with Pareto optimal consumption-leisure allocations. If the stock of outside government liabilities is fixed, the monetary steady states we study are dynamically efficient, but if the stock is growing, they are dynamically inefficient though still Pareto superior to the nonmonetary steady states. The possibility that dynamically inefficient steady states could arise in overlapping generations models with production and capital was noted by Diamond (1965). In deterministic overlapping generations models, steady states with relatively low real interest rates (real rates less than the output growth rate) are necessarily dynamically inefficient. The empirical evidence of relatively low real rates in the U.S. consequently comprises a *prima facie* case for the dynamic inefficiency of the U.S. economy. Abel, Mankiw, Summers, and Zeckhauser (1989) show that in stochastic versions of the Diamond model relatively low real interest rates need not imply dynamic inefficiency and propose an alternative efficiency test that is not based on real interest rates. They apply this test to data for the U.S. economy and conclude that it is dynamically efficient.

We cannot hope to resolve the empirical issues involving dynamic efficiency in this paper. We argue in this appendix, however, that the results of the Abel, *et al.*, (1989) test do not provide a convincing demonstration that the U.S. economy is dynamically efficient.

The Abel, *et al.*, (1989) efficiency test involves a comparison of gross returns to capital to gross investment; if the former invariably exceeds the latter, the relevant economy is dynamically efficient. Any empirical implementation of this test must be based on the

assumption that the “returns” which are aggregated to form the gross returns measure are solely attributable to the “capital” whose stock is being augmented by the gross investment measure. This assumption creates problems for the Abel, *et al.*, (1989) test. The Abel, *et al.*, (1989) estimate of the gross returns to capital is a residual—it is the difference between gross national income and their estimate of the returns to labor. If we assume that aggregate returns to scale are constant, this measure includes the returns to any factors of production whose services are not internalized within labor. Their investment measure, however, is gross private domestic investment<sup>28</sup> as measured by the Bureau of Economic Analysis (BEA), and thus includes only increments to the stocks of business and residual capital.

Abel, *et al.*, (1989) explicitly acknowledge one additional factor that might contribute to gross *ex-labor* returns—land. According to their estimates land rents are quite substantial, amounting, on average, to about 5 percent of GNP. While Abel, *et al.*, (1989) do not deduct estimates of the value of land rents from the return measures they presented in their Tables 1 and 2, they note that doing so would not affect the qualitative results of their efficiency tests.<sup>28</sup>

One factor of production that is entirely omitted from the Abel, *et al.*, (1989) analysis is tangible capital provided by the government. It can be argued that the bulk of the stock of tangible capital provided and maintained by the government contributes, directly or indirectly, to the returns to private business activity. Presumably, the government recovers all or part of the returns to its tangible capital through taxation of capital income.<sup>29</sup>

An omitted factor that may be quite important is intangible capital. Eisner (1985, 1988) has demonstrated the importance of intangible capital in the U.S. economy. According to Eisner (1985), in 1981 gross investment in intangible capital amounted to 28.8 percent of NIPA gross national product. The bulk of this intangible capital is human capital, however, and Eisner (1985) assumes that the returns to human capital are fully internalized within

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<sup>28</sup>The analysis presented here will not challenge the Abel, *et al.*, (1989) estimate of the value of land rents. It is worth noting, however, that their analysis with regard to land rents could be applied to natural resources of other sorts: mineral deposits (including oil and natural gas), water, timber, etc. While we have not attempted to estimate the returns to these sorts of resources, we suspect that they are also quite substantial. Wright (1990) argues that historically U.S. exports have tended to be relatively natural resource intensive, and that exploitation of nonreproducible natural resources (particularly oil and minerals) has played a key role in the growth of U.S. manufacturing output.

<sup>29</sup>Profits tax revenues, it should be noted, are included in the Abel, *et al.*, (1989) returns measure.

labor. Lucas (1988), in contrast, emphasizes the social aspect of human capital investment and argues that a substantial portion of the services of human capital may be external to labor. Presumably, most of the returns generated by these services are captured in the first instance by private firms, though some part of them may again be captured by the government (which provides large subsidies for human capital investment) through capital income taxation.

The assumption that a substantial fraction of human capital acts to augment the stock of external human capital, combined with recognition of the role played by land and government capital, can have profound implications for the results of dynamic efficiency tests of the sort conducted by Abel, *et al.*, (1989). We will now present some informal calculations that illustrate this possibility.

We begin by assuming that the stock of external intangible capital amounts to 50 percent of conventional (NIPA) gross domestic product. We will attempt to justify this assumption, *ex post*, through its implications for the level of investment in external intangible capital.<sup>30</sup> We use data on gross private domestic investment drawn from the NIPA accounts, and data on the government and business capital stocks drawn from BEA estimates of the stock of fixed nonreproducible tangible wealth. These data are expressed as shares of GNP and are averaged over the period 1948-1992. We can use the BEA's estimate of the average real GDP growth rate for the same period to infer the average rate of depreciation of business capital consistent with maintenance of a steady state in which the ratio of the stock of business capital to output is constant. Since Eisner's estimates of government investment in tangible capital are hard to interpret, we simply assume that government tangible capital depreciates at the same rate as its business counterpart and use the steady state relationship to infer the value of government investment in tangible capital.

We use the same steady state relationship, along with Eisner's estimates of the stock of and investment in consumer durables (for 1981), to obtain an estimate of the depreciation rate of consumer durables.<sup>31</sup> We then assume that the depreciation rate on external human

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<sup>30</sup>Eisner (1988) estimates the total stock of human capital in 1981 at 361 percent of NIPA GNP for that year—a number considerably in excess of his estimate of the total stock of tangible capital. In terms of this estimate, we are assuming that external intangible capital amounts to only 14 percent of the total stock of intangible capital. Eisner's (1988) estimate, however, is obtained under the assumption that human capital depreciates at an extremely low rate; we will assume that the relevant depreciation rate is higher.

<sup>31</sup>We prefer Eisner's estimates to those of the BEA because Eisner defines consumer durables more broadly,

capital is equal to the average of our estimated depreciation rate on business capital (which is relatively low) and our estimated depreciation rate on consumer durables (which is relatively high). We also assume that government taxation of capital income exactly captures the returns to external human capital, so that the gross real return to each type of tangible capital is equal to its marginal product times its stock. Finally, we assume that the net real return to each form of capital is exactly zero—as in the baseline calibration of our model—so that the marginal product of each variety of capital is equal to its depreciation rate.

By this calculation, aggregate profits—the sum of the returns to external intangible capital and the returns to business and government tangible capital—amount to 22.4 percent of NIPA GNP. The comparable Abel, *et al.*, (1989) figure is an almost identical 22.6 percent: a 27.6 percent average “profits” share for 1948-1992, less a 5 percent deduction for land rents. Since gross business investment amounted to 16.3 percent of GDP during this period, following Abel, *et al.*, (1989) by misattributing all capital income to business investment would indeed lead to the conclusion that the U.S. economy is dynamically efficient. Under our scenario, however, returns to business capital amount to only 10.8 percent of GDP, so that business capital is in fact overaccumulated and the U.S. economy is not dynamically efficient.

In our scenario, returns to government capital amount to 3.8 percent of NIPA GDP, and returns to external human capital amount to 7.8 percent. Government revenue from taxation of capital income consequently amounts to 11.6 percent of GDP—a bit more than half of gross capital income. This does not seem like an unreasonable figure. Historically, corporate profits taxes alone have amounted to 3.8 percent of NIPA GDP (the 1948-1992 average), which leaves other forms of capital income taxation (taxes on the income of proprietorships, state and federal income taxes, property taxes, etc.) capturing less than 8 percent of GDP and about 35 percent of total capital income.<sup>32</sup>

Our scenario implies that government investment in tangible capital amounts to 5.7 percent of GDP. This is not far from Eisner’s estimate of the volume of total government investment in tangible capital (including research and development spending), which is 5.4

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in a manner more consistent with the BEA definitions of business and government equipment.

<sup>32</sup>Currently, the tax receipts of all forms of government account for about 30 percent of NIPA GDP.



percent of GNP (in 1981). Investment in external human capital amounts to 9.4 percent of GDP in our scenario; this amounts to a bit less than a third of Eisner's estimate of total investment in tangible capital (in 1981).<sup>33</sup>

This scenario can be used to rationalize several aspects of the specification of our model, in which a single capital good can be thought of as a proxy for the four forms of capital identified above (business capital, government capital, external intangible capital, and consumer durables). The average depreciation rate on these forms of capital, weighted by their estimated stocks, is almost exactly 10 percent—the figure we use in our baseline calibration. If we include the return to consumer durables in the total return to capital, and augment GDP by (1) the services of consumer durables, (2) Eisner's estimate of the opportunity cost of higher education (foregone labor income), and (3) 50 percent of Eisner's estimate of the value of uncompensated domestic labor services, we obtain a capital share of 25.2 percent, which is close to the 25 percent share we use in our baseline case.<sup>34</sup>

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<sup>33</sup>Eisner's estimate of total government investment in human capital is 8.6 percent of GNP, but there is no particular reason to identify external human capital investment with government human capital investment.

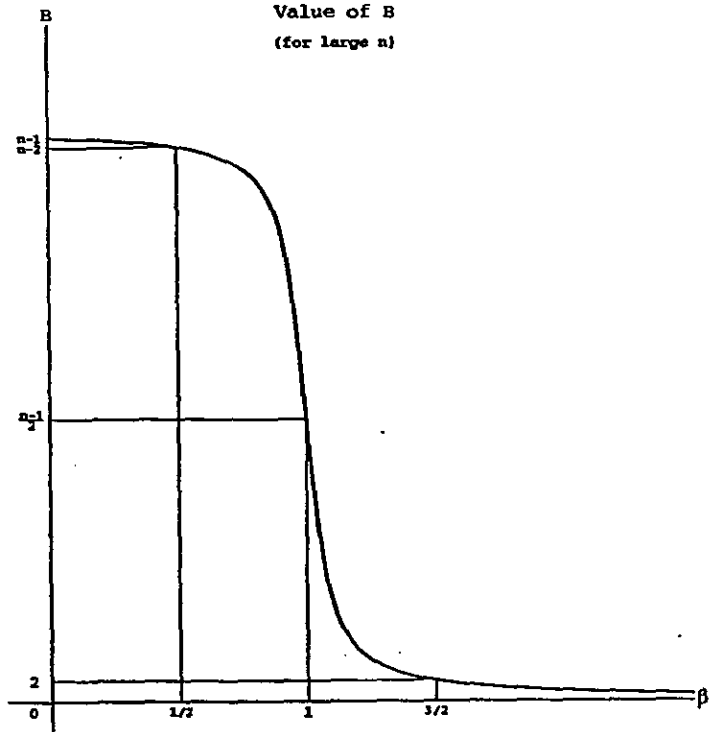
<sup>34</sup>The 50 percent deduction from the uncompensated domestic labor services figure represents our guess regarding the fraction of these services that are realistic substitutes for services the market might otherwise provide. Choosing a lower deduction would reduce the capital share; as we have seen, a lower capital share makes it easier for our model to plausibly generate low real interest rate steady states. If we choose a deduction of zero, the estimated capital share falls to approximately 21.5 percent. This is still within the range of capital shares that appear in the literature on calibrated multiperiod overlapping generations models with capital. Laitner (1987), for instance, imposes a capital share of 18 percent.

If we omit the student and domestic labor adjustments to GDP and impose a real interest rate of 6.5 percent, we obtain a capital share of 36.4 percent. This is close to the figure of 36 percent favored by many equilibrium business cycle theorists, who usually include consumer durables in their concept of capital. In representative agent business cycle models the real interest rate must be equal to the sum of the consumption (and output) growth rate and the rate of time preference, which is typically set to 3 or 4 percent.

Figure 1

Figure 2

Value of B  
(for large n)



Per Capita Savings

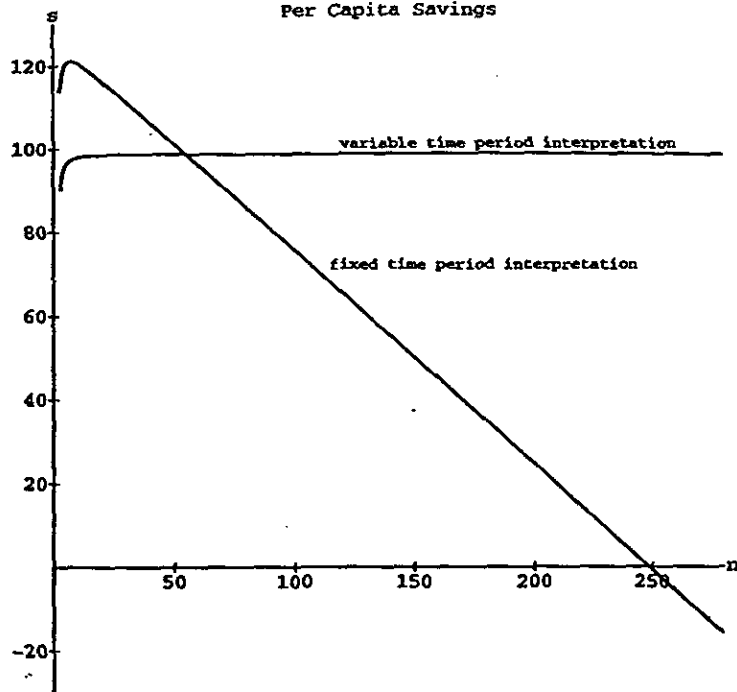
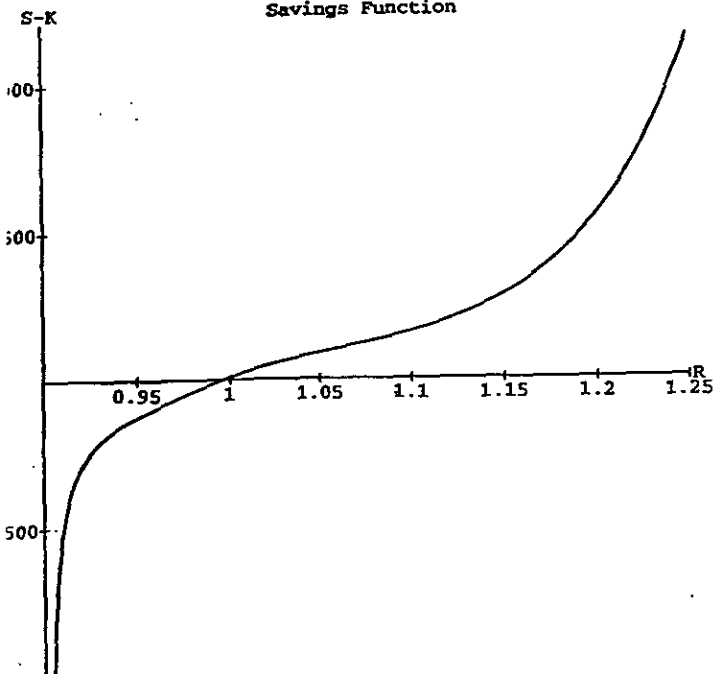


Figure 3

Figure 4

Savings Function



Changing the Number of Periods  
Model with production and capital

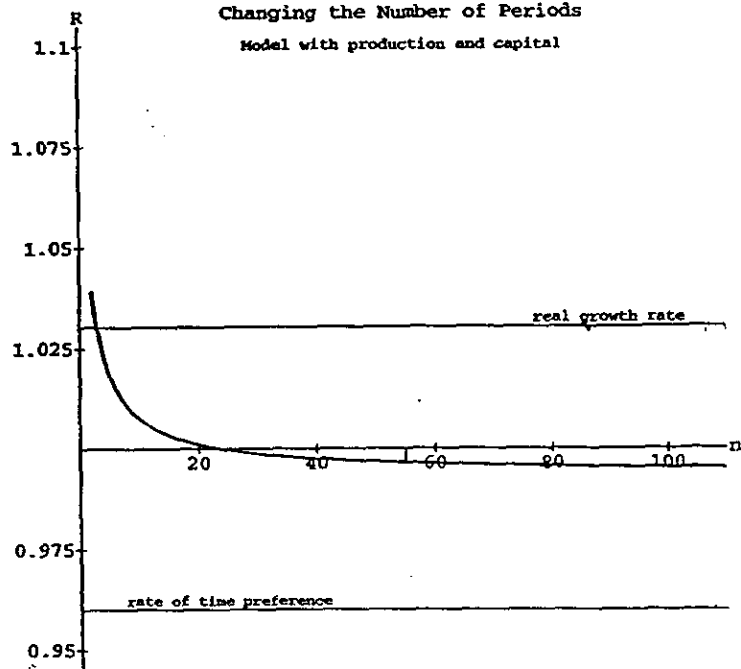


Figure 5

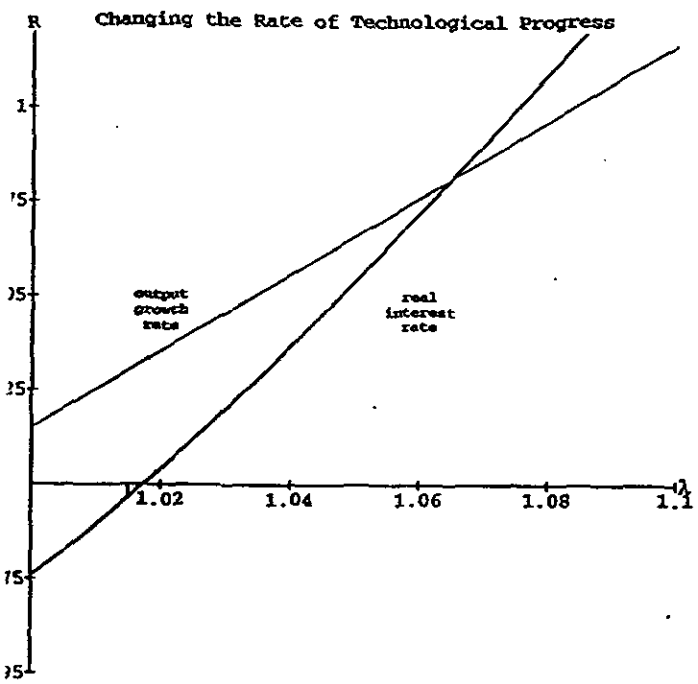


Figure 6

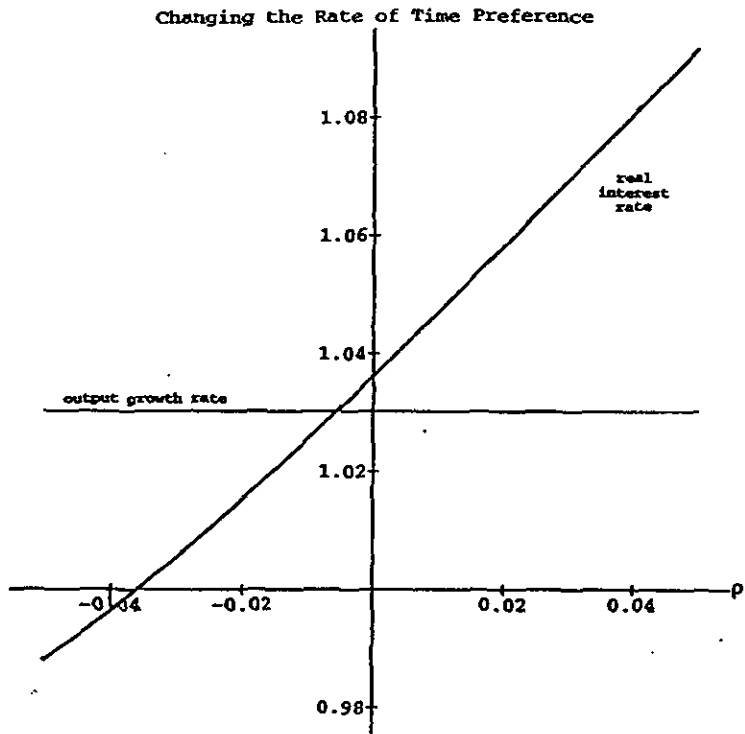


Figure 7

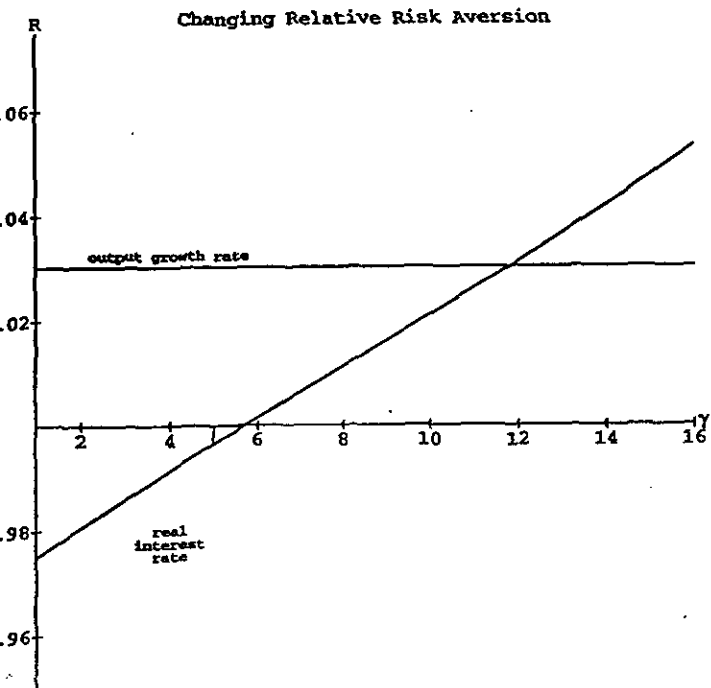


Figure 8

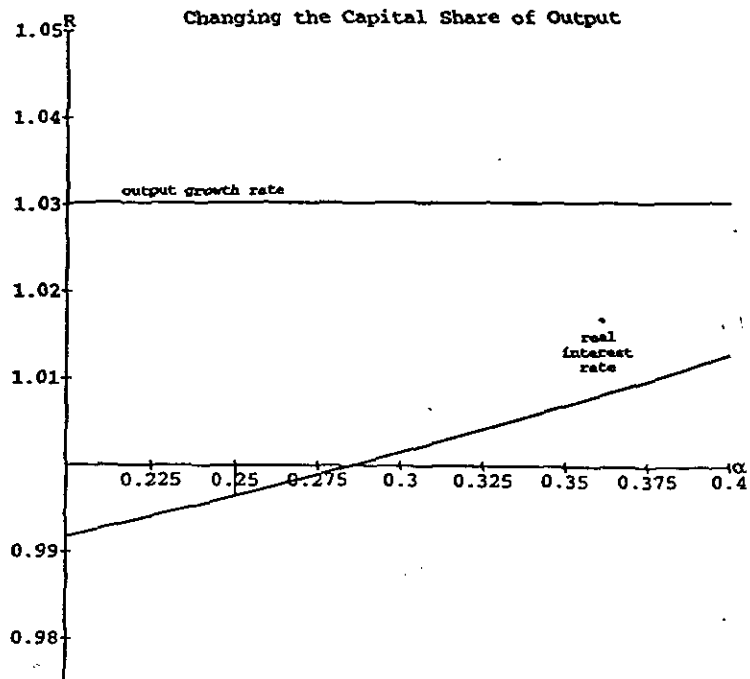


Figure 9

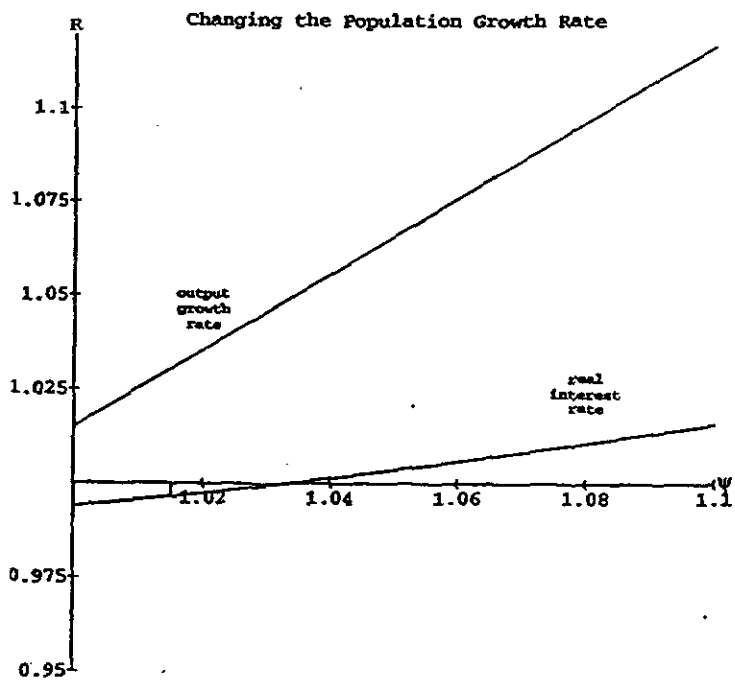


Figure 10

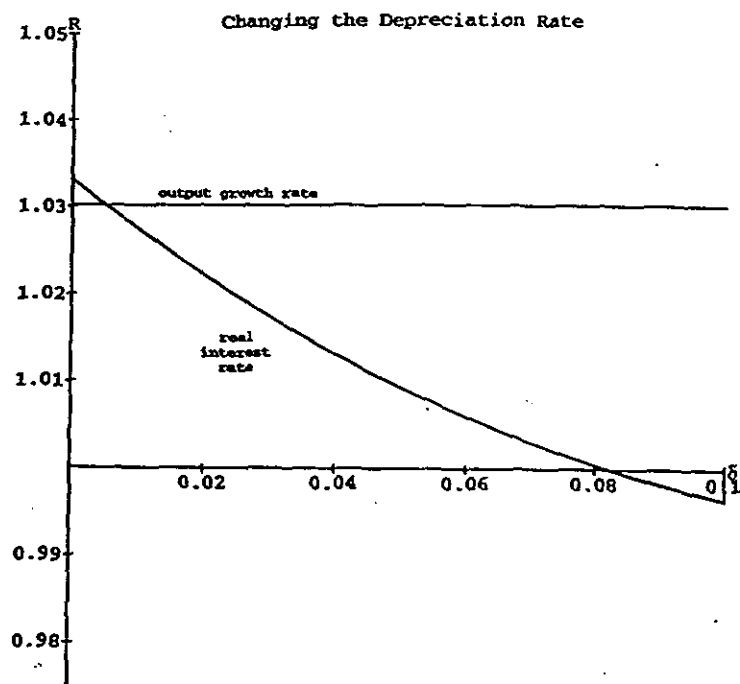


Figure 11

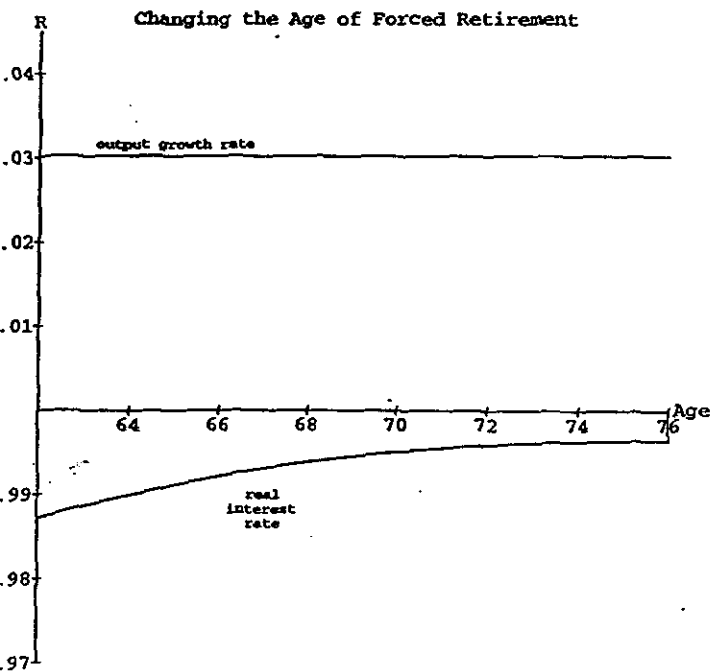


Figure 12

