ON THE FLUCTUATIONS INDUCED BY MAJORITY VOTING

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Scott Freeman, and the seminar participants of the Federal Reserve Bank of Dallas have made useful comments.

ABSTRACT

In this paper a dynamic model is constructed in which labor and capital taxes are determined endogenously through majority voting. The wealth distribution of the economy is shown to influence the voting behavior, and hence the equilibrium levels of the tax rates, which in turn affect the future distribution of wealth. It is shown that the economy exhibits a unique dynamic behavior. Because of the endogenously determined taxes, the asset prices, wealth distribution, and the tax rates can display persistent fluctuations, and even limit cycles, in reaction to exogenous disturbances, or even due to initial conditions. It is also shown that "tax smoothing" does not necessarily appear to naturally arise in such a model, as the economy can display extreme fluctuations in the endogenously determined tax rates.

I. INTRODUCTION

In this paper a dynamic model is constructed in which agents *vote* each period on the desired capital and labor taxes that are to be implemented in order to finance a given level of government consumption. Based on their different wealth levels, agents will have distinctive preferences concerning these tax rates. Given that a majority voting scheme is assumed to be in place, a very interesting dynamic behavior arises in cross sectional wealth levels, as well as in the time-series paths of asset prices and tax rates. It is shown that temporary exogenous disturbances can have not only persistent effects, but also a permanent impact on a variety of endogenous variables. Exogenous disturbances, as well as initial conditions are shown to produce limit cycles in the wealth distribution, asset prices, as well as in the endogenously determined tax rates. This is an important breakthrough because there is a paucity of models, with endogenous policy formulation, in which policies implemented in one period influence the economy in such a manner as to alter the *future* course of policy formulation as well.

There has been a tremendous literature written on both the normative and positive aspects of taxation. The normative aspects usually relate to how tax policy can be formulated so as to maximize some sort of welfare criterion, such as minimizing the tax burden on agents, or possibly maximizing growth. For example, one might ask the question as to what are the appropriate distortional tax rates that should be implemented in order to retire a given amount of government debt. Or, alternatively one might ask whether a capital-gains tax would have a deleterious effect on the growth rate of the economy. The positive analyses of the literature usually examine the impact of actual taxes on welfare, or various endogenous variables. A case in point would be whether the actual levels of capital and labor taxation exacerbate, rather then ameliorate, the cyclical fluctuations of aggregates over the course of the business cycle.[1]

Until very recently, however, what had been missing from most of these analyses of tax policy is a theory of how taxes are actually determined. Presumably in most market economies the policy-makers make decisions based on what they perceive to be the collective desires of a diverse group of agents in the economy. Government policies may then influence the distribution of wealth across the economy. This distribution then influences the manner in which future policy variables are determined, which in turn affects the levels of future policy variables through the policy-making mechanism. Such mechanisms are usually absent from traditional analyses of the affect of government policy. It is the thesis of this paper that it is important to understand how the wealth dynamics produced by an economy help to influence the rules determining public policy parameters, rather than vice versa. Furthermore, this inquiry is also conducted within the context of a dynamic general equilibrium framework. This is important because there have been comparatively few papers that have incorporated endogenous optimal voting decisions into dynamic equilibrium models with utility-maximizing agents, presumably because of the complicated nature of the problem.

There has been a great deal of work done recently in which some government policies are determined endogenously. Tabellini (1991) studies the behavior of government debt in an economy in which policies are determined by majority rule. Tabellini and Alesina (1990) study an economy in which agents vote on the composition of government spending. They then characterize the factors that influence the size of the budget deficit. Tabellini and Persson (1990) provide a comprehensive guide to how credibility and political issues can influence the determination of macroeconomic policy. Alesina (1988) and Perotti (1992) provide a detailed set of references to this growing theoretical as well as applied literature. What is missing from much of the existing literature, and what is the main point of the present paper, is an explanation of how policies implemented in one period influence the distribution of wealth in such a manner as to also influence the policies that are chosen in future periods.[2]

There is a further empirical motivation for utilizing the approach to the determination of policy parameters studied in this paper. One might be *very* hard-pressed to derive an argument for the optimality of the current observed government tax and spending structures based on the solution to some optimal welfare problem, so the reason for such a policy structure may well lie with the political nature in which policy parameters are formulated. One might suggest that it is no coincidence that in the U.S. there has recently been a growing interest in cutting capital taxation, not to mention running higher government budget deficits, and spending more money on social programs for the elderly, at the same time that there is a growing population of elderly citizens. Obviously, these people have a strong incentive to participate in the political process in such a way as to encourage policy-makers to divert more resources in their own direction, and possibly also cut tax rates on their main sources of income. For the elderly, this is more likely to be capital income. Although this particular issue is not addressed in this paper, this observation serves as a motivation for studying or modelling how the economic agents can influence the policy-making mechanism.

The remainder of this paper is organized as follows. In the next section the physical and political structure, as well as the nature of the equilibrium of this very simple economy is described in detail. The economy is populated by a sequence of overlapping generations, each of which lives for three period only. Agents work in their first two periods of life, and hold capital into the last two periods of life. Each period the government must finance a fixed level of real consumption for itself. Each period agents vote on the appropriate levels of capital and labor taxation, while playing a Nash game against future generations. It is assumed that the majority of the voters determine the levels of these parameters. In Section III a series of examples are presented. It is shown how temporary exogenous disturbances can have persistent and permanent effects on the equilibrium, and on the endogenous policy variables. It is shown that there can exist multiple steady-state equilibria, which depend on the initial conditions of the economy. The implications for the wealth distribution, and asset prices are studied. Also, it is shown that the tax rates can appear to fluctuate *dramatically* in the model, with the agents voting to use the labor tax, *or* the capital tax, but apparently never both simultaneously. This provides some motivation for why one might expect to observe the antithesis of the usual tax smoothing behavior. Section IV contains some final remarks.

II. THE ECONOMIC ENVIRONMENT AND THE EQUILIBRIUM

The economy is one in which time is discrete and is indexed by $t = 1,2,\ldots$ Each period there is a generation of agents of size N who enter the economy, and are present there for three periods. An agent who enters the economy in period t will be said to be a member of generation t, and is present in the economy in periods t, t+1, and t+2. Agents have perfect foresight concerning the future.[3] Each member of generation t wishes to consume some of the single consumption good in period t+1, and t+2. That is to say, they do not consume in the first period of life. Such agents have one unit of labor effort to supply inelastically in period t and in t+1, and which will produce $w_{1,t}$ and $w_{2,t+1}$ units of the consumption good in periods t and t+1 respectively.[4] These wages are measured in units of the consumption good.

There is productive capital in the economy which produces a dividend in units of the consumption good each period. As a benchmark, it is assumed that the capital stock does not depreciate, and cannot be augmented. Agents who are members of generation t will wish to consume in future periods, and can do so by purchasing productive capital in period t at a price P_t. For each unit of capital held by the agent, they then receive a dividend of d_t units of the consumption good in period t+1, and can then sell some of their capital or buy more capital in period t+1 at a price P_{t+1}.

A member of generation t has a utility function that will be described as follows

$$\ln(c_{2,t+1}) + \beta \ln(c_{3,t+2}),$$

where $c_{s,t}$ represents consumption by an agent in period t who is currently in period s of his life (s = 2, 3), and $\beta > 0$ is the discount factor. In period t each member of generation t supplies their labor inelastically. The agent has his labor income taxed at a rate τ^{ℓ}_t. The agent will then purchase capital with the remaining income, so that the period t budget constraint for such an agent is then

$$P_t x_{2,t+1} = w_{1,t}(1-\tau_t^\ell). \tag{1}$$

Here $x_{s,t+1}$ represents the number of units of capital purchased in period t by an agent, who is currently in period (s-1) of his life, and which is then taken into period t+1.

A member of generation t who enters period t+1 with $x_{2,t+1}$ units of capital then collects a dividend, in units of the consumption good, in the amount of d_{t+1} per unit of capital. Furthermore capital can then be purchased or sold at a price of P_{t+1}. However, the total return to holding capital - dividend and price of capital - is taxed at a rate of τ^k_{t+1}.[5] The agent also supplies his unit of labor, and collects wage income of $w_{2,t+1}$ and pays taxes on this income. The agent will then wish to take some capital into the final period of his lifetime, and the amount of this capital will be denoted by $x_{3,t+2}$. The budget constraint for such an agent in the second period of his planning horizon is then written as follows

$$c_{2,t+1} = (P_{t+1} + d_{t+1})(1-\tau^k_{t+1})x_{2,t+1} + w_{2,t+1}(1-\tau^\ell_{t+1}) - P_{t+1}x_{3,t+2}. \tag{2}$$

In the final period of this agent's life he brings $x_{3,t+2}$ units of capital into the period. The agent then collects the dividend (d_{t+2}) on the capital, and sells his stock of capital at a price P_{t+2}. The agent then pays the capital tax at a rate τ^k_{t+2} on the total return to capital, and consumes the remaining proceeds. The budget constraint for the agent in this period can then be written as

$$c_{3,t+2} = (P_{t+2} + d_{t+2})(1-\tau^k_{t+2})x_{3,t+2}. \tag{3}$$

It will be assumed that the amount of capital is in fixed supply. This amount is normalized to be equal unity, and it will then also be assumed that N=1. Then the market clearing condition for the capital market in every period t will then be written as

$$x_{2,t+1} + x_{3,t+1} = 1. \tag{4}$$

Each period there is a certain amount of real government consumption g_t that must be financed through taxing labor and/or capital income. This government expenditure provides no utility to agents.

To preserve the simplicity of the environment, it will also be assumed that the exogenous sequence $\{d_t, g_t, w_{1,t}, w_{2,t}\}_{t=1}^{\infty}$ is strictly positive and is known with certainty $\forall$ t.

Parenthetically, it should be noted that at date t=1, there exist the members of generation 0, and -1. At the beginning of this period these agents hold the aggregate capital stock of one unit (i.e. $x_{0,1} + x_{-1,1} = 1$). In period 1 the members of generation (-1) supply all their capital to maximize their period 1 consumption. In this same period the members of generation 0 face budget constraint (2), and constraint (3) in the following period. These agents then maximize their utility function subject to these constraints.

Now, the tax rates that appear in the budget constraints are yet to be determined. The mechanism that sets these parameters is now described. It is assumed that at the beginning of every period t, the members of generation t-2, t-1, and t *vote* on the size of the tax rates $(\tau^k_t\ \tau^\ell_t)$, which are restricted to be non-negative. After the tax rates are then determined, the agents maximize utility subject to their budget constraints while acting as price takers, and taking as given the behavior of other agents, including the behavior of future generations.

It should be apparent now that in any period the members of the young generation will always prefer a capital income tax to a labor income tax, since their sole source of income is labor income. It is also apparent that the members of the old generation will always prefer the labor income tax since their sole source of income derives from capital. Hence, the decision as to what the tax rates will actually be is determined solely by the members of middle-aged generation.

Obviously the middle-aged agents must balance costs of both types of taxation. In particular, they dislike capital income taxation because they hold some capital. But they also dislike labor taxation for two reasons. First, a labor tax is also a tax on their second period labor income, and so hurts them directly. Secondly, this latter tax lowers the labor income of the young and middle-aged agents, and thereby lowers the equilibrium price of capital, and hence lowers the return to holding capital.

Along with these latter consequences are other indirect effects. As will be made obvious below, both these taxes have the impact of lowering the price of capital from what it would otherwise be if there were no taxes. This may be a fortunate effect from the point of view of a middle-aged agent since this may allow them to purchase *more* capital at the reduced price and thereby raise their consumption in the last period of their life. Yet another affect is that a change in the tax rates while an agent is middle aged will influence the amount of capital taken by his generation into the last period of life. This has the potentially unfavorable affect of lowering the rate of return to capital, since these agents will be supplying their capital inelastically, which helps lower the rate of return.

There is one last effect that the middle-aged agents must also take into consideration. Each period t the government must finance a level of expenditures g_t, and the government revenue from capital and labor sources must be sufficient to finance this expenditure level. The middle-aged agents must also take this into consideration when formulating their voting strategy. This means that the consideration of a marginally lower capital tax rate must then necessarily imply a marginally higher labor tax. Agents are assumed to take all of these effects into consideration when formulating their voting strategy.

To illuminate this discussion, it may help to proceed with the solution of the agents' optimization problem. In particular note first that in period t the agents who are members of generation t have a trivial sort of behavior, described by equation (1), in that they buy as much capital as their labor income will permit. Members of generation t-2 consume all their after tax capital earnings, and so their decision bears no more discussion. The interesting problem is then posed by analyzing the decision problem of a member of generation t-1. Now consider this optimization problem from the agents point of view of such an agent after the tax rates in period t have been set. Then such an agent maximizes the following objective function

$$\ln(c_{2,t}) + \beta \ln(c_{3,t+1}), \tag{5}$$

subject to the constraints

$$c_{2,t} = (P_t + d_t)(1-\tau_t^k)x_{2,t} + w_{2,t}(1-\tau_t^\ell) - P_t x_{3,t}$$

$$c_{3,t+1} = (P_{t+1} + d_{t+1})(1-\tau_{t+1}^k)x_{3,t+1}$$

It is easily seen that the solution to this problem is of the following form

$$P_t x_{3,t} = \left(\frac{\beta}{1+\beta}\right)\Big((P_t + d_t)(1 - \tau_t^k)x_{2,t} + w_{2,t}(1-\tau_t^\ell)\Big), \tag{6}$$

$$c_{2,t} = \left(\frac{1}{1+\beta}\right)\Big((P_t + d_t)(1 - \tau_t^k)x_{2,t} + w_{2,t}(1-\tau_t^\ell)\Big). \tag{7}$$

Now by substituting equations (1) and (6) into (4), the equilibrium price of capital can then be derived as follows

$$P_t = \left[\frac{w_{1,t}(1 - \tau_t^\ell) + \Big(d_t(1-\tau_t^k)x_{2,t} + w_{2,t}(1 - \tau_t^\ell)\Big)\left(\frac{\beta}{1+\beta}\right)}{1-x_{2,t}\left(\frac{\beta}{1+\beta}\right)(1 - \tau_t^k)}\right]. \tag{8}$$

There are several important features of this pricing equation to note. First, note that the *distribution* of capital across the population influences the price of capital. The more capital that is held by the middle-aged generation ($x_{2,t}$), as opposed to agents who are in the last period of their life, the higher the price of capital. This should make sense since the agents who are in the third period of life will be *supplying all their capital inelastically*, and the more capital they have, the lower will be the equilibrium price of capital. Consequently, the more capital held by members of the middle-aged generation, the higher will be the price of capital. Secondly, note that the higher the labor income tax the lower will be the price of capital since the members of generation t cannot afford to purchase as much capital. Lastly, the higher is the capital income tax the lower is the price of capital as well. It is also the case that the impact that changes in τ_t^ℓ or τ_t^k have on P_t will depend on the levels of $w_{1,t}$, $w_{2,t}$, d_t, and $x_{2,t}$.

The government is restricted to balancing its budget each period so that it must implement labor and capital tax rates each period to finance its expenditures. Hence its budget constraint is written as follows

$$g_t = \tau_t^\ell (w_{1,t} + w_{2,t}) + \tau_t^k (P_t + d_t). \tag{9}$$

Now a substitution of the optimal decision rules, (3) and (7), for a member of generation t-1 back into their utility function (5) produces the following version of an indirect utility function

$$\ln\left[\left(\frac{1}{1+\beta}\right)\left((P_t + d_t)(1 - \tau_t^k)x_{2,t} + (1 - \tau_t^\ell)w_{2,t}\right)\right] + \beta\ln\left[(P_{t+1} + d_{t+1})(1 - \tau_{t+1}^k)x_{3,t+1}\right]. \tag{10}$$

This is the indirect utility function that the agents, who are middle-aged in period t, seek to maximize. However, these agents also realize that their choice of taxes will influence the price of capital in the present period (P_t), the future price of capital (P_{t+1}), as well as their future asset holdings ($x_{3,t+1}$). Since $x_{3,t+1}$ is determined from equations (4) and (6), and P_t and P_{t+1} are determined by versions of equation (8), by making these substitutions into equation (10), and then after conducting a mammoth amount of algebra, this indirect utility function can then be rewritten as follows.

$$\begin{aligned} V_t(\tau_t^\ell, \tau_t^k) &= (1+\beta)\ln\left[x_{2,t}(1-\tau_t^k)[w_{1,t}(1-\tau_t^\ell) + d_t] + w_{2,t}(1-\tau_t^\ell)\right] - \ln\left[1 - x_{2,t}(1-\tau_t^k)\left(\frac{\beta}{1+\beta}\right)\right] \\ &-\beta\ln\left((1-\tau_t^\ell)[w_{1,t}(1+\beta)+\beta w_{2,t}]+x_{2,t}(1-\tau_t^k)\beta d_t-\beta(1-\tau_{t+1}^k)(1-\tau_t^\ell)w_{1,t}\left[1 - x_{2,t}(1-\tau_t^k)\left(\frac{\beta}{1+\beta}\right)\right]\right) \\ &+ \Omega(w_{1,t}, w_{2,t}, d_t, w_{1,t+1}, w_{2,t+1}, d_{t+1}, \tau_{t+1}^\ell, \tau_{t+1}^k, \beta). \end{aligned} \tag{11}$$

This is the indirect utility function faced by an agent, who is a member of generation t-1, at the beginning of period t, and it reflects the optimal savings behavior for such an agent, given any specified level for the tax parameters (τ^k_t, τ^ℓ_t). Here the third line of this expression is a non-linear combination of the specified variables which are beyond the control of the agents who are members of generation t-1. The point of this is, however, that this latter term is independent of the choice of tax rates (τ^k_t, τ^ℓ_t) which are to be chosen at date t by members of generation t-1. The first two lines of this expression shows a function of variables that are exogenous at the beginning of period t, or are to be chosen at the beginning of period t by the voters. The members of generation t-1 then choose (or vote for) the tax rates that will maximize their indirect utility function V_t subject to the government budget constraint (9).

It is assumed that at the beginning of each period the agents, in choosing the tax rates, are then playing a Nash game against future generations. That is, the middle-aged agents take future tax rates as given when choosing their optimal tax rates. However, they take into account how these tax rates will influence the present and future prices of capital, and asset holdings.[6] [7]

Finally, to make this discussion precise, it is worthwhile to proceed with the following formal definition of the equilibrium under study.

<u>Definition:</u> A *Nash Perfect Foresight Competitive Equilibrium* for this economy is a collection of non-negative sequences $\{d_t, g_t, P_t, w_{1,t}, w_{2,t}, x_{1,t}, x_{2,t}, c_{1,t}, c_{2,t}, \tau^k_t, \tau^\ell_t\}_{t=1}^{\infty}$, such that for $t \geq 1$, the following conditions are satisfied.

i) For members of generation t-1, given the levels of $(d_t, g_t, w_{1,t}, w_{2,t}, x_{2,t}, \tau^k_{t+1})$, the period t taxes (τ^k_t, τ^ℓ_t) are chosen to maximize the value function $V_t(\tau^k_t, \tau^\ell_t)$, as given by equation (11), subject to the government's budget constraint (9).

ii) Given tax rates (τ^k_t, τ^ℓ_t), and the price of capital P_t, the quantities $(x_{1,t}, x_{2,t}, c_{1,t}, c_{2,t})$ maximize the utility function (5), subject to the budget constraints. This implies the decision rules (3), (6), and (7) are satisfied.

iii) The government budget constraint (9) holds for each period.

iv) Equation (4) holds, so that there is equilibrium in the capital market.

v) Given d_t, g_t, $x_{2,t}$, τ^k_t, and τ^ℓ_t, the price of capital P_t is given by equation (8).

It should also be noted that an exogenous constraint that is being imposed is that the tax rates (τ^k_t, τ^ℓ_t) are restricted to being non-negative.

Now equation (11) is a formidable and intimidating expression. Rather than attempting to gain insights directly from this equation, it will be more enlightening to look at a series of examples to obtain a feel for the nature of the equilibria of the economy.

III. SOME SAMPLE ECONOMIES

Much of the work in the optimal taxation literature has the implication that the "optimal" level of distortional taxation is that which minimizes the social deadweight loss. This usually gives rise to optimal tax rates in which all commodities are taxed to some degree so that the *marginal* social costs from all forms of taxation are equated. This would be the case if one followed the Ramsey tax rules. This might be referred to as a tax-smoothing argument.[8] It is then of interest to see if, in the context of the above-specified framework with agents voting on the optimal levels of taxation, the resulting tax levels would display such properties, in the sense that agents will choose to have positive taxation on both labor and capital.

As it happens, and as will be shown, for all the examples that will be presented below, this result does not obtain. In particular, middle-aged agents always prefer to have capital taxation *or* labor taxation, *but not both*. Another way of putting this is to say that the value function displayed in equation (11), rather than being concave in the tax rates (τ^k_t, τ^ℓ_t), turns out to be *convex* in these tax rates. The reason for this will be explained in the first example. This is an illuminating result in that it shows a potential avenue through which there might be divergence between the actual observed taxation rates, and those derived from the solution from some optimal planning problem.

Example #1: This example illustrates the potential instability that exists in the model. In particular, there exists two equilibria which depend upon the initial conditions of the economy. One displays limit cycles of two periods in length, while the other does not.

The parameter values are chosen as follows: $w_{1,t}=11$, $w_{2,t}=7$, $d_t=g_t=5$, and $\beta=3$, $\forall$ t. Two different initial conditions are chosen for capital holdings of the initial middle-aged agents. One is $x_{2,1}=.39$, while the other is $x_{2,1}=.40$. There is no other exogenous uncertainty or changes in the economy. Figure 1 illustrates the resulting paths for the price of capital in each case. The solid line shows the behavior for the price of capital when $x_{2,1}=.40$ while the dashed line is the price of capital when $x_{2,1}=.39$. Obviously, the solid line converges relatively quickly to a constant steady state while the dashed line displays cycles. Figure 2 shows the resulting paths for the capital holdings by the middle-aged agents for the same example. Again the cycles appear in this variable as well as for the case where $x_{2,1}=.39$.

Figure 3 illustrates the behavior of the tax rates that are observed in the cyclic equilibrium, while Figure 4 shows the path of tax rates in the other non-cyclic equilibrium. In this non-cyclic equilibrium the agents choose to use only the labor tax. In Figure 3 the solid line is the path for the capital income tax, while the dashed line is the path for the labor income tax.

What is happening in this example is that when $x_{2,1}=.39$, the initial middle-aged agents begin with relatively little capital, and consequently vote to tax capital heavily in the first period of the economy, while choosing to not tax labor at all. Consequently, they have comparatively little capital in period 2 because they began with little in the previous period, and also because the young agents in the previous period did not have their labor income taxed, and could then afford to purchase plenty of capital. In period 2, the new middle-aged then have plenty of capital, relative to the initial middle-aged in the previous period, and they do the reverse: they vote to tax only labor and not capital at all. This pattern of behavior then repeats itself every two periods. There is no convergence of any of the decision variables, nor for the price of capital, or the distribution of capital. For the case where $x_{2,1}$=.40, the initial middle-aged agents do not own quite enough capital for it to be advantageous for them to vote for a capital income tax. Figure 4 shows that as a result the capital income tax is never chosen, and so the economy only implements the labor tax.

As noted above, it is also of interest to see if the agents in this economy voluntarily choose to implement some variant of what may be called a tax-smoothing policy. As both Figures 3 and 4 show in the cyclic equilibrium, the solutions observed for the tax rates tend to be of the "bang-bang" variety, with the middle-aged agents choosing *either* labor *or* capital income tax, but not both. The reason for this is best seen in Figure 5. This illustrates the shape of the value function, as a function of how much of the government revenue is raised from the labor tax.[9] As can be seen, the agent is likely to choose to tax either labor *or* capital, and in this particular diagram it is the latter. As Figure 5 illustrates, the constraint that the capital and labor taxes be non-negative is indeed binding. Were this constraint not imposed, agents might then vote to expropriate labor *or* capital income in order to subsidize the other factor.

To obtain an understanding for why this result obtains, first of all note that from a voter's point of view, there is not a linear relationship between the two tax rates. This is seen by noting that although equation (9) shows there is a linear trade-off between taxing labor and capital income, capital income is endogenous, as equation (8) demonstrates. To see why the value function may be convex in the tax rates, note that the beginning of period wealth of a middle-aged agent in period t can be written as follows

$$(P_t + d_t)(1 - \tau_t^k)x_{2,t} + (1 - \tau_t^\ell)w_{2,t}.$$

By using equation (8) to substitute for the price of capital, this can then be re-written as

$$\left[\frac{(1-\tau_t^k)x_{2,t}\left[(1-\tau_t^\ell)w_{1,t}+d_t\right]+(1-\tau_t^\ell)w_{2,t}}{1-x_{2,t}\left(\frac{\beta}{1+\beta}\right)(1-\tau_t^k)}\right]. \tag{12}$$

Now obviously this is a highly nonlinear expression in the tax rates. One could additionally substitute the government's budget constraint (9) into this expression to get rid of one of the tax rates. The result would be an expression that is even more non-linear in the remaining tax rate.[10]

The agent's consumption in the final period can then be written as follows

$$(P_{t+1}+d_{t+1})x_{3,t+1}(1-\tau_{t+1}^k)=\left[\frac{(1-\tau_{t+1}^\ell)\left[w_{1,t+1}+w_{2,t+1}\left(\frac{\beta}{1+\beta}\right)\right]+d_{t+1}}{1-x_{2,t+1}\left(\frac{\beta}{1+\beta}\right)(1-\tau_{t+1}^k)}\right]\left[x_{3,t+1}(1-\tau_{t+1}^k)\right]. \tag{13}$$

Now experimentation with equations (12) and (13) reveals that there are two forces making welfare decreasing in the amount of the labor tax, and two forces making it decreasing in the amount of the capital tax. First, the numerator of equation (12) is decreasing in the amount of the labor tax since these labor endowments determine the after-tax labor income directly, and indirectly influence capital income. Additionally the quantity $x_{3,t+1}$ in equation (13) is also decreasing in the amount of the labor tax, since a higher labor tax lowers the amount of capital that these agents can afford to purchase for the remaining period.

But there are also two forces influencing welfare to be decreasing in the amount of the capital tax. First, obviously the denominator of equation (12) is increasing in the amount of the capital tax. Secondly, the denominator of the term in equation (13) is also decreasing in the amount of the capital tax. This can be seen by noting first of all that the members of generation (t-1) can purchase more capital in the second period of life if more of the capital tax is employed in period t. This in turn raises the level of $x_{3,t+1}$, but this in turn must lower the value of $x_{2,t+1}$ (since $x_{2,t+1} + x_{3,t+1} = 1$ (from equation (4)). Hence raising the level of τ^k_t will increase the value of the denominator in equation (13), hence raising the value of the whole right side of this equation. The effect of a change in τ^k_t on the *denominators* of both equations (12) and (13) makes these equations increase *at an increasing rate*, and this is the feature that accounts for the convexity of the value function observed in Figure 5.[11]

Also, as Figure 5 illustrates, there are positive levels of capital and labor taxation that appear to minimize the value function (11). At these tax rates the agents welfare is low because the taxes are chosen to both reduce the (utility) value of wealth in the second period of life, and also reduce the total return to holding capital.

Figure 6 illustrates the transition dynamics for capital holdings in this example. The horizontal axis measures the quantity of capital held by the middle-aged in period t ($x_{2,t}$), while the horizontal axis measures the same variable in the following period.[12] The upward-sloping dashed line in the figure is a 45 degree line which helps pinpoint the stationary equilibria. The downward-sloping line with a break is the line that describes the transitional dynamics of asset holdings. As can be seen in the diagram, there are many equilibria, depending on what the initial asset holdings are. However, there appear to be only two limiting equilibria. One has a constant steady-state for capital holdings equal to .4129. The second equilibria is the one which displays limiting cycles. In any period, for any capital holdings ($x_{2,t}$) less than .393 the middle-aged agents will choose to use only capital taxation to finance government spending. Alternatively, for a capital holdings greater than .393 the middle-aged agents will choose only labor taxation. That is, the upper branch of this line (to the left of this diagram) reflects the amount of capital held by middle-aged agents when the capital tax is imposed, while the lower branch describes the amount of capital held when the labor tax is enacted. For capital holdings just equal to .393, the agent will be indifferent between the two types of taxation.[13]

There is an interesting dynamic behavior in this example. If the initial middle-aged agents hold a capital stock less than .393, or greater than .450, then the economy ultimately converges to the cyclical equilibrium. On the other hand, if the initial middle-aged agents have capital holdings between .393 and .450, then the economy ultimately converges to the non-cyclic equilibrium.[14] Hence both the cyclic and non-cyclic equilibria are locally stable.

If by chance, the solid line describing the transition dynamics for capital holdings does not cross the 45 degree line, then there is no steady-state equilibria without limiting cycles. It will be shown below that indeed this can be the case.

Despite the fact that there is an equilibrium in which all endogenous variables display cycles, this equilibrium is indeed stationary, since the variables do not depend on time, once the capital holdings at the beginning of the period are known. Also, in contrast with other models which produce limiting cycles, the present framework does not employ a backward-bending supply curve for saving, as a function of the interest rate, and neither does it make use of any externality, except to the extent that the voting scheme can be interpreted as one.

Lastly, it is of interest to compare the welfare implications of the two different equilibria in this particular example. In the non-cyclic equilibrium the realized utility level of all agents is 9.2929. In the cyclic equilibrium, the utility level of agents who have relatively plenty of capital when middle-aged, and hence vote for a labor tax in the second period of life, is 9.4537, while the utility level of agents who relatively little capital is 9.1551. Obviously, in the cyclic equilibrium, for capital-rich agents, the benefits of owning plenty of capital and voting to tax labor when middle-aged outweigh the resulting costs imposed by taxing capital when middle-aged and having capital taxed in the following period.[15]

<u>Example #2:</u> The point of this example is to illustrate the instability exhibited by the model in reaction to an expected exogenous shock to the productivity of young agents. Again there exists two equilibria for this economy, and if the exogenous shock is sufficiently large the economy will move from the stationary equilibrium to the non-stationary one.

The parameter values are chosen as follows: $w_{1,10}=13.75$, and otherwise $w_{1,t}=11$, $w_{2,t}=7$, $d_t=g_t=5$, $\beta=3$ $\forall$ t, and $x_{2,1}=.4$. That is, the economy has no exogenous disturbances until period 10 when the young agents are then *temporarily* relatively very productive.

Figure 7 shows the resulting behavior for the capital holdings of the middle-aged agents. Starting from $x_{2,1}=.40$, the capital holdings appear to be converging to a constant. However, the exogenous disturbance is sufficiently large that from period 10 onward, the economy is on a cyclical equilibrium with every second generation holding relatively large quantities of capital.[16] The resulting behavior for the price of capital is shown in Figure 8. Again, this price is converging to a constant until period 10, when it then exhibits cycles as well.

The path for the equilibrium tax rates are shown in Figure 9. Until period 10, the agents choose to only use the labor tax, with the capital tax always being zero. After period 10, the agents choose to alternate these taxes, choosing the capital tax only in even periods and the labor tax in odd periods.

It should be noted that the exogenous shock in this example has been "large enough" to cause the economy to take a path onto the cyclical equilibrium. For smaller disturbances the economy might only gravitate back to the non-cyclic equilibrium after several periods.

Example #3: This example further illustrates the nature of the instability exhibited by the model in reaction to an expected exogenous shock to the productivity of young agents. This example is identical to the previous one with the exception that the discount factor is lower. Again, there will exist two equilibria for this economy, but the exogenous shock will not make the economy go to the non-stationary equilibrium because of the lower discount factor.

The parameter values are chosen as follows: $w_{1,10}=13.75$, and otherwise $w_{1,t}=11$, $w_{2,t}=7$, $d_t=g_t=5$, $\beta=2.7$ $\forall$ t, and $x_{2,1}=.4$. That is, again the economy has no exogenous disturbances until period 10 when the young agents are then *temporarily* relatively productive.

Figure 10 shows that capital holdings by middle aged agents converge to a steady-state until period 10. At this time these capital holdings are temporarily pushed away from this value, but eventually converge back to this same equilibrium. Figure 11 shows the path for the price of capital. It too is temporarily displaced from its steady-state value, but converges back to it steady state value.

To obtain an understanding of how changing the discount factor influences the equilibrium it may be useful to look at Figure 12, where the transitional dynamics for asset holdings is illustrated for different discount factors. Increasing (decreasing) the discount factor moves this function toward (away from) the origin. As can be seen, for the case in which $\beta=15$, there does not even exist a non-cyclic equilibrium since the transitional dynamics line does not cross the 45 degree line.

It should be stated that not all configurations of this economy exhibit the cycles displayed in these examples. To produce these cycles requires the right kind of balance between the ratio of capital to labor income on the one hand, and the discount factor on the other hand. In particular, for a given discount factor (β), if the agent's labor income is too high, relative to the amount of capital income, then agents may always prefer to have labor taxed. Conversely, holding the discount factor constant, if there is very little aggregate labor income, then agents may then always prefer capital taxation. Alternatively, holding constant the levels of endowments and dividends, the agent is more likely to prefer the capital (labor) tax, the lower (higher) is the discount factor since this makes future consumption less (more) important. It should be noted, however, that experimentation has revealed that although not all economies exhibit these fluctuations, neither are the economies that do exhibit this behavior "knife-edge," or extremely special cases. A wide range of economies can be seen to display these features. For example, Figure 12 shows that when $\beta=15$ there is only a cyclical stationary equilibrium, and this equilibrium continues to appear for even much higher levels of β.

It is also easy to incorporate an endogenous labor decision the agents second period. However, in some cases this effect only serves to exacerbate the effects described above.[17] This is because a fall in the agent's wage will then encourage them to work less, which will then lower their wage income even further.

It should also be noted that examples can also be constructed in which a temporarily unusually high or low level of government consumption is also capable of generating these types of cycles as well.

Example #4: A natural question that arises at this point is whether the taxes are indeed playing much of a role in this economy. After all, in economies of this sort, when agents live for more than two periods, fluctuations in the wealth distribution of this sort are known to arise even in he absence of government (see Huffman (1987)). Therefore, it may be that the taxes are sort of a side-show which contribute very little. The present example shows that this is not true. This example illustrates the instability exhibited by the model in Example #2 depends critically on the presence of endogenous taxation.

The parameter values that are chosen are exactly the same as in Example #2: $w_{1,10}=13.75$, and otherwise $w_{1,t}=11$, $w_{2,t}=7$, $d_t=g_t=5$, $\beta=3\ \forall\, t$, and $x_{2,1}=.4$. Again, the economy has no exogenous disturbances until period 10 when the young agents are then *temporarily* relatively unproductive. However, it is of interest to compare the behavior of the equilibrium in this example with that which would arise if there were a *constant* tax regime in place. In particular, in the constant tax regime, the economy utilizes only the labor tax in every period ($\tau^{\ell}_t=.2778$), and never taxes the return to capital ($\tau^k_t=0$).[18]

Figure 13 shows the resulting paths for the capital holdings of the middle-aged. The dashed line in this diagram is the same as that shown in Figure 7, with the economy exhibiting a cyclic equilibrium after period 10. The solid line displays the capital holdings of the exact same economy, but where tax rates are kept constant for all time. There is no cyclical equilibrium for this latter economy.[19] This example shows that the endogenous determination of tax rates is of critical importance in producing the cyclical equilibrium.

Example #5: This example is presented to show how the transition dynamics for capital holdings is affected by the composition of aggregate output into its components of labor and capital income. Consider two alternative economies, one which is identical to that in the first example with $w_{1,t}=11$, $w_{2,t}=7$, $d_t=g_t=5$, and $\beta=3$. The second is identical except that $w_{1,t}=9$. Figure 14 shows how the transitional dynamics for capital holdings behaves in the two cases. Loosely speaking, lowering the agent's labor income has the effect of making capital income relatively less important than it would otherwise be. Consequently, agents are less likely to want to choose the labor tax, and more likely to choose the capital tax. In Figure 14, the break in the line describing the transitional dynamics of asset holdings is moved to the right when the amount of steady-state labor income diminishes. Because of this the non-cyclic equilibria exists in both of these cases, but when $w_{1,t}=11$ this equilibrium has only labor income being taxed. In the equilibrium in which $w_{1,t}=9$, the equilibrium has only capital being taxed.

Example #6: This example is presented to show how the transition dynamics for capital holdings is affected by the level of government spending. Consider two alternative economies, one which is identical to that in the first example with $w_{1,t}=11$, $w_{2,t}=7$, $d_t=g_t=5$, and $\beta=3$. The second is identical except that $g_t = 10$. Clearly, in this new case the tax rates must be higher to finance the government consumption. The lines in Figure 15 describing transitional dynamics displays a larger "break" in this case since the taxes, when imposed, have a much larger impact on the future capital holdings since the taxes are higher. If government consumption were zero, then there would be no break in the transitional dynamics line at all since all taxes would be zero. In the case when $g_t = 10$, there is no non-cyclic equilibrium, and there can only exist equilibria in which there are fluctuations. Of course, in general (but not always), the lower is the level of government spending, the more likely it will be that there will exist a stationary equilibria.

IV. FURTHER REMARKS

Barro (1979) presents some normative reasons why, in a dynamic environment, governments should "smooth" the tax rates so as to minimize the burden of taxes. The present positive analysis illustrates why atomistic agents, behaving in a privately optimal manner, would choose to have a tax structure which would appear to *cause* some fluctuations in endogenous variables. In particular, this analysis points to where the potential divergences might arise among positive and normative tax analyses. The public finance literature is replete with research showing the ways in which observed tax rates may differ from the "optimal" tax policies, for reasons that are usually left unexplained. Nevertheless, presumably an arguable view is that society has arrived at its current tax policies by agents making optimal choices when choosing political representatives who will make policy choices for that will affect society. It is also our task to understand how and why these choices are made.

As with any analysis, the present paper leaves many questions unanswered. Here assumptions were placed on how policy variables were determined, namely through majority voting rather than some other, possibly ad-hoc, mechanism. It was exogenously imposed that government revenue is derived from the taxation of labor or (gross) capital income. It would be better if it could be shown that such a policy mechanism is "optimal" relative to a set of potential mechanisms. This remains a formidable topic for future research.

The present model has a fixed capital stock. It would be enlightening to know how this type of majority voting scheme would influence the level of endogenous investment and output. Presumably higher capital tax rates would deter capital accumulation, and influence the wealth distribution in the future.[20] Huffman (1993) has already studied this issue, and most of the results shown above still obtain when capital accumulation is incorporated. In particular, the agents still choose tax rates of the "bang-bang" variety. It is also possible to show that if agents are permitted to vote on the size of the inflation tax in any period in order to finance government spending, then they might choose a volatile path for the inflation, as this is just another tax. These analyses are currently topics of ongoing research.

Additionally, it is also of interest to know how the results presented above would change if a different utility function were employed. Preliminary work in this area indicates that the existing dynamics still are present with other utility functions, but that other dynamics are also present. By changing the elasticity of substitution of consumption between periods, it appears that the slope of the line describing the transition dynamics of asset holdings (e.g. Figure 6) can be made steeper or flatter. In particular, if this line is made sufficiently steep then the non-cyclic steady-state equilibrium can be made unstable.

Additionally, on a topic that is closely related, research is also being conducted into whether it is possible to produce sunspot equilibria, or more complex cyclical dynamics for this economy by changing the preferences. That is to say, what is being studied is how the behavior of the transition dynamics of the type shown in Figure 6 an be altered so that different cyclical equilibria can arise.

It is possible that this approach could also be utilized to explain why government spending would be increased at some times and not others. Rather than just saying that this spending is wasteful, instead this might be undertaken due to the fact that there would be a significantly large constituency that benefits from such spending. Additionally, it may be possible to use a similar model to explain the level of the deficit that the government may run. Possibly the aforementioned instability of the economy could cause government spending to display an unstable response to a temporary disturbance to the economy.

The forgoing analysis raises obvious questions concerning the manner in which our economic policy-making institutions are designed. Do we choose to have institutions in which citizens can potentially exert unremitting or day-to-day control of government policy based on their own private self interests? Or, on the other hand, do we choose to have institutions which set out policy according to some relatively fixed rules that cannot be easily changed based on the whimsy or vocal protests of groups of citizens?[21] Should we choose to have constitutional amendments prohibiting certain types of taxation, as there effectively is now in the U.S. with the poll tax, or at least put some restrictions on the amount of this taxation? Some countries, such as Britain, have no written constitution and therefore appear to put few prior restrictions on how such policies can be formulated.

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FOOTNOTES

1. On the normative side, Lucas (1990) describes why the desired tax on capital should be zero. In a positive and normative analysis, Barro (1979) shows why the government may wish to "smooth" the levels of distortional taxation over time so as to minimize the deadweight loss from the taxation.

2. Much of this existing literature contains analyses of models which are finite horizon economies. The model studied in the present paper has an infinite horizon, and as such permits an analysis of how the endogenous variables evolve over time in reaction to various disturbances. Alesina and Spear (1988) use the overlapping generations model to construct a model of electoral competition. Boldrin (1993) uses a three-period overlapping generations framework to analyze the impact that public school financing has on the accumulation has on human capital. Krusell and Ríos-Rull (1993a, 1993b) also provide a very interesting analysis of the impact of endogenous policy formulation within a dynamic environment.

As noted by Alesina (1988), much of the extant literature is rather descriptive, and not cast within the context of a general equilibrium optimizing framework. The present paper does, however, fall into this category.

3. Uncertainty could obviously be incorporated, but this feature is not of any importance for the central issues under study, and would only obscure the very simple nature of the dynamics that arises in the perfect-foresight version of the economy.

4. Permitting the agent to have different labor income in the different periods, $w_{1,t} \neq w_{2,t+1}$, or enabling young and middle-agent agents to earn different amounts in a given period, $w_{1,t} \neq w_{2,t}$, enables the model to have the flavor of investment in human capital which might explain why agent's labor earnings would differ in this manner. It would also be possible to incorporate an endogenous labor decision in the agent's second period, but this would add little while complicating the present setup.

5. It could alternatively be assumed that just the dividend was taxable but this would not alter the central qualitative nature of the results, although it would change some of the quantitative results presented in the next section.

6. Agents also take into account how the taxes imposed in the present period will influence *future* asset holdings of the next generation, (since this is just one minus the amount the current middle-aged generation will choose to hold) and hence how these asset holdings will affect the future price of capital.

7. It may be that there would be other equilibria as well with more complex forms of strategic interaction, but the present approach would seem to preserve a sufficient degree of simplicity and tractability, given the complexity of the dynamics in this infinite horizon economy.

8. That is, this is meant to be an intratemporal argument: tax rates in a given period should be set so as to minimize this tax burden. However, as argued by Barro (1979), obviously the same argument has been used to conclude that taxes should also be set intertemporally to minimize this burden as well.

9. In particular, this example illustrates the first two lines of the value function shown in equation (11), so this is the actual value function up to a constant.

10. The *intuitive* reason behind why there exists this nonlinear relationship is that there appears to be a strange hidden type of double taxation in the model. Consider a middle-aged agent who is contemplating voting to tax only capital to finance government spending. He then contemplates what would happen if he voted to tax capital slightly less, and taxed labor a little bit more heavily. This would leave him with somewhat more of *his capital* income, but less of his labor income. If capital income remained the same the agent could merely then determine if the changed taxes left him better off or not. However, capital income does change because equation (8) shows that the price of capital may fall because of the higher labor taxes. Consequently the middle-aged agent can make himself worse off by taxing labor a *little bit* because it lowers his before tax capital income as well.

11. Central to this discussion is how the price of capital changes as the tax rates change. One might then be tempted to observe that this affect would not be present if there were capital accumulation. This is not true. What is important here is that the price of capital is endogenous, which can happen in a model with capital accumulation, not that the capital stock be fixed.

12. Of course both axis should extend from zero to one, but this has not been done since in this example the outer regions are never realized.

13. In this instance, the agent will be indifferent and hence may then choose a mixed strategy between complete labor taxation, and complete capital taxation. However, it is only the initial middle-aged agents who might choose a mixed strategy. It is clear from the diagram that no subsequent generation would then have a capital stock equal to .393 in the second period of their life.

14. This example is somewhat special since the critical value of $x_{2,t}=.393$ that separates the cyclic and non-cyclic equilibria is also the point where the *transition line* "breaks". This is not always the case. As is shown in Figure 12 and 15, there can occur a break in the transition line, but with no non-cyclic equilibrium.

15. Perhaps surprisingly, it turns out that the *steady-state* welfare of agents in these two equilibria is such that the agents are better off in the cyclic equilibria. The reason for this is as follows. Relative to the non-cyclic equilibrium, in the cyclic equilibrium in the periods when capital (labor) tax is imposed, the agents are consuming less (more) in both periods. However, the cyclic equilibrium appears to display "smoother" consumption paths within a generation, *but obviously not across generations.* This is perhaps not as surprising as it

may at first seem since this only considers the steady-state welfare, and ignores the affect on the initial generations, and additionally ignores transitions to the steady-state.

16. Keep in mind that what is being shown in Figure 7 is $x_{2,t+1}$, which is chosen in period t. This is why the "jump" in asset holdings in the diagram appears to occur in period 11, but in fact this occurs in period 10.

17. This would depend on the relative strengths of the wealth and substitution effects. If the latter dominated the former then the economy is likely to display the features described above since a labor tax would then reduce the work effort and further reduce labor income.

18. That is, these are the steady state values of the tax rates that the endogenous-tax economy is converging to before period 10. However, this convergence takes place for *any* constant tax rates.

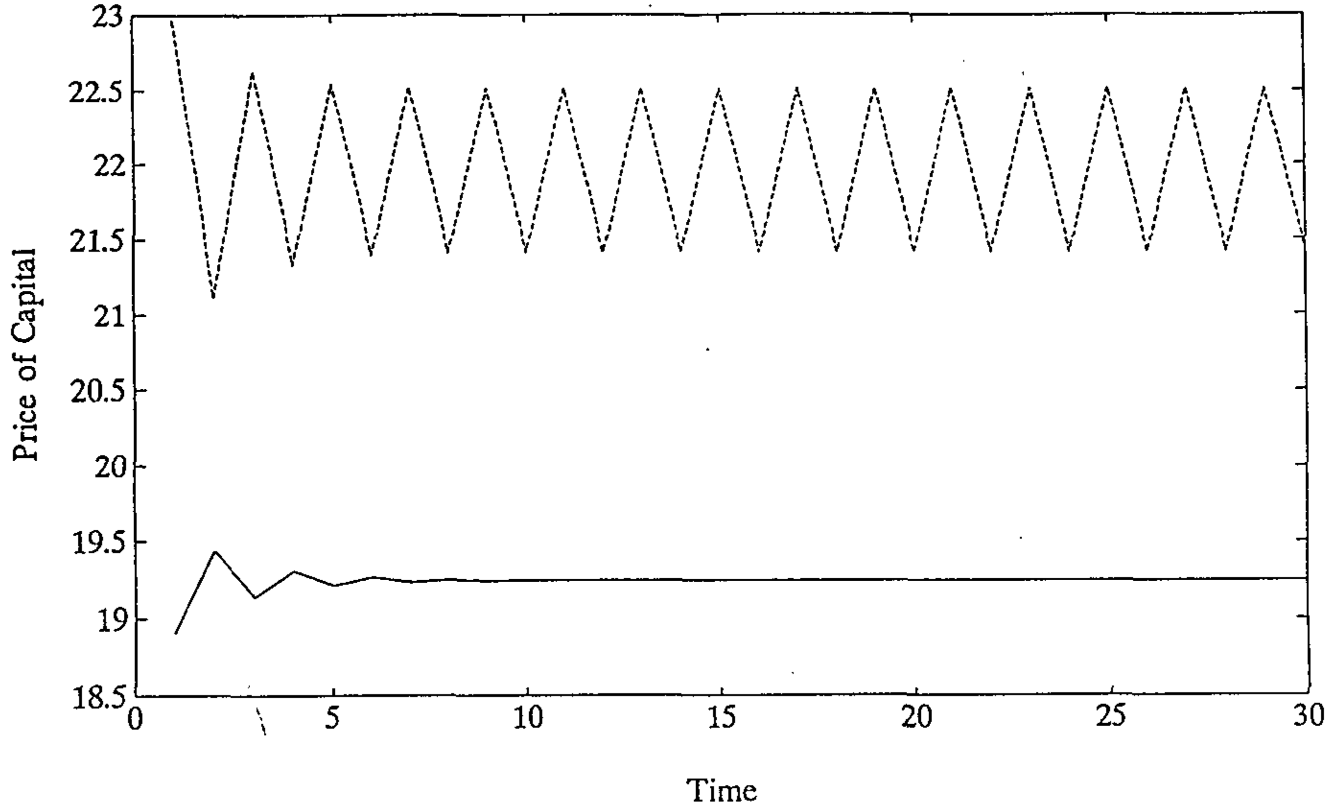
19. However, for the constant tax case, the higher is the value for β, the larger will be the fluctuations that are displayed in reaction to an exogenous disturbance.

20. Hence, this majority voting scheme may be another propagation mechanism whereby temporary technology disturbances would influence future levels of output, although it seems unlikely that these could be used to explain the high-frequency business cycle movements in aggregate time-series.

21. This has everything to do with how central banks are structured in different economies. Some countries, such as the Germany choose to have relatively independent central banks who are supposed to focus primarily on producing price stability. Other countries choose to have central banks that are much less independent of the executive or legislative branches of government, and are more susceptible to political pressure.

This is also related to how government institutions at different levels are designed. For example, what policy forces should be vested in the Federal Government of a country, and which powers should be possessed by the local governments? And how are these powers vested in these different institutions?

Figure 1



Price of Capital
23
22.5
22
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5
10
15
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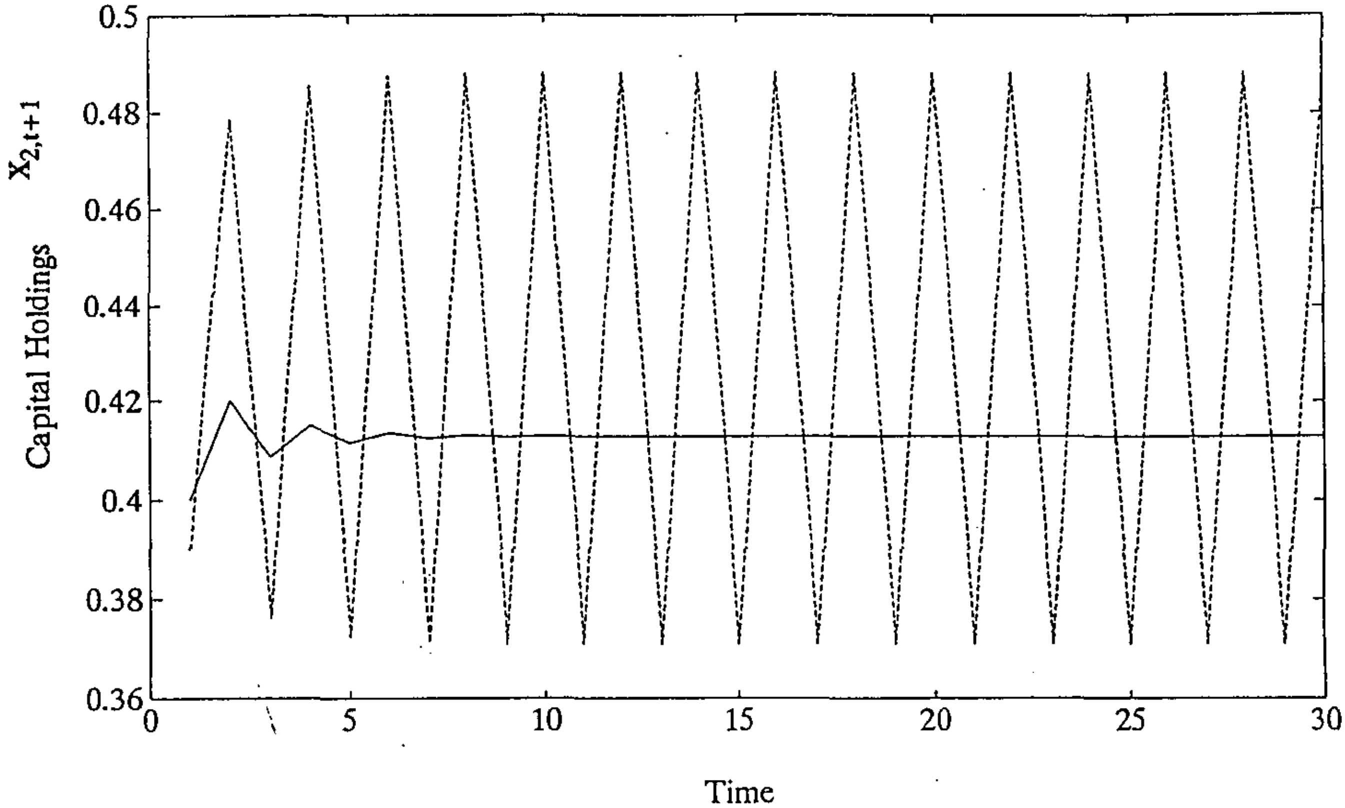
25
30
Time

Figure 2



$x_{2,t+1}$
Capital Holdings
0.5
0.48
0.46
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0.4
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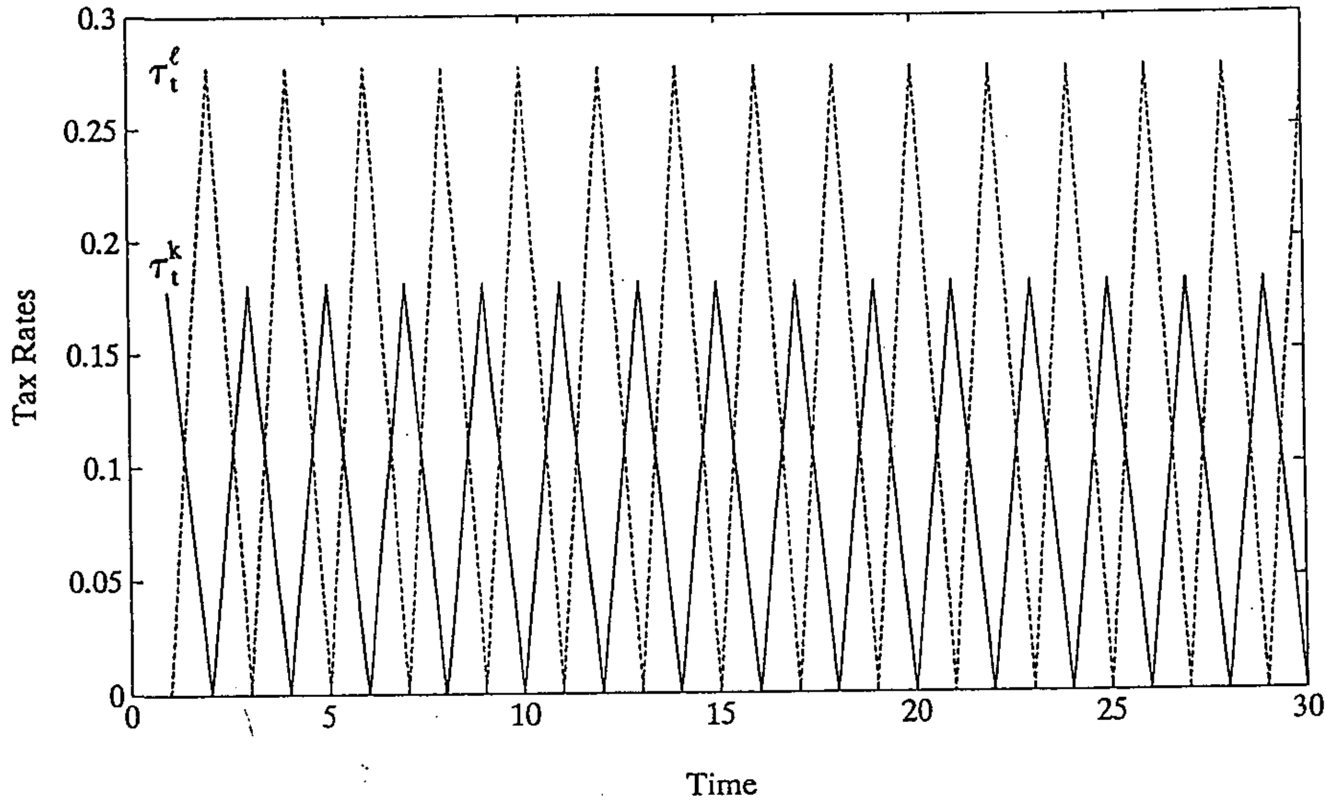
0
5
10
15
20
25
30
Time

Figure 3



0.3
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0.1
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Tax Rates
τ_t^ℓ
τ_t^k
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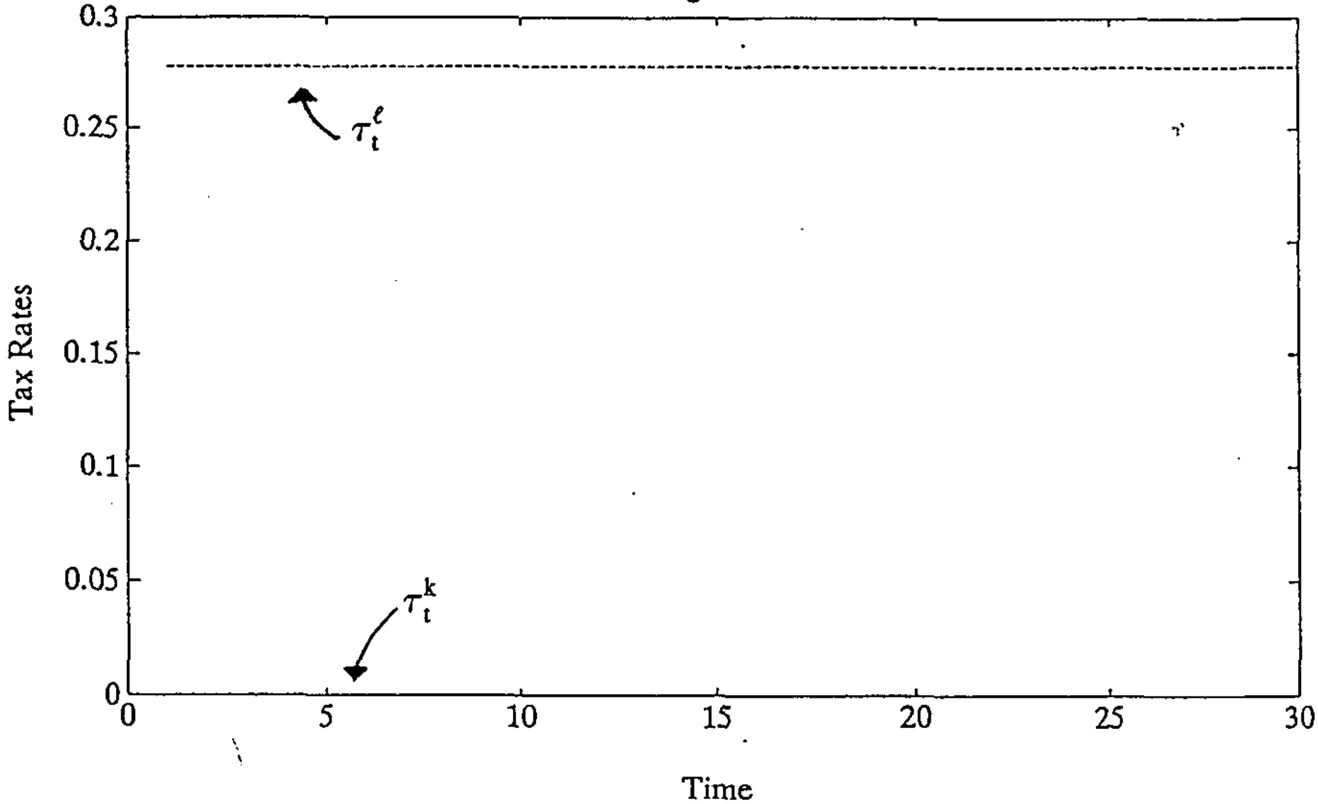
25
30
Time

Figure 4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Tax Rates
τ_t^ℓ
τ_t^k
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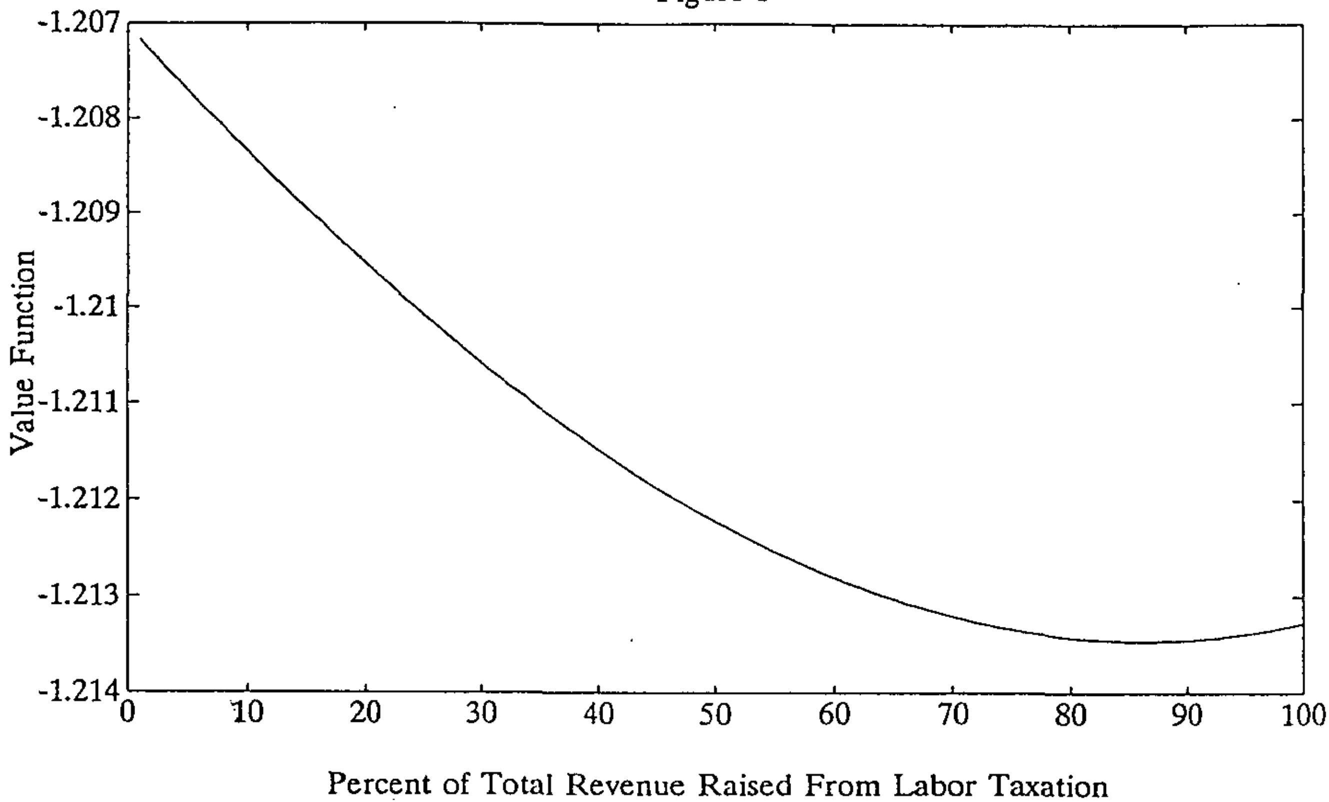
Time

Figure 5


-1.207
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-1.21
-1.211
-1.212
-1.213
-1.214
Value Function
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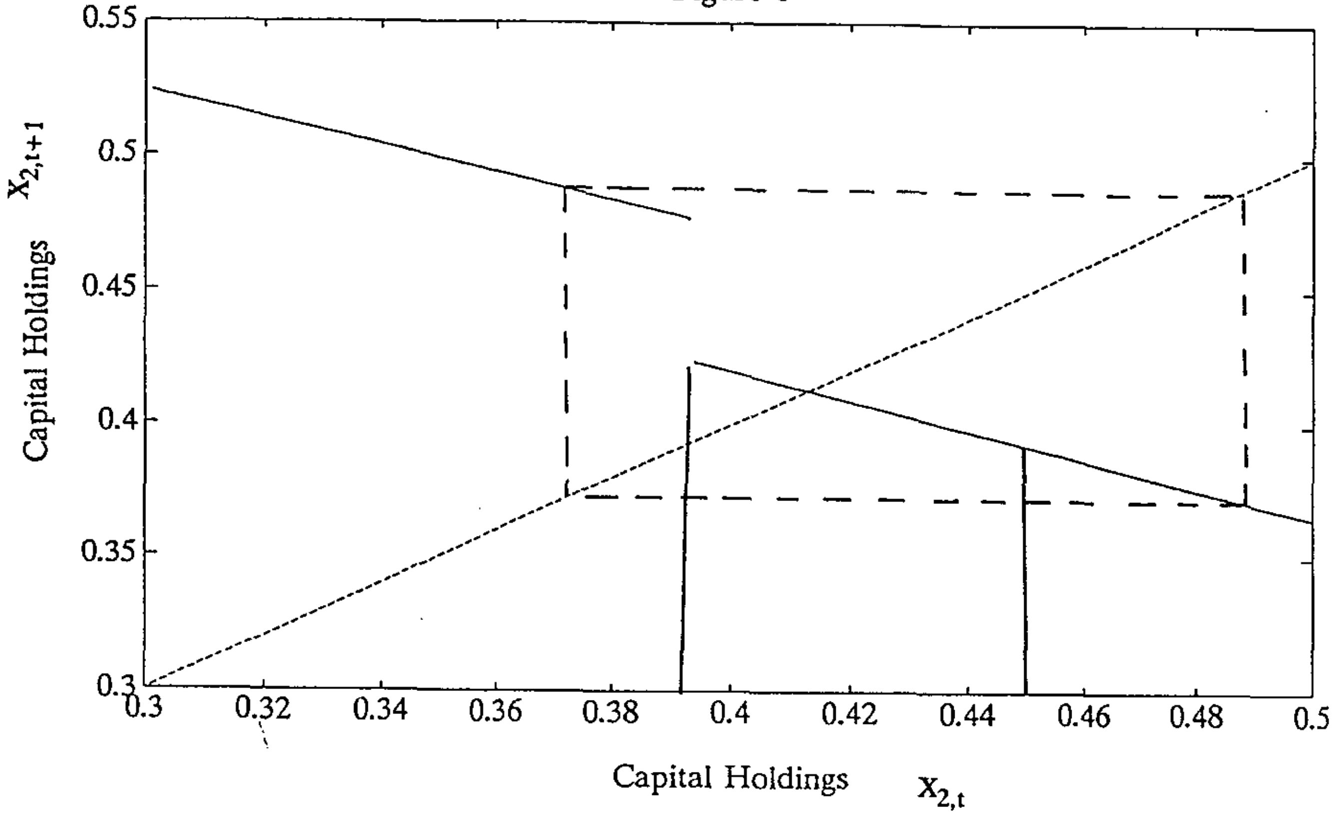
70
80
90
100
Percent of Total Revenue Raised From Labor Taxation

Figure 6


0.55
0.5
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0.3
Capital Holdings $x_{2,t+1}$
0.3
0.32
0.34
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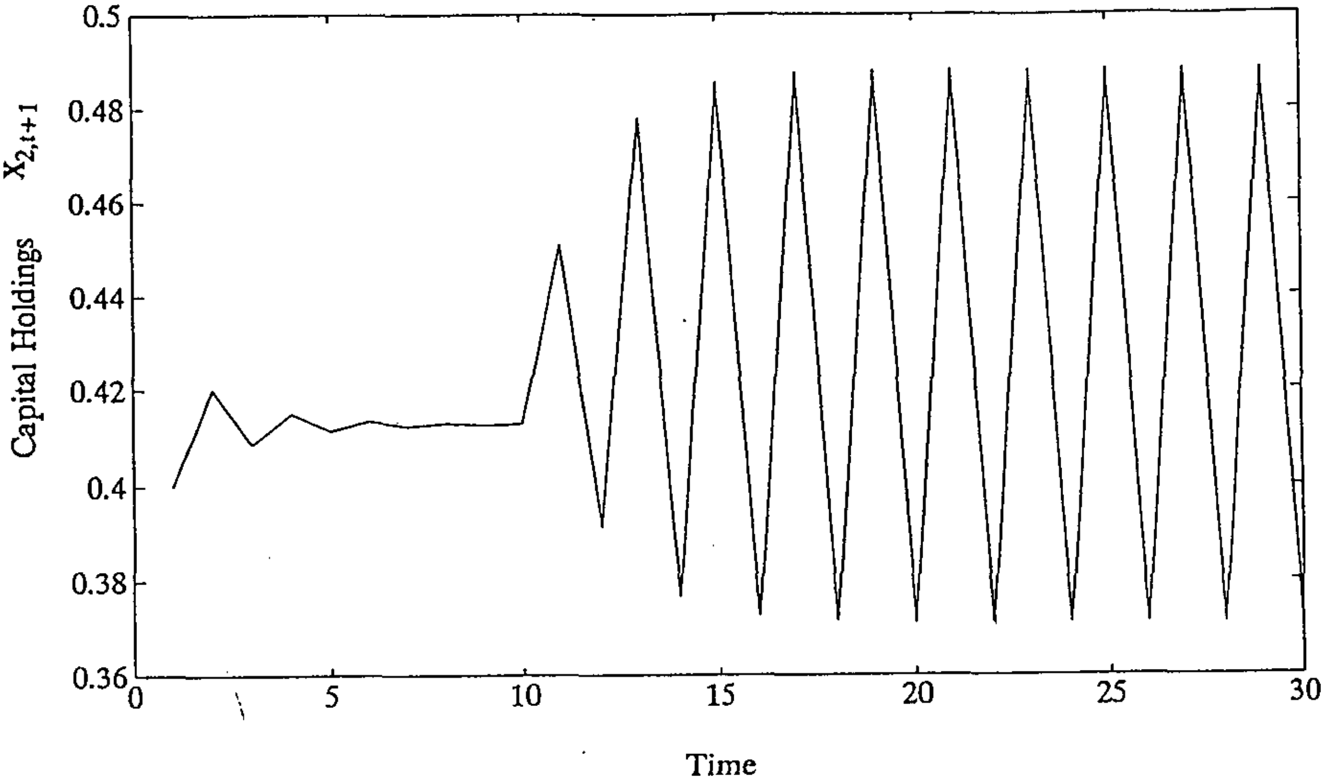
0.38
0.4
0.42
0.44
0.46
0.48
0.5
Capital Holdings $x_{2,t}$

Figure 7



0.5
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Capital Holdings $X_{2,t+1}$
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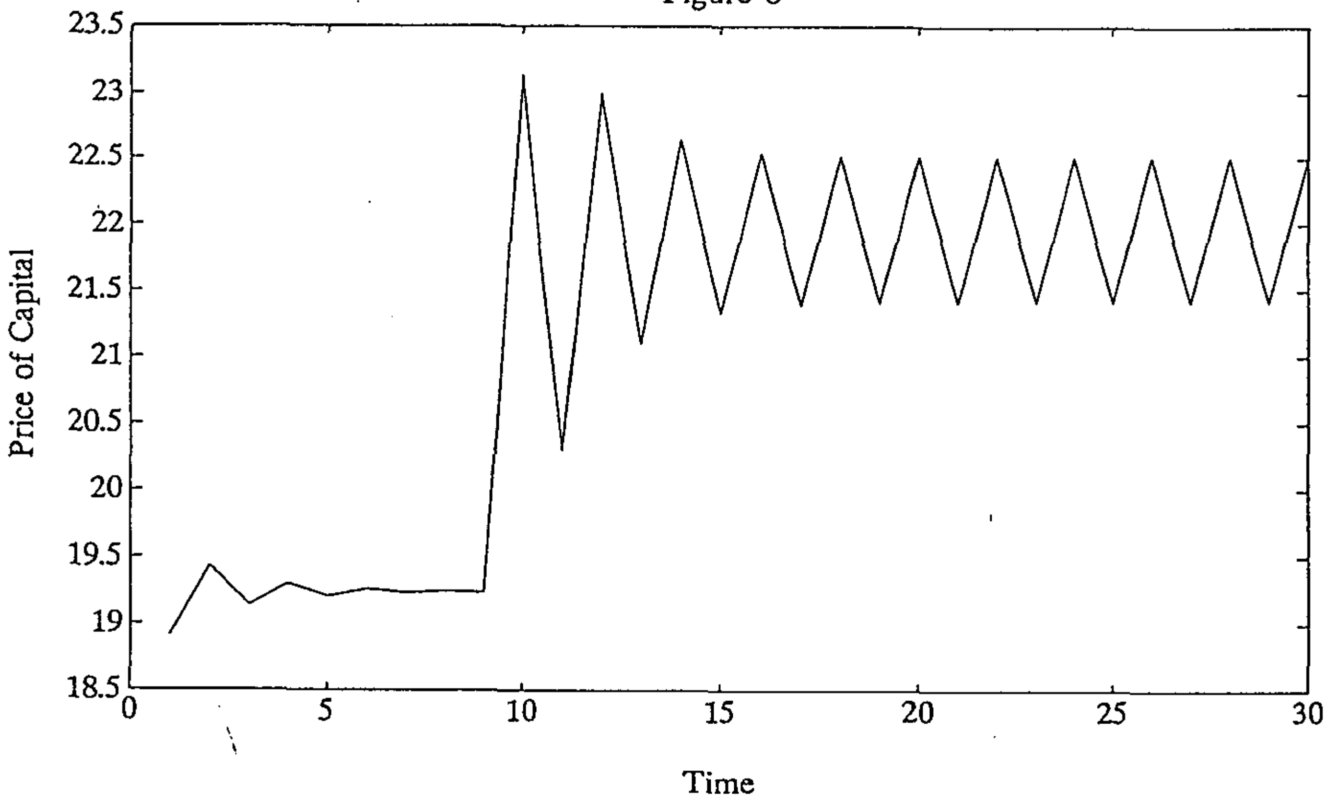
Time

Figure 8



Price of Capital
23.5
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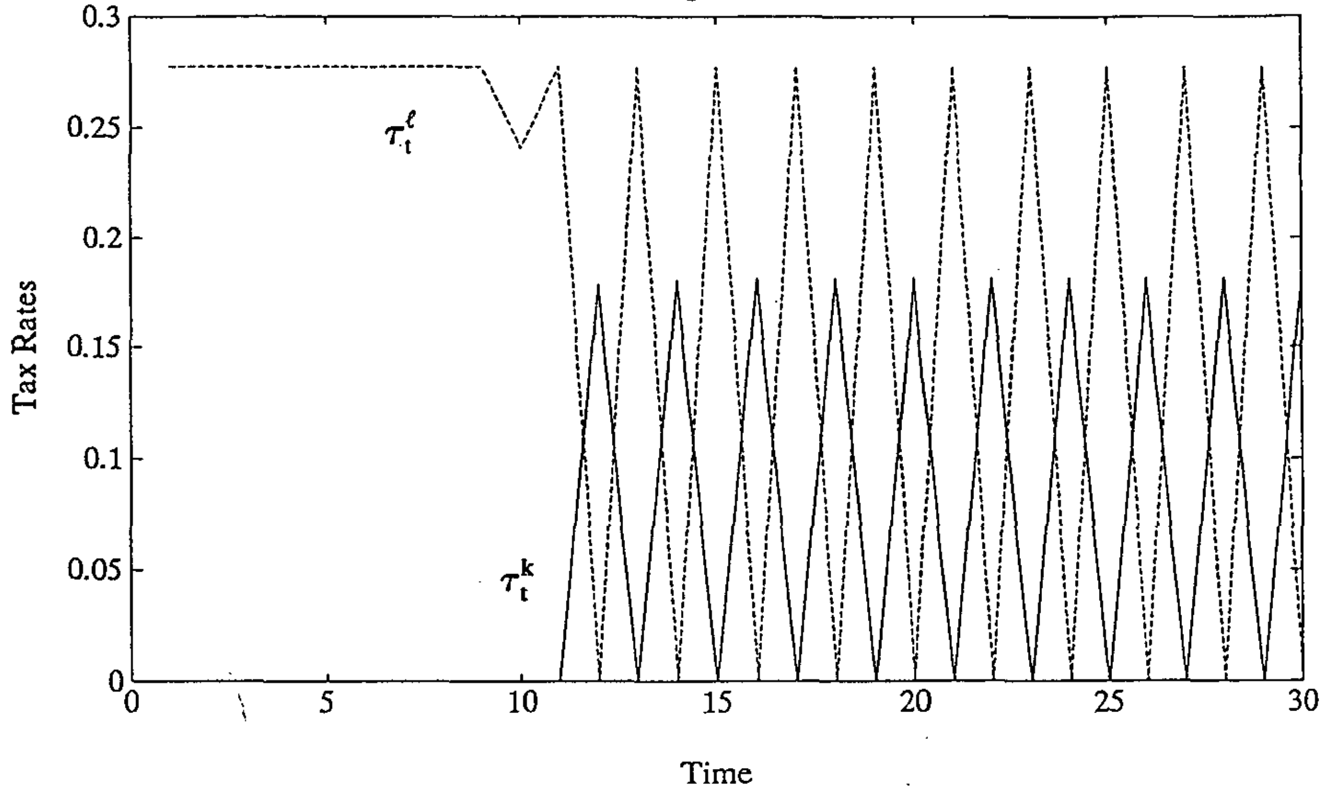
Figure 9



0.3
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Tax Rates
τ_t^ℓ
τ_t^k
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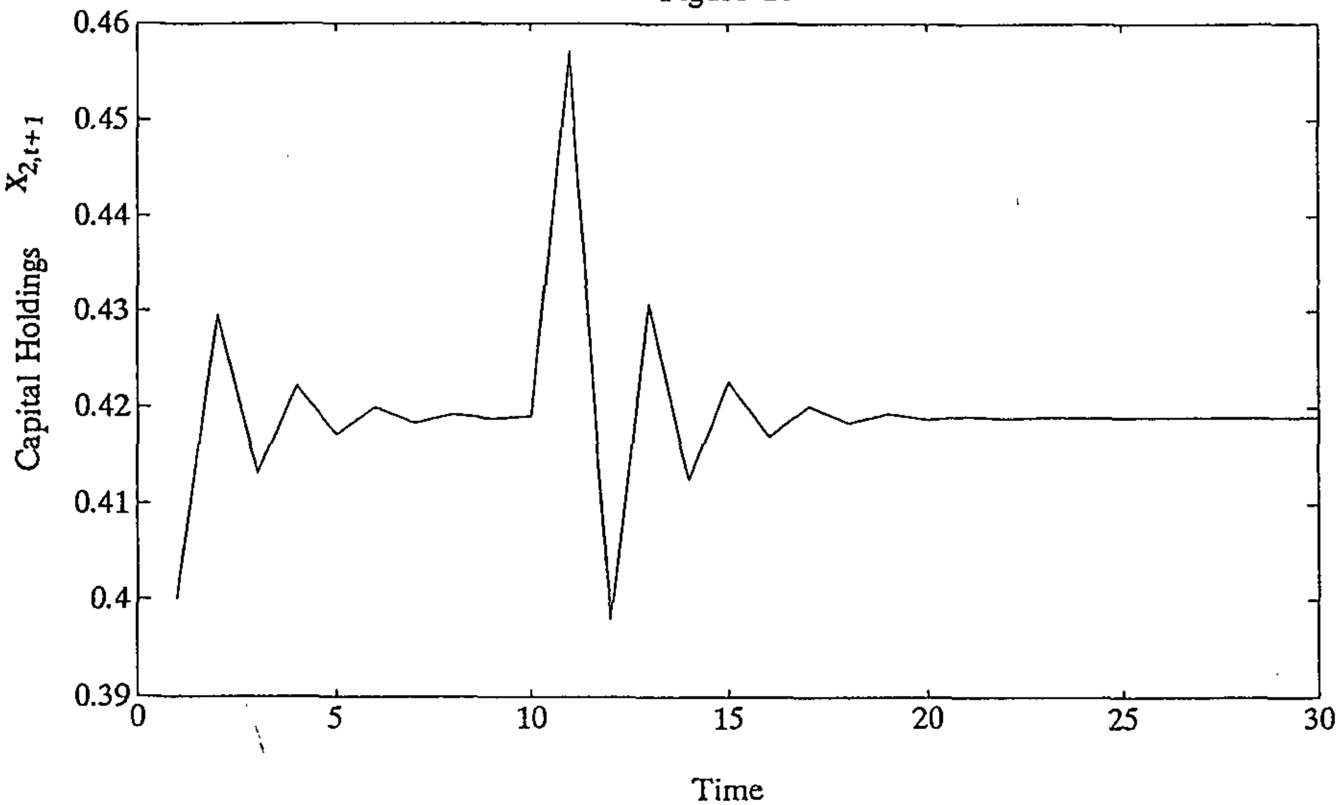
Time

Figure 10


0.46
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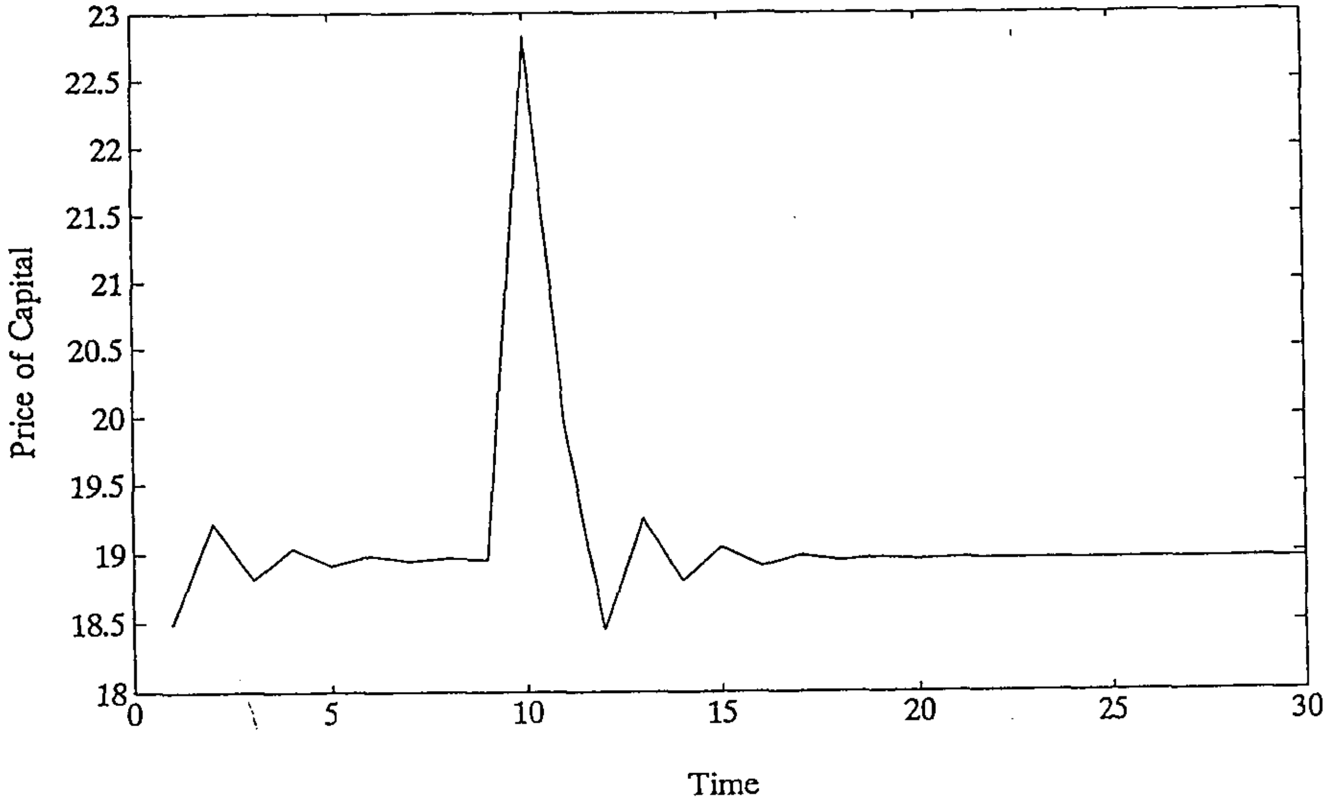
Capital Holdings $x_{2,t+1}$
0
5
10
15
20
25
30
Time

Figure 11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21
20.5
20
19.5
19
18.5
18
Price of Capital
0
5
10
15
20
25
30
Time

Figure 12

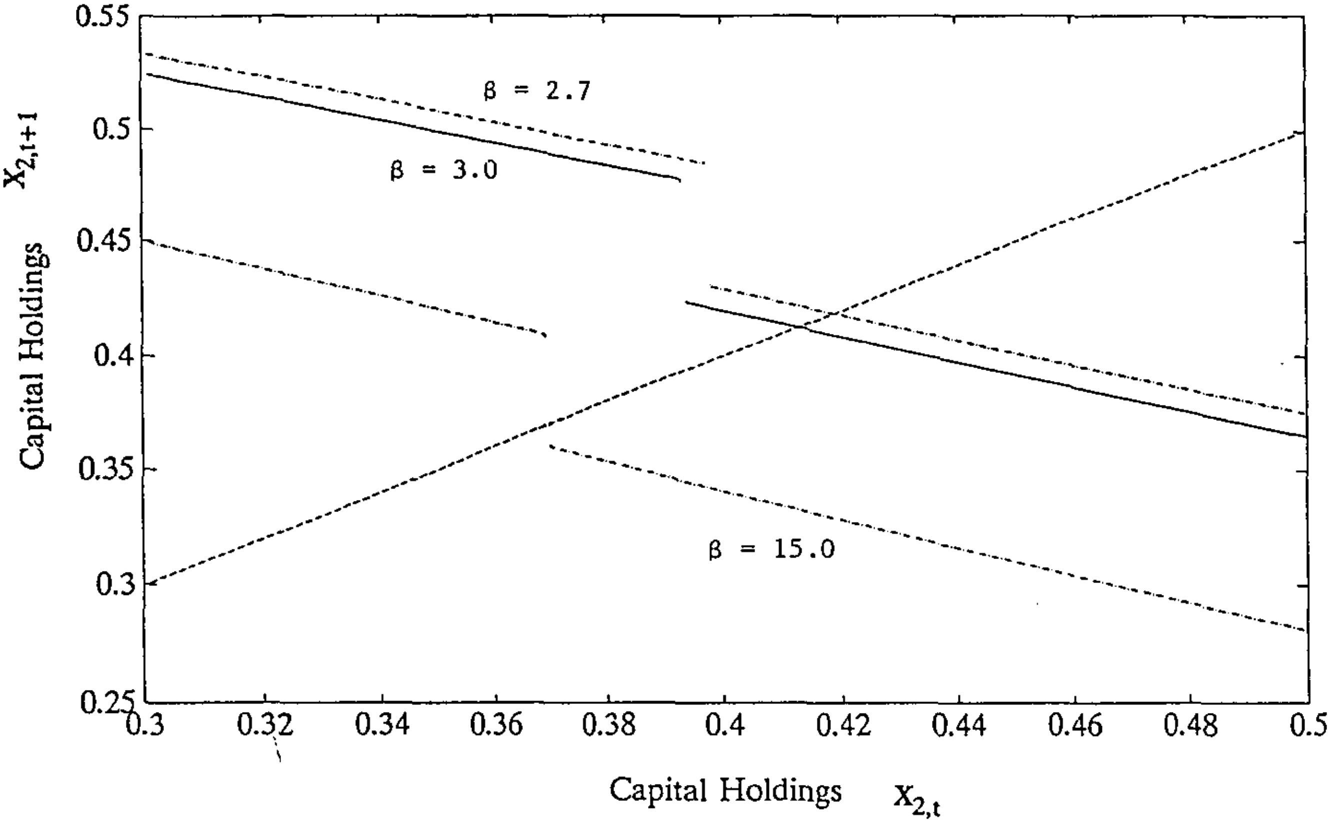
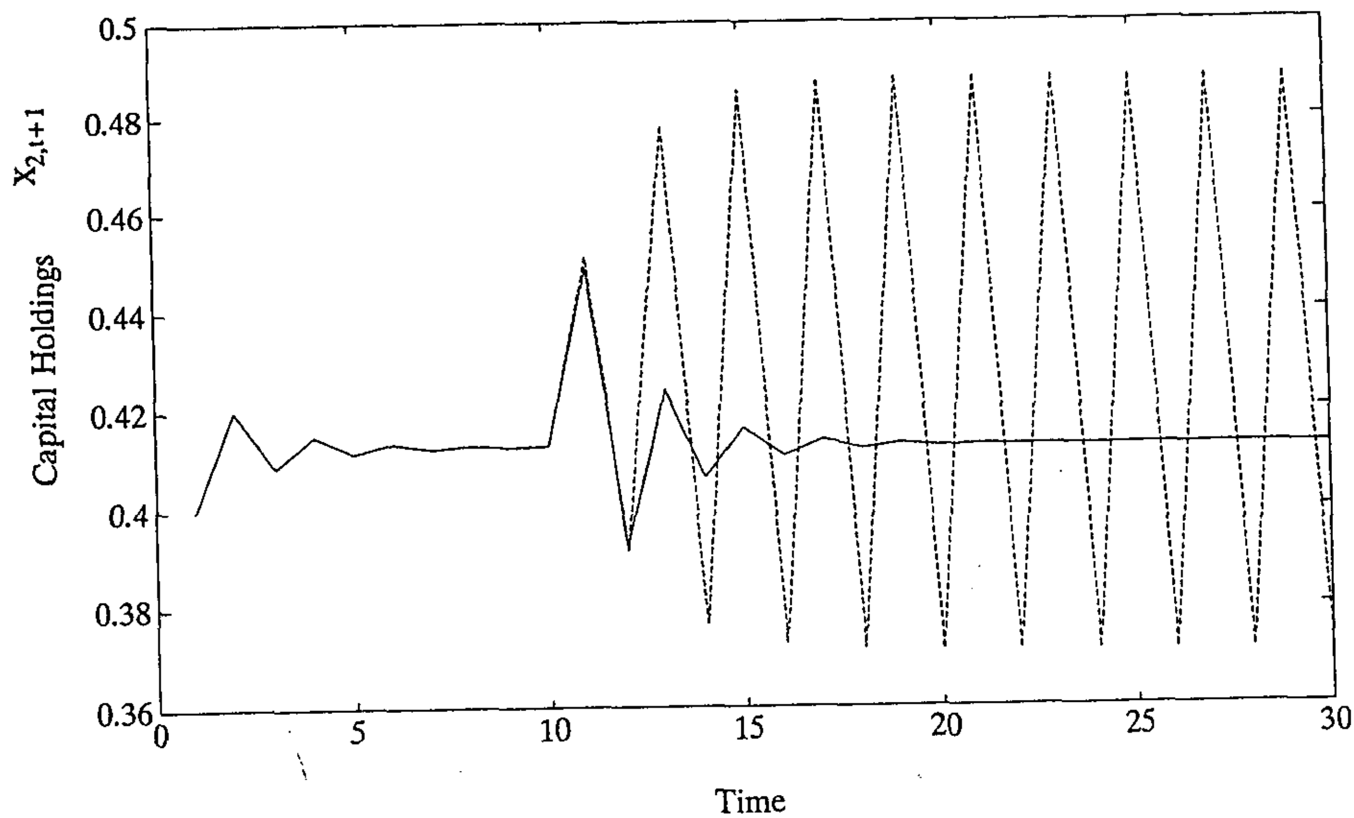

0.55
0.5
0.45
0.4
0.35
0.3
0.25
Capital Holdings $x_{2,t+1}$
β = 2.7
β = 3.0
β = 15.0
0.3
0.32
0.34
0.36
0.38
0.4
0.42
0.44
0.46
0.48
0.5
Capital Holdings $x_{2,t}$

Figure 13



$x_{2,t+1}$
Capital Holdings
0.5
0.48
0.46
0.44
0.42
0.4
0.38
0.36
0
5
10
15
20
25
30
Time

Figure 14

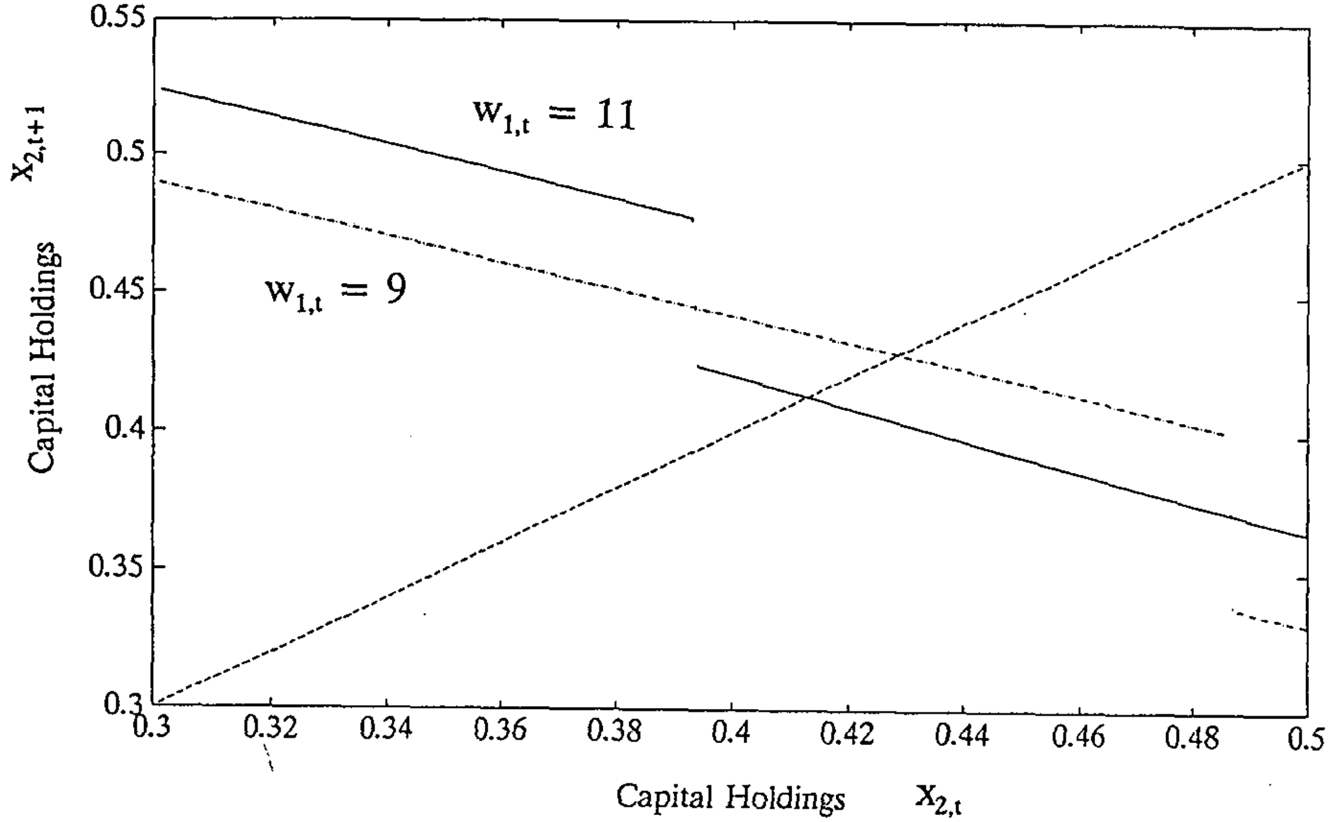
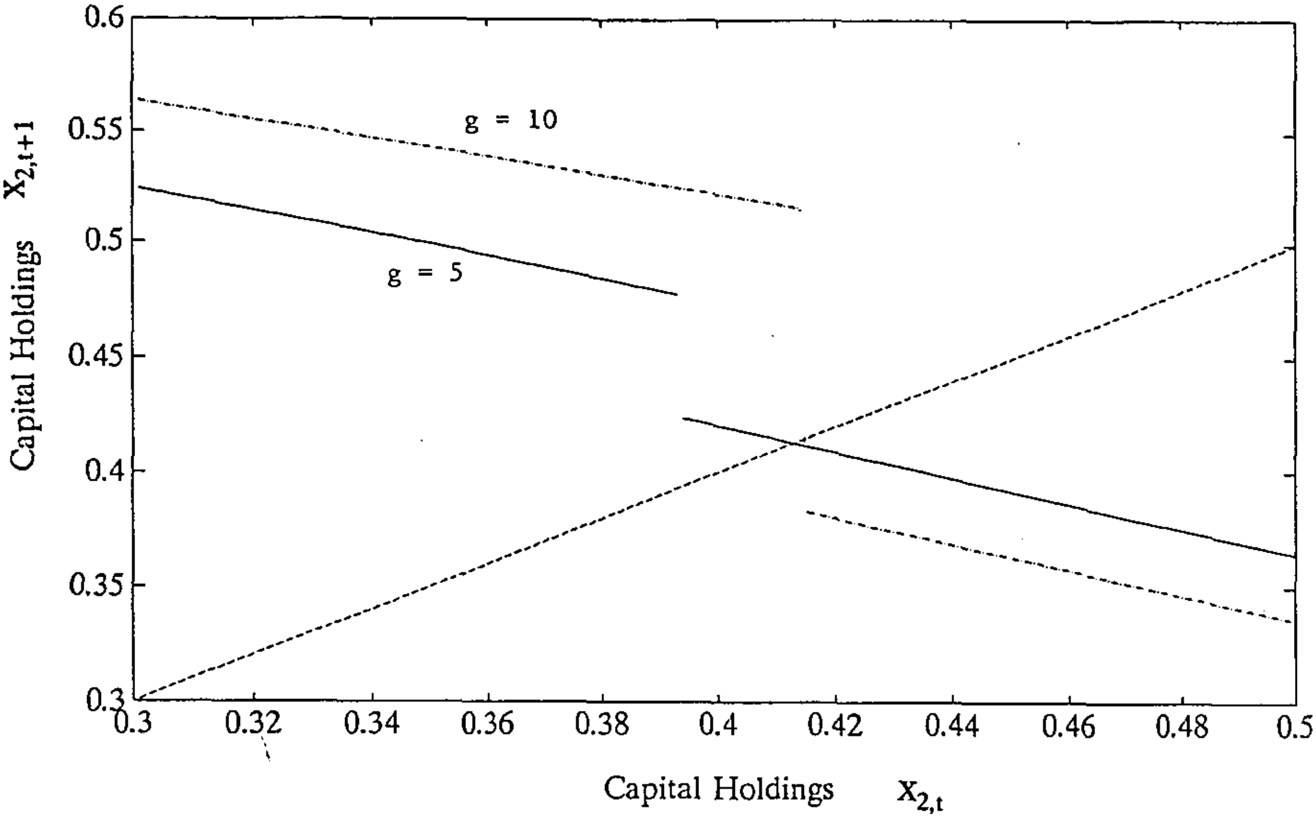

$x_{2,t+1}$
Capital Holdings
$w_{1,t} = 11$
$w_{1,t} = 9$
0.55
0.5
0.45
0.4
0.35
0.3
0.3
0.32
0.34
0.36
0.38
0.4
0.42
0.44
0.46
0.48
0.5
Capital Holdings
$x_{2,t}$

Figure 15



0.6
0.55
0.5
0.45
0.4
0.35
0.3
g = 10
g = 5
Capital Holdings $x_{2,t+1}$
0.3
0.32
0.34
0.36
0.38
0.4
0.42
0.44
0.46
0.48
0.5
Capital Holdings $x_{2,t}$