Is Consumption Variability Across Households Consistent with Efficient Credit Allocation? 
~ A Preliminary Quantitative Study

by

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1. Introduction

Attempts to provide microeconomic foundations for a theory of the business cycle have relied heavily on a model of households as being parametric price takers in competitive financial asset markets. This model in its simple form is implausible, though, because relatively few households own appreciable stocks of financial assets relative to their total wealth. This fact is convincingly documented by Avery, Elliehausen and Kennickell (1988, cited by Aiyagari 1992).

In view of this data, actual discretionary financial-market transactions of typical households could hardly be a significant factor in explaining business cycles. A sophisticated appeal to the model of households as being active, competitive asset-market participants must construe this model as explicating an "as-if" proposition. The standard way of construing this proposition has been to suppose that households make intermediated transactions that are functionally equivalent to asset-market transactions. In particular, households' transactions with banks and other depository and credit intermediaries have been understood to be tantamount to issuance and trading of bonds in a securities market. This understanding of households' intermediated transactions may therefore be called the debt-securities model.

Two alternatives to the debt-securities model have also been proposed to explain the pattern of households' intermediated transactions. One, which may be called the ex-ante inefficiency model, posits that the competitive allocation envisioned according to the debt-securities model is actually distorted by some sort of impediment such as monopoly power of intermediaries or price rigidity of interest rates (in the form of ceilings imposed by usury laws, for example). The other alternative model is the efficient-contract model according to which the financial intermediaries with whom households deal are analogous to competitive providers of long-term insurance contracts than to Walrasian auctioneers in financial-asset markets.

For close to two decades, researchers have been attempting to compare data regarding actual income and consumption to predictions from these various models. Because aggregate fluctuations seem to account for at most one fourth of the variability of a typical household's income stream (cf. Lillard and Willis 1978), convincing work of this sort has to be based on data at the individual household level. Such research has been dominated by contributions that share two common

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1 It may nevertheless be true that nondiscretionary transactions are systematically related to the business cycle. Aggregate new investment in pension funds presumably has positive covariance with the employment level, for example. However, such transactions tend to be determined largely by contractually determined formulae which plausibly are less responsive to business-cycle events than households' discretionary decisions would be.

2 The ex-ante inefficiency model has usually been called the liquidity-constraint model by its proponents. Although we followed that usage in an earlier paper (1991), we abandon it here in recognition of the fact that both it and the efficient-contract model envision that households' consumption allocations will be significantly influenced by non-price rationing.

3 To understand why it is conceptually incorrect to use aggregate data as a basis for examining theories of optimization by individual households in a stochastic environment, consider the extreme case of a closed exchange economy of many households that face independent endowment risks from one another. By the law of large numbers, the level of risk per household in this economy is vanishingly small. However, if each household faces significant individual-level risk, then the gross demand per household for assets or contracts that share this risk will be substantial. (Net demand per household will be zero in a closed exchange economy, so gross demand is what should be
features. First, linear regressions or related estimators have been applied to longitudinal panel data sets on household consumption, and the signs of the estimated coefficients have been evaluated according to rules of thumb derived loosely from the debt-securities model. Second, when those estimates have not been interpreted as corroborating the debt-securities model, they have generally been regarded as providing support for the ex-ante inefficiency model—the efficient contract model has not been considered.

We proposed in a previous paper (1991) that there is a more informative way to compare statistics of longitudinal time-series data on households’ consumption with qualitative implications of alternative models of the household sector. Our proposal involved comparing data generated by actual households to artificial data generated by simulation of equilibrium consumption paths for households in alternative models. We pointed out that there are several types of specific comparison that could be made.

The first sort of comparison that ought to be made between actual and simulated data is a calibration-type comparison. A model is worthwhile to consider only if simulation produces artificial data sets that at least roughly match the moments of actual data. In this paper, we provide such comparisons between an actual data set and the simulation outputs of three models. The actual data to which we refer is from the Panel Study of Income Dynamics (PSID). Specifically, we consider moments of that data that were computed by Hall and Mishkin (1982). Two of the models are endowment-economy versions of the debt-securities and efficient-contract models. By an endowment-economy version of a model, we mean one in which there is a single good at each date and in each state of nature, and in which each household is endowed with a stochastic stream of this good which it divides at each date between current consumption and intertemporal transformation (that is, production of the goods consumable at other dates and states of nature). Such a parametrization contrasts with a labor-economy version of a model in which time and a produced output good can both be consumed at each date and state of nature.

The variance of consumption innovations that these two endowment-economy models imply for a typical household is less than two percent of the corresponding variance of the actual data. In this respect the models are highly unsatisfactory. One interpretation of this finding would be that the debt-security and efficient-contract models are unsatisfactory in general. However, the finding might rather be interpreted to show that the endowment-economy specification is unsatisfactory. We investigate this possibility by formulating a prototypical labor-economy model. This model incorporates an extreme assumption that would prevent it from matching a long-run feature of actual household-consumption data. However, the assumption makes it easy to compute the values that the model implies for the moments with which we will be concerned here. We show that these (studied.) If one tries to explain this significant level of demand per household in terms of the negligible amount of endowment variability per household, a fantastically high level of risk aversion will be “implied” by the data.

For example, the debt-securities model has been interpreted as implying that the covariance of successive consumption innovations should zero. This interpretation is based on the idea that consumption should be a martingale according to the debt-securities model. Strictly speaking, though, that is not an implication of the model. The marginal utility of consumption, not consumption itself, is what plausibly may be approximated by a martingale. Moreover, in careful formulations such as that of Schechtman (1976) in which the lower-bound constraint on feasible consumption is taken into account, the household’s marginal utility is not even exactly a martingale.
moments are mostly of the same sign and order of magnitude as the corresponding moments of
the PSID data. We conclude that less extreme specifications of labor-economy models are worthy
of study. Whether or not such models will be able to account for long-term features of the data
without substantial deterioration of their fit to short-term moments is an open question.

In any event, the present results show that panel-data studies of household's intertemporal
allocation decisions need to be interpreted with great caution. Such panel data typically measure
consumption of goods only, and neglect households' consumption of leisure. Our results show that
the use of goods consumption as a proxy for a consumption aggregate, involving both goods and
leisure, constitutes a measurement problem with serious implications. Baxter and Jervall (1994)
made a closely related point in the context of a model of household production.

2. Features of actual households' consumption

The starting point of most theories of households' consumption patterns is that households use
transactions (made either directly or via intermediaries) to smooth their consumption levels across
dates and across states of nature. If households implicitly attempt with some success to insulate
their consumption streams against both short-run and long-run fluctuations in income, then con-
sumption streams should have two main features. First, the variance of short-run innovations in
consumption should be substantially smaller than the variance of the corresponding short-run inno-
vations in income, and the covariance of consumption with income should be fairly small. Second,
consumption levels should remain roughly constant over time and should not stochastically "fan
out" very much. That is, if the experience of a large community of similar households mirrors well
the stochastic prospects of a typical households, then demographically similar households that be-
gin with similar levels of consumption ought to tend to remain close together in their consumption
levels.

Hall and Mishkin (1982) constructed measures of income and consumption from several years
of longitudinal panel data reported on an annual basis in the PSID. They did show that the two
moments just discussed have the expected properties, although neither of the moments is negligible.
They also reported some additional moments, such as the covariance of innovations in consumption
with lagged innovations in consumption and income.

An important thing to keep in mind about Hall and Mishkin's results is that they have dealt
with econometrically constructed data rather than with "raw" data. There is considerable justifi-
cation for this approach, especially since PSID data are presented on an annual basis while most
households actually receive their income on a monthly basis, and they may also learn about income
innovations such as layoffs with leads that are of substantial length but nevertheless much shorter
than a year. In order to deal with such problems, Hall and Mishkin have constructed income and
consumption series which reallocate the quantities reported in the PSID across annual boundaries
to some extent. Some aspects of this reallocation involve some unavoidable arbitrariness, such as
the use of parametric estimates of households' subjective expectations at one point in the procedure.
This arbitrariness engenders some uncertainty about all of the reported moments, but especially
about covariances between variables that are not contemporaneous. For this reason, as well as for
the obvious reason that it would be a miracle if artificial data derived from highly schematic models were to match actual data with any precision, our main concern is to match the signs and orders of magnitude of variances and contemporaneous covariances.

Deaton and Paxton (1992) have shown that some long-term "fanning out" of consumption levels among initially similar households is evident in longitudinal panel data from several countries. Equilibrium allocations of the endowment-economy versions of the debt-security and efficient-contract models possess this qualitative feature, but it is pointless to compare simulated equilibrium allocations quantitatively with actual data because the comparison of short-run moments has already been so devastating. As we have already mentioned, our prototypical labor-economy model has been specified in a way that makes "fanning out" impossible. This impossibility is a consequence of an extreme assumption that time and the production good are perfect substitutes in consumption at each date. As these pairs of goods are made less substitutable, it is clear (as we will show) that "fanning out" will increasingly occur but also that equilibria allocations of the model will less fully reflect the variability of actual consumption data. Whether or not a permitting a small deterioration in the latter respect will effect a large improvement in the former respect is a matter for further research.

3. Endowment-economy models

In this section we describe the endowment-economy versions of the debt-securities model and the efficient-contract model. This description summarizes more thorough discussions in Green (1987) and Green and Oh (1991).

3.1. The economic environment

We begin by describing the economic environment.

There are a continuum of households that receive stochastically independent, identically distributed income streams. Thus there is no aggregate risk in this economy.

Let \( Y = (Y_t)_{t=1}^{\infty} \) denote the income stream of a typical household. This income stream is a Bernoulli process which is observed contemporaneously and privately by the household. On the basis of this observation, the household makes choices that result in a stochastic consumption stream \( X = (X_t)_{t=1}^{\infty} \). The household's expected discounted utility of this consumption is \( U(X) = \mathbb{E} \sum_{t=1}^{\infty} \beta^t u(X_t) \).

The economic environment that we study can best be regarded as an open economy in which borrowing and lending are available at a gross interest rate \( R = 1/\beta \).

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5 The choice of this value of \( R \) imposes the Fisher equation on the model. Atkeson and Lucas (1992) have noted that the model is thus a partial-equilibrium one. However the imposition of the Fisher equation is not essential to our characterization or computation of equilibrium. We make this assumption because it is a reasonable approximation that reduces by one the number of parameters of the model. We regard it as being an innocuous assumption for our present purpose of studying moments at business-cycle frequencies. The fundamental difference between our model and that of Atkeson and Lucas is that the consumption good can be transformed across time in our model.
3.2. The debt securities model

The traditional way to think about borrowing and lending in the environment just described is in terms of “budget-constrained” trading of bonds—securities with noncontingent returns. If one takes seriously the idea of a budget constraint in this infinite-horizon environment, then that constraint must be equivalent to a variational condition (specifically that discounted marginal utility of a present-value unit of consumption is a martingale) and a transversality condition. However, because all trading must be based on information available at finite dates while the transversality condition depends on the full informational history, it is hard to see how such trading could be feasible subject to the informational constraints imposed by the environment.

So, despite the common description of the debt-securities model as a model of budget-constrained trading, that is not what the model really assumes. Rather it assumes that each household may issue and purchase bonds subject to a constraint that its indebtedness should not exceed the expected discounted (at rate $R$) present value of its future endowment stream.

3.3. The efficient-contract model

When the expected discounted value of the endowment stream is adopted as an indebtedness constraint, the endowment stream is implicitly being treated as collateral which is valued on a complete set of markets for dated, state-contingent consumption. This assumption implies in turn that there must be an intermediary that has access to markets on which the individual households are restricted from participating. Then the question arises, why can an intermediary not trade the endowment stream of every household (so that it bears no risk in per-capita terms) for a guarantee of a constant, risk-free consumption stream? That is, if the intermediary can acquire the endowment stream of a household by default, why can it not acquire it by voluntary negotiation? This interpretive puzzle suggests that the notion of collateralization implicit in the debt-securities model is a suspicious one. The efficient-contract model analyzes financial intermediation in an environment where the intermediary can participate in a complete system markets to which the household lacks access but in which households' endowment streams are inalienable.

This model is based on the idea that households send unverifiable messages to the intermediary regarding their endowment realizations, and that at each point a household’s net trade with the intermediary is based on its history of these messages. Since the messages are unverifiable, the intermediary must act on them in such a way that households are provided with incentives to report truthfully. (Townsend (1988) has proved a version of the revelation principle for an environment similar to this one.) The intermediary can precommit itself to a policy, called a contract, regarding how it will respond to messages. It chooses this policy to maximize profits, and the condition for competitiveness among equilibrium is that these profits are driven to zero. The competitive equilibrium allocation of this model bears strong resemblance to actual intermediary arrangements in which households engage, both in its institutional form (that a credit balance is a sufficient statistic for a household's treatment according to the contract) and in its resemblance to the consumption pattern envisioned by Friedman's (1957) permanent income hypothesis.

A contract is a sequence of mappings from finite histories of reports to current net trades at
each date. Formally,
\[ \Gamma = \{ \Gamma_t : \mathbb{R}^t \rightarrow \mathbb{R} \}_{t=1}^{\infty} \].
(1)

These mappings define consumption levels at each date and state of nature, according to
\[ X_t = Y_t + \Gamma_t(Y_1, \ldots, Y_t). \]
(2)

As emphasized above, \( \Gamma \) must be incentive compatible. That is, given the net trades guaranteed by the contract conditional on various possible reports, it must be in the household's interest to give truthful reports even though truth cannot be verified directly.

3.4. Cost minimization and efficiency

The efficient contract can be characterized in terms of a cost-minimization problem. Define the resource expenditure of contract \( \Gamma \) to be
\[ E \sum_{t=1}^{\infty} \beta^t \Gamma_t(Y_1, \ldots, Y_t). \]
(3)

The cost \( C(v) \) of providing utility level \( v \) to the typical household is the minimum resource expenditure of any incentive-compatible contract \( \Gamma \) such that \( U(Y + \Gamma) = v \). If \( v^* \) is a utility level such that \( C(v^*) = 0 \), then an efficient contract is an incentive-compatible contract that provides utility level \( v^* \) with zero resource expenditure.

4. Simulating households' equilibrium consumption streams

Now we discuss how the consumption streams of typical households in the endowment-economy versions of the debt-securities model and the efficient-contract model can be simulated.

4.1. The debt-securities model

The household's optimization problem in the debt-securities model has a variational condition which takes a rather complicated form essentially equivalent to that derived by Schechtman (1976) for a model where the debt-constraint level is zero. Following Hall (1978), researchers have approximated this variational condition by the martingale condition which is the appropriate variational condition for the model with a true infinite-horizon budget constraint. Here we will also follow Hall by using the martingale condition to simulate a consumption path which we will treat as being implied by a debt-securities trading equilibrium. We believe that this method yields a good approximation except when the household is close to its maximum-indebtedness constraint.

4.2. Endowment economy version of the efficient-contract model: overview

Now we turn to the efficient-contract model. The first step in simulation of a household's consumption stream is to characterize a tractable necessary and sufficient condition for a contract to be incentive compatible. The cost function defined in section 3.4 is approximated using this condition. Finally a simulated sample path of a household's consumption stream is constructed using the approximated cost function.
The prototype labor-economy version of the efficient-contract model will be solvable in direct form. Thus it does not have to be simulated numerically. However, as we have already mentioned, that prototype version fails to reflect an important long-run property of data from actual economies. We anticipate that a more satisfactory labor-economy version of the model will have to be simulated, and that the method developed here will be usable with no essential change.

4.3. Temporary incentive compatibility

The intermediary's optimization problem subject to the constraint of incentive compatibility can be characterized solely in terms of a variational condition, temporary incentive compatibility. That is, the intermediary's profit-maximizing contract imposes the transversality condition for truth-telling on households automatically.

A household's report to the intermediary at any date has implications for both its current and future treatment according to the contract. Temporary incentive compatibility can be understood intuitively by considering the current and future components of the household's treatment separately. For any income report, the household's future treatment according to the contract may be regarded as being specified by a new contract. Such future-treatment contracts are indexed by the current contract (which has been determined by the household's past reporting) and its current report. For simplicity, suppose that the household's endowment stream is a Bernoulli process so that the household's report at each date will be either 0 or 1. Let \( z^i \) denote the current net trade that the intermediary will offer to the household if report \( i \) is given, and let \( \Gamma^i \) denote the contract that will determine the intermediary's future treatment of the household in that case.

For any real \( z^0 \) and \( z^1 \), and for any incentive-compatible contracts \( \Gamma^0 \) and \( \Gamma^1 \), define a new contract \( \Gamma \) by:

\[
\Gamma_1(0) = z^0 \text{ and } \Gamma_1(1) = z^1
\]

For \( t > 1 \),

\[
\Gamma_t(y_1, \ldots, y_t) = \Gamma_{t-1}^w(y_2, \ldots, y_t)
\]

Adopt some convenient notation:

\[
Y' = \langle Y_2, Y_3, \ldots \rangle.
\]

\[
v^0 = U(Y' + \Gamma^0) \text{ and } v^1 = U(Y' + \Gamma^1).
\]

\( \Gamma \) is incentive compatible if and only if the household's expected present discounted value of telling the truth is at least as great as that of misrepresentation for either possible endowment realization. That is,

\[
u(z^0) + \beta v^0 \geq u(z^1) + \beta v^1
\]

and

\[
u(1 + z^1) + \beta v^1 \geq u(1 + z^0) + \beta v^0.
\]
These two constraints (8) and (9) define temporary incentive compatibility at the household's history under consideration. Let $IC$ denote the set of $(z^0, z^1, v^0, v^1)$ satisfying the constraints at every history.

4.4. A functional equation for the cost function

Taking advantage of the equivalence between temporary incentive compatibility and true, infinite-horizon incentive compatibility, the cost function used to characterize the efficient contract can be characterized in turn by a functional equation. Essentially this equation states that the cost of assuring the household of a given level of discounted expected utility is the discounted sum of the cost of the current net trade and the expected cost of future net trades.

$$C(v) = \min \{ \Pr(Y_i = 0)[z^0 + \beta C(v^0)] + \Pr(Y_i = 1)[z^1 + \beta C(v^1)] \}$$
subject to
$$v = \Pr(Y_i = 0)\left[u(z^0) + \beta v^0\right] + \Pr(Y_i = 1)\left[u(1 + z^1) + \beta v^1\right]$$
and
$$(z^0, z^1, v^0, v^1) \in IC.$$  (12)

4.5. Computing the cost function

The functional equation (10) can be used to compute an approximation to the cost function $C$. In order to explain this, suppose that $C$ is an “appropriate” metric space of functions $C$ and that $A \subseteq C$ is a “flexible” subspace of $C$ consisting of computable functions. Define $\Phi : C \rightarrow C$ by

$$[\Phi(C)](v) = \min \{ \Pr(Y_i = 0)[z^0 + \beta C(v^0)] + \Pr(Y_i = 1)[z^1 + \beta C(v^1)] \}$$
subject to the constraints (11) and (12).

Now, what is meant by $C$ being appropriate is that $\Phi$ is a contraction mapping in some metric on $C$ (cf. Thomas and Worrall (1990)). Then one can construct a computable approximation $\Psi : A \rightarrow A$ to $\Phi$ which is also a contraction mapping. Thus, starting with any $\alpha \in A$ and iterating the application of $\Psi$,

$$C \approx \lim_{n \to \infty} \Psi^n(\alpha).$$

That is, the accuracy of the iterative approximation in (14) is limited only by the distance of $C$ from $A$.

In practice, we have begun by solving the minimization problem of equation (10) subject to equation (11), but not subject to equation (12). That is, we have solved the cost function for providing the household with any given level of expected discounted utility in an environment where full insurance is feasible. This is an easy problem to solve in closed form for the classes of utility functions used in consumption theory and macroeconomics. Next, $A$ is taken to be the set of all functions obtained by adding any constant function to the full-insurance cost function. The distance between any two functions in this set is simply the absolute value of difference between the constants that define them. The image of $\Psi$ at any $\alpha \in A$ is obtained by evaluating $\Phi(\alpha)(x)$ at
a finite number of points \( x \in \mathbb{R} \) and then choosing the element of \( A \) which minimizes the sum of squared errors at those points. We have iterated this procedure until reaching a fixed point. Then we have tested the accuracy of our approximation by enlarging \( A \) to include linear combinations of its original elements with scalar multiples of one or more other functions, and seeing how far the solution in this larger space was from the original solution. In the case of negative exponential utility, where Green (1987) proved that the true value function lies in \( A \) as we have defined it narrowly, our numerical approximation reflected this fact. Thus we believe that numerical round-off error per se is negligible. In the case of other parametric families of utility functions, the changes in solutions obtained by enlarging \( A \) to increase “flexibility” were extremely slight.

4.6. Simulating a household

Simulation is based on an approximation of \( C \) computed in the way just described. To begin the simulation, set \( v_1 = C^{-1}(0) \). After \( v_t \) has been set, draw \( y_t \) from Bernoulli process. Set \( \Gamma(y_1, \ldots, y_t) = z^y \) and \( x_t = y_t + \Gamma(y_1, \ldots, y_t) \). Set \( v_{t+1} = v^{y_t} \).

This procedure is continued recursively until as long a sequence as desired has been constructed. The results reported at the end of this paper are based on a simulation of 200 periods. The discount factor \( \beta \) has been set at .96, reflecting an interpretation of the period as being a year. A constant-relative-risk-aversion utility function \( u(x) = -2(x^{-\frac{1}{2}} - 1) \), widely used in calibration of real-business-cycle models, is used. The Bernoulli endowment stream is rescaled to match the sample variance of income innovations reported by Hall and Mishkin (1982). CRRA specification permits such rescaling.

5. A prototype labor-economy efficient-contract model

5.1. The environment

We modify the model described environment in the previous section by introducing a second good, time, at each date and state of nature. At each date, the household is endowed with \( \lambda \) units of time. The stochastic process \( \{Y_t\}_{t=1}^\infty \) is now interpreted as specifying the employment opportunity of the household, rather than an endowment of a good. If \( Y_t = 0 \), the household must consume its time at date \( t \) as leisure. If \( Y_t = 1 \), the household may consume time as leisure or else may use it entirely for production. (Time is indivisible). In that case, the \( \lambda \) units of time produce \( \gamma \) units of the output good. The household’s utility function is \( \mathbb{E} \sum_{t=1}^\infty \beta^t u(g_t + \tau_t) \), where \( g_t \) is the household’s consumption of the output good and \( \tau_t \) is its consumption of time. Assume that \( u \) is strictly increasing and strictly concave. As in the endowment-economy version of the model, realizations of \( Y_t \) are private information of the household.

Note that, in order to make the model tractable to solve, we make an extreme assumption to which we have already alluded. Time and the output good are perfect substitutes in consumption for households in this environment. Assume that

\[
\gamma > \lambda, \tag{15}
\]

so that an autarkic household would work when it had the opportunity to do so.
Let \( p_i \) denote the probability that \( Y_i = i \).

5.2. The efficient contract

Consider a social planner’s problem of providing full insurance without incentive compatibility constraints. Current receipt of an insurance indemnity does not affect a household’s future prospects, so in the notation of section 4.3,

\[
v^1 = v^0.
\]

(16)

Full insurance at the initial date implies that a household that uses its time for production and consumes \( \gamma \) units of the output good plus its net trade will have the same utility level as a household that consumes its time endowment and only the consumption good in its net trade. That is,

\[
u(\gamma + z^1) = u(\lambda + z^0).
\]

(17)

Feasibility requires the average of all households’ net trades to be zero, so

\[
p_0 z^0 + p_1 z^1 = 0.
\]

(18)

Consider the allocation solving this problem in terms of incentive compatibility. That is, would a household with \( X_i = 1 \) be willing to work if its employment opportunity were private information? The answer is affirmative because equations (16) and (17) imply that

\[
u(\gamma + z^1) = u(\lambda + z^0) + \beta v^1.
\]

(19)

Thus the fully-informed social planner’s allocation coincides with the efficient-contract allocation in this environment.

Despite the fact that this is a full-insurance allocation, there can be substantial variability of measured consumption. The reason is that only the household’s consumption of the produced good—not of time—is reflected in measured consumption. According to this reasoning, measured consumption variability may be an artifact of mismeasurement. We now investigate whether this possibility is consistent with the moments reported by Hall and Mishkin (1982).

In order to do so, we solve for the net trades \( z^i \). Because \( u \) is strictly increasing, equation (17) implies that \( \gamma + z^1 = \lambda + z^0 \). This, combined with equation (18), implies that

\[
z^0 = p_1(\gamma - \lambda).
\]

(20)

By inequality (15), then the household without an employment opportunity receives a positive net trade reflecting an insurance indemnity. By equations (18) and (20),

\[
z^1 = -p_0(\gamma - \lambda).
\]

(21)

From equations (20) and (21), we can calculate the moments of measured consumption (that is, the moments of consumption of the produced good) in the efficient-contract allocation.
6. Comparison of model predictions with PSID data

The following table reports comparisons between moments of data constructed from the PSID by Hall and Mishkin (1982) and the predictions of the models discussed here. These predictions were obtained by simulation in the case of the endowment-economy versions of the debt-securities model (EDS) and efficient-contract model (EEC) and by computation of theoretical moments in the case of the prototype labor-economy version of the efficient-contract model (LEC). Rescaling has been used to match the variance of income innovations in the PSID data to that of endowment or output innovations in the model predictions.

<table>
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<th>PSID</th>
<th>EDS</th>
<th>EEC</th>
<th>LEC</th>
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<td>(\text{Var}(\Delta y))</td>
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<td>6.7720</td>
<td>6.7720</td>
<td>6.7720</td>
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References


