Consumption, Commitment and Cycles

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Abstract

There were three important changes in the United States economy during the 1980s. First, from 1982-90, the decade featured the longest consecutive stretch of positive quarterly output growth in United States history. Second, wage inequality expanded greatly as the wages of highly skilled workers grew markedly faster than the wages of less skilled workers (Katz and Murphy (1992)). Finally, consumption inequality also expanded as the consumption of highly skilled workers grew faster than that of less skilled workers (Attanasio and Davis (1994)).

This paper argues that these three aspects of the United States economic experience can be interpreted as being part of an *efficient* response to a macroeconomic shock given the existence of a particular *technological* impediment to full insurance. I examine the properties of efficient allocations of risk in an economic environment in which the outside enforcement of risksharing arrangements is infinitely costly. In these allocations, relative productivity movements have effects on both the current and future distribution of consumption across individuals. If preferences over consumption and leisure are nonhomothetic, these changes in the allocation of consumption will generate persistent cycles in aggregate output that do not occur in efficient allocations when enforcement is costless.

## I. Introduction

There were three important changes in the United States economy during the 1980s. The first two are well-known. First, the decade featured the longest consecutive stretch of positive quarterly output growth in United States history. Second, wage inequality expanded as the wages of highly skilled workers grew markedly faster than the wages of less skilled workers (Katz and Murphy (1992)).

The third change has only been uncovered recently by Attanasio and Davis (1994). They document that just like wages, the consumption of highly skilled workers grew much faster than the consumption of less skilled workers. As they emphasize, a *full insurance* model of risksharing in which individuals can insure themselves against all possible contingencies would predict that the relative consumptions of the two types of workers would be unaffected by the shift in relative wages. Moreover, Attanasio and Davis point out there was nothing private about this shock: the movement of relative wages was a *publicly observable* aggregate disturbance. They conclude that their results represent significant evidence against both full insurance models and models (e.g. Green (1987), Phelan and Townsend (1991), Atkeson and Lucas (1992), Wang (1994)) in which private information is the sole friction disrupting the implementation of full insurance.

This paper argues that these three aspects of the United States economic experience can be interpreted as being part of an *efficient* response to a macroeconomic shock given the existence of a particular *technological* impediment to full insurance. It considers an economic environment in which the outside enforcement of contracts is infinitely costly. In this world, no risksharing arrangement can survive unless individuals find it in their own self-interest to follow its dictates, given that everyone else is going to abide by it.

The infeasibility of outside enforcement has important consequences for the behavior of the cross-sectional distribution of consumption in an efficient allocation of resources. I prove it is typically efficient for society to tilt the distribution of consumption towards individuals whose labor is surprisingly productive relative to others in the economy. This "bribe" keeps the (surprisingly) highly skilled from leaving the societal risksharing arrangement. Moreover, it is efficient for the tilt of the consumption distribution towards those with surprisingly high wages to be persistent: society should spread out over time the "extra" consumption compensation to those who receive a temporarily high wage shock[1]. I show that this persistent response of consumption to wage shocks is consistent with the regression results obtained by Attanasio and Davis (1994).

Besides the absence of outside enforcement, there is a second important feature of the economic environment: preferences are nonhomothetic over consumption and leisure, which implies that Engel curves for leisure are nonlinear. As discussed above, the absence of enforcement implies that it is typically efficient for the cross-sectional distribution of consumption to respond to a shock to the cross-sectional distribution of wages. If the Engel curves for leisure are nonlinear, rich and poor agents adjust their output levels differently in response to the transfer of consumption from the less skilled to the highly skilled, and aggregate output may contract or expand. Cycles in aggregate output thus emerge *endogenously* as an *efficient* response to the impossibility of enforcing risksharing arrangements. Even though the shocks hitting the economy are i.i.d., these cycles are *persistent* because it is efficient to tilt the consumption distribution towards the more highly skilled for many periods, not just for one.

The model thus provides the following interpretation of the effects of the 1980s expansion in wage inequality. If there had been full insurance against this event, aggregate output would not have increased from 1982-90 because the increase in output generated by the high wage individuals would be offset by the decrease in output experienced by the low wage individuals. However, because enforcement is costly, it was efficient to shift consumption towards the high wage earners. This meant that the substitution effects of the wage movements were damped by an income effect. Critically, this income effect was asymmetric because of the nonlinearity (in particular, concavity) of leisure Engel curves. Aggregate output ended up rising because the income effect on the high wage earners of the tilt in the consumption distribution was *smaller* than the income effect on the low wage earners.

While I focus on the experience of the 1980s, the implications of the model should hold for any period in the United States data. Unfortunately, it is more difficult to evaluate the model's predictions because there is little data available on the behavior of consumption inequality for periods previous to the 1980s. Note though that stylized descriptions of the 1920s boom and the recession of 1990-91 also appear consistent with the above story.

The fundamental assumption in this paper is that risksharing agreements, especially ones that mandate a response to widely felt societal shocks, are highly costly to enforce[2]. This view becomes especially compelling when one considers the costs of enforcing progressive income taxation, which (at least in some part) represents an attempt to share risk across individuals. In order to obtain compliance with the terms of this risksharing "agreement", the government uses threats of jail, fines, etc.. Even then, compliance is not perfect as individuals cheat on their income taxes or leave the country in order to escape living up to their end of the societal contract.

In this paper, I take an unrealistic view of enforcement costs by setting them equal to infinity. However, the results mentioned above are likely to survive in models with a more realistic (and more technically demanding) modelling of enforcement technologies. In particular, in any such environment, transitory relative wage disturbances will have effects on both the current and future allocation of consumption; the resultant reallocations will have consequences for the behavior of aggregate output if preferences are nonhomothetic in consumption and leisure.

The paper builds on two different literatures. Thomas and Worrall (1988, 1994), Kocherlakota (1994), and Gauthier and Poitevin (1994) all examine the patterns of risksharing found in infinite horizon environments with exogenous random incomes and costly commitment. The current paper enriches these models by allowing the allocation of time between labor and leisure to be endogenous. Atkeson and Phelan (1994) mention that better technologies of enforcement will lead to more efficient allocations.

The second literature examines how market imperfections can lead to cycles in aggregate output. Scheinkman and Weiss (1986) set up a model in which individuals face diversifiable relative wage shocks, but can only trade a single riskfree asset over time. The wage shocks generate equilibrium movements in the distribution of wealth, which in turn generate cycles in aggregate output because Engel curves for leisure are nonlinear. Kiyotaki and Moore (1993) look at an environment in which entrepreneurs are unable to borrow against their human capital; in this world, small firm-specific shocks are amplified to generate large economy-wide effects.

The rest of the paper is organized as follows. I describe the basic model in the next section, and develop the implications of nonhomothetic preferences. In Section III, I describe the notion of efficiency in a world in which outside enforcement is infinitely costly, and discuss the structure of efficient risksharing arrangements. Section IV compares some statistical implications of the model with the findings of Attanasio and Davis (1994) and others. Section V concludes.

## II. Elements of the Model

### *A. Description*

There are two infinitely-lived individuals. (Alternatively, one can think of the environment as consisting of two types of individuals, with equal numbers of each type.) Each of the individuals are endowed with one unit of time in each period. The state of the world in a given period is determined by the realization of agent 1's marginal productivity of labor w. In any given period, there are S possible realizations for the state variable w; state s occurs with probability $\pi_s$. In any given period, both agents have the same information set which consists of the current and past realizations of the state variable.

When state s occurs, individual 1 is able to convert time into some amount of the perishable consumption good according to the individual specific technology:

$$y_s^1 = w_s n_s^1$$

where $n^1$ is the amount of time spent working by agent 1 and $y^1$ is the amount of consumption good produced by agent 1. Similarly, when state s occurs, individual 2 is able to convert time into consumption according to the formula:

$$y_s^2 = (1-w_s)n_s^2$$

Thus, the two individuals have idiosyncratic productivities; throughout, I refer to these productivities as "wages" (although I impose no market structure). Note that "aggregate productivity" is time and state invariant in the sense that the sum of the two agents' productivities is equal to one in every date and state[3]. I also assume that $w_s$ is i.i.d. and that $\Pr(w = w_s) = \Pr(w = 1-w_s)$ for all s[4].

An allocation is a stochastic vector process $\{(c_t^j,\ n_t^j)_{t=1}^{\infty}\}_{j=1}^{2}$ which is restricted to be measurable with respect to current and past realizations of s; the realizations of the process must lie in the set $([0,1] \times [0,1])^2$. We can model uncertainty using a Debreu tree. In particular, we can think of an allocation as being an element of $C^2$, where $C = \Pi_{t=1}^{\infty} ([0,1] \times [0,1])^{S^t}$. We can endow C with the product topology, and then $C^2$ is a compact space. A feasible allocation is an element of $C^2$ such that $c_t^1 + c_t^2 \leq w_t n_t^1 + (1-w_t)n_t^2$.

In period t, all individuals have the same preferences over future consumption and leisure:

$$E_t \ \Sigma_{\tau=t}^{\infty} \ \beta^{t-\tau} \ \{(c_t)^{1-\alpha}/(1-\alpha) + \ln(1-n_t)\},\ 0 < \beta < 1,\ \ \alpha > 0$$

where $c_t$ is consumption in period t and $n_t$ is the fraction of time spent working. (Setting $\alpha = 1$ corresponds to making the utility function over consumption logarithmic.) Note that these preferences are nonhomothetic over consumption and leisure unless $\alpha = 1$. When I refer to the utility derived by agent j from a given allocation, I mean his ex-ante utility, which is evaluated before any uncertainty has been resolved.

### *B. First Best Allocations*

I first examine the properties of allocations that are efficient when enforcement is costless; I term such allocations *first best*. Any first best allocation is an element $(c^1,\ n^1,\ c^2,\ n^2)$ of $C^2$ that solves the following maximization problem:

$$\max_{\{c_t^j,\ n_t^j\}} \quad E_0 \ \Sigma_{t=1}^{\infty} \beta^{t-1} \{(c_t^1)^{1-\alpha}/(1-\alpha) + \ln(1-n_t^1)\}$$

$$\text{s.t. } E_0 \ \Sigma_{t=1}^{\infty} \beta^{t-1} \{(c_t^2)^{1-\alpha}/(1-\alpha) + \ln(1-n_t^2)\} \geq u_0$$

$$\text{s.t. } c_t^1 + c_t^2 \leq w_t n_t^1 + (1-w_t)n_t^2$$

$$\text{s.t. } n_t^j \geq 0 \text{ for all } t,\ j.$$

for some reservation level of utility $u_0$ for agent 2. Assuming that the lower bound on labor is never binding, the first order conditions to this problem imply that for each $u_0$, agent 1 receives the same fraction $\gamma(u_0)$ of total consumption in every date and state. Hence:

$$c_t^1 = \{\hat{\gamma}(u_0)/(1-\hat{\gamma}(u_0))\}c_t^2$$

$$(c_t^1)^{-\alpha} w_t = 1/(1-n_t^1)$$

$$(c_t^2)^{-\alpha}(1-w_t) = 1/(1-n_t^2)$$

Define aggregate output to be $Y_t = c_t^1 + c_t^2 = w_t n_t^1 + (1-w_t)n_t^2$. Then:

$$(Y_t\hat{\gamma}(u_0))^{-\alpha} w_t = 1/(1-n_t^1)$$

$$(Y_t(1-\hat{\gamma}(u_0)))^{-\alpha}(1-w_t) = 1/(1-n_t^2)$$

which implies that:

$$(Y_t)^{\alpha}\{\hat{\gamma}(u_0)^{\alpha} + (1-\hat{\gamma}(u_0))^{\alpha}\} = w_t + (1-w_t) - Y_t = 1 - Y_t.$$

A reparameterization in terms of $\gamma$ instead of $u_0$ implies that an allocation in $C^2$ is first best if and only if there exists some constant $\gamma \in [0,1]$ such that:

$$c_t^1 = \gamma\hat{Y}(\gamma)$$

$$c_t^2 = (1-\gamma)\hat{Y}(\gamma)$$

$$(1-n_t^1) = (c_t^1)^{\alpha}/w_t$$

$$(1-n_t^2) = (c_t^2)^{\alpha}/(1-w_t)$$

where $\hat{Y}(\gamma)$ is the unique solution $Y$ to the equation $Y^{\alpha}(\gamma^{\alpha} + (1-\gamma)^{\alpha}) + Y = 1$ for a given $\gamma$.

This analysis shows that in a first best allocation there are no cycles in aggregate output. The intuition behind this result is simple. Wage movements typically have both a wealth and substitution effect on individual time allocation. In a first best allocation, the wealth effect is eliminated because of perfect insurance. Because of the form of preferences, the substitution effect of the offsetting movements in wages is symmetric across individuals: hence, one individual's increase in output is exactly the same as the other individual's decrease.

However, if $\alpha \neq 1$, then the level of aggregate output is not the same *across* first best allocations: how the resources are split across the two individuals affects the level of aggregate output. In particular, if $\alpha > 1$, then increasing $|\gamma - 1/2|$ increases $(\gamma^{\alpha} + (1-\gamma)^{\alpha})$, which means that $\hat{Y}(\gamma)$ falls. Similarly, if $\alpha < 1$, then $\hat{Y}(\gamma)$ is decreasing in $|\gamma - 1/2|$.

The intuition behind the differences in the level of aggregate output across first best allocations derives from the shape of the Engel curve for leisure. Suppose the planner picks a different reservation level of utility for agent 2, so that resources are taken from the rich and given to the poor (that is, $\gamma$ is made closer to 1/2). Because leisure is a normal good, it is optimal for the social planner to increase the effort of the rich and decrease the effort of the poor in every state. However, the relative sizes of these changes depends on the shape of the Engel curve for leisure. For example, when $\alpha < 1$, the Engel curve for leisure is strictly concave. In that case, the increase in the output of the rich is smaller than the decrease in the output of the poor: making the distribution of resources more equal reduces aggregate output. Conversely, if $\alpha > 1$, the Engel curve for leisure is convex, and increasing equality increases aggregate output[5].

## III. Efficient Sustainable Allocations

### *A. Sustainable Allocations*

I now want to consider an environment in which it is infinitely costly to force either individual to comply with any risksharing arrangement. In this environment, a risksharing arrangement can only survive if both agents find it optimal to follow its dictates, taking as given the decision of the other agent to abide by the arrangement.

It is immediately clear that no allocation can survive that does not provide both individuals with more utility than what they would receive by reverting to autarky in any date or state. In fact, any allocation that promises more utility than autarky to both agents in every date and state can be supported by a risksharing arrangement which specifies that default in any date and state results in a reversion to an "agreement" to never make transfers again. Since this autarkic "agreement" is self-enforcing (given that the other agent is not going to make insurance transfers to me, why should I make them to him?), any arrangement supported by the punishment of autarky can survive without the benefit of outside enforcement.

To motivate these concepts more formally, define:

$$u_{aut}(w) \equiv \text{Max}_{0 \leq n \leq 1} \ (wn)^{1-\alpha}/(1-\alpha) + \ln(1-n)$$

$$V_{aut} \equiv \Sigma_{s=1}^{S} \ \pi_s u_{aut}(w_s)/(1-\beta)$$

Thus, $u_{aut}(w)$ is the highest amount of utility an individual can achieve within a given period if his marginal productivity equals w; note that $u_{aut}$ is increasing in w. Correspondingly, $V_{aut}$ is the ex-ante discounted expected value of living in autarky forever. Note that the ex-ante utility associated with any sustainable allocation is greater than or equal to $V_{aut}$.

With these definitions in hand, it is possible to fully characterize the set of allocations that can be implemented with the available enforcement technology.

*Definition 1*: A feasible allocation $(c^1, n^1, c^2, n^2)$ in $C^2$ is *sustainable* if and only if:

(1) $E_t \sum_{\tau=0}^{\infty} \beta^{\tau}\{(c^1_{t+\tau})^{1-\alpha}/(1-\alpha) + \ln(1-n^1_{t+\tau})\} \geq u_{aut}(w_t) + \beta V_{aut}$

(2) $E_t \sum_{\tau=0}^{\infty} \beta^{\tau}\{(c^2_{t+\tau})^{1-\alpha}/(1-\alpha) + \ln(1-n^2_{t+\tau})\} \geq u_{aut}(1-w_t) + \beta V_{aut}$

in any date and state.

If an allocation is sustainable, then neither agent has any incentive to deviate from it: sustainable allocations are the ones that are attainable even if individuals cannot be forced to comply with an arrangement that they find against their interests *after* the resolution of uncertainty. Note that default on the arrangement can take one of two forms: failing to make the correct transfer or failing to work the correct amount.

*B. Defining Efficiency: A Recursive Approach*

I now turn to describing the characteristics of efficient allocations in this world with limited enforcement. To do so, let $\Gamma$ be the set of sustainable allocations. Then, an element $(c^1, n^1, c^2, n^2)$ of $C^2$ is *efficient* if it is a solution to the maximization problem:

$$\text{Max} \quad E\{\textstyle\sum_{t=0}^{\infty} \beta^{t}\{(c^1_t)^{1-\alpha}/(1-\alpha) + \ln(1-n^1_t)\}\}$$

$$\text{s.t.} \quad E\{\textstyle\sum_{t=0}^{\infty} \beta^{t}\{(c^2_t)^{1-\alpha}/(1-\alpha) + \ln(1-n^2_t)\}\} = u_0$$

$$\text{s.t.} \quad (c^1, n^1, c^2, n^2) \in \Gamma$$

The set $\Gamma$ is convex and compact; hence, a solution to this maximization problem exists for any $u_0$. As $u_0$ changes, the solutions sketch out the contract curve in this environment with infinitely costly enforcement[6].

This infinite dimensional maximization problem is somewhat unwieldy. However, it turns out that there is an alternative recursive representation for an efficient allocation. In the above maximization problem, define $V(u_0)$ to be the maximized value of agent 1's utility function given that agent 2 receives ex-ante utility $u_0$; also, define $V_{max} = V(V_{aut})$ to be the maximal level of ex-ante utility attainable by either agent. Note that V is concave and decreasing in $u_0$; Kocherlakota (1994) proves that V is differentiable in $u_0$ as well.

Then, following Kocherlakota (1994) and Thomas and Worrall (1988), it is easy to show that V satisfies the following functional equation (FE):

$$V(u_0) = \max_{c_s^1, n_s^1, c_s^2, n_s^2, u_s} \Sigma_{s=1}^{S} \pi_s[(c_s^1)^{1-\alpha}/(1-\alpha) + \ln(1-n_s^1) + \beta V(u_s)]$$

$$\text{s.t. } \Sigma_{s=1}^{S} \pi_s[(c_s^2)^{1-\alpha}/(1-\alpha) + \ln(1-n_s^2) + \beta u_s] \geq u_0 \qquad (C1)$$

$$\text{s.t. } (c_s^1)^{1-\alpha}/(1-\alpha) + \ln(1-n_s^1) + \beta V(u_s) \geq u_{aut}(w_s) + \beta V_{aut} \qquad (C2)$$

$$\text{s.t. } (c_s^2)^{1-\alpha}/(1-\alpha) + \ln(1-n_s^2) + \beta u_s \geq u_{aut}(1-w_s) + \beta V_{aut} \qquad (C3)$$

$$\text{s.t. } c_s^1 + c_s^2 \leq w_s n_s^1 + (1-w_s) n_s^2 \qquad (C4)$$

$$\text{s.t. } V_{aut} \leq u_s \leq V_{max}$$

$$\text{s.t. } n_s^1 \geq 0,\ n_s^2 \geq 0$$

In this problem, the variable $u_s$ represents the total ex-ante utility from future consumption and leisure.

According to this recursive problem, we can think about the evolution of agent 2's utility $u_t$ in an efficient sustainable allocation as follows. In period 0, the social planner fixes the weight placed on each individual by choosing a value for $u_0$ - this is exogenous to the problem. Then he updates this value for agent 2's reservation utility by setting $u_1 = u_s$, where s is the realization of the state in period 1. He then re-solves the problem (FE), treating the value $u_1$ as agent 2's reservation utility.

*C. Structure of Efficient Allocations*

The first order conditions of the maximization problem in (FE) are as follows (assuming that the inequality constraints on $u_s$ and $n_s^j$ are nonbinding):

$$(c_s^1)^{-\alpha}(\pi_s + \mu_s^1) - \nu_s = 0 \qquad \text{(FOC1)}$$

$$(c_s^2)^{-\alpha}(\pi_s\lambda + \mu_s^2) - \nu_s = 0 \qquad \text{(FOC2)}$$

$$(1/(1-n_s^1))(\pi_s + \mu_s^1) - \nu_s w_s = 0 \qquad \text{(FOC3)}$$

$$(1/(1-n_s^2))(\pi_s\lambda + \mu_s^2) - \nu_s(1-w_s) = 0 \qquad \text{(FOC4)}$$

$$\beta V'(u_s)(\pi_s + \mu_s^1) + \beta(\pi_s\lambda + \mu_s^2) = 0 \qquad \text{(FOC5)}$$

Here, $\lambda$ is the multiplier on (C1), $\mu_s^1$ is the multiplier on (C2), $\mu_s^2$ is the multiplier on (C3) and $\nu_s$ is the multiplier on (C4). I will term (C2-C3) the sustainability constraints.

Just as in a first best allocation, knowing the distribution of consumption across agents in a given date and state of an efficient sustainable allocation allows one to figure out all other aspects of the allocation in that date and state. The first step to showing this is to note that from (FOC1), (FOC2) and (FOC5):

$$(c_s^1)^{-\alpha}/(c_s^2)^{-\alpha} = -V'(u_s)$$

Hence, there is a strictly decreasing deterministic function $\hat{u}$ such that $u_s = \hat{u}(\gamma_s)$, where $\gamma_s = c_s^1/(c_s^1 + c_s^2)$.

From (FOC1) and (FOC3), we see that for any $u_0$ and any state s, $(c_s^1)^{-\alpha}w_s = 1/(1-n_s^1)$. Similarly, from (FOC2) and (FOC4), we see that for any $u_0$

and any state s, $(c_s^2)^{-\alpha}(1-w_s) = 1/(1-n_s^2)$. Thus, the absence of commitment does not affect the intratemporal efficiency of the allocation of resources. This result implies that $Y_s = \hat{Y}(\gamma_s)$ in any date and state of an efficient sustainable allocation, where (as in Section IIB) $\hat{Y}$ is defined to be the solution to the equation:

$$\hat{Y}^{\alpha}(\gamma^{\alpha} + (1-\gamma)^{\alpha}) + \hat{Y} = 1$$

Recall from Section IIB that the function $\hat{Y}$ is symmetric around 1/2 in the sense that $\hat{Y}(\gamma) = \hat{Y}(1-\gamma)$ for all $\gamma$; $\hat{Y}$ is increasing in $|\gamma - 1/2|$ if $\alpha < 1$ and decreasing in $|\gamma - 1/2|$ if $\alpha > 1$. Some simple comparative statics confirms the intuition that $\gamma\hat{Y}(\gamma)$ is always increasing in $\gamma$ (even though $\hat{Y}(\gamma)$ is not).

It is easy to construct a function $\hat{n}$ such that:

$$n_s^1 = \hat{n}(\gamma_s, w_s) \text{ and } n_s^2 = \hat{n}(1-\gamma_s, 1-w_s)$$

in every date and state. The function $\hat{n}$ is chosen in such a way so that it solves the equation:

$$\gamma^{-\alpha}\hat{Y}(\gamma)^{-\alpha}w = 1/(1-\hat{n})$$

for any $\gamma$ and $w$. Note that $\hat{n}$ is increasing in $w$ (substitution effect) for any given $\gamma$, and is decreasing in $\gamma$ (wealth effect) for any $w$.

Thus, in any date and state, all of the endogenous variables ($c_s^1$, $c_s^2$, $n_s^1$, $n_s^2$, $u_s$) are time and state invariant functions of $\gamma_s$ and $w_s$. All that is left to determine is how the cross-sectional distribution of consumption varies over dates and states in an efficient allocation. We have seen that in a first best allocation, the distribution of consumption is time and state invariant; however, in general, the presence of the sustainability constraints means that this will not be the case in an efficient sustainable allocation.

Throughout the following analysis, I assume that the discount factor $\beta$ is sufficiently close to one that there exists some nonautarkic sustainable allocation; this immediately implies that $V_{max} > V_{aut}$. (Note that if $V_{max} = V_{aut}$, the characterization of the efficient sustainable allocations is simple but uninteresting: only autarkic allocations are efficient.) To characterize the evolution of $\gamma$ over time, we begin by dividing the S states into three groups.

$$S_1 = \{s|\ \mu_s^1 > 0\},\quad S_2 = \{s|\ \mu_s^2 > 0\},\ S_3 = \{s|\ \mu_s^1 = \mu_s^2 = 0\}$$

Since $V_{max} > V_{aut}$, the two sets $S_1$ and $S_2$ do not intersect. Suppose they did. Then for some state s:

$$(c_s^1)^{1-\alpha}/(1-\alpha) + \ln(1-n_s^1) + \beta u_s^1 = u_{aut}(w_s) + \beta V_{aut}$$

$$(c_s^2)^{1-\alpha}/(1-\alpha) + \ln(1-n_s^2) + \beta V(u_s^2) = u_{aut}(1-w_s) + \beta V_{aut}$$

Since $u_s \geq V_{aut}$ and $V(u_s) \geq V_{aut}$, it follows that:

$$(c_s^1)^{1-\alpha}/(1-\alpha) + \ln(1-n_s^1) \leq u_{aut}(w_s)$$

$$(c_s^2)^{1-\alpha}/(1-\alpha) + \ln(1-n_s^2) \leq u_{aut}(1-w_s)$$

However, the allocation of consumption and leisure is intratemporally first best: neither of these inequalities can be strict. Hence, the two inequalities are actually equalities, which implies $u_s = V(u_s) = V_{aut}$. But this is impossible if $V_{aut} < V_{max}$.

The following four lemmas use the three sets of states to describe the contemporaneous relationship between the distribution of consumption across agents and the realization of wages, given the value of $u_0$ (or equivalently, $\gamma_0 = \hat{u}^{-1}(u_0)$).

*Lemma 1*: Suppose s,r lie in $S_1$. Then $w_s > w_r$ implies $\gamma_s > \gamma_r$ and $w_s = w_r$ implies $\gamma_s = \gamma_r$ and $u_s = u_r$.

*Proof*: Suppose not. Then $\gamma_s \leq \gamma_r$ and $w_s > w_r$. We know that since $\gamma\hat{Y}(\gamma)$ and $(1-\hat{n})$ are increasing in $\gamma$, and $(1-\hat{n})$ is decreasing in w:

$$(c_s^1)^{1-\alpha}/(1-\alpha) + \ln(1-n_s^1) + \beta V(u_s) < (c_r^1)^{1-\alpha}/(1-\alpha) + \ln(1-n_r^2) + \beta V(u_r)$$

But this is impossible, because $u_{aut}(w_s) > u_{aut}(w_r)$.

The proof of the second statement is similar. $\Delta$

Thus, Lemma 1 shows that an individual's share of aggregate consumption is increasing in his wage when his sustainability constraint binds. Intuitively, insurance is only partial when the sustainability constraint binds; hence, when an individual receives a high wage, his consumption is higher.

The following lemma considers what happens when neither agent's sustainability constraint binds.

*Lemma 2*: Suppose s,r lie in $S_3$. Then $\gamma_s = \gamma_r = \gamma_0$.

*Proof*: Since s,r lie in $S_3$, $V'(u_r) = V'(u_s) = \lambda$, which in turn equals $V'(u_0)$ from the Envelope Theorem. The result follows. $\Delta$

Thus, if neither agent's constraint binds, consumption inequality does not depend on the realization of the wage. This is similar to what happens in a first best allocation.

The next lemma says that an individual's constraint binds whenever his wage is higher than some cutoff point.

*Lemma 3*: Suppose s lies in $S_1$ and $w_r \geq w_s$. Then, r lies in $S_1$.

*Proof*: Suppose instead r lies in $S_2$ or $S_3$. Then from (FOC5), $V'(u_s) > V'(u_r)$, and so $u_r > u_s$. This implies that $\gamma_r < \gamma_s$. From the logic in the proof of Lemma 1, we can conclude that:

$$(c_r^1)^{1-\alpha}/(1-\alpha) + \ln(1-n_r^1) + \beta V(u_r) < (c_s^1)^{1-\alpha}/(1-\alpha) + \ln(1-n_s^1) + \beta V(u_s)$$

$$= u_{aut}(w_s) + \beta V_{aut}$$

$$\leq u_{aut}(w_r) + \beta V_{aut}$$

which violates the sustainability constraint (C2). Δ

Finally, Lemma 4 demonstrates that an individual receives a higher fraction of aggregate consumption when his sustainability constraint binds as opposed to when it doesn't.

*Lemma* 4: Suppose s lies in $S_1$ and r lies in $S_3$. Then $\gamma_s > \gamma_r = \gamma_0$.

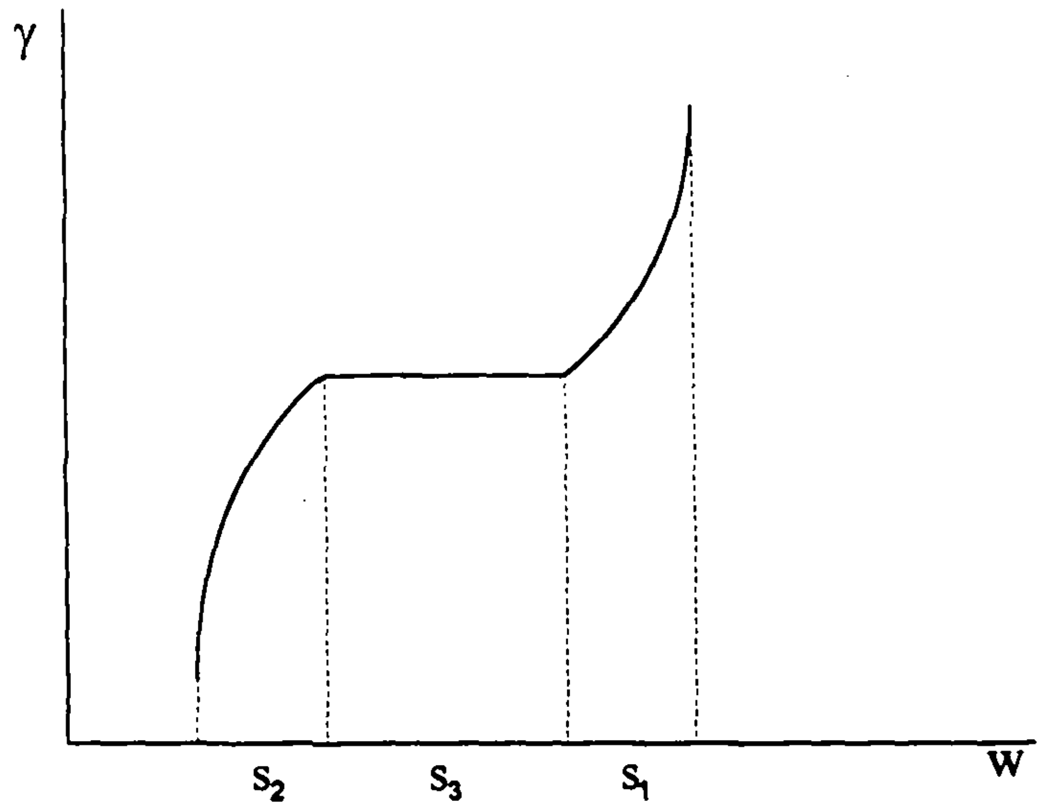
*Proof*: Suppose not and $\gamma_s \leq \gamma_r$. Then, $u_s \geq u_r$, and $V'(u_s) \leq V'(u_r)$. But this violates (FOC5). Δ

Graph 1 uses Lemmas 1-4 to depict the interaction between agent 1's share of aggregate consumption and agent 1's wage in period (t+1) of an efficient allocation, conditional on agent 2's reservation utility level being equal to $u_t$. Lemma 4 implies that when agent 1's wage is low, then agent 2's sustainability constraint binds; from Lemma 1, we know that wages and consumption shares are positively correlated in this region. When agent 1's wage is about average, then neither sustainability constraint is binding: in that region, Lemma 2 implies that the distribution of consumption across agents is the same as last period. Finally, when agent 1's wage is high, his sustainability constraint binds, and his share of aggregate consumption is positively correlated with his wage.

Many types of partial insurance arrangements will give rise to pictures like Graph 1. The interesting feature about efficient sustainable allocations is that this picture moves over time because last period's consumption split, $\gamma_t$, affects the relationship drawn in Graph 1.

Graph 1

Efficient Consumption versus Wages
Conditional on Past History




This graph depicts the relationship between period (t+1) consumption share and wage of agent 1, conditional on information known at time t. $S_1$ is the set of states in which agent 1's sustainability constraint binds, $S_2$ is the set of states in which agent 2's sustainability constraint binds, and $S_3$ is the set of states in which neither agent's sustainability constraint binds.

*Proposition 1*: Suppose the state s lies in $S_1$ when $\gamma_0 = \gamma^*$. Then s lies in $S_1$ when $\gamma_0 = \gamma' < \gamma^*$. Conversely, suppose s lies in $S_2$ when $\gamma_0 = \gamma^*$. Then s lies in $S_2$ when $\gamma_0 = \gamma' > \gamma^*$.

*Proof*: In Appendix.

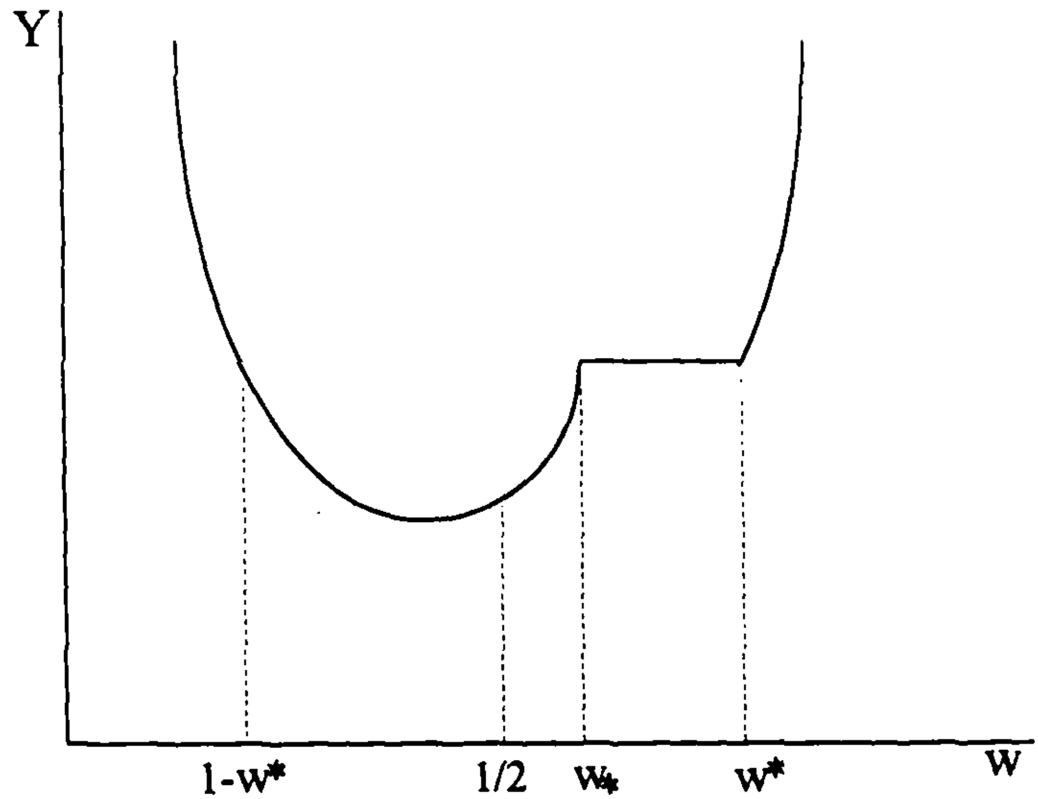
Proposition 1 implies that when $\gamma_t$ increases, then the cutoff points between $S_1$ and $S_3$, and between $S_2$ and $S_3$, shift to the right; also, the consumption split in region $S_3$ is higher. Hence, the response of $\gamma_t$ to a temporary shock to wage inequality persists over time; intuitively, it is efficient to spread the extra consumption compensation to the highly skilled worker over time.

We have seen that in a first best allocation, aggregate output does not fluctuate over dates and states. However, just as changes in consumption inequality cause aggregate output to vary *across* first best allocation, changes in consumption inequality cause aggregate output to vary *within* efficient sustainable allocations. (Mathematically, this can be seen from the fact that $\hat{Y}$ is a nonconstant function of $|\gamma_t - 1/2|$ when $\alpha \neq 1$.) The intuition behind this variability of aggregate output again derives from the shape of Engel curves for leisure. Thus, if the Engel curves are concave, transferring consumption from the rich to the poor will have more impact on the output levels of the latter; hence, aggregate output falls.

Graph 2 uses this intuition to depict $Y_{t+1}$ as a function of $w_{t+1}$, given that $\gamma_t > 1/2$ and $\alpha < 1$. For wage realizations higher than the cutoff point $w^*$, $\gamma_{t+1}$ is larger than $\gamma_t$; since consumption inequality is becoming more pronounced as agent 1's wage increases, $Y_{t+1}$ is an increasing function of wages. For intermediate wage realizations, $\gamma_{t+1}$ is the same as $\gamma_t$ and so $Y_{t+1}$ equals $Y_t$.

Graph 2

Efficient Aggregate Output versus Individual Wages
Conditional on Past History




This graph depicts the relationship between period (t+1) aggregate output and the wage of agent 1, conditional on information known at time t, and assuming that $\gamma_0 > 1/2$ and that $\alpha < 1$. The cutoff points $w_*$ and $w^*$ are described in the text.

The more interesting case occurs when agent 1's wage realization is below $w_*$, so that $\gamma_{t+1}$ falls below $\gamma_t$. If $w_{t+1}$ is slightly below $w_*$, then $Y_{t+1}$ is lower than $Y_t$ and is a decreasing function of agent 2's wage. For low enough realizations of $w_{t+1}$, though, output becomes an increasing function of agent 2's wage (because lower realizations of agent 1's wage make the distribution of consumption less equal). Finally, if the realization of $w_{t+1}$ is less than $(1-w^*)$, then the distribution of consumption is less equal than in the previous period - but is now tilted in favor of agent 2.

## IV. Statistical Characteristics of Efficient Sustainable Allocations

In this section, I discuss some properties of population moments of efficient sustainable allocations and compare these properties with what is known about United States data. The asymptotic validity of the comparison relies on the following result.

*Proposition 2*: The reservation utility level $u_t$ follows a Markov process that satisfies the Feller property. If there exists no sustainable first best allocation, then the distribution of $u_t$, conditional on $u_0$, converges weakly to a nondegenerate limit (called the invariant or unconditional distribution of $u_t$) that is independent of $u_0$. It follows that a strong law of large numbers applies to the Markov process $u_t$.

*Proof*: In Appendix.

According to this proposition, when there is no sustainable first best allocation, sample moments of the endogenous variables will converge almost surely to population moments (calculated with reference to the invariant distribution) as the size of the sample grows larger.

The critical condition in this proposition is that there does not exist any sustainable first best allocation[7]. Mathematically, this is equivalent to:

$$u_{SFB}(w_{max}) + \beta \Sigma_{s=1}^{S} \pi_s u_{SFB}(w_s)/(1-\beta) < u_{aut}(w_{max}) + \beta V_{aut}$$

where $w_{max}$ is the maximal realization of the wage process and $u_{SFB}(w_{max})$ is the within period utility an individual receives in a symmetric first best allocation when the wage realization is $w_{max}$. According to this condition, the individuals want to deviate from the symmetric first best allocation when the wage realization is high. (Note that any other first best allocation makes one of the individuals worse off in every date and state: nonsustainability of the symmetric first best allocation implies nonsustainability of any first best allocation.)

*A. Features of the Insurance Arrangement*

The following proposition points out that insurance is only partial and is a simple consequence of the conditional covariance result depicted in Graph 1.

*Proposition 3*: If there exists no sustainable first best allocation, then $Cov(w_t, \gamma_t) > 0$.

*Proof*: Graph 1 shows that $Cov(\gamma_t, w_t | I_{t-1}) \geq 0$, where $I_{t-1}$ represents the information available to the agents in period (t-1). If $Cov(\gamma_t, w_t | I_{t-1}) = 0$ in any date or state, then the allocation must be first best in all future dates and states because $\gamma_t$ remains constant. Hence, the nonexistence of a sustainable first best allocation implies $Cov(\gamma_t, w_t | I_{t-1}) > 0$ in all dates and states.

It is a well-known consequence of the Law of Iterated Expectations that:

$$Cov(\gamma_t, w_t) = E\{Cov(\gamma_t, w_t | I_{t-1})\} + Cov(E(\gamma_t | I_{t-1}), E(w_t | I_{t-1})).$$

But $E(w_t | I_{t-1}) = E(w_t)$. Hence, the second term on the right is zero; the result follows. $\Delta$

This result shows that insurance is always partial in an efficient sustainable allocation; the split of consumption across individuals tends to tilt towards an individual who receives a high wage realization.

Proposition 3 discusses the contemporaneous covariance between wage shocks and consumption. It is also of interest to analyze the dynamic interaction between wages and consumption. Along these lines, Attanasio and Davis (1994) attempt to identify the effects of "temporary" and "permanent" relative wage shocks on consumption inequality in the United States. They try to isolate the effects of temporary wage shocks by regressing the *annual* growth rate of individual consumption, $\ln(c^j_{t+1}/c^j_t)$, on the *annual* growth rate of individual wages, $\ln(w^j_{t+1}/w^j_t)$. In contrast, they measure the effects of permanent wage shocks by regressing the *decade* growth rate of individual consumption, $\ln(c^j_{t+10}/c^j_t)$ on the *decade* growth rate of individual wages, $\ln(w^j_{t+10}/w^j_t)$. (The first regression conditions on the state of the aggregate economy by including year dummies.) They find that the slope coefficient is positive and much larger in the second regression than in the first. They conclude that it is more difficult for individuals to insure against permanent shocks to wages than temporary ones. (Attanasio and Davis are forced to use differenced data in order to eliminate individual-specific fixed effects.)

In the environment described in this paper, all shocks are temporary (because wages are i.i.d.). Nonetheless, the above difference in the behavior of long run as opposed to short run growth rates in consumption and wages is a characteristic of an efficient sustainable allocation.

*Proposition 4*: Suppose that there exists no sustainable first best allocation. In an efficient sustainable allocation:

a.
$$\frac{\mathrm{Cov}(\ln(\gamma_{t+1}/\gamma_t),\ \ln(w_{t+1}/w_t))}{\mathrm{Var}(\ln(w_{t+1}/w_t))} \le \frac{\mathrm{Cov}(\ln(\gamma_t),\ \ln(w_t))}{\mathrm{Var}(\ln(w_t)}$$

b.
$$\lim_{\tau\to\infty} \frac{\mathrm{Cov}(\ln(\gamma_{t+\tau}/\gamma_t),\ \ln(w_{t+\tau}/w_t))}{\mathrm{Var}(\ln(w_{t+\tau}/w_t))} = \frac{\mathrm{Cov}(\ln(\gamma_t),\ \ln(w_t))}{\mathrm{Var}(\ln(w_t)}$$

*Proof*:

a. Using the fact that $w_t$ is i.i.d. over time, it is easy to see that:

$$\mathrm{Cov}(\ln(\gamma_{t+1}/\gamma_t),\ \ln(w_{t+1}/w_t))/\mathrm{Var}(\ln(w_{t+\tau}/w_t))$$

$$= \mathrm{Cov}(\ln(\gamma_t),\ \ln(w_t))/\mathrm{Var}(\ln(w_t)) - 0.5\mathrm{Cov}(\ln(\gamma_{t+1}),\ \ln(w_t))/\mathrm{Var}(\ln(w_t))$$

The inequality is valid if $\mathrm{Cov}(\ln(\gamma_{t+1}), \ln(w_t))/\mathrm{Var}(\ln(w_t)) > 0$. But we know that there exists an increasing function f such that $\gamma_{t+1} = f(\gamma_t, w_{t+1}) = f(f(\gamma_{t-1}, w_t), w_{t+1})$. Since $w_t$ is independent of both $\gamma_{t-1}$ and $w_{t+1}$, the result follows.

b. Following the above argument, the key is the size of $\lim_{\tau\to\infty} \mathrm{Cov}(\ln(w_t), \ln(\gamma_{t+\tau}))$. Since $\ln(\gamma_{t+\tau})$ is stationary, $\lim_{\tau\to\infty} E_t(\ln(\gamma_{t+\tau})) = E(\ln(\gamma_{t+\tau}))$, and so the limiting covariance is zero. The result follows. $\Delta$

In data from an efficient sustainable allocation, regressing differenced consumption on differenced wages produces a different result from regressing consumption levels on wage levels. If both consumption and wages were i.i.d., as they would be in a first best allocation, then there would be no difference in the regression results. But in an efficient sustainable allocation, even though wages are i.i.d. over time, consumption is not. Because of the infeasibility of outside enforcement, it is efficient to smooth shocks over time: a temporarily high realization for wages has a persistent effect on consumption. This is what generates part (a) of the above proposition. Of course, consumption is stationary, and so the persistent effects of a temporarily high wage realization must eventually die away. This is what generates part (b) of the above proposition: regressing long differences of consumption on long differences of wages is equivalent to doing a contemporaneous regression in the levels of the two variables.

Thus, I have an alternative interpretation for the regression results obtained by Attanasio and Davis for decade growth rates as opposed to annual growth rates. In my view, the difference in the two types of regressions does not tell us directly about differences in the ability of individuals to insure against permanent versus temporary shocks. Instead, I see it as a sign of efficiency: given the technological constraints that prevent them from directly smoothing consumption across states, individuals smooth over time instead.

*B. Inequality and the Cycle*

We have already seen that aggregate output is a deterministic function of consumption inequality: the function is increasing if $\alpha < 1$, decreasing if $\alpha > 1$, and constant if $\alpha = 1$. Thus, if Engel curves for leisure are concave ($\alpha < 1$), then consumption inequality is procyclical. (This implication is consistent with the descriptions of United States data contained in Phelan (1992), for example.) Indeed, according to this model, cycles in aggregate output are best thought of as being generated by movements in consumption inequality.

The following proposition shows that because $\gamma_t$ is persistent, $Y_t$ is also persistent.

*Proposition 5*: If $\alpha \neq 1$, $Pr(Y_t | Y_{t-1} = y')$ first order stochastically dominates $Pr(Y_t | Y_{t-1} = y)$ if $y' > y$.

*Proof*: In Appendix.

In the absence of outside enforcement, it is efficient to translate temporary diversifiable shocks at the individual level into persistent movements in aggregate output.

Consumption inequality and output are both endogenous in this setting. It is also interesting to think about how an exogenous variable, wage inequality, covaries with output.

*Proposition 6*: Suppose there is no sustainable first best allocation, and let $z_t = |w_t - 1/2|$ be a measure of wage inequality. Then, if $\alpha < (>)\ 1$, there exists $z^*$ such that $Cov(Y_t, z_t | z_t \geq z^*) > (<)\ 0$.

*Proof*: In Appendix.

Graph 2 demonstrates that aggregate output is not an increasing function of wage inequality for all realizations of $w_t$. However, conditional on wage inequality being sufficiently high, more wage inequality does generate more consumption inequality; if Engel curves are concave, this increase in consumption inequality is in turn associated with an increase in total output.

## V. Conclusions

In this paper, I examine the properties of efficient allocations of risk in an economy when outside enforcement of contracts is infeasible. I find that in these allocations, temporary relative wage movements have persistent effects on the allocation of consumption across individuals. If preferences over consumption and leisure are nonhomothetic, shifts in the allocation of consumption generate movements in aggregate output that do not occur in first best allocations when enforcement is costless.

Throughout the paper, I treat the 1980s increase in inequality as a *surprise* movement in relative wages. Juhn (1994) documents the existence of an upward trend in wage inequality since 1949, so it is likely that at least part of the increase in wage inequality during the 1980s was anticipated. What kind of effect does this anticipated increase in wage inequality have on efficient allocations? If enforcement is costless, it is efficient for consumption inequality to remain constant despite the growth in wage inequality: this can be accomplished by the high skilled workers "borrowing" from the less skilled during the early part of the period and paying back this "debt" later. In contrast, when enforcement is infinitely costly, there is no mechanism to force the high skilled workers with high wage growth to repay their debt. For this reason, it turns out that the only sustainable (and hence only efficient allocation) is autarky. Thus, in the absence of enforcement, a deterministic trend in wage inequality should generate a deterministic trend in consumption inequality. It is intriguing to speculate that some portion of United States growth since 1948 can be attributed to wealth effects generated trending consumption inequality.

In my model, the nature of the technology of enforcement causes the distribution of consumption to fluctuate over dates and states, and generates persistent fluctuations in aggregate output because preferences over consumption and leisure are nonhomothetic. Like Scheinkman (1984), I believe that these kinds of distributional effects play a role in explaining business cycle fluctuations in the United States: for example, they might well explain why movements in the price of a relatively unimportant input like oil seem to have such a large impact on aggregate output.

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## Appendix

*Proof of Proposition 1:*

Suppose s lies in $S_1$ when $\gamma_0 = \gamma^*$. Then from (FOC5), $V'(u_s) > V'(u_0)$, which means that $u_s < u_0$ or $\gamma_s > \gamma_0$. Since $\gamma\hat{Y}(\gamma)$ and $(1-\hat{n})$ are both increasing in $\gamma$, it follows that $(\gamma_0\hat{Y}(\gamma_0))^{1-\alpha} + \beta\ln(1-\hat{n}(\gamma_0, w_s)) + \beta V(\hat{u}(\gamma_0)) < u_{aut}(w_s) + \beta V_{aut}$. For the same reason, if $\gamma' < \gamma^*$, then $(\gamma'\hat{Y}(\gamma'))^{1-\alpha} + \beta\ln(1-\hat{n}(\gamma', w_s)) + \beta V(\hat{u}(\gamma')) < u_{aut}(w_s) + \beta V_{aut}$. This means that s can't lie in $S_2$ or $S_3$ when $\gamma_0 = \gamma'$, so it must lie in $S_1$.

The proof of the other statement is similar. Δ

*Proof of Proposition 2:* The recursive representation (FE) immediately implies that $u_t$ is a first order Markov process. It satisfies the Feller property because the policy functions $u_s$ are continuous as a function of $u_0$ from the Theorem of the Maximum; also, the policy functions are increasing as a function of $u_0$.

Hence, the only question is whether the process displays enough "mixing" to guarantee that it has a unique invariant measure. From Theorem 12.12 of Stokey, Lucas with Prescott (1989) (SLP), we need only prove that there exists some $u^*$ and t such that $\Pr(u_t \in [V_{aut}, u^*] | u_0 = V_{max})$ is positive and $\Pr(u_t \in [u^*, V_{max}] | u_0 = V_{aut})$ is positive.

Define $u^*$ to be the level of ex-ante utility such $V(u^*) = u^*$. (There exists such a point because V is a continuous function from $[V_{aut}, V_{max}]$ into itself.) Thus, there is an efficient sustainable allocation that provides both agents with the same level of utility. (This fixed point is unique because V is decreasing.) Suppose $S_1$ is empty when $u_0 = u^*$. The symmetry of the problem then says that $S_2$ should be empty also. But this is impossible because there is no sustainable first best allocation. Thus, in the efficient allocation in which both agents receive utility $u^*$, $S_1$ and $S_2$ are both nonempty in period one.

I want to prove that starting from any $u_0 < u^*$, there is some t such that $u_t > u^*$ with positive probability. Define the function $v(u) = \text{Max}_s\, u_s(u)$. Start with an arbitrary $u_0 < u^*$. From Proposition 1 we know that if $S_1$ is nonempty when $u_0 = u^*$, $S_1$ is nonempty for any $u_0 < u^*$. Hence, $v(u_0) > u_0$. Define the sequence $\{\xi_n\}_{n=1}^{\infty}$ recursively by the formula $w_n = v(\xi_{n-1})$ and $\xi_0 = u_0$. The set $S_1$ must be nonempty for any initial level of utility less than $u^*$ so $\xi_n > \xi_{n-1}$ for any $\xi_{n-1} \le u^*$.

Suppose there does not exist any n such that $\xi_n > u^*$. Then, $\{\xi_n\}_{n=1}^{\infty}$ is a strictly increasing sequence that is bounded from above by $u^*$; $\xi_n$ converges to some limit $\xi^*$ that is less than or equal to $u^*$. Since v is continuous (from the Theorem of the Maximum), this limit must satisfy $v(\xi^*) = \xi^*$. But this is impossible because $v(u) > u$ for any $u \le u^*$.

Thus, it is possible to start with any $u_0 < u^*$ and find t such that the probability that $u_t$ exceeds $u^*$ is positive. Similarly, I can prove that starting at any $u_0 > u^*$, there is some t such that $u_t < u^*$ with positive probability.

Theorem 12.12 of SLP therefore implies that $\Pr(u_t | u_0)$ weakly converges to a distribution function that is independent of $u_0$. This, combined with the fact that $u_t$ satisfies the Feller property and that the range of $u_t$ is compact, is sufficient to guarantee that $u_t$ satisfies a strong law of large numbers (Theorem 14.7, SLP). Δ

*Proof of Proposition 5:* Without loss of generality, let $\alpha < 1$. Then $Y_t = \hat{Y}(\gamma_t)$

$= \hat{Y}(f(\gamma_{t-1}, w_t))$, where f is a deterministic function that is increasing in both $\gamma_{t-1}$ and $w_t$, and $\hat{Y}$ is increasing in $|\gamma_t - 1/2|$. It follows that $Pr(Y_t|\gamma_{t-1} = \gamma')$ first order stochastically dominates $Pr(Y_t|\gamma_{t-1} = \gamma)$ if $\gamma' > \gamma \geq 1/2$.

Now, given that $Y_{t-1}$ equals y, we know from the symmetry of the environment that, unconditionally, $\gamma_{t-1}$ is equally likely to be equal to one of the two elements of the set $\hat{Y}^{-1}(Y) = \{\gamma, 1-\gamma\}$. Hence, we can write:

$$Pr(Y_t|Y_{t-1} = y) = 0.5Pr(Y_t|\gamma_{t-1} = \gamma) + 0.5Pr(Y_t|\gamma_{t-1} = (1-\gamma))$$

where $\{\gamma, (1-\gamma)\} = \hat{Y}^{-1}(y)$. But the symmetry of the environment implies that $Pr(Y_t|\gamma_{t-1} = \gamma) = Pr(Y_t|\gamma_{t-1} = (1-\gamma))$: the probability density of output is not affected by who is richer in period (t-1). Hence, there exists $\gamma \geq 1/2$ such that $Pr(Y_t|Y_{t-1} = y) = Pr(Y_t|\gamma_{t-1} = \gamma)$. Further, there exists $\gamma' > \gamma \geq 1/2$ such that $Pr(Y_t|Y_{t-1} = y') = Pr(Y_t|\gamma_{t-1} = \gamma')$. The proposition follows. Δ

*Proof of Proposition 6:* Define $w^*$ as the solution to the equation:

$$(0.5\hat{Y}(0.5))^{1-\alpha}/(1-\alpha) + \ln(1-\hat{n}(1/2, w^*)) + \beta V(\hat{u}(1/2)) = u_{aut}(w^*) + \beta V_{aut}$$

(Note that a solution to this equation is guaranteed by the absence of any sustainable first best allocation.) The solution $w^*$ must be at least as large as 1/2 because:

$$(0.5\hat{Y}(0.5))^{1-\alpha}/(1-\alpha) + \ln(1-\hat{n}(1/2, 1/2)) = u_{aut}(1/2)$$

and $V(\hat{u}(1/2)) \geq V_{aut}$.

From the definition of $w^*$, we know that for any realization of $w_t \geq w^*$, the realization of $\gamma_t \geq 1/2$. Why? Suppose $w_t \geq w^*$ and $\gamma_t < 1/2$. Then:

$$\begin{aligned}&(\gamma_t\hat{Y}(\gamma_t))^{1-\alpha}/(1-\alpha) + \ln(1-\hat{n}(\gamma_t, w_t)) + \beta V(\hat{u}(\gamma_t))\\ &< (0.5\hat{Y}(0.5))^{1-\alpha}/(1-\alpha) + \ln(1-\hat{n}(1/2, w^*)) + \beta V(\hat{u}(1/2))\\ &< u_{aut}(w_{t-1}) + \beta V_{aut}\end{aligned}$$

which violates agent 1's sustainability constraint. This confirms that high realizations of wage inequality mean high realizations for consumption inequality whenever $w_t \geq w^*$.

It follows that $Cov(|\gamma_t - 1/2|, z_t|w_t \geq w^*, I_{t-1}) > 0$. Symmetrically, we can conclude that $Cov(|\gamma_t - 1/2|, z_t|w_t \leq (1-w^*), I_{t-1}) > 0$. Note though that $E(z_t|w_t \geq w^*, I_{t-1}) = E(z_t| w_t \leq (1-w^*))$ because $w_t$ is independent of $I_{t-1}$ and is symmetrically distributed around 1/2. Hence:

$$Cov(|\gamma_t - 1/2|, z_t| z_t \geq |w^* - 1/2|) > 0$$

The theorem follows. Δ

Endnotes

1. In the model, individuals have constant returns to scale technologies in a single input, labor. Hence, in an abuse of terminology, I will refer to their marginal products as "wages" even though there are no explicit markets in the model.

2. In their conclusion, Attanasio and Davis (1994) suggest that costs of enforcement of social risksharing arrangements might be an explanation for their findings. This paper serves to confirm their intuition.

3. The assumption that individual productivities are perfectly negatively correlated is very strong; I impose such a strong restriction on the environment only to make more clear the role of distributional effects in generating cycles.

4. The assumption that the two agents have symmetric productivities is merely for technical convenience; the main results can be obtained in an environment in which one agent's expected wage is higher than the other's.

5. It should be kept in mind that as an economy grows over time, nonhomotheticicities have unpleasant consequences. Concave Engel curves for leisure imply that the fraction of time spent working will grow without bound as the marginal product of labor grows. These implications can potentially be remedied by assuming that there is a secular shift in preferences (perhaps due to relative consumption effects) that is tied in some way to the secular shift in technology.

6. Requiring that the agents revert to infinite horizon autarky after any default by either party means that there are enormous temptations for the agents to get together at some point in the future and "renegotiate" their way out of this arrangement which is bad for both of them. However, Kocherlakota (1994) proves that the efficient sustainable allocations are in fact strongly renegotiation-proof in the sense of Farrell and Maskin (1989): any efficient sustainable allocations can be supported using the threat of reverting to an efficient sustainable allocation which the defector does not like but the other agent does.

7. As in Kocherlakota (1994) and Thomas and Worrall (1994), if there is a sustainable first best allocation, then in an efficient sustainable allocation, $u_t$ converges with probability one to a constant. Hence, any efficient sustainable allocation converges to a first best allocation asymptotically (the limiting allocation depends on the initial distribution of resources).