Optimal Monetary Policy in an Economy with Sequential Service [1]

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[1]The ideas developed here have been considerably refined through years of discussion with Ben Eden and Bob Lucas, neither of whom, however, can be held responsible for the form they have ultimately taken. I would also like to thank Dennis Carlton, Peter Diamond, Julio Rotemberg, and Steve Williamson for helpful discussions, the Institut d'Anàlisi Econòmica (Barcelona) for its hospitality while I wrote the first draft of this paper, and the NSF for research support.

Robert Lucas's celebrated paper, "Expectations and the Neutrality of Money" (Lucas (1972)), set a new standard for elegance and rigor of theoretical analysis in monetary economics. It also presented a new view of the connection between monetary instability and fluctuations in aggregate economic activity, which challenged the foundations of much of the conventional wisdom of the time about the potential role of monetary policy.

In Lucas's analysis, unexpected variations in the money supply affect economic activity because the suppliers of goods have incomplete information about the current state of aggregate demand. (The exact way that this occurs in his model is by now so well-known as not to require summary here.) This type of explanation of the observed non-neutrality of money has the implication that *only* unexpected variations should have significant real effects. This in turn suggested that the possible gains from systematic monetary policy should be small. Sargent and Wallace (1975) argued, on the basis of a model of aggregate supply inspired by Lucas's analysis, that optimal monetary policy required making the money supply each period completely predictable, and that there were no gains to be had from making the growth of the money supply depend upon past economic conditions. Thus a simple "k-percent rule" for the path of the money supply, of the kind long advocated by Milton Friedman, belonged to the class of optimal rules.

This result depends, however, upon an assumption that if a rule for monetary policy is not to rely upon superior information on the part of the monetary authority as to the current state of the economy, relative to that possessed by the suppliers of goods, it must make the money supply a function only of lagged variables (information also possessed by suppliers) and of noise unrelated to economic "fundamentals". Variation of the first kind is argued to have no effect, while variation of the second kind is unambiguously bad.

I wish to reconsider this conclusion, in the context of a model of the real effects of monetary instability that not only aspires to a level of theoretical rigor comparable to that of Lucas's model, but that proposes a broadly similar account of the real effects of monetary instability. In this model – which is even more closely related to models used by Lucas (1979), Lucas and Woodford (1994), Eden (1994), and Williamson (1993) – [1] monetary surprises again have real effects because the suppliers of goods do not learn about the current value of the money supply (or, more generally, about the current state of aggregate demand) sufficiently quickly to incorporate this information into their pricing policy. Here this is modelled through a *sequential service constraint* on transactions in the goods market, as a result of which supply commitments made to individual buyers cannot be contingent upon the overall state of demand. The intention is to model equilibrium in a goods market where goods are allocated through non-price rationing, as seems to be true in many actual markets (Carlton (1991)), due to the prohibitive informational requirements for the organization of a competitive spot market. This device differs from that used by Lucas (1972) in that monetary instability has real effects even if exogenous variation in the money supply is the *only* source of variations in demand, and also in that buyers need not be assumed to have superior information to that of sellers. Nonetheless, it is similar in the crucial respect that it again implies that only unanticipated variations in the money supply have any effect upon economic activity. It also provides a clear justification for the view that unnecessary noise in monetary policy is to be avoided, as it results not only in more variable output but in a lower average level of utilization of productive capacity as well. [2]

Nonetheless, I find that a constant-growth-rate rule for the money supply (or, since I here restrict attention to rules that make the money supply a stationary variable, a constant-money-supply rule) is not generally optimal. Under a certain extreme specification of the nature of the exogenous uncertainty in the economy, a completely elastic money supply is actually optimal (by which I mean that the central bank supplies as much money is demanded, at a fixed nominal interest rate), and more generally, it is usually desirable to allow the money supply to vary to some extent in response to exogenous shocks to aggregate demand.

These results differ from those of Sargent and Wallace (1975) for two important reasons. The first is that while I insist that the monetary policy rule not require the central bank to have superior information to that of suppliers of goods, I do not restrict attention to the class of feedback rules for the money supply that they consider. The money supply is allowed to vary in response to changes in the demand for funds in

[1] The relation of the present model to these others is discussed further at the end of section 1.

[2] The model's consequences in these regards are identical to those of the related model analyzed in Lucas and Woodford (1994).

an interbank market; it is only the central bank's *supply schedule* in this market that is not allowed to vary with the current state of aggregate demand. This allows consideration of varying degrees of elasticity of the endogenous response of the money supply to changes in the demand for funds, that result from exogenous non-monetary disturbances to aggregate demand. A crucial innovation of the present model, relative to that of Lucas (1979) or Lucas and Woodford (1994), is accordingly the introduction of both stochastic variation in consumer preferences (as a simple source of non-monetary demand disturbances) and of a credit market in which buyers must borrow to finance their purchases (in order to provide the channel through which monetary policy may accommodate such disturbances to a greater or lesser degree). Since actual operating procedures for monetary policy in the United States and elsewhere typically involve short-run control of an interest rate (such as the Federal funds rate in the U.S.) rather than a monetary aggregate, [3] such an extension of the analysis seems amply justified. [4]

The second difference is that I do not assume, as Sargent and Wallace do, that stabilization of aggregate output is an end in itself. In fact, in the model presented here, a more elastic funds supply schedule will generally mean greater variation in the money supply in response to shocks, and as a result greater instability of aggregate output. [5] However, this need not imply a lower level of welfare, and this despite the fact that in the present model it also means a greater average quantity of unused capacity. The reason is that the degree of monetary accommodation affects the *allocation* of output across alternative uses, as well as its aggregate quantity. Since critics of policies that accept sharp fluctuations in interest rates in order to meet monetary aggregate targets often complain that the required interest rate fluctuations affect some kinds of spending much more than others, in a way that distorts the overall pattern of spending in the economy, an explicit analysis of the consequences of heterogeneity in borrowers' situation for the welfare effects of alternative policies is called for.

In the case that fluctuations in aggregate demand have mainly to do with variations in the *number* of potential buyers who have productive uses for resources at a given time, rather than in the *intensity* of the demand of the typical buyer who enters the market, a relatively elastic central bank supply of funds (i.e., a policy that smooths the interbank interest rate) increases welfare despite the increase in the volatility of aggregate output, because it results in a lesser concentration of available resources in the supply of goods for those uses that result in the most *stable* demands. Because of the sequential service constraint, it is not possible in equilibrium for stable-demand buyers to obtain all of the output in low-demand states while sharing some of it with volatile-demand buyers in high-demand states. A inelastic money supply policy therefore stabilizes spending, and hence aggregate output, only because it prevents the demands of volatile-demand buyers from being expressed in the market, even in high-demand states. Such "stability", however, is purchased at the expense of efficiency in the allocation of resources. Efficiency (in the sense of maximization of the expected utility of the representative household) would instead require a policy that allows volatile-demand buyers to obtain some resources in the states in which they have productive uses for them, even though the capacity committed for that purpose must as a result remain unused in other states.

The paper proceeds as follows. In section 1, I introduce the model of a goods market subject to a sequential service constraint, and show how this implies effects of monetary instability upon aggregate output. In section 2, I introduce the model of stochastic consumer demand, including the heterogeneity among "types" of buyers upon which the welfare analysis turns. Finally, in section 3, I complete the model by presenting an analysis of the behavior of lenders in the consumer credit market and in the interbank market, which latter market represents the lever through which monetary policy affects aggregate demand.

[3] Even the operating procedure briefly adopted by the Fed in 1979, which targeted non-borrowed reserves, involved a significant degree of endogeneity of the money supply in response to demand shocks, through the endogenous response of discount-window borrowing to such shocks. See, e.g., Anderson *et al* (1986).

[4] Sargent and Wallace do consider a second type of policy rule, in which an interest rate, rather than the money supply is set equal to a predetermined quantity plus noise, but they argue that all such rules should be excluded from consideration on the ground that they lead to indeterminacy of the price level. This issue is addressed below in section 4.

[5] This result, however, depends upon the fact that I do not allow for shocks to money demand, in the sense of stochastic disturbances to the relation between desired spending and desired money balances; such shocks can easily result in greater instability of aggregate demand, and hence of output, under a constant-money-supply rule than with an elastic money supply, as in the analysis of Poole (1970). Extension of the model to allow for such shocks is briefly discussed in footnote xx below.

Section 4 then analyzes the question of optimal monetary policy, showing that completely inelastic and completely elastic central bank supply schedules are each optimal in contrasting polar cases. Section 5 concludes.

1 Sequential Service in the Goods Market and Aggregate Supply

In this section, I begin the development of the model within which I propose to analyze the consequences of alternative monetary policies. I describe the organization of the goods market, and introduce in this context the type of sequential service constraint with which I am concerned. I then discuss the consequences of this constraint for aggregate supply, and show that unexpected variations in the nominal value of aggregate spending will be associated, in equilibrium, with variations in output of the same sign. Thus this aspect of the model is responsible for the real effects of monetary policy.

Each period, sellers have a total productive capacity of $y > 0$ units of a homogeneous good. [6] Each seller seeks to maximize expected sales revenues during the period, subject to its capacity constraint. Unused productive capacity depreciates completely at the end of the period; thus there is no intertemporal decision problem for sellers. [7]

Exchange is assumed to occur each period in two stages. In a first stage, buyers and sellers negotiate supply contracts, that are then executed during the second stage. Prior to the negotiation stage, buyers learn their individual "types", but do not yet know the aggregate state; prior to the execution stage, the value of an aggregate state variable s is revealed to buyers, but not to sellers. Each buyer's demand for the good in the execution stage depends (in a way made precise in the next section) upon its type i and upon the aggregate state s.

The significance of the negotiation stage is that it is assumed that at this stage, buyers and sellers are able to costlessly negotiate supply contracts in a competitive environment with full information. (The types of individual buyers are assumed to be public information.) On the other hand, once the state s is realized, it is assumed not to be possible to organize a centralized Walrasian spot market for the good. Instead, suppliers must be able to fill orders placed by individual buyers without waiting to learn the state of aggregate demand s. (This is the "sequential service" constraint. [8])

Suppliers negotiate in advance the terms under which they will supply goods to particular buyers, should these buyers present them with orders during the execution stage. But the supply commitment made to an individual buyer cannot be contingent upon the purchases made by any other buyers, and so cannot be contingent upon the aggregate state s. The existence of the negotiation stage makes it possible for types that are more certain of their desire for the good to negotiate a secure supply of the good on favorable terms (which sellers will grant them, on the basis of their type, in order to reduce the risk of ending with unused capacity). [9] However, since the state of demand is not known to sellers, buyers do not have an enforceable obligation to buy any amount. Rather, a seller is committed to supply any amount demanded by the buyer, up to a certain quantity limit, at a prearranged price schedule (that may make the supply price depend upon the quantity that the buyer purchases). The actual quantity purchased under this commitment, $c_i(s)$, remains at the discretion of the buyer, and will in general depend upon the realization of the state s. Thus sellers are not able to "sell forward" their output in the negotiation stage, and unused capacity is possible in equilibrium.

[6] Eventually, these "sellers" are identified as firms owned by the same households that enter the goods market as "buyers", but at this stage we need not consider how the interaction here described in a single period's goods market is imbedded in a larger, dynamic general equilibrium model.

[7] Maximization of expected revenues serves the interests of each firm's shareholders, as is explained in the next section.

[8] The terminology follows Diamond and Dybvig (1983), who assume a constraint of this kind on the form of commitment that a financial intermediary can make with its depositors regarding the terms of withdrawal. See Wallace (1987) for further discussion of the kind of communication barriers implicit in such a constraint.

[9] Carlton (1991) discusses evidence that firms do charge different prices to different customers based upon the stochastic properties of their demand. According to this evidence, customers pay more the more positive the correlation between their individual demand and the firm's aggregate demand. Such a relationship is predicted by the model developed here.

Finally, it is assumed that the supply commitments must indicate the price (if any) at which each successive additional purchase by a buyer can be made, simply as a function of the number of units already purchased by that buyer, and not contingent upon the eventual quantity that the buyer might subsequently purchase. This is taken to be a consequence of sequential service, like the requirement that an individual buyer's price cannot depend upon the eventual total purchases of others.

Thus a supply commitment for a buyer of type i takes the form of a quantity limit $\bar{c}_i$ and a right-continuous function $p_i(\cdot)$ defined on the interval $[0, \bar{c}_i)$. Here $p_i(c)$ denotes the price at which the next increment of the good is available to the buyer, after a quantity c has already been purchased. (The assumption of right-continuity means that if $p_i(c) = p$, it is possible for some additional positive quantity of the good to be purchased at an average price that is arbitrarily close to p; in the absence of this, it would not be meaningful to say that further purchases are possible at the price p. Treating purchases as a continuous variable simplifies the analysis of competitive equilibrium, as usual.) Allowing p_i to depend upon the quantity purchased makes it possible for sellers to raise their prices to some extent in high-demand states, but in a way that respects the sequential service constraint.

Competitive equilibrium in the market for supply contracts results in the existence of a single quantity limit $\bar{c}_i$ and a single equilibrium supply function $p_i(\cdot)$ for each type i. I assume that there is competition between sellers to supply the next increment of the good purchased by any buyer of type i who has already purchased quantity c (for any $0 \le c < c_i$); $p_i(c)$ is the market-clearing price for such commitments. The analysis is simplified if we assume that a seller in competing to offer additional supply commitments to such buyers are unable to distinguish whether the buyer obtaining an incremental commitment has obtained his prior supply commitment from the same seller or from someone else, and are similarly unable to observe the terms of the buyer's prior supply commitment; but it is assumed that a seller can verify the total quantity already purchased by the buyer at the time that the buyer places an order under the commitment. Thus there is clearly a single market-clearing price depending only upon the type i and the previously purchased quantity c. The equilibrium quantity limit $\bar{c}_i$ represents the lowest quantity such that no seller is willing to offer further commitments to buyers of type i who have already purchased that amount of the good.

In the negotiation stage, buyers of type i voluntarily accept price commitments up to the quantity limit $\bar{c}_i$, of the form indicated by the equilibrium supply function $p_i(c)$. (Since buyers are not obligated to make any purchases, there is no reason not to keep accepting incremental commitments if they are available, at the lowest prices at which they are available.) Sellers choose the quantity of commitments that they wish to make to buyers of each type, at the competitive prices for each possible type of incremental commitment. Equilibrium requires that sellers choose to supply exactly the quantity of commitments that buyers demand, which is to say exactly one full commitment (allowing purchases up to c_i) per buyer of type i. In addition, equilibrium requires that no seller be willing to offer incremental commitments at prices lower than the equilibrium prices, or be willing to offer incremental commitments beyond the equilibrium quantity limit $\bar{c}_i$ at any finite price.

In the execution stage, each buyer of type i decides what quantity $0 \le c_i \le \bar{c}_i$ to purchase, given the state s that is realized. The purchasing decision $c_i(s)$ depends upon the commitment $(\bar{c}_i, p_i(\cdot))$ obtained during the negotiation stage. Rational anticipation of this decision, in turn, determines the willingness of sellers to extend such commitments during the negotiation stage. Individual sellers take as given the equilibrium values $\{c_i(s)\}$, just as they do the equilibrium supply terms $\{\bar{c}_i, p_i(\cdot)\}$, in deciding which supply commitments to offer themselves. Expected revenues per unit of capacity committed, in the case of incremental supply commitments to buyers of type i who have already purchased quantity c, are equal to

$$p_i(c) \sum_s \pi_s I(c_i(s) > c),$$

where $\pi_s > 0$ is the probability that state s occurs (conditional upon information available as of the negotiation stage), and $I(x)$ is the function whose value is 1 if statement x is true, and 0 otherwise.

A supply commitment to a given buyer can be made only if it is feasible for the seller to honor the commitment with certainty. It follows that optimization by a seller requires that each unit of capacity be committed in the way that yields the highest expected revenues. As in equilibrium it is necessary that supply commitments be made to each type of buyer, up to the quantity limit $\bar{c}_i$, it follows that one must have

$$p_i(c) \sum_s \pi_s I(c_i(s) > c) = \lambda \tag{1.1}$$

for each i and each $0 \leq c < \bar{c}_i$. Here λ indicates the maximum available rate of expected revenues per unit of capacity. Furthermore, if $\lambda > 0$ (i.e., if orders are placed at positive prices by some kind of buyer in some state), optimization by a seller requires that all capacity be committed. (I assume that the same communication barriers that during the execution stage that prevent organization of a centralized market also preclude purchases from being made other than under previously negotiated commitments.) Then equilibrium requires that if $\lambda > 0$,

$$\sum_i n_i \bar{c}_i = y, \tag{1.2}$$

where $n_i > 0$ denotes the fraction of buyers that are of type i.

But equilibrium in the negotiation stage requires more than mere indifference on the part of sellers between the various types of supply commitments that are actually offered in equilibrium; sellers must also be content *not* to offer any of those that are not offered in equilibrium. There is plainly no point in a seller's offering an incremental supply commitment to buyers of type i after a quantity c has been purchased at a price $p > p_i(c)$, since no buyer will accept such a commitment, given the availability of supply commitments at a lower price. In the case of a quantity c such that in every state, either $c_i(s) < c$ or $c_i(s) > c$, there is equally plainly no point in offering an incremental supply commitment at a price $p < p_i(c)$. For in states s in which $c_i(s) < c$ without the price reduction, the incremental commitment will not be drawn upon despite the lower price, while in states s in which $c_i(s) > c$ without the price reduction, it would be drawn upon even if the price were $p_i(c)$. Hence such a price reduction can only reduce expected revenues per unit of capacity so committed.

A more complex case, however, is that of a quantity c such that $c_i(s) = c$ for some state s. This means that at the equilibrium incremental supply price $p_i(c)$, buyers choose not to draw upon the commitment in state s. However, it may be that they *would* purchase more in state s in the case of an incremental supply commitment at some lower price, and so one must consider whether a seller could increase expected revenues by offering such a commitment. Consideration of this requires that we specify what sellers expect about whether additional commitments, not offered in equilibrium, would be drawn upon in various states. Given the equilibrium commitment $(\bar{c}_i, p_i(\cdot))$, the determinants of type i demand (further specified in the next section) determine not only the quantity $c_i(s)$ that is purchased in state s under that commitment, but also the lowest price $\hat{p}_i(s)$ at which type i would be willing to buy an additional positive quantity, after having purchased $c_i(s)$ under the equilibrium commitment. (Technically, I will use $\hat{p}_i(s)$ to denote the greatest lower bound of the set of such prices.) An incremental commitment to buyers of type i after purchases of a quantity c at a price $p \leq p_i(c)$ will thus be accepted by such buyers in the negotiation stage, and drawn upon in the execution stage with probability

$$\sum_s \pi_s[I(c_i(s) > c) + I(c_i(s) = c)I(\hat{p}_i(s) > p)].$$

(Note that this reduces to $\sum_s \pi_s I(c_i(s) > c)$, as assumed in (1.1), in the case that $p = p_i(c)$, for one must have $\hat{p}_i(s) \leq p_i(c_i(s))$ in every state.)

Expected revenues per unit of capacity committed under incremental commitments of this kind are just p times this probability. It follows that a further requirement for equilibrium is that

$$p \sum_s \pi_s[I(c_i(s) > c) + I(c_i(s) = c)I(\hat{p}_i(s) > p)] \leq \lambda \tag{1.3}$$

for all types i and all prices $p \leq p_i(c)$, in the case of any quantity $0 \leq c < \bar{c}_i$. Furthermore, condition (1.3) must hold for all types i and all prices p, in the case that $c = \bar{c}_i$; for only if this is so will sellers not wish to increase the quantity limit $\bar{c}_i$.

Conditions (1.1) – (1.3) determine the equilibrium supply commitments $\{\bar{c}_i, p_i(\,)\}$, given a specification of the determinants of buyer demand in each state s. Detailed characterization of such an equilibrium is not possible without further specification of buyer behavior. Still, several observations are already possible as to the nature of aggregate supply in such a model.

I shall assume that states are ordered so that for each buyer type i, $s' \geq s$ implies $c_i(s') \geq c_i(s)$; and if $c_i(s') = c_i(s)$, it implies that $\hat{p}_i(s') \geq \hat{p}_i(s)$.[10] Thus higher s corresponds to a state of uniformly higher demand. The lowest-demand state is numbered $s = 1$. Then it follows from (1.1) that each equilibrium supply schedule $p_i(\cdot)$ is a non-decreasing, piecewise constant function,[11] of the form

$$p_i(c) = p_s \qquad \text{for all} \quad c_i(s-1) \leq c < c_i(s) \tag{1.4}$$

for each s such that $c_i(s) > c_i(s-1)$, where $c_i(0) \equiv 0$. The sequence of prices $\{p_s\}$ is furthermore given by

$$p_s = \lambda \left[\sum_{s' \geq s} \pi_{s'} \right]^{-1} \tag{1.5}$$

for each state s'. Assuming $\lambda > 0$, this is an increasing sequence of positive prices. (I shall assume below properties for buyer demand that preclude (1.3) holding with $\lambda = 0$.) Note that these prices are the same for all types i. Condition (1.1) also implies that for any $c < \bar{c}_i$, there exists a state in which $c_i(s) > c$, so that

$$\bar{c}_i = sup_s\{c_i(s)\}. \tag{1.6}$$

If there is a highest-demand state $\bar{s}$, $\bar{c}_i = c_i(\bar{s})$. Thus (1.4) specifies $p_i(\cdot)$ over its entire domain $[0, \bar{c}_i)$.

Condition (1.4) implies that every order that is placed if and only if state s occurs is made at the same price p_s, regardless of the buyer's type. If we rank orders so that orders that are placed in a larger number of states have an earlier rank, then the price associated with each order is a function solely of its rank (independent of the type of buyer). This price is given by the *aggregate* supply schedule $p(\cdot)$, a function defined on $[0, y)$, such that $p(c)$ is the next price at which orders are placed when aggregate purchases in excess of quantity c are made. Conditions (1.4)–(1.5) imply that $p(\cdot)$ is a non-decreasing, piecewise constant function, of the form

$$p(c) = p_s \qquad \text{for all} \quad c(s-1) \leq c < c(s) \tag{1.7}$$

for each s such that $c(s) > c(s-1)$, where $c(s) \equiv \sum_i c_i(s)$. The aggregate supply schedule indicates the distribution of transaction prices as a function of the total quantity purchased. Note that the schedule (1.7) is defined for all quantities $0 \leq c < y$, since (1.6) implies that

$$sup_s\{c(s)\} = y. \tag{1.8}$$

It is now apparent that unexpected variations in the nominal value of aggregate spending – unexpected, that is, relative to information available during the negotiation stage – *must* be associated with variations in capacity utilization. For aggregate spending is given by

$$Y(s) = \sum_{s \leq s'} p_s[c(s) - c(s-1)] = R(c(s')),$$

where

$$R(c) \equiv \int_0^c p(c')dc' \tag{1.9}$$

[10] In the next section, these results are derived from an assumption stated in terms of primitives of the model.

[11] If the formalism were extended to allow for a continuous state space, rather than assuming a countable set of possible states, the functions would not in general be piecewise constant. See the characterization of aggregate supply in Lucas and Woodford (1994).

indicates the total cost of the first c orders, under the ranking described above. The function $R(\cdot)$ also gives total revenues of sellers, as a function of the aggregate quantity purchased. If $\lambda > 0$, $R(\cdot)$ is a continuous, strictly increasing function. One can therefore invert it, to determine the equilibrium quantity purchased as

$$c(s) = R^{-1}(Y(s)). \tag{1.10}$$

This implies that if $Y(s') > Y(s)$, one must have $c(s') > c(s)$ as well, so that the fraction of total capacity y that is unused must be lower in state s'.

If, as in the analysis of Lucas and Woodford (1994), aggregate spending is determined (through a cash-in-advance constraint on purchases) by the size of a random monetary injection, it is evident that unexpected variations in the money supply will result in variations in capacity utilization. On the other hand, expected variations in the money supply have no such effect. For if the money supply (and hence the nominal value of aggregate spending) is known with certainty at the negotiation stage, it follows from (1.10) that the real quantity purchased is known with certainty as well. Condition (1.8) then implies that the certain value must be y; that is, capacity is fully utilized with certainty. Thus even if the money supply depends upon a random state variable z, that is revealed prior to the negotiation stage, equilibrium output will equal capacity y regardless of the state z that is realized; the equilibrium price level will instead differ across states z in proportion to the money supply.

Thus unexpected variations in the money supply, and *only* unexpected variations, affect output in this model, as in Lucas (1972). The reason, as in that model, depends upon incomplete information on the part of sellers as to the state of aggregate demand; but here this matters because of the sequential service constraint on transactions in the execution stage. Note that it is not the mere fact that supply commitments are negotiated in advance that makes unexpected variations in the money supply non-neutral. For if it were possible to negotiate (and enforce) state-contingent forward contracts, as assumed in the Arrow-Debreu model, unexpected variation in the money supply would have no effect. Suppliers would sell forward their entire capacity in each state, and the supply prices in the different states would vary as necessary to clear the market for each of the state-contingent forward contracts. Hence there would be no variation across states in the quantity sold, and transaction prices would vary across states in proportion to the money supply.

[Add here further discussion of the previous literature]

2 Preference Shocks as a Source of Aggregate Demand Variation

I now further specify the demand side of the model, indicating the source of the variations in aggregate demand to which monetary policy may be more or less accommodative. The economy consists of a continuum (normalized at length one) of infinite-lived households, which are *ex ante* identical (i.e., prior to the random assignment of "types"). Each household seeks to maximize the *ex ante* expected value of its lifetime utility

$$\sum_{t=0}^{\infty} \beta^t \delta_t u(c_t).$$

Here $u(\cdot)$ is a strictly increasing, strictly concave, continuously differentiable period utility function, defined for all non-negative levels of consumption, and $0 < \beta < 1$ is a discount factor; both are common to all households. The process $\{\delta_t\}$ is a random preference shock that is household-specific. It is assumed that each period's value of the preference shock is given by

$$\delta_t = \delta(s_t; i_t), \tag{2.1}$$

where the function $\delta(\cdot;\cdot)$ is time-invariant, s_t is the aggregate demand state variable referred to in the previous section, and i_t is a household-specific random variable (the household's "type" in period t). The random variable s_t represents an exogenous disturbance to aggregate demand through a change in the population distribution of values for the factor δ. (The population distribution of values of i is the same each period, only the assignment of types to households is random.) An individual household's type affects its demand behavior through its effect on the household's value of δ_t. The function $\delta(\cdot;\cdot)$ is further specialized in examples discussed below.

As explained in the previous section, in each period there is a negotiation stage followed by an execution stage. The types $\{i_t\}$ are randomly assigned to households at the beginning of the period t negotiation stage, while the state s_t is revealed to households (in their capacity as buyers) only at the beginning of the execution stage. (It is actually only necessary to suppose that each household learns its current value of δ_t at that time.) For simplicity, it is assumed that both $\{i_t\}$ and s_t are independently and identically distributed each period. Each period, each household has a probability n_i of being assigned type i; the total fraction of households with type i each period is deterministic and equal to n_i. Each period, the aggregate state s_t takes the value s with probability π_s.

Purchases each period are subject to a cash-in-advance constraint, which requires that

$$Y_{it}(s) \leq M_{it}(s) \tag{2.2}$$

for each type i in each possible state s. [12] Here $Y_{it}(s)$ represents the nominal value of type i's expenditure on goods in period t if the state is s, which is to say $R_t(c_{it}(s))$, where

$$R_t(c) = \int_0^c p_t(c')dc' \tag{2.3}$$

by analogy with (1.9), and $M_{it}(s)$ is the amount of money held by type i in that state. Money holdings may vary depending upon the realization of s_t, due to the existence of a credit market during the execution stage.

The credit market, like the goods market, is assumed to be subject to a sequential service constraint. This means that during the negotiation stage, buyers negotiate loan commitments, specified by a quantity limit $\bar{M}_i$ and a supply function $r_i(\cdot)$, indicating the terms upon which that buyer can borrow. Here $\bar{M}_t$ is the maximum quantity that may be borrowed during the execution stage, and $r_t(M)$ indicates the interest rate at which the next incremental loan is available, after a quantity M has already been borrowed. As before, $r_t(\cdot)$ is assumed to be a right-continuous function defined on the interval $[0, \bar{M}_t)$. The quantity $r_t(M)$ is a gross nominal interest rate on loans of cash, so that if the quantity borrowed is $M_{it}(s)$, the repayment obligation $B_{it}(s)$ due at the end of the period is given by $B_t(M_{it}(s))$, where

$$B_t(M) = \int_0^M r_t(M')dM'. \tag{2.4}$$

Note that this notation assumes that the quantity of cash available to finance purchases (in constraint (2.2)) is equal to the quantity borrowed in the credit market. This is because all money holdings of a household at the beginning of a period are assumed to be deposited with financial intermediaries, which intermediaries then lend the funds back to households in the credit market to finance their purchases of goods.

The complete sequence of events during period t is as follows. Let a household begin the period with nominal wealth W_t, held in the form of cash. (For simplicity, all financial claims are assumed to mature at the end of the period, so that a household begins the next period with no financial assets other than cash.) There is first a securities trading stage, before household types $\{i_t\}$ are revealed. During this stage, three types of transactions occur, in perfectly competitive markets. First, the household deposits a quantity D_t of cash with intermediaries, in exchange for a promise of repayment with interest at the end of the period. (What

[12] One could easily generalize the model, to allow a utility benefit from the use of money in transactions, without requiring a fixed proportion between money holdings and spending. In that case, the simple case for a policy of fixing the money supply as a means of stabilizing aggregate demand is complicated in familiar ways. A fixed money supply would no longer suffice to ensure predictability of aggregate spending, and in the case of exogenous random variation in the desired ratio of money holdings to spending, monetary accommodation would serve to stabilize aggregate demand, rather than destabilizing it, as in the analysis of "LM shocks" by Poole (1970). These complications, despite their obvious relevance, are ignored here to simplify the model, and allow concentration upon a different reason for monetary accommodation to be welfare-improving than the one stressed by Poole.

the intermediaries do with these deposits is taken up in the next section.) Second, households exchange contingent claims that pay off at the end of the period, depending upon which aggregate state s_t is realized. And third, households exchange insurance contracts that pay off at the end of the period, depending upon which type i_t the household has drawn. The position that a household takes in these latter two markets can be represented by a vector $\{A_t(i,s)\}$, where $A_t(i,s)$ indicates the cash payment that the household receives at the end of the period if its type $i_t = i$ and the aggregate state $s_t = s$. The household's budget constraint in the securities trading stage is given by

$$D_t + \sum_i \sum_s n_i a_t(s) A_t(i,s) \leq W_t, \tag{2.5}$$

where the vector $\{a_t(s)\}$ is an asset pricing kernel. Note that prices of payments contingent upon a household's being of type i are proportional to n_i, because this risk is completely diversifiable.

Next the household's type i_t is revealed, and the negotiation stage occurs. In this stage, a household of type i negotiates a goods supply commitment $(\bar{c}_i, p_i(\cdot))$ and a loan supply commitment $(\bar{M}_i, r_i(\cdot))$. After this, the aggregate state s_t is realized, and the execution stage occurs. In this stage, if the state is s, the household chooses its borrowing $M_{it}(s)$ and its purchases $c_{it}(s)$, subject to (2.2). Finally, at the end of the period, the household receives payments due on its portfolio, and repays the amount $B_{it}(s)$ due as a result of its borrowing. It also receives a distribution of profits $\Pi_t(s)$ from its ownership share in the firms that supply goods and in the financial intermediaries that supply credit, and pays its lump-sum tax obligation $T_t(s)$ to the government. Both profit distributions and tax obligations are assumed to be independent of a household's type.

The household thus begins the next period with cash holdings W_{t+1} given by

$$W^i_{t+1}(s) = r^d_t D_t + A_t(i,s) + [M_{it}(s) - Y_{it}(s)] - B_{it}(s) + \Pi_t(s) - T_t(s), \tag{2.6}$$

where r^d_t is the gross nominal interest rate paid by intermediaries on deposits. The household is subject to a *borrowing limit*, that requires that its portfolio, borrowing, and spending decisions in period t be such that

$$W_{t+1} \quad \geq \quad -\sum_{j=1}^{\infty} \sum_{S^j} q_{t+1,t+j}(s^j_{t+j})[\Pi_{t+j}(s_{t+j}) - T_{t+j}(s_{t+j})] \tag{2.7}$$

under every possible realization of the random variables (i_t, s_t). Here $\sum_{S^j}$ denotes summation over all possible histories $s^j_{t+j} \equiv (s_{t+1}, \ldots, s_{t+j})$, and for each such history,

$$q_{t+1,t+j}(s^j_{t+j}) \equiv a_{t+1}(s_{t+1}) \quad \cdots \quad a_{t+j}(s_{t+j})$$

defines present values in terms of the prices of the contingent claims traded in the securities trading stage each period. Constraint (2.7) states that beginning-of-period wealth must be no lower than the negative of the present value of all subsequent profit distributions net of taxes. [13] The sequences of constraints (2.5) and (2.7) imply the existence of an intertemporal budget constraint of the form

$$\sum_{j=0}^{\infty} \sum_{S^{j+1}} q_{t,t+j}(s^{j+1}_{t+j}) \sum_{I^{j+1}} n(i^{j+1}_{t+j})[B^i_{t+j}(s_{t+j}) + Y^i_{t+j}(s_{t+j}) - M^i_{t+j}(s_{t+j})]$$

$$leq \quad W_t \quad + \quad \sum_{j=0}^{\infty} \sum_{S^{j+1}} q_{t,t+j}(s^{j+1}_{t+j})[\Pi_{t+j}(s_{t+j}) - T_{t+j}(s_{t+j})] \tag{2.8}$$

restricting a household's spending from period t onward, where $\sum_{I^{j+1}}$ denotes summation over all possible histories $i^{j+1}_{t+j} \equiv (i_t, \ldots, i_{t+j})$, and

$$n(i^{j+1}_{t+j}) = n(i_t) \quad \cdots \quad n(i_{t+j})$$

[13] See Woodford (1994) for further discussion of a similar borrowing limit in the context of a standard cash-in-advance model.

is the probability of such a history.

The borrowing limit (2.7) is assumed to apply to each household, in addition to the quantity limits $(\bar{c}_t^i, \bar{M}_t^i)$. This is because the latter quantity limits (and the associated supply schedules) depend solely upon a household's type, and not upon its wealth. (It is assumed, for simplicity, that individual households' wealths are not observed by sellers and lenders during the negotiation stage; the assumptions that sellers and lenders make about the extent to which their commitments will be drawn upon are based upon their knowledge of the *typical* wealth of households of each type, but not upon each household's individual wealth. It is for this reason that there exist competitive markets for the service of each type of buyer, rather than an individual market for each household. In equilibrium, the wealths of households of a given type are in fact always the same.) At the execution stage, instead, the wealths of individual borrowers are verified, so as to ensure that credit is extended only when repayment is possible with certainty.

I now consider household trading, given preference shocks and budget constraints of the kind just described. I here take as given the loan commitments $(\bar{M}_t, r_t(\cdot))$ offered by lenders; the determination of lender behavior is taken up in the next section. Let us first consider optimal portfolio choice in the securities trading stage. The government is assumed not to intervene in the market for contingent securities, as a result of which there is zero net supply of such securities. Equilibrium thus requires that aggregate demand for them be zero, so that

$$\sum_i n_i A_t(i, s) = 0 \tag{2.9}$$

for each possible state s. Since optimization requires that (2.5) hold with equality, it is evident that in equilibrium one must have $D_t = W_t$ as well. This is consistent with optimization only if

$$r_t^d = [\sum_s a_t(s)]^{-1}, \tag{2.10}$$

so that a household is indifferent between holding a greater or lesser level of deposits, given the availability of the contingent claims.

Risk aversion implies that households choose to perfectly insure one another against the risk associated with random drawing of types. Thus the portfolio $\{A_t(i, s)\}$ is chosen so as to make W_{t+1} independent of the household's drawing of i_t. Given (2.6) and (2.9), this implies that

$$A_t(i, s) = [B_{it}(s) - B_t(s)] + [Y_{it}(s) - Y_t(s)] - [M_{it}(s) - M_t(s)] \tag{2.11}$$

where $B_{it}(s)$ and so on represent the anticipated decisions of households of type i, while $B_t(s)$ and so on represent the corresponding aggregate variables, defined as in (2.9). This portfolio represents an optimal choice for a household if and only if, in addition to (2.10), the series of state prices $\{a_t(s)\}$ satisfies

$$\nu_t a_t(s) = \beta \pi(s) \nu_{t+1}(s) \tag{2.12}$$

for each possible state s. Here ν_t represents the household's marginal utility of nominal income in the securities trading phase of period t, and $\nu_{t+1}(s)$ its marginal utility of nominal income in the corresponding phase of period $t+1$, in the case that $s_t = s$. (Note that ν_{t+1} is independent of the household's drawing of i_t, as a result of the perfect insurance implied by (2.11).)

The analysis is simplified if we consider only *stationary* equilibria, in which both the real allocation of resources and the distribution of transaction prices (including interest rates) depend each period only upon the current aggregate state s_t, and depend upon the current state in the same way at all dates. [14] Since optimization requires that (2.8) hold with equality, stationarity of the variables $(a, B^i, Y^i, M^i, \Pi, T)$ implies

[14] The restriction of attention to equilibria in which the price level has no trend does not restrict the attainable level of expected utility, since the form of cash-in-advance constraint assumed here does not introduce any distortions that follow from the absolute level of interest rates, as opposed to differentials between the interest rates charged for different transactions. This is because there is here assumed to be no opportunity to economize on money holdings by substituting "credit" purchases or leisure for purchases subject to the cash-in-advance constraint. There thus appears to be no loss of generality, for purposes of the welfare analysis, in restricting attention to stationary equilibria (and hence a zero average rate of inflation).

that W_t must be constant over time. The existence of an equilibrium of this kind is ensured by an appropriate fiscal policy rule. Let $W > 0$ be the constant value of beginning-of-period nominal wealth, and assume as an initial condition that $W_0 = W$. Then the fiscal policy rule $T_t = T(s_t)$, where

$$T(s) = \Pi(s) + [M(s) - Y(s) - B(s)] + (r^d - 1)W, \tag{2.13}$$

implies that in equilibrium $W_t = W$ forever, independently of the sequence of aggregate states $\{s_t\}$. (This follows from substitution of (2.11) and (2.13) into (2.6).)

In the case of a stationary equilibrium, each household faces an identical infinite-horizon decision problem, looking forward from the beginning of any period t, regardless of the date t and of the history of aggregate states prior to date t. It follows that the marginal utility of nominal income in the securities trading stage is equal to the same positive constant, $\nu_t = \nu > 0$, for all dates t and all histories to that date. Condition (2.12) then implies that $a_t(s) = \beta\pi(s)$ at all dates t, and for each possible state s. This in turn implies, using (2.10), that $r^d = \beta^{-1}$ in a stationary equilibrium. It also implies that firms maximize the present value of the profits distributed to their shareholders ($\sum_s a_t(s)\Pi_t(s)$) by maximizing expected profits, which in the case of goods suppliers means maximization of expected sales revenues, as assumed in the previous section.

This result also implies that

$$\lim_{T\to\infty} \beta^T E_t[\nu_T W_T] = 0,$$

so that the usual transversality condition for intertemporal optimization is satisfied. [15] It thus suffices to show that a candidate time-invariant decision rule $\{A(i,s), c_i(s), M_i(s)\}$ would be optimal during a single period t, given beginning-of-period wealth $W_t = W$ and assuming a marginal utility of income $\nu_{t+1} = \nu$ at the beginning of the following period, regardless of the realizations of (i_t, s_t). If so, the rule represents an optimal program for the household's stationary infinite-horizon problem. [16]

Using this result, we can turn to the characterization of optimal borrowing and purchasing decisions during the execution stage. Given a goods supply commitment $(\bar{c}_i, p_i(\cdot))$ and a loan supply commitment $(\bar{M}_i, r_i(\cdot))$, and the realization of state s, a buyer of type i chooses purchases borrowing $M_i(s)$ and purchases $c_i(s)$ to maximize

$$\delta(s;i)u(c_i(s)) + \beta\nu[M_i(s) - Y_i(s) - B_i(s)] \tag{2.14}$$

subject to constraint (2.2), and the constraints $Y_i(s) = R_i(c_i(s))$, $B_i(s) = B_i(M_i(s))$, where the functions $R_i(\cdot)$ and $B_i(\cdot)$ are defined in (2.3) - (2.4).

Let us assume that $(p_i(\cdot), r_i(\cdot))$ are both non-decreasing functions over their respective domains, with $p_i(c) > 0$ for all c and $r_i(M) > 0$ for all M. (The assumed properties for the goods supply schedule were shown to be necessary for equilibrium in the previous section, and the corresponding properties for the loan supply schedule are established in the next. Note that $r_i(M) > 0$ means a non-negative cost of incremental borrowing.) Then let $E_i(c)$ denote the total cost for type i of purchasing a quantity c, counting both the direct cost of the purchases and the interest paid for the money borrowed to make the purchases, for any c such that $0 \leq c \leq \bar{c}_i$, $0 \leq R_i(c) \leq \bar{M}_i$. This domain can be written as $[0, c_i*]$, where $c_i* = \bar{c}_i$ if $\bar{M}_i \geq R_i(\bar{c}_i)$, and $c_i* = R_i^{-1}(\bar{M}_i)$ otherwise. Then on this domain, $E_i(c) = B_i(R_i(c))$. It follows from the assumptions just made, and definitions (2.3) - (2.4), that $E_i(c)$ is an increasing, convex function, with $E_i(0) = 0$.

Maximization of (2.14) subject to (2.2) then requires that $c_i(s)$ be chosen so that

$$(\beta\nu)^{-1}\delta(s;i)u'(c_i(s)) \quad \in \quad \partial E_i(c_i(s)), \tag{2.15}$$

where ∂E_i denotes the subdifferential of $E_i(\cdot)$ (Rockafellar (1970)). That is, for any $0 \leq \tilde{c} \leq c_i*$,

$$\partial E_i(\tilde{c}) \equiv \{\lambda \geq 0 | E_i(c) \geq E_i(\tilde{c}) + \lambda(c - \tilde{c}) \quad \text{for all} \quad 0 \leq c \leq c_i*\}.$$

[15] This condition implies that if (2.5) holds with equality each period, and (2.6) and (2.11) hold each period, then (2.8) holds with equality each period, so that the household exhausts its intertemporal budget constraint.

[16] The sort of dynamic programming argument that may be used to demonstrate this rigorously is illustrated in Lucas and Woodford (1994). It involves demonstration of the existence of a value function for the household's continuation problem, $v(W_{t+1})$, that is differentiable at the value $W_{t+1} = W$, with derivative $v'(W) = \nu$.

It is evident that ∂E_i is a non-decreasing, closed and convex-valued, upper-hemicontinuous correspondence, such that $0 \in \partial E_i(0)$, that $0 \notin \partial E_i(c)$ for any $c > 0$, and that $\sup \partial E_i(c_{i*}) = \infty$. It then follows (given that $u'(\cdot)$ is positive-valued, continuous, and strictly decreasing) that (2.15) is satisfied by exactly one value of $c_i(s)$. [17]

Borrowing must then satisfy the cash-in-advance constraint (2.2), together with the complementary slackness condition

$$[r_i(R_i(c_i(s))) - 1][M_i(s) - R_i(c_i(s))] = 0. \tag{2.16}$$

In the event that $r_i(M) > 1$ for all M, this implies that

$$M_i(s) = R_i(c_i(s)), \tag{2.17}$$

so that $M_i(s)$ is uniquely determined as well.

Condition (2.15) indicates how each household's demand depends upon its type and the state s that is realized. It follows from this (and the monotonicity of u' and ∂E_i that a state with a higher value of $\delta(s;i)$ must result in a higher value of $c_i(s)$. Hence it suffices to assume that states are ordered so that for each type i, $s' \geq s$ implies that $\delta(s';i) \geq \delta(s;i)$, in order for all types' demands to be non-decreasing in the index of the state s, as assumed in the previous section. This assumption will be maintained in what follows.

These relations determine equilibrium purchases, given the supply commitments offered by sellers and lenders. However, as noted earlier, equilibrium supply behavior depends upon suppliers' understanding of the conditions under which buyers would make use of additional supply commitments (not actually offered them in equilibrium). It remains to specify this aspect of buyer behavior.

Consider a buyer of type i in the case of state s. Given the equilibrium supply commitments, the buyer chooses to purchase $c_i(s)$. It is evident from (2.15), however, that the buyer *would* have made use of an incremental supply commitment beyond this point, had additional goods been available beyond the quantity $c_i(s)$ at a price p, and loans been available beyond the quantity $R_i(c_i(s))$ at an interest rate r, such that $pr < \hat{e}_i(s)$, where

$$\hat{e}_i(s) = (\beta\nu)^{-1}\delta(s;i)u'(c_i(s)). \tag{2.18}$$

Note that (2.15) implies that $\hat{e}_i(s) \in \partial E_i(c_i(s))$, and hence that

$$\hat{e}_i(s) \leq p_i(c_i(s))r_i(R_i(c_i(s))). \tag{2.19}$$

If we take as given the loan supply commitments, and consider only incentives to deviate from the equilibrium supply commitments on the part of sellers (as in the previous section), then an incremental supply commitment beyond the quantity $c_i(s)$ at a price p would be made use of in state s if and only if $p < \hat{p}_i(s)$, where

$$\hat{p}_i(s) = \hat{e}_i(s)/r_i(R_i(c_i(s))). \tag{2.20}$$

(Note that given (2.20), (2.11) implies that $\hat{p}_i(s) \leq p_i(s)$, as asserted earlier. Furthermore, if $s' \geq s$ and $c_i(s') = c_i(s)$, then given (2.20), (2.18) implies that $\hat{p}_i(s') \geq \hat{p}_i(s)$, as indicated earlier.) This description of buyer behavior in the case of counterfactual supply commitments then enters into the determination of equilibrium supply commitments, through equilibrium condition (1.3).

3 Monetary Policy and Equilibrium Loan Commitments

It remains to specify the determination of the equilibrium loan commitments $\{\bar{M}_i, r_i(\cdot)\}$; it is at this point that monetary policy affects the nature of equilibrium. The nature of the sequential service constraint in the

[17] Note that if $p_i(\cdot)$ is continuous at the value $c_i(s)$, and $r_i(\cdot)$ is continuous at the value $R_i(c_i(s))$, then $E_i(\cdot)$ is differentiable at the value $c_i(s)$, and $\partial E_i(c_i(s))$ consists of a single value, the derivative. In this case (not, unfortunately, the typical one in equilibrium, under the assumption of a countable number of states), (2.15) reduces to the first-order condition $\delta(s;i)u'(c_i(s)) = \beta\nu p_i(c_i(s))r_i(R_i(c_i(s)))$. More generally, $\sup \partial E_i(c) = p_i(c)r_i(R_i(c))$, for any $0 \leq c < c_{i*}$.

credit market has already been discussed in the previous section. Here I describe the objectives of lenders and complete the development of the requirements for a stationary equilibrium.

Lenders are intermediaries, that hold no cash or other financial claims at the beginning of a period, and seek to maximize the expected profits that are distributed to their shareholders at the end of the period. They offer loan commitments to buyers in the negotiation stage of each period; cash is actually lent to the buyers under these commitments in the execution stage, and is then repaid (with interest) at the end of the period. Lenders must themselves obtain the cash that they lend out in the execution stage, in an interbank market for funds that must be repaid with interest at the end of the period. Their profits at the end of the period are given by the difference between the amount that buyers repay and the amount that the lenders must repay for the funds they have borrowed in the interbank market.

Letting $\rho(s)$ denote the interest rate paid for funds in the interbank market, contingent upon the aggregate state s, aggregate profits of lenders are thus equal to $B(s) - \rho(s)M(s)$. The intermediaries that accept the deposits of households in the securities market stage lend these funds in the interbank market, and thus obtain aggregate profits of $[\rho(s) - r^d]D$. (It does not matter whether we assume that these depository institutions are themselves the lenders, or a separate category of competitive intermediaries.) Since the revenues of sellers equal total expenditure on goods, $Y(s)$, the total profits distributed per household at the end of the period are given by

$$\Pi(s) = Y(s) + [B(s) - \rho(s)M(s)] + [\rho(s) - r^d]D. \tag{3.1}$$

(This is the quantity referred to in the household's budget constraints (2.6) – (2.7).)

The interbank market is a competitive spot market, assumed to involve no sequential service constraint, as only specialists participate in it. It is held following the conclusion of the execution stage; thus the market-clearing cost of funds depends upon the aggregate state s. (I stipulate that it is held *after* the execution stage so that no issue arises of the state s being revealed to lenders prior to the execution stage, and hence of loan commitments being contingent upon the aggregate state.) The participants in this market are the lenders in the credit market, the depository institutions, and the central bank. The lenders demand the quantity of funds needed to cover the loans that they have made in the execution stage (that need not be as large as the *commitments* extended in the negotiation stage). The depository institutions supply the funds deposited with them by households. The net demand for funds by these two categories of participants is thus completely inelastic. The central bank's supply schedule for funds then determines the interest rate at which the market clears. Market clearing requires that

$$M(s) = W + F(s), \tag{3.2}$$

where $F(s)$ denotes the net funds supplied by the central bank in state s. (Recall that in equilibrium the nominal quantity of deposits equals W, the nominal value of beginning-of-period wealth.)

Monetary policy in this model consists of the central bank's supply schedule in the interbank funds market. This policy can be described by a *funds supply correspondence* Γ, indicating the pairs of interest rates ρ and net supplies of funds F that are consistent with the bank's supply rule. (Because of (3.2), this is equivalent to a rule that determines the money supply as a function of the interest rate in the interbank market.) This correspondence is a non-empty subset of the Euclidean plane, assumed to satisfy the following properties:

(i) if $(\rho, F), (\rho', F') \in \Gamma$, and $\rho' > \rho$, then $F' \geq F$; similarly, if $F' > F$, then $\rho' \geq \rho$; and

(ii) for any $(\rho, F) \in \Gamma$, there exists another point $(\rho', F') \in Gamma$, with $\rho' \geq \rho$, $F' \geq F$, and at least one inequality strict; similarly, there exists another point $(\rho', F') \in Gamma$, with $\rho' \leq \rho$, $F' \leq F$, and at least one inequality strict.

Condition (i) is a weak monotonicity requirement; alternative monetary policies thus amount to different decisions as to how rapidly or slowly interest rates will be allowed to rise in the interbank market in response to an increase in the net demand for funds. Condition (ii) implies that the correspondence gives a complete description of the central bank's stance in the face of an arbitrarily high or low net demand for funds.

It is useful to distinguish between two possible classes of policies, that I shall call *direct* and *indirect* monetary policies. Under a direct policy, the central bank directly controls the volume of credit extended to households by the lenders, and hence the money supply, whereas under an indirect policy the central bank restricts itself to control simply of the supply of funds to lenders in the interbank market. In terms of the formalism just introduced, a direct policy corresponds to the choice of a correspondence with properties (i) - (ii), whereas in the case of an indirect policy the funds supply correspondence must satisfy an additional property:

(iii) if $(\rho, F) \in \Gamma$, $\rho \geq 1$.

Condition (iii) must be satisfied in the case of an indirect policy, because if the central bank does not monitor the lending of the lenders, then it cannot prevent them, in the case of a negative interest rate in the interbank market ($\rho < 1$), from borrowing an arbitrarily large quantity of funds and simply holding them rather than lending them out. Hence equilibrium in the interbank market cannot involve an interest rate less than $\rho = 1$, and we can represent this as a constraint on the possible supply correspondences of the central bank (in which case we need not introduce notation for a distinction between lender's lending $M(s)$ and the quantity that they borrow in the interbank market). In the case of a direct policy, by contrast, the central bank can provide incentives for lenders to actually lend, and not simply to borrow from the central bank, and so equilibria are possible with negative interest rates in the interbank market. Thus the class of possible indirect policies is a proper subset of the class of possible direct policies.

Simple examples of such policy correspondences include the case of a completely *elastic* supply of funds at some interest rate $\rho*$,

$$\Gamma = \{(\rho, F) | \rho = \rho*\} \tag{3.3a}$$

and the case of a completely *inelastic* supply of funds at some net supply $F*$,

$$\Gamma = \{(\rho, F) | F = F*\}. \tag{3.3b}$$

Each of these corresponds to a possible direct policy, and given condition (i), one can say that all possible direct policies lie between these two extremes. Example (3.3a) also corresponds to an indirect policy, in the case of an interest rate $\rho* \geq 1$. On the other hand, (3.3b) does not correspond to an indirect policy; the closest approximation to it, within the class of indirect policies, would be

$$\Gamma = \{(\rho, F) | \rho = 1 \text{and} F \leq F*, \quad \text{or} \rho \geq 1 \text{and} F = F*\}. \tag{3.3c}$$

This represents a policy in which the central bank insists upon a net supply of funds $F*$, but does not control whether they are actually lent out to households. Then, in the case that the market clears at a zero interest rate, lenders may be lending any quantity $M(s)$ such that $M(s) \leq W + F*$.

It will be observed that a description of monetary policy in these terms does not allow policy to be stochastic. This is because arbitrary randomization of the terms on which funds are supplied by the central bank disrupts the exchange process in this model (as in Lucas and Woodford (1994)), and so is not plausibly part of an optimal policy. [18] A more important restriction is that Γ is assumed to be independent of the aggregate state s; that is, the point $(\rho(s), F(s)) \in \Gamma$ that is selected as the spot equilibrium of the interbank market may depend upon s, but the set Γ may not. This is intended to capture the idea that the central bank is not able to respond to the revelation of the aggregate state s any faster than can sellers of goods. Thus

[18] The real effects of monetary policy shocks could be analyzed in the present framework by allowing the correspondence Γ to depend upon a random state z, realized at the beginning of the negotiation stage, and revealed at that time to lenders and to buyers, but not to sellers. Sellers would then negotiate supply commitments taking into account the *ex ante* probability distribution for the state z, which would affect the degree to which buyers plan to draw upon the commitments, just as with the random state s. In the case of a single preference state s (and hence a single "type" each period), the random variation in the supply correspondence Γ would be the only source of uncertainty on the part of sellers as to their sales, and this random variation would result in less than full utilization of capacity in some states. With a fixed correspondence Γ, by contrast, regardless of its nature, there would be full utilization of capacity in equilibrium, and hence the maximum possible level of expected utility for the representative household, just as in Lucas and Woodford. For the sake of simplicity, I omit any analysis of the interaction between random variation in both states z and s.

the possible benefits of an accommodative monetary policy discussed in the next section do not depend upon any informational advantage assumed for the central bank. However, the restriction of the central bank to act upon the basis of no more information than sellers have when they enter into their supply commitments does not mean that there is no non-trivial choice to be made with regard to monetary policy, for the central bank can choose a flatter or steeper supply schedule Γ, implying a greater or lesser degree of accommodation of shocks to aggregate demand from other sources.

I turn now to the characterization of equilibrium interest rates and the equilibrium supply of loan commitments. Maximization of expected profits by depository institutions implies that in the securities trading stage, any such institution will be willing to offer an interest rate equal to the expected value of the interest rate $\rho(s)$ in the interbank market, so that in equilibrium

$$r^d = \sum_s \pi_s \rho(s). \tag{3.4}$$

Similarly, maximization of expected profits by lenders implies that in the negotiation stage, any lender will be willing to supply incremental loan commitments at an interest rate equal to the expected value of $\rho(s)$ in the interbank market, conditional upon the loan commitment's being drawn upon. Thus each loan supply schedule $r_i(\cdot)$ must satisfy

$$r_i(M) = \frac{\sum_s \pi_s \rho(s) I(M_i(s) > M)}{\sum_s \pi_s I(M_i(s) > M)} \tag{3.5}$$

for all $0 \leq M < sup_s M_i(s)$. We may without loss of generality assume that lenders do not bother to offer commitments that they know will never be drawn upon, so that

$$\bar{M}_i = sup_s M_i(s) \tag{3.6}$$

for each i, by analogy with (1.6). Then (3.5) defines $r_i(\cdot)$ over its entire domain.

Another necessary condition for equilibrium in the credit market during the negotiation stage is that $r_i(M) \geq 1$ over the entire domain of the function, as asserted in the previous section. In the case of an indirect monetary policy, this follows from (3.5). However, it is necessary for equilibrium even if negative interest rates are possible in the interbank market. This is because no lender could expect to gain by offering an incremental loan commitment at an interest rate $r < 1$, rather than some other rate r' with $r < r' < 1$. For a buyer with either commitment will draw upon it with certainty, in any state in which he borrows quantity M and thus becomes eligible to do so. (The money thus borrowed need not be spent; part could simply be held in order to repay the loan at the end of the period.) Furthermore, there is no possibility that competition between lenders could force them to offer commitments at the rate r. for even For if other lenders offer incremental commitments at that rate, a lender can offer his own commitment at the rate r', to apply after the buyer has already borrowed M', where $M' - M$ is the quantity of commitments obtained at the lower interest rate. Such an incremental commitment will still be accepted (as it does not preclude acceptance of the lower-rate loan commitments first), and it is exercised in the same states as the other ones (since whenever the buyer borrows M, he will borrow M', given the negative cost of the additional borrowing); therefore it results in higher expected revenues. This conclusion does not contradict the necessity of (3.5); it simply means that the equilibrium interest rates in the interbank market must be such that the right hand side of (3.5) is never lower than 1, lest an arbitrage opportunity be created for lenders.

I shall impose as an additional regularity condition on the class of equilibria under consideration the assumption that for each type i, $\{M_i(s)\}$ is a non-decreasing series. This assumption amounts simply to a particular assumption about how buyers decide among alternative levels of borrowing among which they are indifferent. The characterization of the buyer's problem in the previous section implies that his level of borrowing $M_i(s)$ can be any level in the closed interval with lower bound $R_i(c_i(s))$ and upper bound

$$\inf R_i(c_i(s)) \leq M < \bar{M}_i | r_i(M) > 1,$$

where the infimum is defined as $\bar{M}_i$ if the set is empty. It has been shown in the previous section that $\{c_i(s)\}$ is a non-decreasing series, and this implies that the lower bound of the interval optimal values is non-decreasing with s. It is thus possible to select series of optimal choices $\{M_i(s)\}$ that are non-decreasing, and I assume that each household chooses a borrowing plan of this kind.

It then follows from (3.5) that each equilibrium loan supply schedule is a piecewise constant function, of the form

$$r_i(M) = r_s \qquad \text{for all} M_i(s-1) \le M < M_i(s) \tag{3.7}$$

for each state s such that $M_i(s) > M_i(s-1)$, where $M_i(0) \equiv 0$. The sequence of interest rates $\{r_s\}$ referred to in (3.7) is given by

$$r_s = \frac{\sum_{s' \geq s} \pi_{s'} \rho(s')}{\sum_{s' \geq s} \pi_{s'}}. \tag{3.8}$$

Furthermore, aggregate borrowing $\{M(s)\}$ is also a non-decreasing series. Given the monotonicity assumption on Γ (condition (i) above), this implies that the series $\{\rho(s)\}$ must be non-decreasing, except possibly in the case of successive states over which $M(s)$ remains constant, at a value corresponding to a vertical segment of the correspondence Γ. I shall impose as a further regularity condition on equilibria that $\{\rho(s)\}$ be a non-decreasing series in such a case as well. (This amounts to consideration only of equilibria that can be approximated arbitrarily closely by equilibria associated with monetary policies which make the money supply a well-defined, increasing function of ρ.) Then (3.8) implies that the series $\{r_s\}$ is non-decreasing as well.

Finally, comparison of (3.8) with (3.1) indicates that $r_1 = r^d$, so that $r_i(M) \geq r^d$ for all i and all $M \geq 0$. In the case of a stationary equilibrium, as shown in the previous section, $r^d = \beta^{-1}$, and so one has $r_i(M) > 0$ for all i and all M. Then equilibrium borrowing must be given by (2.17) in each state and for each type.

As with sellers, optimal behavior by lenders requires also that expected profits be non-positive in the case of all loan commitments that are *not* offered in equilibrium. I shall strengthen the requirements for equilibrium, by demanding not merely that no seller wish to offer any other supply commitments *given* the equilibrium loan commitments, and that no lender wish to offer any other loan commitments *given* the equilibrium supply commitments of sellers, but furthermore that there be no *joint* deviation by a seller and a lender that would be jointly profitable. Because increased borrowing occurs if and only if a buyer increases his purchases of goods, by (3.9), it suffices to consider the condition under which the combination of an incremental goods supply commitment and an incremental loan commitment would induce an increase an increase in the goods purchased in some state, and a corresponding increase in the money borrowed to pay for those additional purchases.

It has been shown in the previous section that a buyer of type i would make use of an incremental supply commitment beyond the point $c = c_i(s)$ and an incremental loan commitment beyond the point $M = R_i(c_i(s))$, in state s, if and only if the incremental supply price p and the incremental interest rate r are such that $p \leq p_i(c)$, $r \leq r_i(M)$, and $pr < \hat{e}_i(s)$. It is then a requirement for equilibrium that for each type i, and for any quantity $0 \leq c < \bar{c}_i$, there exists no price $p \leq p_i(c)$ and interest rate $r \leq r_i(R_i(c))$ such that

$$p \sum_s \pi_s [I(c_i(s) > c) + I(c_i(s) = c) I(\hat{e}_i(s) > pr)] \geq \lambda \tag{3.9}$$

and

$$\sum_s \pi_s (r - \rho(s)) [I(c_i(s) > c) + I(c_i(s) = c) I(\hat{e}_i(s) > pr)] \geq 0, \tag{3.10}$$

unless both (3.9) - (3.10) hold with equality. For if such a pair (p, r) were to exist, it would represent a joint incremental commitment that would increase the expected profits of at least one party (the seller or the lender), without reducing the expected profits of the other. Similarly, in the case that $c = \bar{c}_i$, there must exist no pair (p, r) that satisfy (3.9) - (3.10), unless both hold with equality.

Note that in the case that $c = c_i(s)$ and $r = r_i(R_i(c))$, the left hand side of (3.10) is equal to zero when $p = p_i(c)$, using (2.19) and (3.5). ~~It then follows that (3.10) holds for all $p \leq p_i(c)$. Hence for this choice~~ of r, there must exist no $p \leq p_i(c)$ for which (3.9) holds, unless it holds with equality. But, using (2.20), this requirement is seen to be equivalent to (1.3). Thus requirements (3.9) - (3.10) imply (1.3), and can be regarded as an extension of the previous requirement.

Now in the case that c does not equal $c_i(s)$ for any state s, requirements (3.9) – (3.10) are implied by (1.4) – (1.5) and (3.7) – (3.8). Thus we need only to consider the case $c = c_i(s)$ for some state s. There are three subcases to consider. These consist of states s and pairs (p, r) such that (i) $s = \bar{s}$ or $c_i(s+1) > c_i(s)$, and $pr \geq \hat{e}^i(s)$; (ii) $s = 1$ or $c_i(s) > c_i(s-1)$, and $pr < \hat{e}_i(s)$; or (iii) $c_i(s) = c_i(s-1)$, but $\hat{e}^i(s) > \hat{e}^i(s-1)$, and $\hat{e}^i(s-1) \leq pr < \hat{e}_i(s)$. Every pair (p, r) belongs to one of these three subcases. In subcase (i), (3.9) becomes $p \geq p_i(c)$ and (3.10) becomes $r \geq r_i(R_i(c))$, so that the requirements are trivially satisfied. In subcases (ii) or (iii), (3.9) becomes $p \geq p_s$ and (3.10) becomes $r \geq r_s$, as a result of which the requirements are satisfied if and only if

$$\hat{e}_i(s) \leq p_s r_s \tag{3.11}$$

for each state s belonging to either of those two classes (i.e., such that $s = 1$, $c_i(s) > c_i(s-1)$, or $c_i(s) = c_i(s-1)$ but $\hat{e}_i(s) > \hat{e}^i(s-1)$). Thus (3.11) is necessary and sufficient for requirements (3.9) – (3.10) to be satisfied.

Furthermore, the fact that (3.7) holds for all states in the classes just listed implies that it must hold at all states. For in the case of any sequence of states $(s', \ldots, s'+j)$ such that $c_i(s'+k) = c_i(s')$ and $\hat{e}_i(s'+k) = \hat{e}_i(s')$ for all $0 \leq k \leq j$, (3.7) must hold for state s' as long as $s'-1$ does not also have the same property. But then the fact that series $\{p_s\}$ and $\{r_s\}$ are both non-decreasing implies that (3.7) holds as well for each of the states $(s', \ldots, s'+j)$. Thus (3.7) must hold for all states.

It is now possible to dispose of the possibility of equilibrium in which (1.5) holds with $\lambda = 0$. Let us assume that for each type i, there exists a state s for which $\delta(s;i) > 0$, and let $\underline{s}_i$ be the lowest such state. (The monotonicity assumption made earlier implies that $\delta(s;i) > 0$ for all $s \geq \underline{s}_i$.) Then (2.18) implies that $\hat{e}^i(s) > 0$ for all $s \geq \underline{s}_i$. But if $\lambda = 0$, (1.5) would imply that the right hand side of (3.11) equals zero for all s, so that (3.11) would be violated in the states just mentioned. Thus equilibrium necessarily involves $\lambda > 0$, as announced earlier.

Let us return to the characterization of the equilibrium relations between prices and quantities. Using the characterizations (1.4) – (1.5) and (3.7) – (3.8) of the equilibrium supply schedules, (2.15) becomes

$$p_{s^-(s)} r_{s^-(s)} \leq (\beta\nu)^{-1} \delta(s;i) u'(c_i(s)) \leq p_{s^+(s)} r_{s^+(s)}, \tag{3.12}$$

where for each state s, $s^-(s)$ denotes the lowest state s' for which $c_i(s') = c_i(s)$, and $s^+(s)$ denotes the immediate successor to the highest state s' with this property. (If there is no higher state for which the property does not hold, then the second inequality in (3.12) is vacuous. Also, the first inequality does not apply if $c_i(s) = 0$.) Substitution of (2.18) into (3.11) and comparison with (3.12) shows that the upper bound in (3.12) can be tightened, to yield

$$p_{s^-(s)} r_{s^-(s)} \leq (\beta\nu)^{-1} \delta(s;i) u'(c_i(s)) \leq p_s r_s, \tag{3.13}$$

where the first inequality again must hold only if $c_i(s) > 0$. (Note that $p_s r_s$ cannot exceed $p_{s^+(s)} r_{s^+(s)}$, because of the monotonicity of the series $\{p_s\}$ and $\{r_s\}$.)

It follows from (3.12) (or equivalently, (3.13)) that in any state $s < \underline{s}_i$, so that $\delta(s;i) = 0$, one must have $c_i(s) = 0$. (Note that (1.5) and (3.8) imply that $p_s r_s > 0$ in all states, given that $\lambda > 0$ as just shown.) It is also necessary that $c_i(s) \geq c_i(s-1)$ for all $s \geq \underline{s}_i$ (where in the case $s = 1$, we again define $c_i(0) \equiv 0$). If $c_i(s) > c_i(s-1)$, $s^-(s) = s$, and (3.13) implies that

$$u'(c_i(s)) = \beta\nu\delta(s;i)^{-1} p_s r_s.$$

On the other hand, if the second inequality in (3.9) is strict, one must have $s > s^-(s)$, so that one must have $c_i(s) = c_i(s-1)$, and hence $u'(c_i(s)) = u'(c_i(s-1))$. Thus for all $s \geq \underline{s}_i$,

$$u'(c_i(s)) = \min[u'(c_i(s-1), \beta\nu\delta(s;i)^{-1} p_s r_s],$$

from which it follows that

$$u'(c_i(s)) = \min\left[u'(0), \min_{\underline{s}_i \leq s' \leq s} \{\beta\nu\delta(s';i)^{-1} p_{s'} r_{s'}\}\right]. \tag{3.14}$$

Given sequences $\{p_s, r_s\}$ and a value $\nu > 0$, (3.14) uniquely determines the sequence $\{c_i(s)\}$.

We may now collect our results characterizing stationary equilibrium. A stationary equilibrium is described by non-decreasing series of prices $\{p_s, r_s, \rho(s)\}$, collections of purchasing and borrowing plans $\{c_i(s), M_i(s)\}$ (each of which consists of two non-decreasing series), and quantities $\lambda, \nu > 0$, such that

(i) given $\lambda > 0$, the prices $\{p_s\}$ are given by (1.5);

(ii) given the interbank market rates $\{\rho(s)\}$, the credit market rates $\{r_s\}$ are given by (3.8);

(iii) given the prices $\{p_s, r_s\}$ and $\nu > 0$, the consumption plans $\{c_i(s)\}$ are given by (3.14), together with the requirement that $c_i(s) = 0$ for all $s < \underline{s}_i$, and the borrowing plans $\{M_i(s)\}$ are given by (2.17), where the function $H_i(\cdot)$ is defined by (1.4) and (2.3);

(iv) given the plans $\{c_i(s), M_i(s)\}$, the quantity limits $\{\bar{c}_i, \bar{M}_i\}$ are given by (1.6) and (3.6);

(v) the quantity limits $\{\bar{c}_i\}$ satisfy (1.2);

(vi) the interbank market rates $\{\rho(s)\}$ and the plans $\{M_i(s)\}$ are such that

$$(\rho(s), M(s) - W) \in \Gamma \tag{3.15}$$

for each state s, where $M(s) = \sum_i M_i(s)$; and

(vii) the interbank market rates $\{\rho(s)\}$ are such that

$$\sum_s \pi_s \rho(s) = \beta^{-1}. \tag{3.16}$$

Here (3.15) follows from (3.2), and (3.16) follows from the requirement that $r_1 = r^d = \beta^{-1}$ in a stationary equilbrium, using (3.8). Given series satisfying conditions (i) – (vii), it is then straightforward to compute the equilibrium values of $\{H(s), T(s)\}$, $\{A(i,s)\}$, and so on, using the other equilibrium conditions set out earlier.

For arbitarily chosen values for $\lambda, \nu > 0$ and an arbitary series $\rho = \{\rho(s)\}$, conditions (i) – (iv) indicate how one may compute unique equilibrium values for the aggregate supply commitment $\bar{c} \equiv \sum_i n_i \bar{c}_i$, the expected interbank interest rate $\bar{\rho} \equiv \sum_s \pi_s \rho(s)$, and aggregate borrowing $M(s)$ in each state. Then such values for (λ, ν, ρ) describe a stationary equilibrium if and only if

$$\bar{c}(\lambda, \nu, \rho) = y, \tag{3.17a}$$

$$\bar{\rho}(\rho) = \beta^{-1}, \tag{3.17b}$$

and

$$(\rho(s), M(s; \lambda, \nu, \rho) - W) \in \Gamma \tag{3.17c}$$

for each state s. The number of independent equilibrium conditions in (3.17) is equal to the number of endogenous variables (λ, ν, ρ). This does not in itself prove that equilibrium is either possible or uniquely determined in the case of any particular monetary policy Γ, but should suggest that the proposed definition of equilibrium satisfies at least minimal standards of coherence. Rather than pursue further questions of existence or uniqueness of stationary equilibrium for general policies, I turn instead to the welfare properties of the equilibria associated with certain policies.

4 Optimal Monetary Policy in Two Polar Cases

I now turn to the question of optimal monetary policy in the context of the model just set out. An obvious welfare criterion in the present context is the *ex ante* expected utility of the representative household. In a stationary equilibrium of the kind described above, this is proportional to the quantity

$$W = \sum_i \sum_s n_i \pi(s) u(c_i(s)). \tag{4.1}$$

I thus wish to compare the level of W obtained in the alternative stationary equilibria associated with alternative possible central bank supply correspondences Γ.

In a certain extreme case, a simple answer is possible, and it follows directly from the discussion in section 1. Suppose that all households are identical; that is,

$$\delta(s;i) = \delta(s) \tag{4.2}$$

for each type i, where $\{\delta(s)\}$ is an increasing series. Any feasible allocation of resources clearly must satisfy the constraint

$$\sum_i n_i c_i(s) \leq y \tag{4.3}$$

in each state s. Given (4.2), the allocation that maximizes (4.1) subject to constraint (4.3) is clearly given by $c_i(s) = y$ *for each type in each state*. Then since there exists a direct monetary policy that achieves this, it is plainly an optimal policy, among the class of direct policies.

A policy that achieves this first-best outcome is a constant-money-supply policy, i.e., a correspondence Γ of the form (3.3b), for any $F* > -W$ (so that the constant money supply is equal to $M* = W + F* > 0$). The associated stationary equilibrium is given by $c_i(s) = y$, $M_i(s) = M*$ for each type i in each state s; $\bar{c}_i = y$, $\bar{M}_i = M*$ for each type;

$$\lambda = \frac{M*}{y}, \tag{4.4a}$$

$$\nu = \frac{\delta(1) y u'(y)}{M*}; \tag{4.4b}$$

and the sequence $\{p_s\}$ implied by (4.4a), using (1.5). The sequence $\{r_s\}$ is chosen so that $r_1 = \beta^{-1}$, and

$$r_s \geq \frac{\delta(s)}{\beta\delta(1)} \sum_{s' \geq s} \pi_{s'} \tag{4.4c}$$

for each $s > 1$. The sequence $\{\rho(s)\}$ is then given by

$$\rho(\bar{s}) = r_{\bar{s}}, \tag{4.4d}$$

and

$$\rho(s) = \frac{r_s[\sum_{s' \geq s} \pi_{s'}] - r_{s+1}[\sum_{s' \geq s+1} \pi_{s'}]}{\pi(s)} \tag{4.4e}$$

for all $s \leq \bar{s}$. (If there exists no terminal state $\bar{s}$, then (4.4e) applies to all states s.) Substitution of these values into the equilibrium conditions listed at the end of the previous section allows one to verify that all conditions for a stationary equilibrium are satisfied.

Furthermore, *any* policy that achieves the first-best outcome in case (2.2) must be essentially of this kind. For, as explained in section 1, it is only possible to have a common value of $c(s)$ in all states if aggregate expenditure, and hence the money supply $M(s)$, is the same in all states. Letting the common value for the money supply be denoted $M*$, then the fact that all purchases occur at price p, requires that $p_1 y = M*$, which implies (4.4a), using (1.5). Given that $r_1 = \beta^{-1}$ in any stationary equilibrium, condition (3.14) for state $s = 1$ implies (4.4b). Using these values for λ, ν, and substituting (1.5), condition (3.14) for any state

$s > 1$ then implies (4.4c). Finally, conditions (4.4d) - (4.4e) simply invert (3.8). Thus all of conditions (4.4) necessarily hold in any first-best equilibrium.

If for any state $s > 1$,

$$\delta(s) \sum_{s' \geq s} \pi_{s'} > \delta(1),$$

then (4.4c) cannot be satisfied by a series $\{r_s\}$ with $r_s = \beta^{-1}$ for all s. But if $r_s > r_1 = \beta^{-1}$, then it follows from (4.4d) - (4.4e) that the series $\{p(s)\}$ cannot take a single value for all states s either. But then (3.15) implies that the correspondence Γ must possess a vertical segment. And while the *entire* correspondence need not be vertical, it is only the vertical segment that plays any role in equilibrium determination.

This conclusion, however, is dependent upon assuming that direct control of the volume of lending (and of the money supply) is possible. There need not be any *indirect* policy that can achieve the first-best allocation. I have just argued that any first-best stationary equilibrium must satisfy conditions (4.4). But there need not be any series $\{r_s\}$ consistent with (4.4c), that is also consistent with (4.4d) - (4.4e), if one is to have $\rho(s) \geq 1$ for all s (as must be the case for an indirect policy).

Multiplying the right hand side of (4.4e) by π_s, and summing over s between the values of 1 and $k - 1$ (for some $k > 1$), one obtains $r_1 - r_k[\sum_{s' > k} \pi_{s'}]$. If $\rho(s) \geq 1$ for all $1 \leq s \leq k - 1$, this quantity must not exceed $\sum_{s' \leq k-1} \pi_{s'}$. Thus one obtains an upper bound on the possible value of r_k in any stationary equilibrium associated with an indirect policy, namely

$$r_k \leq \frac{\beta^{-1} - \sum_{s' < k-1} \pi_{s'}}{\sum_{s' \geq k} \pi_{s'}}. \tag{4.5}$$

This means that if for any $s > 1$,

$$\frac{\delta(s)}{\delta(1)} \quad [\sum_{s' \geq s} \pi_{s'}]^2 + \beta \sum_{s' \leq s-1} \pi_{s'} \quad > \quad 1,$$

there is a contradiction between (4.4c) and (4.5), so that no indirect policy is consistent with the first-best outcome.

A policy of the form (3.3c), for example, results in fluctuations in spending and hence in output, despite the inelastic supply of funds by the central bank, because lenders choose to commit some of the available funds under loan commitments that are drawn upon only in high-demand states. This is tempting because if no loan commitments of this kind were made, the interest rates at which buyers would be willing to borrow additional amounts in the high-demand states would be high enough to make incremental loan commitments more profitable than the loan commitments that are drawn upon in state $s = 1$. (The interest rates in question are given by the right hand side of (4.4c).)

Rather than develop further the question of what can be achieved by indirect policy in such a case, I wish to point out that even in the case of direct policies, the above conclusion holds only for the special type of preference variations indicated by (4.2). If the demand of some types of buyers is more "cyclical" than that of others, an inelastic money supply is generally not optimal. As an opposite extreme to (4.2), suppose that the preference variations are given by

$$\delta(s; i) = I(s \geq \underline{s}_i), \tag{4.6}$$

and suppose that $\underline{s}_i = 1$ for at least one type, while $\underline{s}_i > 1$ for others. In this contrasting extreme case, preferences are the same, for any given type, across all the states in which that type desires to consume at all; what varies across states is the *number* of types with any desire to consume.

In case (4.6), the first-best allocation is no longer attainable under *any* monetary policy, for it is inconsistent with sequential service. Maximization of (4.1) subject to (4.3) in this case would imply choosing

$$c_i(s) = \frac{y}{\sum_i n_i I(s \geq \underline{s}_i)}$$

for each i and each s. But this makes $\{c_i(s)\}$ non-increasing in s for each type i, and decreasing at each successive state $s > \underline{s}_i$ at which one or more additional types of buyers enter the market (i.e., obtain positive utility from consumption). If $\underline{s}_i$ is not the same for all types, then $c_i(s)$ must decrease at some states, for some type i. This violates the *sequential service constraint*, which, as argued earlier, results in a non-decreasing series $\{c_i(s)\}$ for each type.

Let us define the second-best allocation of resources as the allocation that maximizes (4.1) subject to constraints (4.3) and

$$c_i(s) \geq c_i(s-1) \tag{4.7}$$

for each type i and each state s (again defining $c_i(0) \equiv 0$). This amounts to restricting allocation rules that do not violate the sequential service constraint. In case (4.6), the second-best allocation is given by $c_i(s) = 0$ if $s < \underline{s}_i$, [19] while $c_i(s) = \bar{c}_i$ if $s \geq \underline{s}_i$, for each type i. Furthermore, the quantity limits $\{\bar{c}_i\}$ satisfy

$$u'(\bar{c}_i) = \alpha[\sum_{s \geq \underline{s}_i} \pi(s)]^{-1}, \tag{4.8a}$$

where $\alpha > 0$ is determined by the requirement that the solution to (4.8a) satisfy (1.2) as well.

In case (4.6), *the second-best allocation can be achieved by an appropriate choice of monetary policy rule*. A rule that achieves it, and is therefore optimal, is the perfectly elastic supply of funds described by (3.3a), where the nominal interest rate $r*$ is equal to β^{-1}. (This is the only rule of the form (3.3a) that is consistent with a stationary equilibrium, because it is the only one that satisfies (3.16). [20]) The associated stationary equilibrium is given by the second-best allocation described above; borrowing plans according to which, for each type, $M_i(s) = 0$ for all $s < \underline{s}_i$, and $M_i(s) = \bar{M}_i$ for all $s \geq \underline{s}_i$, where

$$\bar{M}_i = \nu^{-1}\bar{c}_i u'(\bar{c}_i) \tag{4.8b}$$

for some $\nu > 0$ (independent of i); and the quantity limits $\{\bar{c}_i, \bar{M}_i\}$ are defined by (4.8a) – (4.8b). The series $\{p_s\}$ is given by (1.5), using the value $\lambda = \alpha\nu^{-1}$, while the series $\{r_s, \rho(s)\}$ are given by $r_s = \rho(s) = \beta^{-1}$ for each s. Again, substitution of these values into the equilibrium conditions at the end of the previous section allows one to verify that all conditions for a stationary equilibrium are satisfied.

Furthermore, *any* policy rule that achieves the second-best outcome in case (4.6) must be essentially of this kind. For given the second-best allocation, as characterized above, (3.14) requires that for each type i,

$$u'(\bar{c}_i) = \beta\nu p_{\underline{s}_i} r_{\underline{s}_i}.$$

Substitution of (1.5) into this and comparison with (4.8) then implies that

$$r_{\underline{s}_i} = \alpha(\beta\nu\lambda)^{-1}$$

for each i. Furthermore, *any state s such that $M(s) > M(s-1)$ is a state in which $\underline{s}_i = s$ for some type i*, and hence a state for which one must have $r_s = \alpha(\beta\nu\lambda)^{-1}$. As there is assumed to be a type for which $\underline{s}_i = 1$, $r_1 = \alpha(\beta\nu\lambda)^{-1}$, as a result of which one must have $\lambda\nu = \alpha$, so that $r_s = \beta^{-1}$ for each state such that $M(s) > M(s-1)$. Then (3.8), together with the assumed monotonicity of Γ, implies that $\rho(s) = \beta^{-1}$ for all s. Thus, even if Γ is not globally horizontal (as specified in (3.3a)), it must have a horizontal segment at height β^{-1}, which segment includes all of the points at which equilibria of the interbank market occur in any state. One consequence is that a constant-money-supply rule is clearly *not* optimal, as it is inconsistent with the second-best allocation in this case.

[19]Actually, the specification of $\{c_i(s)\}$ in the states $s < \underline{s}_i$ is indeterminate; it may be any non-decreasing sequence taking values between the bounds of 0 and $\bar{c}_i$. The second-best allocation described in the text is the unique one that also satisfies the constraint that if $\delta(s;i) = 0$, $c_i(s) = 0$. Since the latter property necessarily obtains in any stationary equilibrium, this is the obvious second-best allocation to consider.

[20]The result that a policy rule of the form (3.3a) makes the second-best allocation an equilibrium allocation does not, however, depend upon this particular choice of r*. Other values of r* are associated with other equilibria in which the allocation of resources is the same, but the *average rate of inflation differs from zero*. Equilibria of this kind are not treated here because of the extension of the notation that would be required.

The elastic money supply (or interest rate smoothing) rule (3.3a) *does* increase the volatility of aggregate demand, in this case, as in traditional analyses like that of Poole (1970). And, as explained in section 1, the resulting uncertainty about aggregate demand results in equilibrium supply commitments that cause some capacity to be left unused in some states. But this does not imply that such a policy leads to a less efficient allocation of resources. Idle capacity is inconsistent with attainment of the first-best allocation, but it may be consistent with attainment of the second-best, as it is in this case. [21] Given the sequential service constraint, the idle capacity in low-demand states is a necessary consequence of the fact that an optimal fraction of capacity is committed to the supply of goods to types of buyers that consume only in high-demand states.

The efficiency result obtained here is closely related to Prescott's (1975) argument for the efficiency of an equilibrium with idle capacity. Prescott's model is one in which prices are fixed in advance for individual units of capacity, but these are not committed to the supply of goods to individual buyers. Yet, as noted in section 1, the equilibrium in his model is formally analogous to the goods market equilibrium described here; the crucial difference is that as he assumes no cash-in-advance constraint on purchases, his buyers behave in the way that buyers do in the present model when facing a perfectly elastic supply of credit at a zero interest rate. However, in the present model, an elastic supply of credit at a positive interest rate results in the same allocation of resources as would a zero interest rate; equilibrium goods prices and borrowing are simply all rescaled by a certain positive factor. Thus the equilibrium allocation here in the case of an elastic supply of credit at the interest rate $r* = \beta^{-1}$ is the same as in Prescott's model, which he shows to be efficient in the case of certain preferences that constitute a special case of (4.6).

[Further discussion to be added.]

[21] This depends upon a relatively special feature of case (4.6), namely, the fact that some types have $\delta(s;i) = 0$ in low states, rather than merely having a low positive value. If $\delta(s;i) > 0$ for all types in all states, then the second-best allocation also must involve full utilization of capacity in all states; and in all cases, at least *one* second-best allocation involves full utilization in all states (see footnote xx). However, in the more general case, the second-best allocation (in the sense proposed above) is not attainable under any monetary policy. The class of allocations attainable in stationary equilibria is a more restrictive class than that defined by constraints (4.3) and (4.7), so that a "third-best" allocation (not defined here) is in general the best that can be achieved. And this "third-best" allocation does not in general involve full utilization of capacity in all states. It is in this sense that the situation illustrated by case (4.6) is robust.

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