Rediscounting under Aggregate Risk

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This paper compares three institutions that offer an elastic currency - open market operations, a discount window, a private, banknote-issuing clearinghouse - in an economy with financial markets otherwise hampered by a lack of liquidity. When this economy is subject to aggregate financial shocks, these alternative forms of central banking are shown to differ in their implications for risk-sharing and moral hazard. Interesting implications include i) central bank losses and monetary innovations that are part of an efficient equilibrium; and ii) desirable quantity restrictions at the discount window.

1 Introduction

An Act to provide for the establishment of Federal Reserve Banks, to furnish an elastic currency, to afford means of rediscounting commercial paper, to establish a more effective supervision of banking in the United States, and for other purposes.

The title of the act establishing the Federal Reserve System (above) states its goals confidently. But what is an elastic currency? Why should the government involve itself in the rediscounting of commercial paper?

To address these questions, this paper constructs a model of the payments system, the means by which debts are repaid. In particular, the model is designed to capture the following basic features of the payments system: i) people make some purchases with debt; ii) the repayment of debt requires a final payment in the form of fiat money; and iii) there is an active market in second-hand debt (i.e., at least some debt is cleared through third parties). In the context of such a model of the payments system one can examine the effects of central bank interventions like an "elastic" supply of currency and the rediscounting of private debt.[1]

The basic version of the model was introduced in Freeman (1996a) and used to show how the provision of liquidity through temporary expansions of the money supply may reduce fluctuations in the short term interest rate, improving agent welfare and the functioning of credit markets. This is consistent with observations cited by Friedman and Schwartz (1963, pp. 292-3) that with the establishment of the Federal Reserve seasonal fluctuations in the stock of money were increased, while seasonal fluctuations in short term interest rates were reduced.[2]

A limiting feature of the policy suggestions of this earlier analysis was the assumed absence of aggregate risk. In the presence of aggregate risk, it can no longer be guaranteed that liquidity-providing interventions such as central bank lending and open market operations are without risk to the central bank. Should the central bank lend funds or purchase debt even

[1]Other notable approaches to studying the need for an elastic currency are related but do not focus on the payments system. Sargent and Wallace (1982) see an elastic currency as an equilibrium response to fluctuations in interest rates and asset demands. Champ, Smith, and Williamson (1996) focus on the need for an elastic currency in response to changes in the relative demand for currency and deposits.

[2]See Champ, Smith, and Williamson (1996) for a model of an elastic currency need that pays greater attention to the specific circumstances of that era.

at the risk of losses? If so, which interventions provide liquidity with the least, or the most desirable, central bank exposure to risk?

The purpose of this paper is therefore to examine the implications of aggregate risk for the welfare effects of alternative methods of providing liquidity. A general equilibrium model in which credit markets are exposed to aggregate default risk is used to show that the provision of liquidity by the central bank results in a more even and efficient distribution of default risk. The paper then compares the welfare properties of open market operations (section 3) with those of the rediscounting of private debt (section 4). It shows that open market operations do more to reduce the risk in the markets for the resale of debt. In the case of symmetric information about default risks, this spreads aggregate risk more broadly and efficiently (if agents are risk-averse).

If, however, central banks have less information than private agents, open market operations are more likely to encourage loans to bad risks. This moral hazard may in some cases also be present in the rediscounting of private debt. For some range of the model's parameters, a central bank policy of rediscounting private debt has two equilibrium outcomes, one with moral hazard and one without. The central bank can select between these two equilibrium outcomes only if it imposes appropriate quantity restrictions on the amount it lends.

Finally, the paper illustrates how liquidity may be provided through private clearinghouses, even in the case of aggregate default risk (section 5). The private clearinghouses must be permitted to issue their own bank notes to acquire debt. The notes will trade below par in the event of a default.

2 The Basic Model

2.1 The environment

The model is based on that of Freeman (1996a), an offshoot of Freeman and Tabellini (1992). A countably infinite, even number I of outer islands are arranged in pairs around a central island. Each pair contains both of two types of islands, which will be called "creditor" and "debtor" islands (in anticipation of their equilibrium trading behavior). On each island, N two-period-lived agents are born in each period $t \geq 1$. In the first period each island also has N agents (the initial old) who live only in the first period. For simplicity N is normalized to 1.

Each agent born on a creditor island ("creditor") is endowed at birth with 1 unit of a non-storable good specific to his island (and with nothing when old). Each chooses to consume c_t^c units of his home (creditor) island good when young and d_{t+1}^c units from a debtor island when old. No other consumption is desired. The utility of a creditor is given by the function $u_c(c_t^c) + u_d(d_{t+1}^c)$. (When applied to choice variables, the superscripts c and d will denote an individual's type, and the subscripts on the utility functions will denote the type of good consumed.) The functions $u_c(.)$ and $u_d(.)$ are continuous, continuously differentiable, at least weakly concave, and strictly increasing.

Each agent born on a debtor island (each "debtor") is endowed at birth with 1 unit of a non-storable good specific to his island (and with nothing when old). Each wishes to consume the goods of both debtor and creditor islands when young and nothing when old. At the beginning of the period, young debtors travel to the creditor island with which they are paired, where they may consume creditor island goods. They return home later in the period.

With probability $1 - \theta$, all creditors and debtors in a period travel to the central island when old. With probability θ, η debtors in a period do not go to the central island but instead are equally distributed to all debtor islands where they are free to consume. This "default shock" is independently distributed over time and its realizations are not known in advance.

The expected utility of a debtor is represented by the function $v_c(c_t^d) + v_d(d_t^d) + \theta\eta v_e(e_{t+1}^d)$, where c_t^d, d_t^d, and e_{t+1}^d respectively represent his consumption of debtor and creditor island goods when young and of debtor island goods when old. The functions $v_c(.)$, $v_d(.)$, and $v_e(.)$ are continuous, continuously differentiable, strictly increasing and concave, and $v_c(.)$ and $v_d(.)$ satisfy the Inada conditions.

After the visits to the central island, old creditors and old debtors who have not visited the central island go to a randomly selected debtor island where they may trade with young debtors. The arrivals are evenly divided among debtor islands, each with an equal chance of going to any given debtor island. The actual destination is not known until arrival. They arrive at their final destination after all travel by the young has been completed.

Arrival at the central island takes place in two stages. In the first, all old creditors and a fraction λ of old debtors arrive (with $0 \leq \lambda \leq 1$ and $1-\lambda \geq \eta$). At the end of the first stage, a fraction 1 - α of the old creditors leave for their final destination while the rest stay until the end of the second stage. The remaining debtors arrive in the second stage: $1 - \lambda - \eta$ if a default shock has occurred and $1 - \lambda$ otherwise. Notice that because in all circumstances the same number of debtors (λ) arrive early, at the end of the first stage it is not possible for a creditor to infer whether a default shock has occurred. All creditors face the same chances of leaving early or late, and all debtors face the same chances of arriving early, late, or not at all. Each learns his arrival or departure time as soon as he turns old but not before. The sequence of travel can be summarized as follows:

1. Young debtors visit neighboring islands. All old creditors and λ old debtors visit the central island.
2. $1 - \alpha$ old creditors leave the central island.
3. $1 - \lambda$ or $1 - \lambda - \eta$ old debtors visit the central island.
4. All old go to debtor islands.
5. Young debtors return from the neighboring islands.

All agents are able to issue unfalsifiable IOUs that identify the issuer. A legal authority exists on the central island that can enforce agreements between parties currently on that island. No such authority exists to enforce agreements at agents' final destinations.

There exists on the central island a monetary authority able to issue fiat money, which is non-counterfeitable, unbacked, intrinsically useless, and costlessly exchanged. This authority issues an initial stock of M dollars to each initial old creditor.

2.2 Equilibrium

2.2.1 Trading patterns

To consume when old, agents must bring something of value to the debtor islands. Fiat money will be accepted by young debtors if it helps them to acquire the goods they desire. If it is accepted in equilibrium, creditors will require that debts be repaid with fiat money.

The young debtors wish to consume goods from creditor islands but own no goods valued by the young creditors that can be offered in immediate direct exchange. Nor do the debtors have any money at the time of this visit. The only thing a debtor can offer creditors is a promise to pay a sum of money in the next period on the central island. The young debtor will acquire this money by selling some of his endowment to those bringing money to the island later in the period. Money is essential in this model as the means by which final payment is made to retire debt – without repayment in valued money, creditors will not accept debt.

Debts are cleared at the central island but not always bilaterally. Because $\lambda < 1$, all debts cannot be repaid before some creditors must leave the central island. The creditors leaving early will therefore offer to sell their yet-unredeemed debt to those leaving later, who will be on the central island when the remaining borrowers arrive to redeem their debt in stage 2. The nominal amount of debt that can be redeemed in this resale market is limited by the size of the cash balances on the central island in the first stage. If this is insufficient to cover the shortfall, creditors will be forced to sell their unredeemed debt at a discount.

2.2.2 The debtors' problem

Let p_t represent the dollar price of a good on a debtor island at t. Since only debtor goods are sold for current period money, we can also call this the "price level." Let π_t represent the number of creditor island goods at t that can be acquired for a promise to pay \$1 on the central island at $t+1$. Let m_t represent a debtor's nominal acquisition of fiat money and let π_t represent the price in creditor island goods at t of a promise to pay \$1 on the central island at $t+1$. The budget constraints of a debtor born at t may now be written as follows:

$$p_t = p_t^d d_t^d + m_t \tag{1}$$

$$m_t = h_t \tag{2}$$

$$c_t^d = h_t \pi_t \tag{3}$$

$$p_{t+1} e_{t+1}^d = m_t \tag{4}$$

where m_t denotes the debtor's nominal demand for currency, and h_t denotes the nominal value at t of his indebtedness. Using these budget constraints we can simplify a debtor's problem to the choice of m_t to maximize

$$v_c\left(m_t\pi_t\right) + v_d\left(1 - \frac{m_t}{p_t}\right) + \theta\eta v_e\left(\frac{m_t}{p_{t+1}}\right) \tag{5}$$

The resulting first order condition for debtor utility maximization is

$$-v_c'\pi_t + \frac{v_d'}{p_t} + \theta\eta\frac{v_e'}{p_{t+1}} = 0 \tag{6}$$

Let l_t represent the nominal value at t of a creditor's loans to debtors. Let q_{t+1} represent the par value of nominal debt purchased by those leaving late at $t+1$. Let $\rho_{t+1} \leq 1$ represent the nominal price at which one dollar of that debt is exchanged in the first stage of visits on the central island at $t+1$. Consumption when old of those leaving late and its marginal utility will be marked with a star (*i.e.*, d_{t+1}^{c*} if a default shock does not occur and both a star and tilde $\tilde{d}_{t+1}^{c*}$ if it does occur). Note that if defaults occur, $\frac{1-\lambda-\eta}{1-\lambda}$ is the fraction of debts repaid.

2.2.3 The creditors' problem

The budget constraints of a creditor born at t when young and when old leaving early are respectively

$$1 = c_t^c + l_t\pi_t \tag{7}$$

$$\rho_{t+1}\left(1-\lambda\right)l_t + \lambda l_t = p_{t+1}d_{t+1}^c. \tag{8}$$

The budget constraints of a creditor when old leaving late without and with a default shock are respectively

$$l_t + \left(1 - \rho_{t+1}\right)q_{t+1} = p_{t+1}d_{t+1}^{c*} \tag{9}$$

$$\left(1-\eta\right)l_t + q_{t+1}\frac{1-\lambda-\eta}{1-\lambda} - \rho_{t+1}q_{t+1} = p_{t+1}\tilde{d}_{t+1}^{c*}. \tag{10}$$

Using the budget constraints to substitute for consumption we can express the creditor problem as the choice of l_t and $q_{t+1}$to maximize

$$u_c(1-\pi_t l_t) + (1-\alpha)\,u_d\left(\frac{\rho_{t+1}(1-\lambda)\,l_t + \lambda l_t}{p_{t+1}}\right)$$
$$+\alpha(1-\theta)\,u_d\left(\frac{l_t + \left(1-\rho_{t+1}\right)q_{t+1}}{p_{t+1}}\right)$$
$$+\alpha\theta u_d\left(\frac{(1-\eta)\,l_t + q_{t+1}\frac{1-\lambda-\eta}{1-\lambda} - \rho_{t+1}q_{t+1}}{p_{t+1}}\right). \tag{11}$$

The first order conditions with respect to l_t and q_{t+1} can now be written respectively as

$$-u_c'\pi_t + (1-\alpha)\,u_d'\frac{\rho_{t+1}(1-\lambda)+\lambda}{p_{t+1}}$$
$$+\alpha(1-\theta)\,u_d'^{*}\frac{1}{p_{t+1}} + \alpha\theta\bar{u}_d'^{*}\frac{(1-\eta)}{p_{t+1}} = 0 \tag{13}$$

$$(1-\theta)\left(1-\rho_{t+1}\right)u_d'^{*} + \theta\left(1-\frac{\eta}{1-\lambda}-\rho_{t+1}\right)\bar{u}_d'^{*} = 0. \tag{14}$$

2.2.4 Market clearing conditions

The clearing of the market of goods for money on debtor islands

$$M = p_t\left(1-d_t^d\right) \tag{15}$$

and the clearing of the market for loans

$$h_t = l_t \tag{16}$$

will determine the nominal price of debtor island goods, p_t, and the period t goods price of debt payable in period $t+1$, π_t.

The clearing of the resale market for loans requires that the debt purchased by late-leaving creditors equal the unredeemed debt owned by early-leaving creditors:

$$\alpha q_{t+1} = (1-\alpha)(1-\lambda)l_t. \tag{17}$$

The resale market for debt is constrained in another way: only fiat money is useful to creditors, but the amount of fiat money available on the central island when the early-leaving creditors depart is limited to those cash balances that have already arrived. Therefore, the nominal value of debt purchased by a late-leaving creditor, $\rho_{t+1}q_{t+1}$, is limited by the cash balances available to a creditor at the end of the first stage, λl_t:

$$\lambda l_t - \rho_{t+1}q_{t+1} \geq 0. \tag{18}$$

If this "liquidity constraint" is not binding, debt will sell at a price ρ^*that reflects only the debt's risk of default. Creditors who learn they will leave early will inelastically sell their all their unredeemed debt at any price so the price of that debt will be determined by the demand for that debt by late-leaving creditors (14), which yields

$$\rho^* = 1 - \frac{\theta \frac{\eta}{1-\lambda} \tilde{u}_d'^*}{(1-\theta)\, u_d'^* + \theta \tilde{u}_d'^*}. \tag{19}$$

If late-leaving creditors are have equal marginal utility in both states (for example, if they have risk-neutral preferences), ρ^* is simply the expected value of a late-arriving debt promising one dollar, or $1-\frac{\theta\eta}{1-\lambda}$. More generally, since $u_d'^* \leq \tilde{u}_d'^*$,

$$\rho^* \leq 1 - \frac{\theta\eta}{1-\lambda}, \tag{20}$$

which holds with strict inequality if creditors are risk-averse.

If the liquidity constraint is binding, the clearing of the resale market requires that

$$\rho_{t+1} = \frac{\alpha\lambda}{(1-\alpha)(1-\lambda)}. \tag{21}$$

A binding liquidity constraint implies that the price of unredeemed debt, ρ_{t+1}, is determined by the liquidity constraint (21) rather than the creditors' first order condition (14), and requires that $\frac{\alpha\lambda}{(1-\alpha)(1-\lambda)} < \rho^*$. In the range the price of unredeemed debt is increasing in α and λ , that is, in the extent to which debtors and creditors overlap on the central island. A binding liquidity constraint implies that the short-run interest rate $\left(1/\rho_{t+1}\right)$ exceeds the interest rate that is necessary to induce creditors to purchase unredeemed debt $(1/\rho^*)$.

Lemma 1 *If $\eta < 1-\lambda$, there are values of parameters such that the liquidity constraint binds.*

[A proof of this lemma follows from the observation that the unrestricted demand for unredeemed debt (from equation 14) does not depend on the value of α, implying that $\rho^*$is independent of α . If $\eta < 1-\lambda$, ρ^*will be bounded above 0. It is therefore possible to find a value of α sufficiently small that $0 < \rho_{t+1} < \rho^*$.]

3 Open Market Operations

3.1 Motives

The liquidity constraint is essentially a restriction of the quantity of funds that late-leaving creditors may supply to creditors who must leave early. It has the usual welfare effects of such a supply restriction — early-leaving creditors experience lower consumption and utility, and late-leaving creditors greater consumption and utility, than would have resulted in an equilibrium not so constrained. This is obvious from the creditor budget constraints (8), 9), and (10). Because late-leaving creditors can always choose to behave like early-leaving creditors, their expected utility in the absence of a binding liquidity constraint must in equilibrium be no less than that of an early-leaving creditor. From that starting point a binding liquidity constraint must necessarily create or increase the difference in the *ex post* utilities of late and early leavers.

Given its power to alter the nominal stock of fiat money, the monetary authority located on the central island may ask whether it may usefully provide additional fiat money balances that can make the liquidity constraint less binding. In other words, can it usefully intervene in the resale market for debt by acting as a liquidity-providing "central bank?"

A second difference in the experience of early- and late-leaving creditors is that wealth losses from the default shock fall entirely on late-leaving creditors because the realization of the shock is not known when early-leavers must depart from the central island. If creditors are risk-averse, one must therefore examine alternative central bank policies not only for their provision of liquidity but also for their effect on the distribution of risk.

3.2 Implementing open market operations

Consider first an "open market" purchase in which the monetary authority or central bank prints additional units of fiat money using them in the first stage of central island visits to purchase unredeemed debt (conduct an "open market" purchase). Let z_t denote the discounted nominal value of unredeemed debt purchased on the central island by the central bank at time t. Then the clearing of the market at $t+1$ for unredeemed debt becomes

$$\frac{z_{t+1}}{\rho_{t+1}} + \alpha q_{t+1} = (1-\alpha)(1-\lambda)l_t \tag{22}$$

with the implied liquidity constraint

$$\alpha\lambda l_t + z_{t+1} \geq \rho_{t+1}(1-\alpha)(1-\lambda)l_t \tag{23}$$

For any given nominal stock of unredeemed debt, the central bank's purchases of unredeemed debt can raise its price, ρ_{t+1}, until the liquidity constraint is no longer binding.

Open market operations, although temporary, have effects on the money supply and price level whether or not a default shock occurs. An open market purchase of debt implies that in the second stage the government receives and retires z_{t+1}/ρ_{t+1} dollars if a default shock does not occur and $\left(1-\frac{\eta}{1-\lambda}\right) z_{t+1}/\rho_{t+1}$ if a default shock does occur. Since $\rho_{t+1} < 1$, the central bank earns profits [of $z_{t+1}/\rho_{t+1} - z_{t+1}$] if defaults do not occur but may suffer losses if they do. The effect of such open market operations on the price level will depend on the disposition of these profits and losses. If the central bank simply absorbs profits and losses by allowing the stock of money in public hands to fluctuate, the stock of money and thus the price level will rise when defaults cause losses and fall when defaults do not occur. To see this notice that the demand for money comes from young debtors and is unaffected by the current default shock; see (15). It follows that any changes in the price level will come from changes in the total end-of-period stock of money arriving at debtor islands. If we let M_t represent the stock of fiat money at the beginning of period t, the end-of-period money stock resulting from open market operations is:

$$M_{t+1} = M_t + z_{t+1}\left(1 - \frac{1}{\rho_{t+1}}\right) \tag{24}$$

if a default does not occur and

$$M_{t+1} = M_t + z_{t+1}\left(1 - \frac{1}{\rho_{t+1}}\left(1 - \frac{\eta}{1-\lambda}\right)\right) \tag{25}$$

if it does.

Under this policy central bank revenue losses caused by defaults cause increases in the fiat money stock and price level that reduce the real value of all money balances, those of "dishonest" debtors and both early- and late-leaving creditors. Similarly, any gains in central bank revenue when defaults do not occur decrease the fiat money stock and thus also the price level, thereby increasing the value of the money balances held by early- and late-leaving creditors alike.

In the absence of price level changes, a default shock reduces the wealth of late-leaving creditors, but early-leaving creditors are unaffected because they sell their unredeemed debt before the realization of the default shock. Therefore, a policy redistributing central bank revenue through price level changes transfers risk from those who otherwise face all the risk (late-leavers) to those who otherwise face no risk (early-leavers). For given borrower utility, better risk-sharing among creditors will result in greater expected utility for risk-averse creditors (recall that creditors do not know in advance whether they will leave early or late). It will also reduce the risk premium that lowers the price of unredeemed debt (see19).

Consider in particular a central bank commitment to purchase the entire stock of late-arriving debt. This is more than sufficient to render the liquidity constraint non-binding. Under such an open market policy, late leaving creditors purchase no unredeemed debt ($q_t = 0$), giving late-leavers the same assets and consumption as early-leavers. Under this policy default losses are shared equally by early and late leavers through the inflation that results when defaults occur. The resulting price of unredeemed debt will rise to

$$\rho^* = 1 - \frac{\theta \frac{\eta}{1-\lambda} \tilde{u}_d'^*}{(1-\theta)\, u_d'^* + \theta \tilde{u}_d'^*} = 1 - \frac{\theta\eta}{1-\lambda}, \tag{26}$$

the expected value of late-arriving debt. (Note that $u_d'^* = \tilde{u}_d'^*$ because late-leavers enjoy the same consumption as early-leavers.) From (25) and (24) we see that the stock of fiat money will rise by $\frac{\eta}{1-\lambda} z_{t+1} = \frac{\eta}{1-\lambda}(1-\lambda) l_t$ dollars if a default occurs and remain unchanged if a default does not occur.[3]

[3]If it has powers of taxation on the central island, the central bank may overcome the liquidity constraint while avoiding price level changes by redistributing its profits or losses

3.3 Moral Hazard

The desirability of the risk-spreading properties of open market purchases of debt may be undermined if there is asymmetric information that induces moral hazard in creditors' lending decisions. Suppose, for instance, that creditors alone know in advance which debtors will default if a default shock occurs. In the absence of central bank actions, late arriving debt will sell for two different, risk-related prices. (For the case of risk-neutral creditors these prices would be par for the safe debt and $1 - \frac{\theta\eta}{1-\lambda}$ for risky debt.) If the monetary authority has all the information available to creditors, it can purchase each type of debt at a distinct, risk-related price.

If instead the monetary authority cannot distinguish between the two types of debt, it can only offer a single price for debt. One central bank option is to offer to purchase only ηl units of debt, the quantity of risky debt. Because creditors will sell risky debt before safe debt, only risky debt is then purchased and the central bank will be able to obtain the (low) price that accurately reflects the risky nature of this debt, $1 - \frac{\theta\eta}{1-\lambda}$. In this case the maximum possible temporary expansion of the money stock is $\left(1 - \frac{\theta\eta}{1-\lambda}\right)\eta l$. For small enough values of α or a value of $\frac{\theta\eta}{1-\lambda}$ close enough to one, however, this will not be enough to render the liquidity constraint nonbinding. To provide additional liquidity by purchasing safe as well as risky debt requires that the central bank pay a higher price on all debt, safe or risky. If the monetary authority purchases all debt, creditors will lend to likely defaulters just as readily as to safe borrowers. Clearly, this subsidization of bad risk will increase creditor losses in times of default.

4 A discount window

An alternative means of overcoming the liquidity constraint is the operation of a discount window. At a discount window the monetary authority stands ready to lend at some announced interest rate ("discount rate") to late-leaving creditors who can demonstrate that they are using the loan to purchase "real bills," in this model the debt of late-arriving borrowers. It is important for the operation of the discount window that these real bills be presented as security for the central bank loans. A central bank offering

through subsidies or taxes among the late-leaving creditors in a way that leaves the total money stock unchanged. Under this redistribution scheme, the risk introduced by the default shock is faced entirely by late-leaving creditors.

unlimited unsecured loans would be inundated by early-leaving creditors with no intention of repaying the loans.

The effects of a discount window offering uncontingent loans will depend in important ways on whether late-leaving creditors are able to repay their loans from the central bank even when a default has occurred. Let us first examine the case in which late-leaving creditors are always able to enjoy positive consumption after repaying central bank loans in full.

4.1 No defaults on central bank loans

Arbitrage will induce late-leaving creditors to borrow from the discount window until the market return to unredeemed debt, adjusted for risk, equals the discount rate. Let r_{t+1} represent the gross discount rate. Then arbitrage requires that the market price of unredeemed debt rise until

$$r_{t+1} = \frac{1}{\rho_{t+1}}\left(1 - \frac{\theta \frac{\eta}{1-\lambda} \tilde{u}_d'^*}{(1-\theta)\, u_d'^* + \theta \tilde{u}_d'^*}\right) \tag{27}$$

In the case of risk-neutral creditors, for example, arbitrage requires

$$r_{t+1} = \frac{1}{\rho_{t+1}}\left(1 - \theta \frac{\eta}{1-\lambda}\right) \tag{28}$$

By setting the (gross) discount rate r_{t+1} to 1, the central bank can bring the price of unredeemed debt to its unconstrained level

$$\rho_{t+1} = \left(1 - \frac{\theta \frac{\eta}{1-\lambda} \tilde{u}_d'^*}{(1-\theta)\, u_d'^* + \theta \tilde{u}_d'^*}\right) = \rho^*. \tag{29}$$

The nominal amount lent by the central bank will be Z_{t+1}, the difference between the discounted nominal stock of unredeemed loans and stock of money in the hands of late-leaving creditors in the first stage:

$$Z_{t+1} = \rho_{t+1}(1-\alpha)(1-\lambda)l_t - \alpha\lambda l_t. \tag{30}$$

A discount window that offers the short-term rate $r_{t+1} = 1$, will in this way supply to late-leaving creditors enough additional fiat money balances, Z^*, to make the liquidity constraint no longer bind:

$$Z^* = \rho^*(1-\alpha)(1-\lambda)l_t - \alpha\lambda l_t. \tag{31}$$

The market clearing condition for unredeemed debt when the central bank makes loans in an otherwise liquidity constrained market is

$$\rho_{t+1} q_{t+1} = \alpha\lambda l_t + Z_{t+1} \tag{32}$$

To this point we have maintained the assumption that creditors are able to repay their discount window loans with something left over for their own consumption even when defaults occur. This will be true if the amount they owe is strictly less than the total after-default return from borrowers:

$$r_{t+1} Z_{t+1} < \left(1 - \frac{\eta}{1-\lambda}\right) q_{t+1} \tag{33}$$

Using successively (32) and (30), the inequality (33) may be simplified as follows:

$$r_{t+1} Z_{t+1} < \left(1 - \frac{\eta}{1-\lambda}\right) \frac{\alpha\lambda l_t + Z_{t+1}}{\rho_{t+1}} \tag{34}$$

$$r_{t+1}\left[\rho_{t+1}(1-\alpha)(1-\lambda)l_t - \alpha\lambda l_t\right] < \left(1 - \frac{\eta}{1-\lambda}\right)(1-\alpha)(1-\lambda)l_t \tag{35}$$

For the benchmark case of full liquidity ($r_{t+1} = 1$) and risk-neutral preferences ($\rho_{t+1} = 1 - \frac{\theta\eta}{1-\lambda}$), the condition may be further simplified to

$$(1-\alpha)(1-\theta)\eta < \alpha\lambda. \tag{36}$$

Central bank profits in this case are

$$r_{t+1} Z_{t+1} - Z_{t+1} \tag{37}$$

If $r_{t+1} = 1$ (and central bank loans are always repaid in full), central bank profits are always zero because the end-of-period fiat money stock is not affected by the operation of the discount window. As a result, the operation of the discount window does not affect the price level, implying that all default risk falls on late-leaving creditors. In this case uncontingent discount window loans offer less risk-sharing than the open market operations discussed above. Under some circumstances, this may not be a problem. For example, one can easily imagine a model economy with late-leaving creditors as banking institutions with risk-neutral preferences and early-leaving creditors as risk-averse depositors. In this case efficient policy would leave the entire risk on their shoulders.

The informational requirements of using the discount window to remove the liquidity constraint are low. The central bank does not need to know the value of any parameter or endogenous variable in order to render the liquidity constraint nonbinding -- the desired policy is always to lend at $r_t = 1$.

Discount window loans, when always repaid introduce less moral hazard than do open market operations because the price of debt will be determined by those with the best information. This ensures that in equilibrium risky debt will always sell for less than safe debt, avoiding any subsidization of loans to known bad risks.

4.2 Defaults on central bank loans

Suppose instead that defaults are so large that late-leaving creditors do not have the resources to repay central bank loans in full when defaults occur. (A necessary condition for the existence of such an equilibrium is that the marginal utility of consumption is finite even when consumption is zero.) Assuming that consumption when old cannot be negative, a marginal increase in a creditor's purchases of debt cannot reduce his consumption in the case of default. In this case removing the liquidity constraint by setting $r_{t+1} = 1$ results in an unlimited demand for unredeemed debt if $\rho_{t+1} < 1$. This drives the price of late-arriving debt up to par ($\rho_{t+1} = 1$).

In this case the central bank suffers losses of $-\frac{\eta}{1-\lambda}Z_{t+1}$ if borrowers default on their debts. This leaves the central bank unable to retire the full amount that it lent at the discount window, resulting in an expansion of the end-of-period fiat money stock of $\frac{\eta}{1-\lambda}Z_{t+1}$ dollars. This increase in the price level lowers the real value of the money balances held by borrowers and early-leaving creditors. In this way a discount window policy that provides liquidity has the secondary effect of redistributing wealth in the event of a default. This risk-spreading redistribution is still less than that of open market operations, given that late-leaving creditors consume nothing.

From (33) we see that the consumption of late-leaving creditors is zero if the amount they owe is strictly greater than the total after-default return from borrowers:

$$r_{t+1}Z_{t+1} > \left(1 - \frac{\eta}{1-\lambda}\right)q_{t+1} \tag{38}$$

Repeating substitutions used above, this can be written as

$$r_{t+1}\left[\rho_{t+1}(1-\alpha)(1-\lambda)l_t - \alpha\lambda l_t\right] > \left(1-\frac{\eta}{1-\lambda}\right)(1-\alpha)(1-\lambda)l_t, \quad (39)$$

the reverse of the inequality (35). When $r_{t+1} = 1$ and $\rho_{t+1} = 1$, this reduces to

$$(1-\alpha)\eta > \alpha\lambda. \quad (40)$$

Let us raise the spectre of moral hazard in this case by temporarily reintroducing the assumption that creditors (but not the central bank) know which debtors will default if there is a default. If a late-leaving creditor knows that his losses will exceed his assets in the event of a default (his consumption will be zero), he is only interested in an asset's return in the event there are no defaults. Both safe and risky borrower debt pay the same return if no default occurs and thus a late-leaving creditor will pay the same amount, par value, for safe and risky debt. Knowing that all late-arriving debt will sell at par in the next period, young creditors will lend as readily to borrowers they know are likely to default as to borrowers they know will never default. This has the obvious consequence of increasing loans to risky borrowers and thus creditor losses during defaults.

4.3 Multiple equilibria at the discount window

Whether or not creditors default on their central bank loans, the central bank's discount window policy for the provision of liquidity is to offer short-term loans at the nominal rate $r_t = 1$. The resulting price of late-arriving debt is then $\rho^* < 1$ if in equilibrium $d_t^{c*} > 0$ and 1 if in equilibrium $d_t^{c*} = 0$. This raises the question of whether both cases may be equilibria for the same parameter values.

Proposition 2 *For some range of parameter values, there are exists both an equilibrium with $\rho_t < 1$ and $d_t^{c*} > 0$ and an equilibrium with $\rho_t = 1$ and $d_t^{c*} = 0$.*

To prove this proposition, let us turn to the case of risk-neutral preferences.

We know that in this case $\rho^* = 1 - \frac{\theta\eta}{1-\lambda}$ if in equilibrium $d_t^{c*} > 0$. Recall from (36) that in this case $d_t^{c*} > 0$ if

$$(1-\alpha)(1-\theta)\eta < \alpha\lambda. \tag{41}$$

Recall also that if $r_{t+1} = 1$ and $\rho_{t+1} = 1$, then $d_t^{c*} = 0$ if

$$(1-\alpha)\eta > \alpha\lambda. \tag{42}$$

Both inequalities are satisfied for the same parameters if

$$1 > \frac{\alpha\lambda}{(1-\alpha)\eta} > 1-\theta \tag{43}$$

implying that for any parameters in this range there exist both an equilibrium with $\rho_{t+1} = 1$ and one with $\rho_{t+1} = 1 - \frac{\theta\eta}{1-\lambda}$. The proposition follows directly.

To establish that this proposition is not vacuous, one must show that (43) are satisfied for liquidity constrained economics. We know from (30) that an economy is liquidity constrained if

$$\rho^*(1-\alpha)(1-\lambda) - \alpha\lambda = (1 - \frac{\theta\eta}{1-\lambda})(1-\alpha)(1-\lambda) - \alpha\lambda > 0. \tag{44}$$

or

$$1 > \frac{\alpha\lambda}{(1-\alpha)(1-\lambda)} + \frac{\theta\eta}{1-\lambda} \tag{45}$$

To see that the inequalities (41), (42), and (45) can be satisfied simultaneously, consider the extreme case of $\theta = 1$. In this case, (41) is always satisfied and α can be chosen sufficiently small that (42), and (45) are also satisfied for $\eta < 1 - \lambda$. [4]

To select the equilibrium without defaults to the central bank requires that the central bank limit the quantity of central bank loans to a number less than the total after-default return from borrowers, $\left(1 - \frac{\eta}{1-\lambda}\right) q_{t+1}$. This requires the central bank to know the default rate, $\frac{\eta}{1-\lambda}$, diminishing one apparent advantage of the discount window, the extent of the information required for its implementation.

[4] A specific numerical example satisfying all three inequalities is $\alpha = \lambda = .2$, $\eta = .1$, and $\theta = .9$.

5 Private provision of liquidity

In an important way, the model environment considered thus far has been rigged in favor of action by the monetary authority. In assuming a monetary authority able to act in every period, the model implicitly assumes the existence of an agent who stays on the central island from one period to the next [see the discussion in Green (1996)]. Let us therefore examine the financial institutions that may emerge when there exist selfish private agents who stay on the central island from one period to the next.[5]

Assume that in every period a continuum of agents is born on the central island. Let's call them "bankers." Bankers live their entire two-period lives on the central island. They have no endowments of goods. At the end of life they may travel to randomly determined debtor islands to consume any wealth they have accumulated. Assume that their preferences are monotonically increasing in their lifetime consumption.

Living on the central island bankers are able to make written commitments that can be enforced by the central island's legal authority. The age of a banker is costlessly known to other agents and the legal authority on the central island.

The presence of bankers on the central island offers another solution to the liquidity problem. Early-leaving creditors may now sell their debt for promissory notes issued by young bankers. To be useful, these notes must be payable to the bearer, as the early-leaving creditors will not themselves return to the central island. The early-leaving creditors want goods from young debtors, who want an asset they can use in the next period on the central to retire their personal debt. A bearer note promising redemption next period in fiat money will be acceptable to young debtors. Bankers who issue such notes in return for late-arriving debt in period t will receive fiat money when the late-arriving debt is redeemed; this fiat money can be used in period $t+1$ to retire the bearer notes. Let B_t represent the total number of dollars that bankers will owe in period $t+1$. Bankers are required to keep reserves equal to the nominal value of their outstanding bank notes..

The notes would have to bear some noncounterfeitable witness from the legal authority on the central island that the notes are issued by a young

[5]The case of privately provided liquidity is also taken up (in environments without defaults) by Freeman (1996b) and Green (1996).

banker, old bankers being unable to keep promises involving actions in the next period. The technology that permits the certification of these notes seems little different than that required for the issuance of noncounterfeitable fiat money.

In the absence of default risk, ($\theta = 0$) these bearer notes promising payment in fiat money are perfect substitutes for fiat money itself and will trade at par. They can be supplied elastically to clear all debt on the central island, rendering the liquidity constraint nonbinding. In this way they exactly duplicate the liquidity effect of a central bank's temporary expansion of fiat money through open market operations or a discount window. The only difference is that the elastic part of the money supply is inside money in the form of bank notes instead of outside (fiat) money. The total money supply arriving on debtor islands in period t will equal the stock of fiat money that arrives early to the central island, call it M_t^*, plus the stock of privately issued bank notes, B_t. The total nominal reserves of banks, denoted by N_t, equal the late-arriving fiat money stock:

$$N_t = M - M_t^*. \tag{46}$$

These reserves equal the stock of bank notes, implying that the total stock of money offered for goods on debtor islands is

$$M_t^* + B_t = M_t^* + (M - M_t^*) = M \tag{47}$$

Regardless of the stock of bank notes, the total stock of money spent on goods is the same, implying that the issuance of bank notes has no effect on the price level, determined by the clearing of the market of money for goods:

$$M_t^* + B_t = M = p_t\left(1 - d_t^d\right) \tag{48}$$

If late-arriving borrower debt carries some default risk ($\theta > 0$), the banker debt that it backs is also a risky asset. Assume first that a default at t is known at t even on the debtor islands. (One might imagine that the large quantity of people and money arriving on the debtor islands will reveal a default.) In the absence of a default, a bank note will trade at par on the debtor islands. In the event of a default the nominal price of a bank note on the debtor islands will fall to $1 - \frac{\eta}{1-\lambda} \equiv \phi^*$, the dollar value of the reserves behind each bank note..

In the event of a default the total money stock will equal the sum of money arriving in the hands of creditors and "dishonest" debtors. Because the amount of reserves lost by creditors will exactly equal the money brought to the island by the dishonest debtors, the nominal value of the total money stock remains at M. The extra dollars brought by the debtors is offset by the lower nominal value of the bank notes brought by creditors. Because there is no change in the dollar value of the money stock, the price level in dollars of fiat money remains unchanged. However, bank notes no longer trade at par, and the price of a good if measured in units of the current issue of bank notes will have risen.

Because bank notes are risky, they will not be accepted at par with fiat money on the central island. Early- and late-leaving creditors, having the same risk preferences, will choose to hold identical portfolios of fiat money and risky bank notes.. The creditors' default losses are in this way spread evenly among all creditors through a drop in the value of bank notes issued in the current period. Notice, however, that "dishonest" debtors do not hold any currently issued bank notes and so are not affected by changes in the value of these bank notes.. (The money balances of the debtors consists of fiat money and bank notes acquired in the previous period.[6]) In this way, although this equilibrium shares risks evenly among creditors, its risk-sharing differs from that of open market operations financed by changes in the fiat money stock. When open market operations increase the fiat money stock in response to a default, the resulting price increase lowers the value of the money held by the "dishonest" debtors, in effect rebating some of the default. (Such price increases will also reduce the value of money balances held by anyone else, "honest"or not, including those in no way connected with credit markets, although for such people have not been explicitly included in the model presented.)

6 Conclusion

It was shown in previous work [Freeman (1996a)] that either open market operations or a discount window can efficiently provide the liquidity needed

[6]Banknotes held by dishonest debtors will not be redeemed in the period after their issue, leaving old bankers with positive assets and liabilities. These however, can be sold to the next generation of bankers at their equilibrium price of zero, the bank's net worth. The banknotes can then be retired in a later period.

for the repayment of debt in the absence of aggregate risk. The model of this paper suggests that the presence of aggregate risk does not alter the desirability of providing liquidity even if the central bank sometimes suffers losses. Indeed, it shows that central bank losses that result in increases in the fiat money stock and the price level distribute default risk more broadly, increasing the willingness of creditors to lend. Indeed, open market operations that provide full liquidity also provide full risk-sharing between early- and late-leaving creditors. The discount window, which leaves late-leaving creditors responsible for all losses due to defaults does less to share the risk of default. A private provision of liquid bearer notes, whose value is tied to the realization of the default shock can also supply full risk-sharing.

If the monetary authority has less information about the default risk than creditors have, moral hazard becomes an issue. In this case, the discount window, which does less to distribute risk, seems to offer an advantage: if those who borrow from the central bank are not forced to default on their central bank loans, the central bank may provide all needed liquidity without introducing moral hazard. The discount window is then the preferred policy if late leaving creditors are risk neutral and thus indifferent to the distribution of risk. The discount window, however, is an imperfect protection against moral hazard: there exist equilibria in which those who borrow from the central bank default, introducing moral hazard. Indeed for some parameter values, there exist both an equilibrium with defaults to the central bank and an equilibrium without such defaults. To select the equilibrium without defaults on central bank loans requires that the discount window be subject to limits on the quantity of debt rediscounted.

The choice of the means by which a central bank provides liquidity therefore depends on whether the risk faced by those who purchase in the resale market for debt or moral hazard is the greater problem. This choice can be left to the private sector if private clearinghouses are permitted to issue bank notes whose value is contingent on the value of the reserves that back the notes.

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