

Federal Reserve Bank of Minneapolis
Research Department

Efficient Borrowing and Savings Constraints*

Harold L. Cole
Narayana Kocherlakota

May 5, 1998

*The views expressed herein are the those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. See the Minneapolis Fed's home page on the World Wide Web at <http://woodrow.mpls.frb.fed.us> for further revisions, as well as other papers.

1. Introduction

All existing dynamic models with informational frictions imply that it is efficient for poor individuals to be *savings-constrained*: given prevailing rates of return, they would like to save more. (See, for example, Townsend (1982), Green and Oh (1991), and Rogerson (1985)). This implication is counter to the usual intuition about the kinds of constraints that individuals actually face in asset markets.

In this paper, we demonstrate that there is a simple and plausible specification of informational frictions that implies that borrowing constraints, not savings constraints, are efficient. Virtually all previous work on dynamic environments with private information assumes that income shocks are independently distributed over time¹. We relax this assumption in the context of a dynamic environment with hidden income; we place special emphasis on the case in which all income shocks are permanent. The standard intuition is that with permanent shocks, no insurance is possible because differential realizations of income shocks do not cause individuals to have different preferences over current and future consumption. We prove that this intuition is correct when individuals have exponential utility. However, the intuition is wrong when individuals have CRRA utility and at least some income is publicly observable. We prove that under these assumptions, if income shocks are permanent, it is possible to insure individuals against income shocks, and the optimal insurance scheme features borrowing constraints upon the poor.

Our result is based on the following key result about individual behavior. When agents have CRRA utility, and income shocks are permanent, we show that currently poor individuals

¹See Green (1987), Thomas and Worrall (1990), Phelan and Townsend (1991), Atkeson and Lucas (1992), Phelan (1995), Wang (1995), and Aiyagari and Alvarez (1995) for examples of the literature in which income shocks are assumed to be iid over time. See Fernandes and Phelan for an exception.

have a greater desire to save out of any extra wealth than do currently rich individuals. This is the exact opposite of what happens when shocks are purely transitory; then, the desire to smooth consumption over time implies that the currently poor have a lower savings demand than the currently rich. In both instances, the efficient transfer scheme is designed so as to exploit differences in savings demand across rich and poor; this consideration leads to savings constraints in the purely transitory case and borrowing constraints in the permanent shock case.

We view our work as complementary to the existing literature on enforcement constraints (Kehoe and Levine (1993); Alvarez and Jermann (1997)). Undoubtedly, individuals are unable to commit to repay some kinds of loans, and this plays a substantial role in their facing borrowing constraints. What we show is that informational frictions may also play a role in generating such constraints.

2. Environment

We consider the following environment. There are two periods and a single perishable consumption good. There is a continuum of agents, who have identical preferences representable by the utility function:

$$\sum_{t=1}^2 \beta^{t-1} u(c_t), \text{ where } u \text{ is } C^2, \text{ strictly increasing, and strictly concave over } \mathbb{R}.$$

Agents are able to borrow and lend freely at a given rate R . Their borrowing and lending is observable.

Agents have both observable income and unobservable income. In period 1, all agents receive y_1 units of publicly observable income and in period 2, they all receive y_2 units of publicly observable income; we define $W = y_1 + R^{-1}y_2$. The assumption that all individuals

receive the same public income is unimportant for the results that follow; the key is that their public income is stochastically independent of their private income.

In terms of unobservable income, in period 1, an agent receives income θ_1 , where θ_1 equals θ_1^h with probability π^h or θ_1^l with probability $\pi^l = 1 - \pi^h$ ($\theta_1^h > \theta_1^l$). In period 2, he receives income $\phi(\theta_1, \theta_2)$, where (θ_1, θ_2) are independent and identically distributed and ϕ is increasing in both its arguments. Hence, period 2 and period 1 private incomes are, in general, positively correlated.

3. Efficiency: General Analysis

We seek to characterize the set of efficient transfer schemes. We denote a transfer scheme by $\tau = \{\tau^i\}_{i=l,h}$, where $\tau^i = (\tau_1^i, \tau_2^i)$ denotes the transfers received by an agent who says that his first period private income shock was θ_1^i . (Throughout, we use the term “private income” to refer to the unobservable income of the agents.) For this transfer scheme to be incentive-compatible, agents who receive high private income in period one must weakly prefer τ^h to τ^l and agents who receive low private income in period one must have the reverse ordering. Transfers are only contingent on first period income; because all agents prefer more income to less, there is no point in trying to separate agents based on their period two income realizations.

In defining the set of incentive-compatible transfer schemes, it is useful to define reduced-form preferences over transfer schemes conditional on first period income as

$$U(\theta_1, \tau^i) = u(\theta_1 + \tau_1^i) + \beta E_{\theta_2} \left\{ u(\phi(\theta_1, \theta_2) + \tau_2^i) \right\}.$$

With this definition, we can represent the set of efficient transfer schemes as being the solu-

tions to the social planner's problem:

$$\max_{\{\tau\}} \sum_{i=\{l,h\}} \pi_i U(\theta_1^i, \tau^i)$$

subject to

$$\sum_{i=\{l,h\}} \sum_{t=1}^2 \pi_i \tau_t^i / R^{t-1} \leq W$$

$$U(\theta_1^l, \tau^l) \geq U(\theta_1^l, \tau^h)$$

$$U(\theta_1^h, \tau^h) \geq U(\theta_1^h, \tau^l)$$

$$\tau_1^i + \theta_1^i \geq 0, \tau_2^i + \phi(\theta_1^i, \theta_2^i) \geq 0 \text{ for all } i, j$$

The first constraint is the resource constraint of the planner (as in Green (1987), he is able to borrow and lend at rate R). The second and third constraints are the self-selection constraints of the poor and rich agents, respectively.

As in any social planner's problem with self-selection constraints, what allows the planner to treat the rich and poor agents differently is that the two types of agents have different preferences over transfer schemes. We can represent these preferences using the following formula for an individual's marginal rate of substitution between first period and second period transfers, given that he takes a given contract τ :

$$MRS(\theta_1^i, \tau) = \beta E_{\theta^2} \{u'(\phi(\theta_1^i, \theta_2) + \tau_2)\} / u'(\theta_1^i + \tau_1)$$

The following Proposition uses this formula to provide a set of necessary conditions for efficiency in this general environment. The first condition guarantees that the planner takes from the rich and gives to the poor. The second condition guarantees that the rich agent and the planner have the same MRS. The final condition says that the MRS of the low income

individual is a weighted average of the planner's MRS and the MRS of the high income individual (where the latter is evaluated at the low income individual's transfer scheme).

PROPOSITION 1. *Any efficient τ satisfies three conditions.*

- i) $\sum_{t=1}^2 \tau_t^l / R^{t-1} \geq \sum_{t=1}^2 \tau_t^h / R^{t-1}$; the inequality is strict if $MRS(\theta_1^l, \tau^l) \neq R^{-1}$
- ii) $MRS(\theta_1^h, \tau^h) = R^{-1}$
- iii) $MRS(\theta_1^l, \tau^l) = \alpha R^{-1} + (1 - \alpha)MRS(\theta_1^h, \tau^h)$ for some α in $(0, 1]$

Proof. See the appendix.

The intuition behind these conditions is simple. Standard risk-sharing arguments imply that as long as the planner can discriminate between the rich and poor, he wants to give more resources to the poor agents. This leads immediately to condition (i). Similarly, because the planner is always trying to take from the rich individual and give to the poor, there is no need to worry about the poor agent trying to pretend to be rich. It follows that given the net present value of the resources being given to the rich agent, the planner wants to maximize the rich agent's utility, and condition (ii) is simply the first order condition of this maximization problem.

The third condition is more subtle. Suppose it is not satisfied; for example, suppose the shadow interest rate of the poor agent is higher than the shadow interest rate of the rich agent, which is in turn higher than R . (Here, the shadow interest rate of an agent is evaluated at the poor agent's transfer profile and is equal to $MRS(\theta_1^i, \tau^i)^{-1}$). Then, consider the following change to the poor individual's transfer scheme. The planner makes a small incremental loan to the poor agent, where the loan has an interest rate higher than R , but

lower than the poor agent's shadow rate. This change makes the poor agent better off, because he is borrowing at a rate lower than his shadow rate. The planner can afford this loan because he is able to borrow at R . The rich agent gets less utility from the poor agent's new transfer profile (because the loan rate is higher than the rich agent's shadow rate), and so is less inclined to switch to the poor agent's contract. It follows that if the third condition is not satisfied, we can construct another feasible and incentive-compatible transfer profile that improves the planner's objective; thus, any optimal transfer scheme must satisfy the third condition.

4. Efficiency: Special Cases

We now focus our attention on three special cases of the above general analysis. In the first, private income shocks are purely transitory, so $\phi(\theta_1, \theta_2) = \theta_2$. In the second, utility is exponential, and private income follows an AR(1), with a positive coefficient on lagged income. In the last case, utility is CRRA, and private income is a logarithmic random walk.

A. Purely Transitory Shocks

Suppose $\phi(\theta_1, \theta_2) = \theta_2$. Then, we know that for all τ :

$$\begin{aligned} MRS(\theta_1^h, \tau) &= \beta E_{\theta^2} \{u'(\theta_2 + \tau_2)\} / u'(\theta_1^h + \tau_1) \\ &> \beta E_{\theta^2} \{u'(\theta_2 + \tau_2)\} / u'(\theta_1^l + \tau_1) \\ &= MRS(\theta_1^l, \tau) \end{aligned}$$

because individuals with temporarily high period 1 private income have a lower marginal utility of current consumption. This result, together with condition (iii) of Proposition 1,

implies that for an efficient transfer scheme τ :

$$MRS(\theta_1^l, \tau^l) > R^{-1}$$

We can then use condition (i) of Proposition 1 to conclude that:

$$\sum_{t=1}^2 (\tau_t^l - \tau_t^h) / R^{t-1} > 0$$

Thus, if all shocks are transitory, it is possible to provide insurance to the temporarily poor agents (so that the net present value of their transfers is higher than the net present value of the high-income agents). Also, the poor agents are savings-constrained: on the margin, they would like to be able to transfer resources from period 1 to period 2 at the interest rate R . These results are similar to those derived in Green and Oh (1991).

B. Exponential Utility

Suppose $u(c) = -\exp(-c)$ and $\phi(\theta_1, \theta_2) = a + \rho_1 \theta_1 + \rho_2 \theta_2$. Then, it is easy to see that:

$$\begin{aligned} MRS(\theta_1^i, \tau) &= \beta E_{\theta^2} \exp(-a - \rho_1 \theta_1^i - \rho_2 \theta_2 - \tau_2) / \exp(-a - \theta_1^i - \tau_1) \\ &= \beta \exp((1 - \rho_1) \theta_1^i - \tau_2 + \tau_1) E_{\theta^2} \exp(-\rho_2 \theta_2) \end{aligned}$$

It follows that if $0 \leq \rho_1 < 1$, then for any τ , $MRS(\theta_1^h, \tau) > MRS(\theta_1^l, \tau)$. As in the purely transitory shock case, we can use Proposition 1 to conclude:

$$\begin{aligned} MRS(\theta_1^l, \tau^l) &> R^{-1} \\ \sum_{t=1}^2 \tau_t^l / R^{t-1} &> \sum_{t=1}^2 \tau_t^h / R^{t-1} \end{aligned}$$

Thus, if utility is exponential and income shocks display any mean reversion, then in an efficient transfer scheme, poor agents are savings-constrained and poor agents receive more

resources than rich agents.

Now suppose $\rho_1 = 1$, so that private income is a random walk with drift. In this case, preference orderings over transfer profiles τ are independent of θ_1^i . Hence, for any incentive-compatible τ , it must be true that:

$$U(\theta_1^i, \tau^h) = U(\theta_1^i, \tau^l) \text{ for } i = h, l$$

Moreover, because preference orderings over τ are independent of θ_1^i , $MRS(\theta_1^i, \tau)$ is the same for all i . From conditions (ii) and (iii) of Proposition 3, any efficient τ must satisfy:

$$MRS(\theta_1^i, \tau^i) = R^{-1}$$

Thus, when $\rho_1 = 1$, in an efficient τ , agents are indifferent between taking the high contract and low contract; also, both contracts “smooth” their consumption profiles over time. Hence, the efficient τ is the solution to the problem:

$$\bar{\tau} = \arg \max_{\tau} -\exp(-\tau_1) - \beta E_{\theta^2} \exp(-\tau_2 - \rho_2 \theta_2)$$

$$s.t. W \geq \tau_1 + \tau_2/R$$

With permanent shocks, no insurance is possible: the efficient transfer profile is achieved by just letting the agents borrow and lend at the market interest rate R . Intuitively, when shocks are temporary, the planner is able to sort between the rich and poor agents by their differential attitudes towards the slope of their consumption profile. This difference disappears when shocks are permanent.

C. CRRA Utility and Permanent Shocks

We assume that u satisfies the requirement that $u'(c) = c^{-\gamma}$ ($\gamma > 0$), and that $\phi(\theta_1, \theta_2) \approx \theta_1 \theta_2^{\rho_2}$, where $\rho_2 > 0$. Note that this means that private income follows a logarithmic random walk. As in Constantinides and Duffie (1996), we assume that the private income shocks do not induce individuals to borrow or lend; that is, we require that:

$$\beta RE_{\theta^2} \{(\theta_2^{\rho_2})^{-\gamma}\} = 1$$

We first prove a result similar to the one derived for exponential utility: when shocks are permanent, no insurance is possible.

PROPOSITION 2. *Suppose $W = 0$. The unique efficient transfer scheme is $\tau = 0$.*

Proof. See the appendix.

In this permanent shock environment, neither the poor nor rich agent have any desire to save or borrow at the interest rate R , and so autarchy is the competitive equilibrium. Proposition 2 shows that as long as there is no publicly observable income, it is impossible for society to improve upon this competitive equilibrium.

However, unlike in the exponential utility or temporary shock cases, this qualitative characterization of the efficient transfer scheme changes when $W > 0$. We show this by first proving that if the rich person is savings-constrained for a nonzero transfer scheme, then the poor person is even more savings-constrained by that transfer scheme.

PROPOSITION 3. *If $\tau \neq 0$ and $MRS(\theta_1^h, \tau) \geq R^{-1}$, then $MRS(\theta_1^l, \tau) > MRS(\theta_1^h, \tau)$.*

Proof. See the appendix.

Intuitively, the poor agents have a positive probability of enduring an extremely low realization of income in the second period. This means given any nonzero transfer sequence, they have a larger desire to shift resources into the second period than do the rich agents. This is different from the temporary shock case, in which the poor agents have a smaller desire to shift resources into the second period.

Condition (iii) of Proposition 1 tells us that in an efficient transfer scheme, a poor agent's shadow interest rate lies between the market interest rate and a rich agent's shadow interest rate, where all shadow rates are evaluated at the poor agent's transfer profile. Proposition 3 implies that this ordering of shadow interest rates is impossible if the poor agent is savings-constrained. Hence, for an efficient τ , we know that:

$$\begin{aligned} MRS(\theta_1^l, \tau^l) &< R^{-1} \\ \sum_{t=1}^2 (\tau_t^l - \tau_t^h) / R^{t-1} &> 0 \end{aligned}$$

Even though all private income shocks are permanent, it is still possible to provide some insurance against the shocks. The planner screens the agents by exploiting the fact that the poor agents' demand for savings is larger than the rich agents' demand for savings. This means that in an efficient transfer scheme, the poor agents are borrowing-constrained, not savings-constrained.

5. Conclusion

We prove that a general characteristic of efficient transfer schemes in settings with hidden income is that the shadow interest rate of poor agents lies between the market interest rate and the shadow interest rate of the rich agents, evaluated at the poor agents'

contract. Specifics of the environment affect the implications of this general property for whether poor agents are savings-constrained or borrowing-constrained. However, we show that if agents have CRRA utility, have some publicly observable income, and face permanent privately observable income shocks, the efficient arrangement requires the poor to be borrowing-constrained. Thus, informational frictions could explain two key features of the consumption data: the low level of consumption risk sharing and poor individuals being borrowing constrained..

Appendix

A1. Proof of Proposition 1

In our proof, we implicitly look at a problem with a larger constraint set by ignoring the incentive constraint on the poor agents (those with low income realizations in period 1). At the end of the proof, we show that any transfer scheme that is optimal relative to this larger constraint set in fact satisfies the poor agents' incentive constraint, and so is optimal relative to the original constraint set.

Given that we are ignoring the poor agents' incentive constraint, the second condition follows immediately from the first order conditions with respect to (τ_1^h, τ_2^h) .

An efficient transfer scheme τ must satisfy:

$$\begin{aligned}\beta E_{\theta^2} \{u'(\phi(\theta_1^l, \theta_2) + \tau_2^l)\} - \lambda R^{-1}/2 - \mu \beta E_{\theta^2} \{u'(\phi(\theta_1^h, \theta_2) + \tau_2^l)\} &= 0 \\ u'(\theta_1^l + \tau_1^l) - \lambda/2 - \mu u'(\theta_1^h + \tau_1^l) &= 0\end{aligned}$$

where μ is the multiplier on the high income person's incentive constraint, and λ is the multiplier on the resource constraint. It follows that:

$$MRS(\theta_1^l, \tau^l) = \alpha R^{-1} + (1 - \alpha) MRS(\theta_1^h, \tau^l)$$

for some α such that $1 \geq \alpha \geq 0$, which proves the third condition.

To prove the first condition, define $\bar{\tau}^i$ to solve the problem:

$$\begin{aligned}\bar{\tau}^i &= \arg \max_{\tau} U(\theta_1^i, \tau) \\ s.t \ W &\geq \tau_1 + \tau_2/R\end{aligned}$$

The transfer profile $\bar{\tau}$ consists of the transfers that result from the rich and poor agents simply borrowing and lending at rate R . Clearly, $(\bar{\tau}^i)$ is incentive-compatible and resource-feasible. Since τ yields a higher ex ante payoff than $\bar{\tau}$, it follows that, because of the concavity of U :

$$\begin{aligned} 0 &\leq \pi_h \nabla U(\theta_1^h, \bar{\tau}^h)(\tau^h - \bar{\tau}^h) + \pi_l \nabla U(\theta_1^l, \bar{\tau}^l)(\tau^l - \bar{\tau}^l) \\ &= \pi_h u'(\theta_1^h + \bar{\tau}_1^h)(\tau_1^h + R^{-1}\tau_2^h - W) + \pi_l u'(\theta_1^l + \bar{\tau}_1^l)(\tau_1^l + R^{-1}\tau_2^l - W) \end{aligned}$$

Since $u'(\theta_1^h + \bar{\tau}_1^h) < u'(\theta_1^l + \bar{\tau}_1^l)$, it follows that $(\tau_1^l + R^{-1}\tau_2^l) \geq W \geq (\tau_1^h + R^{-1}\tau_2^h)$. The inequalities become strict if τ is not equal to $\bar{\tau}$, which is certainly true if $MRS(\theta_1^l, \tau^l) \neq R^{-1}$.

All that remains to be shown is that ignoring the poor agents' incentive constraint is without loss of generality. Suppose ignoring it is not correct; then, τ solves the optimization problem without the poor agents' incentive constraint, and yet:

$$U(\theta_1^l, \tau^l) < U(\theta_1^l, \tau^h)$$

Consider a new transfer scheme (τ_h, τ_h) in which poor agents are given the same transfer scheme as the rich agents. This transfer profile satisfies the rich agent's incentive constraint (obviously). Also, because $(\tau_1^h + \tau_2^h/R) \leq W$, this new transfer profile is resource-feasible. But this new transfer profile gives strictly more utility to the poor agent, and so improves the objective; this contradicts the supposition that τ solves the optimization problem ignoring the poor agent's incentive constraint. ■

A2. Proof of Proposition 2

We actually prove a stronger result: even if we ignore the poor individual's incentive constraint, the unique efficient transfer scheme is autarchy. The proof has two parts. First, we

show that if autarchy is suboptimal, the rich individuals are strictly worse off than autarchy.. Second, we show that in any allocation, if poor individuals are strictly better off than autarchy, so are rich individuals. The theorem follows from these two demonstrations.

Suppose τ is strictly better than 0. Then:

$$\begin{aligned} 0 &< \sum_{i=h,l} \pi_i [u'(\theta_1^i) \tau_1^i + \beta E_{\theta^2} \{u'(\theta_1^i \theta_2^{\rho_2})\} \tau_2^i] \\ &= \sum_{i=h,l} \pi_i u'(\theta_1^i) [\tau_1^i + R^{-1} \tau_2^i] \end{aligned}$$

This implies that $\tau_1^h + \tau_2^h/R < 0$. Now suppose τ is efficient. Since we are ignoring the poor individuals' incentive constraint, we know that $MRS(\theta_1^h, \tau^h) = R^{-1}$. Hence, given an efficient transfer scheme that is strictly better than autarchy, the rich individual is smoothed as he is in autarchy, but the rich individual receives less net present value than in autarchy.. It follows that the rich individual is strictly worse off than in autarchy.. This completes the first part of the proof.

The second part of the proof works as follows. Suppose a transfer scheme τ provides more utility to the poor agents than does no transfers:

$$u(\theta_1^l + \tau_1^l) + \beta E_{\theta^2} \{u(\theta_1^l \theta_2^{\rho_2} + \tau_2^l)\} > u(\theta_1^l) + \beta E_{\theta^2} \{u(\theta_1^l \theta_2^{\rho_2})\}$$

This implies that:

$$u(\theta_1^l + \tau_1^l) + \beta E_{\theta^2} \{u(\theta_1^l + \tau_2^l/\theta_2^{\rho_2})(\theta_2^{\rho_2})^{-\gamma}\} > u(\theta_1^l) + \beta E_{\theta^2} \{u(\theta_1^l)(\theta_2^{\rho_2})^{-\gamma}\},$$

since $u(c) = -\gamma c^{-\gamma}$, and hence that

$$\pi_1^* u(\theta_1^l + \tau_1^l) + \pi_h^* u(\theta_1^l + \tau_2^l/(\theta_2^h)^{\rho_2}) + \pi_l^* u(\theta_1^l + \tau_2^l/(\theta_2^l)^{\rho_2}) > u(\theta_1^l)$$

where:

$$\pi_1^* = (1 + \beta\pi_h u((\theta_2^h)^{\rho_2}) + \beta\pi_l u((\theta_2^l)^{\rho_2}))^{-1}$$

$$\pi_l^* = \beta\pi_l u((\theta_2^l)^{\rho_2})\pi_1^*$$

$$\pi_h^* = \beta\pi_h u((\theta_2^h)^{\rho_2})\pi_1^*$$

But since u exhibits strictly decreasing absolute risk aversion, it follows that:

$$\pi_1^* u(\theta_1^h + \tau_1^l) + \pi_h^* u(\theta_1^h + \tau_2^l/(\theta_2^h)^{\rho_2}) + \pi_l^* u(\theta_1^h + \tau_2^l/(\theta_2^l)^{\rho_2}) > u(\theta_1^h)$$

which in turn says that:

$$U(\theta_1^h, \tau^l) > U(\theta_1^h, 0).$$

This completes the second part of the proof, since $U(\theta_1^h, \tau^h) \geq U(\theta_1^h, \tau^l)$. ■

A3. Proof of Proposition 3

It is first useful to prove the following simple result.

LEMMA 1. Suppose h is a convex function and $\xi > 1$. If $h(1 + a_1)\pi + h(1 + a_2)(1 - \pi) \geq h(1)$

and $a_i \neq 0$ for $i = 1$ or 2 , then $h(1 + \xi a_1)\pi + h(1 + \xi a_2)(1 - \pi) > h(1 + a_1)\pi + h(1 + a_2)(1 - \pi)$.

Proof. Since h is convex, $0 \leq h(1 + a_1)\pi + h(1 + a_2)(1 - \pi) - h(1) < h'(1 + a_1)a_1\pi + h'(1 + a_2)a_2(1 - \pi)$. Now define:

$$g(\xi) = h(1 + \xi a_1)\pi + h(1 + \xi a_2)(1 - \pi) - h(1)$$

It follows that $g(1) \geq 0$ and $g'(1) > 0$. Moreover:

$$g''(\xi) = h''(1 + \xi a_1)(a_1)^2\pi + h''(1 + \xi a_2)(a_2)^2(1 - \pi) \geq 0$$

and so $g'(\phi) > 0$ for all $\phi \geq 1$. But this implies that $g(\phi) > 0$ for all $\phi > 1$, which proves the lemma. ■

Given this lemma, we can proceed to the proof of the Proposition. Define:

$$\pi_l^* = \pi_l u'((\theta_2^l)^{\rho_2})$$

$$\pi_h^* = \pi_h u'((\theta_2^h)^{\rho_2})$$

Remember that by assumption

$$1 = \frac{\{\beta R u'(\theta_1^l (\theta_2^l)^{\rho_2}) \pi_l + \beta R u'(\theta_1^h (\theta_2^h)^{\rho_2}) \pi_h\}}{u'(\theta_1^l)}.$$

Now, if we suppose the Proposition is false for some nonzero τ^l , then it must be true that:

$$\begin{aligned} 1 &= \frac{\{\beta R u'(\theta_1^l) \pi_l^* + \beta R u'(\theta_1^l) \pi_h^*\}}{u'(\theta_1^l)} \\ &\leq \frac{\{\beta R u'(\theta_1^l + \tau_1^l / (\theta_2^l)^{\rho_2}) \pi_l^* + \beta R u'(\theta_1^l + \tau_1^l / (\theta_2^l)^{\rho_2}) \pi_h^*\}}{u'(\theta_1^l + \tau_1^l)} \\ &\leq \frac{\{\beta R u'(\theta_1^h + \tau_1^l / (\theta_2^l)^{\rho_2}) \pi_l^* + \beta R u'(\theta_1^h + \tau_1^l / (\theta_2^l)^{\rho_2}) \pi_h^*\}}{u'(\theta_1^h + \tau_1^l)} \end{aligned}$$

so that the rich individual is more savings-constrained than the poor individual. We can

rewrite this as:

$$\begin{aligned} &\beta R u'(1)(\pi_l^* + \pi_h^*) \\ &\leq \beta R u' \left(1 + \frac{\tau_1^l / (\theta_2^l)^{\rho_2} - \tau_1^l}{\theta_1^l + \tau_1^l} \right) \pi_l^* + \beta R u' \left(1 + \frac{\tau_1^l / (\theta_2^h)^{\rho_2} - \tau_1^l}{\theta_1^l + \tau_1^l} \right) \pi_h^* \\ &\quad \beta R u' \left(1 + \frac{\tau_1^l / (\theta_2^l)^{\rho_2} - \tau_1^l}{\theta_1^h + \tau_1^l} \right) \pi_l^* + \beta R u' \left(1 + \frac{\tau_1^l / (\theta_2^h)^{\rho_2} - \tau_1^l}{\theta_1^h + \tau_1^l} \right) \pi_h^*, \end{aligned}$$

where we divided the numerators and denominators by θ_1^l , $\theta_1^l + \tau_1^l$, $\theta_1^h + \tau_1^l$ respectively. The resulting expression implies that:

$$\begin{aligned} u'(1) &\leq \{u'(1 + a_1\xi)\pi_l^*/(\pi_l^* + \pi_h^*) + u'(1 + a_2\xi)\pi_h^*/(\pi_l^* + \pi_h^*)\} \\ &\leq \{u'(1 + a_1)\pi_l^*/(\pi_l^* + \pi_h^*) + u'(1 + a_2)\pi_h^*/(\pi_l^* + \pi_h^*)\} \end{aligned}$$

where

$$\begin{aligned} a_1 &= (\tau_2^l/(\theta_2^l)^{\rho_2} - \tau_1^l)/(\theta_1^h + \tau_1^l) \\ a_2 &= (\tau_2^l/(\theta_2^l)^{\rho_2} - \tau_1^l)/(\theta_1^h + \tau_1^l) \\ \xi &= (\theta_1^h + \tau_1^l)/(\theta_1^l + \tau_1^l) \end{aligned}$$

But since u' is convex, and $(\theta_1^h + \tau_1^l)/(\theta_1^l + \tau_1^l) > 1$, this restriction violates the lemma. The proposition follows. ■

References

- [1] Aiyagari, S. Rao, and Fernando Alvarez. “Stationary Efficient Distributions With Private Information and Monitoring: A Tale of Kings and Slaves.” Manuscript, Federal Reserve Bank of Minneapolis, 1995.
- [2] Alvarez, Fernando, and Urban Jermann. “Asset Pricing When Risk Sharing is Limited by Default: A Theoretical Framework,” University of Chicago working paper, 1997.
- [3] Atkeson, Andrew, and Robert E. Lucas, Jr. “On Efficient Distribution With Private Information.” *Review of Economic Studies* 59 (July 1992): 427–453.
- [4] Atkeson, Andrew, and Robert E. Lucas, Jr. “Efficiency and Equality in a Simple Model of Efficient Unemployment Insurance.” *Journal of Economic Theory* 66 (June 1995): 64–88.
- [5] Green, Edward J. “Lending and the Smoothing of Uninsurable Income.” In Edward C. Prescott and Neil Wallace, eds., *Contractual Arrangements for Intertemporal Trade*, pp. 3–25. Minneapolis: University of Minnesota Press, 1987.
- [6] Green, Edward J., and Soo-Nam Oh. “Contracts, Constraints and Consumption.” *Review of Economic Studies* 58 (October 1991): 883–899.
- [7] Kehoe, Timothy, and David Levine. “Debt-Constrained Asset Markets,” *Review of Economic Studies* 60 (October 1993): 865–888.
- [8] Phelan, Christopher. “Repeated Moral Hazard and One-Sided Commitment.” *Journal of Economic Theory* 66 (August 1995): 488–506.

- [9] Phelan, Christopher, and Robert M. Townsend. "Computing Multi-Period, Information Constrained Optima." *Review of Economic Studies* 58 (October 1991): 853–881.
- [10] Taub, Bart. "Dynamic Mechanisms on a Continuum." Manuscript, University of Illinois, 1990.
- [11] Thomas, Jonathan, and Tim Worrall. "Income Fluctuation and Asymmetric Information: An Example of a Repeated Principal-Agent Problem." *Journal of Economic Theory* 51 (August 1990): 367–390.
- [12] Townsend, Robert M. "Optimal Multiperiod Contracts and the Gain From Enduring Relationships Under Private Information." *Journal of Political Economy* 90 (December 1982): 1166–1186.
- [13] Wang, Cheng. "Dynamic Insurance With Private Information and Balanced Budgets." *Review of Economic Studies* 62 (October 1995), 577–595.