

Lotteries, Sunspots and Incentive Constraints¹

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Abstract:

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1 Introduction

It is widely agreed that frictions play an important role in asset markets and that contrary to the complete market model individuals bear substantial idiosyncratic risk. Why, for example, is there no private insurance for workers against industry failure? In earlier work, Kehoe and Levine [1990] argued that this is likely due to individual rationality constraints: workers in other industries would simply not honor a contract requiring them to pay a substantial fraction of their wages to workers in other industries. Kocherlakota [1997] and Alvarez and Jermann [1997] and others have used this argument to study issues such as the equity premium puzzle. However, while the lack of private insurance against industry failure can scarcely be explained by moral hazard, the strong concentration of individual portfolios in a narrow range of assets cannot be easily explained by individual rationality constraints. Why, for example, does Bill Gates hold largely of Microsoft stock, or does a small shopkeeper's portfolio consisting largely of the shop itself? While individuals with such undiversified portfolios may be small in number, they are large in the percentage of wealth they control.

Recently there has been a resurgence of interest in incorporating moral hazard and adverse selection into general equilibrium theory in an effort to model asset market frictions of this sort. Bisin and Guaitoli [1995], Bernardo and Chiappori [1997] and Bernardo [1997] have been such efforts. The point of departure has been Prescott and Townsend [1984], who introduce both the idea that incentive constraints can be introduced into general equilibrium theory in a sensible way, and that lotteries play a potentially important role in the resulting theory. Although their theory has been widely used to study indivisibilities in the aggregate economy by Hansen [1985], Rogerson [1985] and others, until recently little effort has been made to study incentive constraints from this point of view, and the idea of using lotteries to study asset markets remains controversial. The goal of this paper is to review the theory from Prescott and Townsend [1984], as well as subsequent research in the area. We formalize the argument that lottery economies are equivalent to sunspot economies, and discuss the fact that in many practical applications involving decreasing absolute risk aversion, lotteries are not needed in equilibrium. We develop the notion of the stand-in consumer economy as a tool for proving theorems about lottery economies. Overall, we argue that the Prescott/Townsend framework

represents a sensible and progressive framework for analyzing moral hazard and adverse selection in general equilibrium theory. Our goal is not to prove new results, but rather discuss existing knowledge about lottery and moral hazard economies in a systematic way.

2 A Simple Insurance Problem: A Motivating Example

There is a continuum of households who are ex ante identical. There are two goods $j=1,2$. Let c_j denote consumption of good j . Utility is given by $\tilde{u}_1(c_1) + \tilde{u}_2(c_2)$, where $\tilde{u}_i(\cdot)$ is concave and strictly increasing. The endowment of good 1 is risky, while good 2 has a certain endowment. Each household has an independent 50% chance of being in one of two states, $s=1,2$. So the endowment of good 1 is state dependent, and can take on one of the two values $\omega_1(1)$ and $\omega_1(2)$, where $\omega_1(2) > \omega_1(1)$, while the endowment of good 2 is fixed at ω_2 .

Viewed in the aggregate, after the state is realized, half of the population has the high endowment, and half the low endowment. After the state is realized, there are gains from trade, as the low endowment households will wish to purchase good 1 and sell good 2. But before the state is realized, there are additional gains from trade, as households will wish to purchase insurance against the bad state. In fact, since all households are identical and utility is concave, it is obvious that there is a unique first best allocation in which all households consume $(\omega_1(1) + \omega_1(2))/2$ of good 1, and ω_2 of good 2.

Suppose, however, that the realization of the idiosyncratic risk is private information, known only to the individual household. In this case, the first best solution is not incentive compatible. In the first best arrangement, low endowment households receive an insurance payment of $(\omega_1(2) - \omega_1(1))/2$, while high endowment households must make a payment of the same amount. So high endowment households will misrepresent their endowment in order to receive a payment rather than make one.

One approach to this problem is to prohibit trading in insurance contracts, and consider only trading that takes place ex post after the state is realized. The resulting competitive equilibrium leads to an equalization of marginal rates of substitution between the two goods for the two types of households, but the low endowment households will have less utility than the high endowment households and there are unrealized gains from ex ante trade in insurance contracts.

A second approach to this problem is to observe that it is possible to trade in insurance contracts, provided that no household buys a contract that would later lead it to misrepresent its state. If endowments can be made public, but only voluntarily, then the high endowment household can imitate the low endowment type, but not conversely. Suppose that a household attempts to purchase $x(1)$ in state one in exchange for $x(2)$ in state 2. In the high endowment state, utility will be $\tilde{u}_1(\omega_1(2) + x_1(2)) + \tilde{u}_2(\omega_2 + x_2(2))$. In this case, the household may wish to pretend that the state is actually state 1. In order to avoid detection, it must make the same spot market purchases that a household in state 1 would make, $x_1(1), x_2(1)$. This results in utility $\tilde{u}_1(\omega_1(2) + x_1(1)) + \tilde{u}_2(\omega_2 + x_2(1))$. So the incentive compatibility constraint is

$$\tilde{u}_1(\omega_1(2) + x_1(2)) + \tilde{u}_2(\omega_2 + x_2(2)) \geq \tilde{u}_1(\omega_1(2) + x_1(1)) + \tilde{u}_2(\omega_2 + x_2(1)).$$

If this constraint is satisfied, the household will have no incentive to lie about its private information.

Since the household strictly prefers its own trade to that of the low endowment household

$$\tilde{u}_1(c_1(2)) + \tilde{u}_2(c_2(2)) > \tilde{u}_1(c_1(1) - \omega_1(1) + \omega_1(2)) + \tilde{u}_2(c_2(1) - \omega_2).$$

It follows that if the insurance purchase $x_1(1), x_1(2)$ is sufficiently close to the incomplete markets contract, the high endowment household will have no incentive to misrepresent. So there are additional incentive compatible gains to trade that are not realized in the incomplete markets model.

Suppose, more generally, that households trade goods contingent on announcements. No household will ever deliver a bundle that is not incentive compatible. If, in addition, there are rational expectations, then every household must know this fact, and so only incentive compatible bundles can be traded. Notice, however, that this stronger argument does not guarantee that all incentive compatible bundles actually will be traded. However, if it is possible to write and enforce contracts that specify net trades in every state, then all incentive compatible bundles will be traded. Notice, however, that these contracts must prohibit some ex post trade. If a high endowment household can receive an insurance payment by claiming a low endowment, and then turn around and trade the insurance payment of good 1 for additional units of good 2, it certainly will

prefer this to admitting a high endowment. The contract must specifically prohibit households claiming to have a low endowment from trading good 1 for good 2. Contracts of this type are not uncommon in insurance markets. It is quite common for insurance contracts to specify that the insurance payment may be used only for a specific purpose, such as replacing a structure on a specific location.

If ex post trade cannot be controlled, then the set of contracts that can be enforced is smaller than the set of all incentive compatible contracts. This does not lead to great complication in the results below. However, the set of contracts that can be enforced will also in general depend on ex post prices, since it is these prices that determine the value of trading following misrepresentation. The complications of the theory that ensue in a setting of individual rationality constraints without incentive constraints are examined in Kehoe and Levine [1990]. The main conclusion of that paper is that typically the welfare theorems fail when spot market trading cannot be contractually controlled. We expect similar results to hold in the setting of incentive constraints. However, for the remainder of the paper we ignore this complication, and consider only incentive (and related) constraints that are independent of market prices.

Let X denote the space of all triples of net trades that satisfy the incentive constraint. If net trades can be contractually specified, these represent incentive feasible net trades. Our program is to restrict households to trading plans in X and then do ordinary competitive equilibrium theory. There are two complications with this program. First, fixing $x_i(2)$, the set of $x_i(1)$ that satisfies the incentive constraint obviously fails to be convex, so X itself is not a convex set. This means that despite the fact that we have taken the underlying preferences to be risk averse, Pareto improvements may be possible by using lotteries. The second complication is that we can use lotteries to weaken the incentive constraints. If a household receives a random allocation contingent on its announcement, then the incentive constraint need only hold in expected value. If we let E_i denote the expectation conditional on the announcement of state i then the incentive constraint becomes

$$E_1 \tilde{u}_1 (\omega_1(2) + x_1(2)) + \tilde{u}_2 (\omega_2 + x_2(2)) \geq E_1 \tilde{u}_1 (\omega_1(2) + x_1(1)) + \tilde{u}_2 (\omega_2 + x_2(1)).$$

For these two reasons, once we introduce incentive constraints into general equilibrium it is desirable also to consider lotteries.

3 The Static Economy

Households may be of I types $i = 1, \dots, I$. There is a continuum of ex ante identical households of each type. An individual household is denoted by $h \in H^i = [0, 1]$. A household's type is commonly known. There are J traded goods $j = 1, \dots, J$. There are also two sources of uncertainty: a commonly observed sunspot, and household specific idiosyncratic risk. A "sunspot" is a random variable σ uniformly distributed on $[0, 1]$. Idiosyncratic risk is represented by specifying that each household of type i may consume in any one of a finite number of states $s \in S^i$. Each state has probability $\pi^i(s)$ where $\sum_{s \in S^i} \pi^i(s) = 1$.

We assume that households can contract for delivery of goods contingent on the sunspot and the individual state of the household.³ We write $x_j^i(s, \sigma, h) \in \mathfrak{R}$ for the net amount of good j delivered to household h of type i when the idiosyncratic state is s and the sunspot state is σ . The distribution of idiosyncratic shocks and sunspots are assumed to be independent. The idiosyncratic shocks are such that the average net trade of all type i households of good j when the sunspot is σ is given by

$$\bar{x}_j^i(\sigma) \equiv \sum_{s \in S^i} \int \pi^i(s) x_j^i(s, \sigma, h) dh.$$

There are several justifications for this assumption. The easiest assumption is that idiosyncratic shocks are independent across households. However, it is known that this is inconsistent with average net trades defined by Lebesgue integration and a space of consumers on the unit interval; this is discussed in ???. Alternatively, we could define average net trades by the Pettis integral, as in ???. Or we could simply assume that idiosyncratic states are correlated across individuals. However, we prefer to avoid these technical issues and simply justify the definition of aggregate consumption given as the limit of average net trades in finite household economies, which, after all, is the purpose in introducing continuum economies in the first place.

³ Strictly speaking households should be allowed to base contracts on the idiosyncratic states of other households. However, utility depends only on household's own idiosyncratic states not on the idiosyncratic states of other households. With idiosyncratic risk assumed to average out over the entire economy, contracts based on other household's idiosyncratic states do not serve any purpose. We omit them to avoid notational complication.

Trading takes place before any uncertainty is realized. Then the idiosyncratic states are realized and announcements of types are made. Next, sunspots are realized. Finally, deliveries are made, and consumption takes place.

Fix a type i and household h . For each realization of the sunspot σ this household realizes a net *trading plan* $x^i(\sigma, h) \in \mathfrak{R}^{J^i}$. This trading plan specifies net trades contingent on announcements of type. We assume that this trading plan must belong to the *feasible trading set* X^i . Notice endowments are incorporated directly into the feasible net trade set and are not specified separately. In many examples the trading set has the special form

$$X^i = \times_{s \in S^i} X^i(s).$$

Utility is given by $u^i: X^i \rightarrow \mathfrak{R}$.

In addition to restricting households to the consumption set X^i , we wish to consider incentive constraints that have an expected value form. We assume for each type i that there is a vector of k^i continuous *incentive functions* $g^i(x^i)$. A sunspot contingent trading plan $x^i(\sigma)$ is called *incentive compatible* if

$$\int g^i(x^i(\sigma)) d\sigma \leq 0.$$

In the insurance example if we take $k^i = 1$ and

$$g^i(x^i) = \tilde{u}_1(\omega_1(2) + x_1(1)) + \tilde{u}_2(\omega_2 + x_2(1)) - \tilde{u}_1(\omega_1(2) + x_1(2)) - \tilde{u}_2(\omega_2 + x_2(2)),$$

to measure the utility gain to the high endowment type from lying when the net trading plan is x^i then this corresponds to the ex ante expected utility form of the incentive constraint.

We do not assume that X^i is convex, or that u^i is concave or non-decreasing. We do assume:

Closed and Bounded Trades X^i is closed and bounded below.

Voluntary Trade $0 \in X^i$.

Continuity u^i is continuous.

Nonsatiation for all $x^i \in X^i$ there exists $\hat{x}^i \in X^i$, $g^i(\hat{x}^i) \leq 0$ such that $u^i(\hat{x}^i) > u^i(x^i)$.

Utility Boundary if $\|x^i\| \rightarrow \infty$ then $\limsup u^i(x^i)/\|x^i\| \leq 0$.⁴

Incentive Boundary if $\|x^i\| \rightarrow \infty$ then $\liminf g^i(x^i)/\|x^i\| \geq 0$

With the exception of the boundary conditions, these assumptions are self-explanatory. The utility boundary condition is kind of an asymptotic diminishing marginal utility or asymptotic aversion to risk: it says that eventually utility increases less fast than a linear function. The incentive boundary condition follows from the utility boundary condition in cases where the incentive constraints are derived from the utility function.

There are several points to emphasize about the model

- Types are commonly known; the idiosyncratic states may or may not be private information. It is important that contracting takes place prior to learning any private information. If contracting is possible only after learning private information, or, what amounts to the same thing, if types are private information, then incentives to misrepresent information will depend on the net trades of rival households. This represents an externality that will generally invalidate the welfare theorems.
- Households do not care about the private information of rival households. This assumption could be relaxed, but it would then be necessary to allow contracting based upon the announcements of the relevant rivals.
- Sunspots take place after idiosyncratic states are realized. We will show below that there is no loss of generality in this assumption – adding an additional sunspot variable after idiosyncratic states are realized does not change the set of equilibria.

3.1 Perfect Competition with Sunspots

A *sunspot allocation* is a measurable map for each type from households to individual trading plans; that is $\chi^i(h, \sigma) \in X^i$. An allocation is *socially feasible* if for each sunspot realization σ

$$\sum_{i=1}^I \sum_{s \in S^i} \int \pi^i(s) \chi^i(h, \sigma)[s] dh \leq 0.$$

Note that this definition incorporates free disposal. We say that an allocation has *equal utility* if $\int u^i(\chi^i(h, \sigma)) d\sigma = \int u^i(\chi^i(\tilde{h}, \sigma)) d\sigma$ almost surely.

⁴ Notice the use of $\lim \sup$. This is necessary because we have not imposed sufficient monotonicity conditions to assure that the limits exist.

A *price function* is a non-zero measurable function $p(\sigma) \in \mathfrak{R}_+^I$. Notice that the price of a delivery contingent on an idiosyncratic state is $\pi^i(s)p(\sigma)$ since in the aggregate $\pi^i(s)$ represents the fraction of the population to whom the delivery will actually be made.

A *sunspot equilibrium* with transfer payments consists of a socially feasible sunspot allocation χ together with a price function p . For all types i and almost all $h \in [0,1]$ $\chi^i(h, \cdot)$ must maximize $\int u^i(x^i(\sigma))d\sigma$ over sunspot contingent trading plans $x^i(\sigma)$ satisfying the *sunspot budget constraint*

$$\int \sum_{s \in S^i} \pi^i(s)p(\sigma)x^i(\sigma)[s]d\sigma \leq \int \sum_{s \in S^i} \pi^i(s)p(\sigma)\chi^i(h, \sigma)[s]d\sigma,$$

and the incentive constraints $\int g^i(x^i(\sigma))d\sigma \leq 0$. The transfer payments themselves must satisfy the equal treatment condition that they depend only on types

$$\int \sum_{s \in S^i} \pi^i(s)p(\sigma)\chi^i(h, \sigma)[s]d\sigma = \int \sum_{s \in S^i} \pi^i(s)p(\sigma)\chi^i(\hat{h}, \sigma)[s]d\sigma \text{ almost surely.}$$

A *sunspot equilibrium with endowments* is a sunspot equilibrium in which the transfer payments are zero

$$\int \sum_{s \in S^i} \pi^i(s)p(\sigma)\chi^i(h, \sigma)[s]d\sigma = 0.$$

Finally, a sunspot allocation is *Pareto efficient* if no socially feasible allocation satisfying the incentive constraints is Pareto preferred.

An immediate consequence of the fact that the transfer payments satisfy the equal treatment condition, is the conclusion that the equilibrium allocation must be an equal utility allocation. If it were not, then a positive measure of type i could increase their utility by switching to a consumption plan used by others of the same type.

Lemma 3.1.1 *A sunspot equilibrium allocation is an equal utility allocation.*

Our main goal is to establish the main theorems of competitive general equilibrium theory for the sunspot economy

Theorem 3.1.2 First Welfare Theorem *Every sunspot equilibrium allocation is Pareto efficient.*

Theorem 3.1.3 Second Welfare Theorem *For every Pareto efficient allocation with equal utility there are prices forming a sunspot equilibrium.*

Theorem 3.1.4 Existence Theorem *There is at least one sunspot equilibrium with endowments.*

The first welfare theorem is a relatively direct consequence of the nonsatiation assumption and the standard proof of the first welfare theorem. The remaining results follow are proven from equivalence theorems below.

3.2 Perfect Competition with Lottery Tickets

We now consider an alternative view of how the economy might operate. Notice that the sunspot random variable and a trading plan $x^i(\sigma) \in X^i$ induce a probability distribution over μ^i over X^i . We refer to this probability distribution as a *lottery*. From the point of view of individual utility, all trading plans that induce the same lottery yield the same utility, and the incentive constraints may also be computed directly from the lottery. We now consider the Prescott and Townsend [1984] perspective, in which households trade directly in lotteries. Our goal is to show that this formulation is exactly equivalent to the sunspot formulation.

We should first note that in many applications utility and the incentive constraints depend only on the idiosyncratic state contingent lotteries. That is, a trading plan can be written as $x^i(s, \sigma) \in \mathfrak{X}^i$. Typically utility and the incentive constraints can be derived from the random variables $x^i(s, \cdot)$; for example, when $u^i(x^i) = \sum_{s \in S^i} \pi^i(s) \tilde{u}^i(x^i(s))$. The distribution of the random variables $x^i(s, \cdot)$ can easily be computed from the lottery μ^i . In addition, when the net trading sets have the special form $X^i = \times_{s \in S^i} X^i(s)$, an equivalent definition of a lottery is as a probability distribution over $\bigcup_{s \in S^i} X^i(x)$ that satisfies the constraint that the marginal distribution over S^i is equal to π^i . Much of the literature on lotteries has used this formulation.

A *lottery ticket allocation* is simply a lottery μ^i for each type. It is said to be *socially feasible* if

$$\sum_{i=1}^N \int \sum_{s \in S^i} \pi^i(s) x^i(s) d\mu^i(x^i) \leq 0.$$

This simply says that in the aggregate the expected net trades used by the lottery allocation is non-positive. Unlike a sunspot equilibrium, a lottery allocation is non-specific about the particular circumstances under which a particular household gets a particular net trade.

A lottery ticket allocation is *Pareto efficient* if no socially feasible, incentive compatible Pareto improvement is possible. A *lottery ticket equilibrium* with transfer payments consists of a socially feasible lottery ticket allocation μ together non-zero price vector $p \in \mathfrak{R}_+^M$. For all types i , μ^i must maximize $u^i(\tilde{\mu}^i)$ over lotteries $\tilde{\mu}^i$ satisfying the *lottery ticket budget constraint*

$$p \int \sum_{s \in S^i} \pi^i(s) x^i[s] d\tilde{\mu}^i(x^i) \leq p \int \sum_{s \in S^i} \pi^i(s) x^i[s] d\mu^i(x^i),$$

and the incentive constraints $\int g^i(x^i) d\tilde{\mu}^i(x^i) \leq 0$. A *lottery ticket equilibrium with endowments* is a lottery ticket equilibrium in which the transfer payments are zero

$$p \int \sum_{s \in S^i} \pi^i(s) x^i[s] d\mu^i(x^i) = 0.$$

Notice that in this formulation, lotteries are priced according to the aggregate resources they use. This is a non-arbitrage condition: two lotteries that use the same aggregate resources must have the same price. If one lottery uses aggregate resources y and another \hat{y} , if the cost of buying y and \hat{y} separately exceeds the cost of buying $y + \hat{y}$, it would be profitable to buy the joint lottery $y + \hat{y}$ and sell the pieces, while in the opposite case, the pieces should be bought separately, then packaged and sold. Only linear pricing in the aggregate resources guarantees that there are no arbitrage opportunities.

We will establish the main theorems of competitive general equilibrium theory for the lottery ticket economy, as well as the sunspot economy

Theorem 3.2.1 First Welfare Theorem *Every lottery ticket equilibrium allocation is Pareto efficient.*

Theorem 3.2.2 Second Welfare Theorem *For every Pareto efficient allocation there are prices forming a lottery ticket equilibrium.*

Theorem 3.2.3 Existence Theorem *There is at least one lottery ticket equilibrium with endowments.*

In Prescott and Townsend [1984] these theorems were proven directly; we will give an alternative proof below. Our results on sunspot equilibria will then follow from showing that lottery ticket and sunspot allocations are equivalent.

3.3 Sunspot Allocations versus Lottery Allocations

Sunspot allocations and lottery ticket allocations are rather different descriptions of the possibilities of randomizing in the base economy. For example, suppose that there are two identical types, and one good, automobiles, for which the consumption set is either one automobile or zero. Suppose moreover, that each type is endowed with $\frac{1}{2}$ an automobile per capita. From the perspective of lotteries, the situation is rather simple; there can be no trade between the two types, so each household should get a 50% chance of an automobile. In other words, in equilibrium, each household of each type purchases a lottery with a $\frac{1}{2}$ chance of 1 automobile, and a $\frac{1}{2}$ chance of 0 automobiles.

This description by means of lotteries can be implemented in many ways by means of sunspots. For example, we could imagine that the individual lotteries are independent,⁵ and that in the aggregate the strong law of large number leads to social feasibility. But an alternative formulation would be to have a simple sunspot allocation in which when the sunspot variable $\sigma \leq 1/2$ the first type receives all the cars, and when $\sigma > 1/2$ the second type receives all the cars. From an individual point of view it makes no difference which of these methods is used to allocate cars. However, the how the economy operates in the large is quite different.

Begin with a sunspot allocation $\chi(h, \sigma)$. For each household, there corresponds a lottery $\bar{\mu}^i(\chi^i(h, \cdot))$. We can then average these lotteries over households to get a single *mean lottery* for the entire type $\mu^i[\chi] = \int \bar{\mu}^i(\chi^i(h, \cdot)) dh$. Notice that the resources used by this lottery are obviously equal to the expected resources used by the sunspot allocation; that is

$$\int x^i d\mu^i[\chi](x^i) = \int \chi^i(h, \sigma) dh d\sigma.$$

Moreover, by definition, in an equal utility sunspot allocation households of type i must be indifferent between the lotteries $\chi^i(h, \cdot), \chi^i(h', \cdot)$ for almost all h, h' . Since their utility is linear in probabilities, this means they must be indifferent between $\chi^i(h, \cdot)$ and the mean lottery $\mu^i[\chi]$ for almost all h . In a similar vein, since the incentive constraints hold for almost all individual lotteries, and are also linear in probabilities, the mean lottery must satisfy the incentive constraint. Consequently, the mean lottery corresponding to a

⁵ Subject to usual caveats about a continuum of independent random variables; see the discussion above.

sunspot equilibrium allocation is a natural candidate to be an equilibrium of the lottery ticket economy. If $p(\sigma)$ is a price function in the sunspot economy, we can in a similar way define the *mean price* $\bar{p} = \int p(\sigma)d\sigma$. Although it is not transparent, we will show below that the mean price is in fact the correct way to price the mean lottery in the lottery ticket economy

We define a sunspot allocation to be *equivalent* to a lottery ticket allocation if the mean lottery of the sunspot allocation is equal to the corresponding lottery in the lottery ticket allocation. We define a sunspot price function to be equivalent to a lottery ticket price if the mean price of the sunspot price function is equal to the lottery ticket price. By definition, there is only one lottery ticket allocation and price that is equivalent to a given sunspot allocation and price function. However, as we have already noted, there is not a unique way to construct a sunspot allocation (or price function) from a lottery ticket allocation. However, there is one important construction that plays a key theoretical role in moving from lottery ticket economies to sunspot economies. For a given lottery ticket price \bar{p} we define the *canonical sunspot price function* to be the constant function $p(\sigma) = \bar{p}$. For a given lottery ticket allocation μ we define the *canonical sunspot allocation* to be a particular allocation in which the aggregate resources used by each type are independent of the sunspot state. Specifically, corresponding to the lottery μ^i is a random variable $\chi^i(\sigma)$. We then define the canonical sunspot allocation by $\chi^i(h, \sigma) = \chi^i((\sigma + h) \bmod 1)$.

These simple constructions show that for every lottery ticket allocation and price there is at least one equivalent sunspot allocation and price. Because the constructions preserve utility, social feasibility and the incentive constraints, we can draw an immediate conclusion about Pareto efficiency.

Theorem 3.3.1 *An equal utility allocation is Pareto efficient in the sunspot economy if and only if the equivalent allocation in the lottery ticket economy is Pareto efficient. Moreover, the socially feasible, incentive compatible equal utility set in the sunspot economy is exactly the same as the socially feasible incentive compatible utility set in the lottery ticket economy.*

Less immediately obvious is the equivalence of equilibria in the two economies.

Theorem 3.3.2 *An allocation and price are a transfer payment equilibrium in the lottery ticket economy if the equivalent allocation and price function are a transfer payment equilibrium in the sunspot economy. An allocation and constant price function are a transfer payment equilibrium in the sunspot economy if the equivalent allocation and price function are a transfer payment equilibrium in the lottery ticket economy. In both cases the size of the transfer payments are the same in the two economies.*

Proof: Consider a sunspot allocation χ and price function p , and an equivalent lottery ticket allocation μ and price \bar{p} . Suppose first that p is constant (so in particular $p(\sigma) = \bar{p}$) and that μ, \bar{p} are a transfer payment equilibrium. Then it is transparent from the fact that households care only about their individual lottery, and the fact that $p(\sigma) = \bar{p}$ that χ, p is a transfer payment equilibrium. Since in both cases, each type pays only for the aggregate resources used, which is the same in both economies, the transfer payments must be the same in both cases.

Now suppose that instead χ, p are a transfer payment equilibrium, and that possibly p is not constant. By definition these transfer payments have to satisfy the equal treatment condition that net income is the same for almost all households; letting h be a typical household that receives the same net income as almost all other households, we denote the transfer payment by

$$R^i \equiv \int \sum_{s \in S^i} \pi^i(s) p(\sigma) \chi^i(h, \sigma)[s] d\sigma.$$

The key observation is that if

$$\bar{p} \int \sum_{s \in S^i} \pi^i(s) x^i[s] d\hat{\mu}^i(x^i) \leq R^i$$

so that a lottery is affordable with the transfer payment, then it can yield no more utility than the common amount χ^i gives almost all households. For suppose not. Consider the canonical sunspot allocation $\hat{\chi}^i$ corresponding to $\hat{\mu}^i$. This gives every household in i more utility than χ^i . Moreover,

$$\int \int \sum_{s \in S^i} \pi^i(s) p(\sigma) \hat{\chi}^i(h, \sigma)[s] d\sigma dh \leq R^i$$

so $\hat{\chi}^i$ is affordable for some positive measure set of type i households, contradicting the fact that χ^i was optimal at prices p equilibrium.

This observation shows that at prices \bar{p} type i cannot get more utility than μ^i . So to complete the proof that μ, \bar{p} is an equilibrium transfer payments R^i , we need only show that μ^i is affordable. If for some household

$$\bar{p} \int \sum_{s \in S^i} \pi^i(s) x^i[s] d\mu^i(x^i) < R^i,$$

this household could find an affordable and utility improving $\hat{\mu}^i$, which we already showed is impossible. So for all households,

$$\bar{p} \int \sum_{s \in S^i} \pi^i(s) x^i[s] d\mu^i(x^i) \geq R^i.$$

Since from social feasibility $\sum_i R^i = 0$, and $\sum_i \int \sum_{s \in S^i} \pi^i(s) x^i[s] d\mu^i(x^i) = 0$, we conclude that

$$\bar{p} \int \sum_{s \in S^i} \pi^i(s) x^i[s] d\mu^i(x^i) = R^i,$$

so μ^i is in fact affordable.

□

3.4 The Stand-in Consumer Economy

We now wish to prove the welfare theorems and the existence of an endowment equilibrium; from the equivalence of the sunspot and lottery ticket equilibria, it is sufficient to do so in either of the two types of economies. Each, however, poses its own complications. The sunspot economy has a net trade set that is complicated and non-convex. The lottery ticket economy has a net trade set that is convex but infinite dimensional. One approach is that of Prescott and Townsend [1984], which is to work directly with theorems for infinite dimensional economies. An alternative that leads to finite dimensional and mathematically simpler proofs is to observe that the household problem of maximizing utility subject to a budget constraint can be broken in two parts. Since the cost of a lottery is simply the cost of the expected net trades it uses, we can think of the household as first purchasing an expected net trade vector. Having done so it then chooses the lottery that maximizes utility subject to the constraint of expected net trades. This utility depends only on the expected net trade vector, which is finite dimensional, so in effect reduces the economy to a finite one.

Specifically, we consider net trade vectors $y^i \in \mathfrak{R}^I$. The set of interest are those net trade vectors that are consistent with feasible trading plans of type i households:

$$Y^i \equiv \text{Closure}(\text{ConvexHull}\{y^i \in \mathfrak{R}^M \mid \exists x^i \in X^i, y^i = \sum_{s \in S^i} \pi^i(s) x^i[s]\}).$$

Given that a bundle $y^i \in Y^i$ has been purchased, how much utility may a type i household get? The answer is given by

$$v^i(y^i) = \sup \int u^i(x^i) d\mu^i(x^i)$$

$$\text{subject to support } \mu^i \subseteq X^i, \int \sum_{s \in S^i} \pi^i(s) x^i d\mu^i(x^i) \leq y^i, \int g^i(x^i) d\mu^i(x^i) \leq 0.$$

This construct will be most useful if we can replace the sup with a max, so that there is at least one lottery that actually yields the utility $v^i(y^i)$.

Lemma 3.4.1 *If the consumption and production boundary condition hold, then $v^i(y^i) = \max \int u^i(x^i) d\mu^i(x^i)$ subject to support $\mu^i \subseteq X^i$, $\int \sum_{s \in S^i} \pi^i(s) x^i[s] d\mu^i(x^i) \leq y^i$, $\int g^i(x^i) d\mu^i(x^i) \leq 0$.*

Sketch of Proof: There is at least a sequence of lotteries converging to the sup; by Caratheodory's Theorem we may assume that these lotteries have support at $m^i + k^i + 2$ points. Let z^i, p^i be the points and probabilities in this sequence of finite lotteries. This has a convergent subsequence on the extended real line. Because X^i is bounded below, any component of z^i that converges to $\pm\infty$ has corresponding probability converge to zero. By the consumption boundary condition the limit of expected utility for such a point is also zero. So the limit lottery places weight only on finite points, and gives the same utility and satisfies the same feasibility condition. It is the optimal lottery.

□

We may now study trade in the economy, by considering I consumers with utility functions v^i and consumption sets Y^i . We refer to consumer i as the *stand-in consumer*, as he represents all households of type i . In effect the stand-in consumer makes purchases on behalf of the ex ante identical households he represents, then allocates the purchases to individual households by means of an optimal lottery. Notice the role played here by the assumption the all households of a given type are ex ante identical: there is no ambiguity about how a lottery should be chosen to allocate resources among individual households.

In the *stand-in consumer economy*, an allocation y is a vector $y^i \in Y^i$ for each type. The allocation is socially feasible if $\sum_i y^i \leq 0$. An *stand-in consumer equilibrium*

with transfer payments consists of a non-zero price vector $\bar{p} \in \mathfrak{R}_+^M$, and a socially feasible allocation y . For each type i y^i should maximize $v^i(\tilde{y}^i)$ subject to $\bar{p} \cdot \tilde{y}^i \leq \bar{p} \cdot y^i$, $y^i \in Y^i$. An endowment equilibrium and Pareto efficiency are defined in the obvious way. Notice that equilibria in the stand-in consumer economy are equivalent to equilibria in the lottery ticket economy in a direct and simple way. Given a lottery ticket equilibrium μ, \bar{p} , we can compute the expected resources used by the equilibrium lottery $y^i = \int \sum_{s \in S^i} \pi^i(s) x^i(s) d\mu^i(x^i)$. Clearly y, \bar{p} is a stand-in consumer equilibrium. Conversely, given a stand-in consumer equilibrium y, \bar{p} , we can by Lemma 2.6.1 find for each stand-in consumer an optimal lottery μ^i , and it is clear that μ, \bar{p} is a lottery ticket equilibrium.

To prove the welfare and existence theorems for the sunspot economy and lottery ticket economy, it suffices to prove them for the stand-in consumer economy. As this is a finite dimensional pure exchange economy, this follows from verifying standard properties of utility functions and consumption sets.

Lemma 3.4.2 *Utility v^i is continuous, concave and if **non-satiation** holds, strictly increasing. The net trade set has $0 \in Y^i$ and is convex and bounded below.*

4 The Role of Lotteries and Incentive Constraints

4.1 The Insurance Problem

Recall our motivating example. There are two goods $j = 1, 2$, one type $i = 1$ and two states $s = 1, 2$. Utility for net trades is

$$u^1(x^1) = 5\tilde{u}_1(\omega_1(1) + x_1^1(1)) + 5\tilde{u}_2(\omega_2 + x_2^1(1)) + 5\tilde{u}_1(\omega_1(2) + x_1^1(2)) + 5\tilde{u}_2(\omega_2 + x_2^1(2)),$$

where $\omega_1(1) < \omega_1(2)$. The net trade set is $X^1 \equiv \{x^1 | x^1 \geq -\omega^1\}$ and the incentive function is

$$g^1(x^1) \equiv u_1(\omega_1(2) + x_1^1(1)) + u_2(\omega_2 + x_2^1(1)) - u_1(\omega_1(2) + x_1^1(2)) - u_2(\omega_2 + x_2^1(2))$$

The aggregation operator is given by $A^1 x^1 = 5x^1(1) + 5x^1(2)$.

Proposition 4.1.1 *Suppose that \tilde{u}_1 exhibits declining absolute risk aversion, and that \tilde{u}_2 is strictly concave. If μ^i solves the stand-in consumer problem*

$$v^1(y^1) = \max \int u^1(x^1) d\mu^1(x^1)$$

subject to support $\mu^1 \subseteq X^1$, $\int \sum_{s \in S^1} \pi^1(s) x^1 d\mu^1(x^1) \leq y^1$, $\int g^i(x^i) d\mu^i(x^i) \leq 0$,

then μ^1 is a point mass on a single point.

Proof: Let μ^1 be a non-degenerate solution to the stand-in consumer problem corresponding to y^1 . Consider the allocation

$$\begin{aligned}\hat{x}_1^1(1) &= \int x^1(1) d\mu^1(x^1(1)) \\ \hat{x}_2^1(1) &= \int x^1(1) d\mu^1(x^1(1)) - z \\ \hat{x}^1(2) &= \int x^1(2) d\mu^1(x^1(2))\end{aligned}$$

where z is chosen so that

$$\int [u_1(\omega_1(2) + x_1^1(1)) + u_2(\omega_2 + x_2^1(1))] d\mu^1(x^1) = u_1(\omega_1(2) + \hat{x}_1^1(1)) + u_2(\omega_2 + \hat{x}_2^1(1)).$$

Since u_2 is strictly concave, z is strictly positive, so \hat{x}^1 satisfies the resource constraint. Moreover, it also satisfies the incentive constraint, since replacing μ^1 with its expected value increases utility in the good state. Indeed, not only does utility in the good state increase, but because u_1 exhibits declining absolute risk aversion, it increases in the bad state as well. So \hat{x}^1 satisfies the constraints of the stand-in consumer problem, this contradicts the fact that μ^1 solves the problem. □

Under the assumption that there are no lotteries involved, we find the competitive equilibrium (since there is only one type) by solving the optimization problem

$$\max \bar{u}_1(\omega_1(1) + x_1^1(1)) + \bar{u}_2(\omega_2 + x_2^1(1)) + \tilde{u}_1(\omega_1(2) - x_1^1(1)) + \tilde{u}_2(\omega_2 - x_2^1(1))$$

$$\text{subject to } \tilde{u}_1(\omega_1(2) + x_1^1(1)) + \tilde{u}_2(\omega_2 + x_2^1(1)) - \tilde{u}_1(\omega_1(2) - x_1^1(1)) - \tilde{u}_2(\omega_2 - x_2^1(1)) \leq 0,$$

where we have substituted out the social feasibility constraint $x_j^1(1) = -x_j^1(s)$. From the first order conditions, we can find

$$\frac{D\bar{u}_1(\omega_1(1) + x_1^1(1))}{D\bar{u}_2(\omega_2 + x_2^1(1))} = (1 - \lambda) \frac{D\tilde{u}_1(\omega_1(2) - x_1^1(1))}{D\tilde{u}_2(\omega_2 - x_2^1(1))} + \lambda \frac{D\tilde{u}_1(\omega_1(2) + x_1^1(1))}{D\tilde{u}_2(\omega_2 + x_2^1(1))},$$

where λ is the Lagrange multiplier on the incentive constraint. Since the constraint must bind, and

$$\frac{D\tilde{u}_1(\omega_1(2) - x_1^1(1))}{D\tilde{u}_2(\omega_2 - x_2^1(1))} > \frac{D\tilde{u}_1(\omega_1(2) + x_1^1(1))}{D\tilde{u}_2(\omega_2 + x_2^1(1))},$$

we conclude that

$$\frac{D\tilde{u}_1(\omega_1(1) + x_1^1(1))}{D\tilde{u}_2(\omega_2 + x_2^1(1))} < \frac{D\tilde{u}_1(\omega_1(2) - x_1^1(1))}{D\tilde{u}_2(\omega_2 - x_2^1(1))}.$$

This means that at the equilibrium there are unexploited gains from trade between the two types. If, for example, a spot market were to open, the two types would carry out trade with each other. So the equilibrium contracts must specify, for example, that additional amount of good 1 acquired by the poor type beyond that paid as an insurance settlement can be claimed by the insurer.

4.2 Ex Post Trading

Lemma 4.2.1 *Suppose that \bar{p} supports v^i at y^i , that $\text{support } \mu^i \subseteq X^i$, that $\int \sum_{s \in S^i} \pi^i(s) x^i[s] d\mu^i(x^i) \leq y^i$, that $\int v^i(x^i) d\mu^i(x^i) = v^i(y^i)$ and that $\int g^i(x^i) d\mu^i(x^i) < 0$. Let $z^i \in \text{ConvexHull } X^i$ be such that for every $\varepsilon > 0$ $\mu^i(B_\varepsilon(z^i)) > 0$. Then there is a solution $\hat{\mu}^i$ to the problem of maximizing $u^i(\tilde{\mu}^i)$ subject to $\text{support } \tilde{\mu}^i \subseteq X^i$, $\bar{p} \int \sum_{s \in S^i} \pi^i(s) x^i[s] d\tilde{\mu}^i(x^i) \leq \bar{p} \sum_{s \in S^i} \pi^i(s) z^i[s]$ in which $\int x^i d\hat{\mu}^i(x^i) \leq z^i$.*

Proof: Let $\tilde{v}^i(x^i) = \max \int u^i(x^i) d\tilde{\mu}^i(x^i)$ subject to $\text{support } \tilde{\mu}^i \subseteq X^i$, $\int x^i d\tilde{\mu}^i(x^i) \leq x^i$. This is well defined by the proof of Lemma 2.6.1, and also satisfies the conclusion of Lemma 3.1.1. Suppose conversely to the Lemma that there is $\tilde{x}^i \in \text{ConvexHull } X^i$ with $\tilde{v}^i(\tilde{x}^i) > \tilde{v}^i(z^i)$, and $pA^i(\tilde{x}^i - z^i) \leq 0$. Since \tilde{v}^i is continuous, for some sufficiently small ε $\tilde{v}^i(B_\varepsilon(\tilde{x}^i)) > \tilde{v}^i(B_\varepsilon(z^i))$. Consider $\hat{\mu}^i$ constructed by replacing $\mu^i|_{B_\varepsilon(z^i)}$ by that same measure translated by $\tilde{x}^i - z^i$. Notice that for \tilde{x}^i sufficiently close to z^i the support of the translated measure is contained in $\text{ConvexHull } X^i$. Moreover by translating a small enough probability, since the incentive constraints were strictly satisfied, they will remain satisfied. Set $w^i = \int x^i d\hat{\mu}^i(x^i)$. Then $p \cdot (A^i w^i - y^i) \leq 0$ and $\tilde{v}^i(w^i) > \tilde{v}^i(y^i)$, a contradiction. □

Notice that this result holds only if the incentive constraints do not bind. We will comment further on this below.

4.3 Risk Aversion and Lotteries

We now specialize to the case where there are no indivisibilities in an effort to generalize the idea that with declining absolute risk aversion incentive constraints do not require lotteries. We do this in the setting of the stand-in consumer economy. We assume that $X^i = \times_s X^i(s)$ $X^i(s) \subset \mathfrak{R}^M$ is a closed convex set bounded below. For every pair of states (s, \hat{s}) we are given a continuous, concave, strictly increasing utility function $U^i[s, \hat{s}]$. This is to be interpreted as the utility of a type s masquerading as a type \hat{s} . Then type i 's utility function in the base economy is given by the expected utility over states

$$u^i(x^i) = \sum_s U^i[s, s](x^i(s))\pi^i(s),$$

and the incentive functions are indexed by pairs of states (s, \hat{s}) where $s \neq \hat{s}$; and have the special form

$$g^i[s, \hat{s}](x^i) = \hat{U}^i[s, \hat{s}](x^i(\hat{s})) - U^i[s, s](x^i(s)).$$

We further assume the existence of certainty equivalents in the following sense: if $\mu^i[\hat{s}]$ is a lottery on $X^i(\hat{s})$, we say that $\bar{x}^i(\mu^i[\hat{s}])$ is the *certainty equivalent* of $\mu^i[\hat{s}]$ for $U^i[s, \hat{s}]$ if for some $\lambda > 0$

$$\bar{x}^i(\mu^i[\hat{s}]) = \lambda \int x^i d\mu^i[\hat{s}](x^i) \in X^i(\hat{s}), \text{ and}$$

$$U^i[s, \hat{s}](\bar{x}^i(\mu^i[\hat{s}])) = \int U^i[s, \hat{s}](x) d\mu^i[s](x^i)$$

Notice that since the utility functions U^i are concave and strictly increasing, it must be that $\lambda \leq 1$.

We now consider the stand-in consumer problem for fixed y^i

$$\max \int u^i(x^i) d\mu^i(x^i)$$

$$\text{subject to } \text{support } \mu^i \subseteq X^i, \int A^i x^i d\mu^i(x^i) \leq y^i, \int g^i(x^i) d\mu^i(x^i) \leq 0.$$

We say that the constraint (s, \hat{s}) is potentially binding, if for some y^i and some solution to this problem $\int g^i[s, \hat{s}](x^i) d\mu^i(x^i) = 0$. We say that utility exhibits *declining absolute risk aversion* if for every potentially binding constraint (s, \hat{s}) the certainty equivalent of any lottery for $U^i[s, \hat{s}]$ is larger than that for $U^i[\hat{s}, \hat{s}]$. That is, any type s who is constrained from masquerading as a type \hat{s} will be less risk averse than \hat{s} .

Theorem 4.1.1 *With declining absolute risk aversion, there exists a solution to the stand-in consumer problem that is a point mass on a single point.*

Sketch of proof: We start with a lottery that solves the consumer problem. We replace it with a lottery that gives each state the certainty equivalent for that state. By assumption this is in X^i , and by construction gives exactly the same utility as the optimum. Moreover, the utility from truth telling is unchanged. For potentially binding incentive constraints, the assumption of declining absolute risk aversion and strictly increasing utility shows that the utility from the masquerade has not been increased, so the constraint is still satisfied. Consider the collection of constraints that are not potentially binding, but are violated by the certainty equivalent, and consider lotteries that are a convex combination of the original lottery and the certainty equivalent. If sufficient weight is placed on the original lottery, none of these potentially non-binding constraints are violated. So consider the supremum of weights with this property. For this lottery, all incentive constraints are satisfied, utility is optimal, but one of the constraints that is not potentially binding does bind, contradicting the fact that it was not potentially binding.

☑

5 Time and Production

In this section we consider extensions of the basic *ex post* pure exchange framework.

We have assumed so far that sunspots occur after households discover and announce their state. An alternative assumption is that the sunspots occur first, as in [??]. However, for any lottery that occurs prior to the announcement of states, we can find an equivalent lottery that occurs after the announcement of states – just let the lottery be independent of the state. More specifically, let $\sigma_0, \sigma_1 \in [0,1]$ be uniform i.i.d. sunspots occurring before and after the realization of the idiosyncratic state. So an allocation is now $x_j^i(s, \sigma_0, \sigma_1, h) \in \mathfrak{R}$, and the incentive constraints

$$\int g^i(x^i(\sigma_0, \sigma_1)) d\sigma_1 \leq 0 \text{ for all } \sigma_0$$

must hold for each realization of the *ex ante* sunspot.

Theorem 5.1.1 An allocation and price are a transfer payment equilibrium in the lottery ticket economy if the equivalent allocation and price function are a transfer payment

equilibrium in the two-period sunspot economy. An allocation and constant price function are a transfer payment equilibrium in the sunspot economy if the equivalent allocation and price function are a transfer payment equilibrium in the two-period lottery ticket economy. In both cases the size of the transfer payments are the same in the two economies.

Next we consider production. All production is assumed to take place at the level of the household. Of critical importance to the model is whether production takes place before or after the realization of idiosyncratic shocks. If it takes place after the realization of shocks, then it can be incorporated directly into the consumption set X^i , but allowing net trades that can be achieved through a feasible household production plan. However, the traditional model of moral hazard involves production that takes place prior to the realization of private information. In the standard moral hazard model, observed output is produced from unobserved effort; more effort increases the probability of producing high levels of output. Notice that the assumption that inputs change the probability of states is not consistent with the usual general equilibrium formulation in which states and their probabilities are exogenously fixed. This can be understood through a simple example: effort may be between 0 and 1; output may be either 0 or 1, and the probability of getting a unit of output is equal to the amount of effort expended. To eliminate probabilities from the production function, we use a standard trick of imagining that there are a continuum of states $s \in [0,1]$. If the effort level is e then output in states $s \leq e$ is 1 and in states $s > e$ is 0.

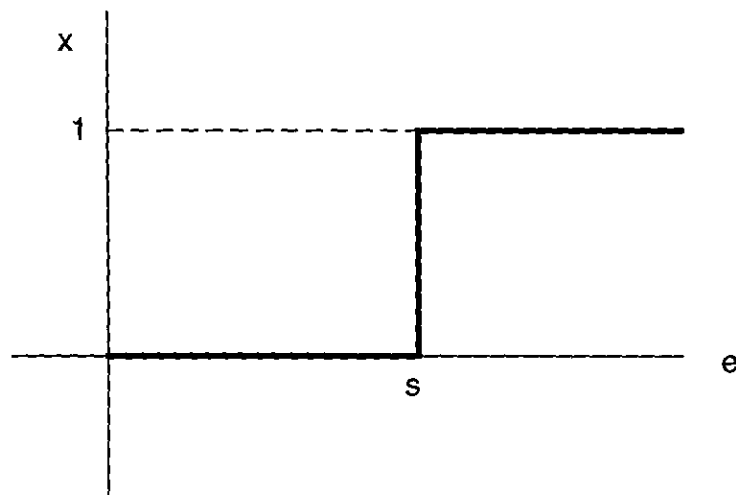


Figure 5.1: Output as a Function of Effort in State s

Notice that this production technology is not convex, which is why lotteries may play an important role with production. Since effort is unobservable, and can be inferred from the state, we should assume here that the state is private information. Here an announcement of the state is equivalent to an announcement of the effort level.

Notice that in this setting with production *ex ante* lotteries play a distinct role from *ex post* lotteries: *ex ante* lotteries can be used to coordinate production decisions, while *ex post* lotteries can be used to weaken the incentive constraints; in the previous setting *ex post* lotteries can substitute for *ex ante* lotteries but not conversely. Here, neither type of lottery can substitute for the other.

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