# Population, Technology, and Growth: From the Malthusian Regime to the Demographic Transition

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# Population, Technology, and Growth: From the Malthusian Regime to the Demographic Transition

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Abstract

This paper develops a unified model of growth, population, and technological progress that is consistent with long-term historical evidence. The economy endogenously evolves through three phases. In the Malthusian regime, population growth is positively related to the level of income per capita. Technological progress is slow and is matched by proportional increases in population, so that output per capita is stable around a constant level. In the post-Malthusian regime, the growth rates of technology and total output increase. Population growth absorbs much of the growth of output, but income per capita does rise slowly. The economy endogenously undergoes a demographic transition in which the traditionally positive relationship between income per capita and population growth is reversed. In the Modern Growth regime, population growth is moderate or even negative, and income per capita rises rapidly. Two forces drive the transitions between regimes: First, technological progress is driven both by increases in the size of the population and by increases in the average level of education. Second, technological progress creates a state of disequilibrium, which raises the return to human capital and induces parents to substitute child quality for quantity.

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## 1. Introduction

This paper examines the evolution of the relationship between population growth, technological change, and the standard of living. It develops a unified model that encompasses three distinct regimes that have characterized the process of economic development. We call these regimes "Malthusian," "Post-Malthusian," and "Modern Growth." The analysis focuses on two differences between these regimes: first, in the behavior of income per capita, and second, in the relationship between the level of income per capita and the growth rate of population.

The modern growth regime is characterized by steady growth in both income per capita and the level of technology. In this regime there is a negative relationship between the level of output and the growth rate of population: the highest rates of population growth are found in the poorest countries, and many rich countries have population growth rates near zero.

At the other end of the spectrum is the Malthusian regime. Technological progress and population growth were glacial by modern standards, and income per capita was roughly constant. Further, the relationship between income per capita and population growth was the opposite of that which exists today: "The most decisive mark of the prosperity of any country," observed Smith (1776), "is the increase in the number of its inhabitants."

The Post Malthusian regime, which fell between the two just described, shared one characteristic with each of them. Income per capita grew during this period, although not as rapidly as it would during the Modern Growth regime. At the same time, the Malthusian relationship between income per capita and population growth was still in place. Rising income was reflected in rising population growth rates. The key event that separates the Malthusian and Post-Malthusian regimes is the acceleration in the pace of technological progress, while the event that separates the Post-Malthusian and Modern

Growth eras is the demographic transition.

The most basic description of the relation between population growth and income was proposed by Malthus (1798). The Malthusian model has two key components. The first is the existence of some factor of production, such as land, which is in fixed supply, implying decreasing returns to scale for all other factors. The second is a positive effect of the standard of living on the growth rate of population. According to Malthus when population size is small, the standard of living will be high, and population will grow as a natural result of passion between the sexes. When population size is large, the standard of living will be low, and population will be reduced by either the "preventive check" (intentional reduction of fertility) or by the "positive check" (malnutrition, disease, and famine).

The Malthusian model implies that, in the absence of changes in the technology or in the availability of land, the population will be stable around a constant level. Further, improvements in technology will, in the long run, be offset by increases in the size of the population. Countries with superior technology will have denser populations, but the standard of living will not be related to the level of technology, either over time or across countries.

The Malthusian model's predictions are consistent with the evolution of technology, population, and output per capita for most of human history. First, the standard of living was roughly constant. Maddison (1982) estimates that the growth rate of GDP per capita in Europe between 500 and 1500 was zero. Lee (1980) reports that the real wage in England was roughly the same in 1800 as it had been in 1300. Clark (1957) concludes that income per capita in Greece in 400 BC was roughly equivalent to that in Britain in 1850 or Germany and France in 1870. According to Chao's (1986) analysis, real wages in China were lower at the end of the 18th century than they had been at beginning of the first century. Mokyr (1990), Lucas (1996), and Pritchett (1997) argue that even in the richest countries, the phenomenon of trend growth in living standards is only a few centuries old. Similarly, population growth was nearly zero, reflecting the slow pace of technological progress. For example, Livi-Bacci (1997) estimates the growth rate of world population from the year 1 to 1750 at 0.064 percent per year. And yet this growth represented a great increase over the rates in earlier periods (Coale, 1974).[1]

Fluctuations in population and wages also bear out the predictions of the Malthusian model. Lee (1997) reports positive income elasticity of fertility and negative income elasticity of mortality from studies examining a wide range of pre-industrial countries. Similarly, Wrigley and Schofield (1981) find that there was a strong positive correlation between real wages and marriage rates in England over the period 1551-1801. Negative shocks to population, such as the Black Death, were reflected in higher real wages and a lower age of marriage (Livi-Bacci, 1997). In North-West Europe, the Malthusian "preventive check" was enforced by late marriage, as couples were forced to inherit or save up to buy a shop, cottage, or farm before marrying. But settlers from that region who came to the American colonies, where land was abundant, married early and bred prolifically. (Stone, 1977; Haines, 1997).

Finally, the prediction of the Malthusian model that differences in technology should be reflected in population density but not in standards of living is also borne out. As argued by Easterlin (1981), Lucas (1996), and Pritchett (1997), prior to 1800 differences in standards of living among countries were quite small by today's standards. And yet there did exist wide differences in technology. China's sophisticated agricultural technologies, for example, allowed high per-acre yields, but failed to raise the standard of living above subsistence. Similarly in Ireland a new productive technology – the potato – allowed a large increase in population over the century prior to the Great Famine

[1]Note that in the absence of a fixed factor of production, this near constancy of population would require an unlikely specification of preference parameters.

without any improvement in standards of living.[2] Using this interpretation, Kremer (1993) argues that changes in the size of population can be taken as a direct measure of technological improvement.

Ironically, it was only shortly before the time that Malthus wrote that humanity began to emerge from the trap that he described. Figure 1 shows the growth rate of total output in Western Europe between the years 500 and 1990, as well as the breakdown between growth of output per capita and growth of population.[3] The figure demonstrates that the process of emergence from the Malthusian trap was a slow one. The growth rate of total output in Europe was 0.3 percent per year between 1500 and 1700, and 0.6 percent per year between 1700 and 1820. In both periods, two-thirds of the increase in total output was matched by increased population growth, so that the growth of income per capita was only 0.1 percent per year in the earlier period and 0.2 percent per year in the later one. In the United Kingdom, where growth was the fastest, the same rough division between total output growth and population growth can be observed: Total output grew at an annual rate of 1.1 percent in the 120 years after 1700, while population grew at an annual rate of 0.7 percent.

Thus the initial effect of faster income growth in Europe was to increase population. Income per capita rose much more slowly than did total output. And as income per capita rose, population grew ever more quickly. Only the fact that output growth accelerated allowed income per capita to continue rising. During this Post-Malthusian Regime, the Malthusian mechanism linking higher income to higher population growth continued to function, but the effect of higher population on diluting resources per capita, and thus lowering income per capita, was counteracted by technological progress, which allowed income to keep rising.

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[2]Livi-Bacci (1997).

[3]Data for 500-1820 are from Maddison (1982) and apply to all of Europe. Data for 1820-1990 are from Maddison (1995), table G, and apply to Western Europe.

Both population and income per capita continued to grow after 1820, but increasingly the growth of total output was expressed as growth of income per capita. Indeed, while the rate of total output growth increased, the rate of growth of population peaked in the 19th century and then began to fall. Population growth was 40 percent as large as total output growth over the period 1820-1870, but only 20 percent as large as total output growth over the period 1929- 1990. Over the next several decades much of Western Europe is forecast to have negative population growth.

The dynamics of population growth reflected both changes in constraints and qualitative changes in household behavior induced by the economic environment. The Malthusian demographic regime had been characterized by high levels of both fertility and mortality. As living standards rose, mortality fell. Between the 1740s and the 1840s, life expectancy at birth rose from 33 to 40 in England and from 25 to 40 in France (Livi-Bacci, 1997). Fogel (1997) estimates that 85 percent of the decline in mortality in France between 1785 and 1870 was due simply to better nutrition. Mortality reductions led to growth of the population both because more children reached breeding age and because each person lived for a larger number of years. The initial effect of higher income was also to raise fertility directly, primarily by raising the propensity to marry. Fertility rates increased in most of Western Europe until the second half of the nineteenth century, peaking in England and Wales in 1871 and in Germany in 1875.[4] Thus, in Malthusian terms, the positive check was being weakened and the preventive check was being less assiduously enforced. But as income continued to rise, population growth fell further below the maximum rate that could be sustained given the mortality regime. The reduction in fertility was at its most rapid in Europe around the turn of the century. In England, for example, live births per 1000 women aged 15-44 fell from 153.6 in 1871-80 to 109.0

[4]See Dyson and Murphy (1985), and Coale and Treadway (1986). The exception was France, where fertility started to decline in the early 19th century.

in 1901-10.[5] The reversal of the Malthusian relation between income and population growth corresponded to an increase in the level of resources invested in each child. For example, the average number of years of schooling in England and Wales rose from 2.3 for the cohort born between 1801 and 1805 to 5.2 for the cohort born 1852-56 and 9.1 for the cohort born 1897-1906. [6]

The emergence from the Malthusian trap raises intriguing questions. How is it that the link between income per capita and population growth, which had for so long been a constant of human existence, was so dramatically severed? And how does one account for the sudden spurt in growth rates?

The existing literature on the relation between population growth and output has tended to focus on only one of the regimes described above. The majority of the literature has been oriented toward the modern regime, trying to explain the negative relation between income and population growth either cross-sectionally or within a single country over time. Among the mechanisms highlighted in this literature are that higher returns to child quality in developed economies induce a substitution of quality for quantity (Becker, Murphy, and Tamura, 1990); that developed economies pay higher relative wages of women, thus raising the opportunity cost of children (Galor and Weil, 1996); and that the net flow of transfers from parents to children grows (and possibly switches from negative to positive) as countries develop (Caldwell, 1976).[7] The negative effect of high income on fertility is often examined in conjunction with a model in which high fertility has a negative effect on income due to capital dilution.[8]

Two recent papers concerned with the Malthusian regime are Lucas (1996) and Kremer (1993). The former presents a Malthusian model in which households make

[5]Wrigley (1969).

[6]Matthews, Feinstein, and Odling-Smee (1982), Table E.1.

[7]See Birdsall (1988), Ehrlich and Lui (1997), and Schultz (1997) and for surveys of the literature in this area.

[8]It is interesting to note that the effect of population growth on the level of natural resources per capita, which is at the center of the Malthusian model, is absent in current growth literature.

optimizing choices over fertility and consumption, but it does not model the transition out of this regime. The latter develops a model in which the rate of technological progress is proportional to the size of the population. The model produces an acceleration in the growth rate of total output, as in the Post-Malthusian regime, but the level of output per capita remains constant and the demographic transition does not follow.[9]

The goal of this paper is to describe the history described above - from the Malthusian regime, through the Post-Malthusian regime and the demographic transition, to the Modern Growth regime - in a single, unified model. At the heart of our model is a novel explanation for the reduction in fertility that has allowed income per capita to rise so far above subsistence. Most studies of the demographic transition focus on the effect of a high *level* of income in inducing parents to switch to having fewer, higher quality children. In our model, parents also switch out of quantity and into quality, but do so not in response the level of income but rather in response to technological progress. In particular, we argue that the "disequilibrium" brought about by technological change raises the rate of return to human capital, and thus induces the substitution of quality for quantity.

The argument that technological progress itself raises the return to human capital was most clearly stated by Schultz (1964). Examining agriculture, Schultz argued that when productive technology has been constant for a long period of time, farmers will have learned to use their resources efficiently. Children will acquire knowledge of how to deal with this environment directly from observing their parents, and formal schooling

[9]See also Eckstein, Stern, and Wolpin (1988). Another strand of literature has attempted to model the acceleration of output growth at the time of the Industrial Revolution without considering the determinants of population growth. Goodfriend and McDermott (1995) examine a model in which the economy endogenously moves through periods of primitive home production and pre-industrial specialization before experiencing an industrial revolution and a quickening in the growth of income per capita. The driving force behind these transitions is the growth of population, which is taken as exogenous. In the model of Acemoglu and Zilibotti (1997), there is a long period of slow, uneven growth before an economy finally takes off into a regime of steady growth, but population growth plays no role in the process.

will have little economic value. But when technology is changing rapidly, the knowledge gained from observing the previous generation will be less valuable, and the trial-and-error process which led to a high degree of efficiency under static conditions will not have had time to function. New technology will create a demand for the ability to analyze and evaluate new production possibilities, which will raise the return to education. Schultz (1975) cites a wide range of evidence in support of this theory. Similarly, Foster and Rosenzweig (1996) find that technological change during the green revolution in India raised the return to schooling, and that school enrollment rates responded positively to this higher return. Such an effect would be a natural explanation for the dramatic rise in schooling in Europe over the course of the 19th century.

The effect of technology on the return to human capital in which we are most interested is the short run impact of a new technology. In the long run, technologies may either be "skill biased" or "skill saving." But we would argue that the introduction of new technologies is mostly skill biased.[10] For example, Williamson (1985; Table 3.7) concludes that early industrialization raised the return to skills. The ratio of average wages of skilled workers to unskilled workers in Britain rose from 2.45 in 1815 to 3.77 in 1851, whereas the 60 years after 1851 saw a significant reduction in wage inequality.[11] If technological changes are skill-biased in the long run, then the effect on which we focus will be enhanced, while if technology is skill-saving then our effect will be diluted.

The second piece of the model is more straightforward: the choice of parents regarding the education level of their children has implication for the speed of technological progress. Children with high levels of human capital are in turn more likely to advance the technological frontier. For example, Cameron (1989) finds a high correlation between the level of education and the speed of industrialization in Nineteenth century Europe.[12]

[10] See Galor and Tsiddon (1997) as well.

[11] The issue of whether technology has been skill-complementing is far from settled - See Goldin and Katz (1996) for a discussion.

[12] Easterlin (1981) and Bartel and Lichtenberg (1987) find that educated individuals have a compar-

We also allow the overall size of the population to positively influence the growth rate of technology, as in Kremer (1993) and Jones (1995).

The final piece of our model is the most Classical: as population rises, the land to population ratio falls, and the wage falls. If technology is static, then the size of the population is self-equilibrating. But technological progress can undo this mechanism, allowing wages to rise.

The model produces a Malthusian "pseudo steady state" that will be stable over long periods of time, but will vanish endogenously in the long run. In this Malthusian regime output per capita is stationary. Technology progresses only slowly, and is reflected in proportional increases in output and population. Shocks to the land to labor ratio will induce temporary changes in the real wage and fertility, which will in turn drive income per capita back to its stationary, equilibrium level. Because technological progress is slow, the return to human capital is low, and so parents have little incentive to substitute child quality for quantity.

The key effect which makes the Malthusian pseudo steady state vanish in the long run is the impact of population size on the rate of technological progress. At a sufficiently high level of population, the rate of population-induced technological progress will be high enough that parents will find it optimal to provide their children with some human capital. At this point, a virtuous circle develops: higher human capital raises technological progress, which in turn raises the value of human capital.

Increased technological progress initially has two effects on population growth. On the one hand, improved technology eases households' budget constraints, allowing them to spend more resources on raising children. On the other hand, it induces a reallocation of these increased resources toward child quality. In the Post-Malthusian regime, the former effect dominates, and so population growth rises. Eventually, however, more

ative advantage in implementing new technologies. Growth models in which a higher level of education raises the speed of technological progress include Lucas (1988) and Azariades and Drazen (1990).

rapid technological progress due to the increase in the level of human capital triggers a demographic transition: wages and the return to child quality continue to rise, the shift away from child quantity becomes more significant, and population growth declines. In the Modern Growth regime, technology and output per capita increase rapidly, while population growth is moderate.

The rest of this paper is organized as follows. In Section 2, we formalize the assumptions about the determinants of fertility and relative wages presented above, and incorporate them into an overlapping generations model. Section 3 derives the dynamical system implied by the model, and analyzes the evolution of the economy along transitions to the steady state. Section 4 concludes by discussing possible extensions of the model.

## 2. The Basic Structure of the Model

Consider a small, open, overlapping-generations economy that operates in a perfectly competitive world where international capital movements are unrestricted and economic activity extends over infinite discrete time.[13] In every period the economy produces a single homogeneous good that can be used for either consumption or investment. The good is produced using physical capital, efficiency units of labor, and land.

In every period the three factors of production are supplied in competitive factor markets. The supply of capital and labor are endogenously determined while the supply of land is exogenous and fixed over time. The stock of physical capital in every period is given by the sum of the economy's aggregate saving and international borrowing, net of the aggregate value of land purchases. The number of efficiency units of labor is determined by households' decisions in the preceding period regarding the number and level of human capital of their children.

[13] The assumption of capital mobility and an exogenous, constant world interest rate is made for analytic tractability. As will become apparent, capital has no role in the mechanism that we examine and the qualitative results would not be changed if the supply of capital were endogenously determined in a closed economy.

## 2.1 Production of Final Output

Production occurs within a period according to a constant-returns-to-scale neoclassical production technology that is subject to endogenous technological progress. The output produced at time $t$, $Y_t$, is

$$Y_t = B_t K_t^{\beta} H_t^{\alpha(1-\beta)} X^{(1-\alpha)(1-\beta)}, \tag{1}$$

where $K_t$, $H_t$, and $X$ are the quantities of capital, efficiency units of labor, and land, employed in production at time $t$, $\alpha \in (0,1)$ and $\beta \in (0,1)$ are parameters which are fixed over time, and $B_t > 0$, represents the endogenously determined technological level at time $t$. The production function is therefore strictly increasing and concave, satisfying the neoclassical boundary conditions which assure the existence of an interior solution to the producer's profit-maximization problem.

Producers operate in a perfectly competitive environment. Given the wage rate per efficiency unit of labor, the interest rate on capital, and the rent on land, producers determine the level of employment of labor, capital, and land so as to maximize profits. Suppose that world interest rate is constant at a level $\bar{r} > 0$. Since the small economy permits unrestricted international lending and borrowing, its interest rate will also be $\bar{r}$. The amount of capital employed in production at time $t$ is therefore a function of $B_t, H_t, X, \bar{r}$, $\alpha$ and $\beta$. Substituting the level of capital into the production function yields:

$$Y_t = a_t H_t^{\alpha} X^{1-\alpha} \equiv H_t^{\alpha}(A_t X)^{(1-\alpha)}, \tag{2}$$

where the state of technology at time $t$ is represented by the technological parameter, $A_t \equiv a_t^{1/(1-\alpha)} \equiv [(\beta/\bar{r})^{\beta/(1-\beta)} B_t^{1/(1-\beta)}]^{1/(1-\alpha)}$.[14] The multiplicative form in which technology, $A_t$, and land, $X_t$, appear in the production function implies that the relevant

[14]The parameter $A_t$ is a positive and stationary transformation of the original technological parameter $B_t$. Thus changes in $A_t$ can be used to measure changes in technology. In particular,

$$\frac{A_{t+1}}{A_t} = \left(\frac{B_{t+1}}{B_t}\right)^{1/(1-\beta)(1-\alpha)}.$$

factor for the output produced is the product of the two, which we define as "effective resources."

Output per worker produced at time $t$, $y_t$, is therefore

$$y_t = h_t^{\alpha} x_t^{(1-\alpha)} \equiv y(h_t, x_t), \tag{3}$$

where

$$y_h(h_t, x_t) > 0 \quad and \quad y_x(h_t, x_t) > 0, \quad \forall (h_t, x_t) >> 0. \tag{4}$$

$h_t \equiv H_t/L_t$ is the number of efficiency units of labor per-worker, and $x_t \equiv (A_t X)/L_t$ is effective resources per worker at time $t$.

Given the structure of the production technology and the competitiveness of markets, the return to an efficiency unit of labor at time $t$, $w_t$, is

$$w_t = \alpha(1-\beta)(x_t/h_t)^{1-\alpha} \equiv w(h_t, x_t), \tag{5}$$

where

$$w_h(h_t, x_t) < 0 \quad and \quad w_x(h_t, x_t) > 0, \quad \forall (h_t, x_t) >> 0. \tag{6}$$

The total return to land (including appreciation) at time $t$, $\rho_t$, and the rate of return to capital at time $t$, $r_t$, are equal to one another, since individuals may save either by purchasing capital or land. Hence, given the constancy of world interest rate at the level $\bar{r}$, it follows that $\rho_t = r_t = \bar{r}$.[15]

## 2.2 Individuals: Fertility, Human Capital, Saving, and Consumption

In each period $t$ a generation that consists of $L_t$ individuals joins the labor force. Each individual has a single parent. Individuals within a generation are identical in

In the rest of the paper we specify technological progress in terms of $A_t$, but we could have done so in terms of $B_t$.

[15]The price of land is determined implicitly so as to assure that, given the marginal productivity of land, the rate of return on land will be equal to $\bar{r}$. As will become apparent, an explicit analysis of the evolution of the price of land is not necessary.

their preferences and their level of human capital. Members of generation $t$ live for three periods.[16] In the first period of life (childhood), $t-1$, individuals consume a fraction of their parent's time. The required time increases with children's quality. In the second period of life (parenthood), $t$, individuals are endowed with one unit of time, which they allocate between childrearing and labor force participation. They choose the optimal mixture of quantity and quality of children and supply their remaining efficiency units of labor in the labor market. They earn the competitive market wage per each efficiency unit of labor and save their income for future consumption. In the third period of life (old age), $t+1$, individuals do not work. They consume their savings from the previous period along with accrued interest.

### 2.2.1 Preferences

The preferences of members of generation $t$, which are defined over consumption in old age, above a subsistence level $\tilde{c} > 0$, as well as over the potential aggregate income of their children are depicted in Figure 2.[17] They are represented by the utility function[18]

$$u^t = (c_{t+1})^{(1-\gamma)}(w_{t+1}n_t h_{t+1})^{\gamma} \tag{7}$$

where $n_t$ is the number of children of individual $t$, $h_{t+1}$ is the level of human capital of each child, and $w_{t+1}$ is the wage per efficiency unit of labor at time $t+1$.[19]

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[16]In order to simplify the notation, a generation is indexed by the period in which it participates in the labor market (i.e., parenthood).

[17]The consumption set, i.e., the set of (physiologically) feasible consumption bundles, is: $\{(c_{t+1}, w_{t+1}n_t h_{t+1}) \in R^2_+ : \ c_{t+1} \geq \tilde{c}.\}$

[18]The second component of the utility function may represent either intergenerational altruism, or implicit concern about potential support from children in old age along the lines of the old-age security hypothesis. The interpretation that emphasizes intergenerational altruism reflects an implicit bounded rationality on the part of the parent. Alternative formulations according to which individuals generate utility from the utility of their children, or from the *actual* aggregate income of their offspring would require parental predictions about the chosen fertility mixture of their dynasty or offspring. These approaches would greatly complicate the model without changing the qualitative results.

[19]The introduction of consumption in the second period of life, $t$, does not change the qualitative results. Maintaining the Cobb-Douglas specification, the fraction of output saved in period $t$ would be constant. Thus the dynamical system that governs the evolution of the economy would be altered only

The utility function is strictly monotonically increasing and strictly quasi-concave, satisfying the conventional boundary conditions that assure that, for sufficiently high income, there exists an interior solution for the utility maximization problem. However, as depicted in Figure 3, for a sufficiently low level of income the subsistence consumption constraint is binding and there is a corner solution with respect to the consumption level.[20]

### 2.2.2 Budget Constraint: Quantity-Quality of Children Vs. Consumption

Following the standard model of household fertility behavior (Becker, 1960) it is assumed that the household chooses the number of children and their quality in the face of a constraint on the total amount of time that can be devoted to child-raising and labor market activities. We further assume that the only input required to produce both child quantity and child quality is time.[21] Since all members of a generation are identical in their endowments, the budget constraint is not affected if child quality is produced by professional educators rather than by parents.

Let $\tau^q + \tau^e e_{t+1}$ be the time cost for a member of generation $t$ of raising a child with an education level $e_{t+1}$.[22] That is, $\tau^q$ is the fraction of the individual's unit time endowment that is required in order to raise a child, regardless of quality, and $\tau^e$ is the fraction of the individual's (or of an equally educated teacher's) unit time endowment that is required per each unit of education of each child.[23]

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by a multiplicative constant.

[20]As will become clear below, the presence of a subsistence consumption constraint provides the Malthusian piece of our model. The formulation that we use implicitly stresses a "demand" explanation for the positive income elasticity of population growth at low income levels, since higher income will allow individuals to afford more children. However, one could also cite "supply" factors, such as declining infant mortality and increased natural fertility, to explain the same phenomenon. See Birdsall (1988) and Olsen (1994).

[21]If both time and goods are required in order to produce child quality, the process we describe would be intensified. As the economy develops and wages increase, the relative cost of a quality child will diminish and individuals will substitute quality for quantity of children

[22]$e_{t+1}$ measures the level of education (quality) of the child in the second period of life $t+1$.

[23]The existence of economies of scale in raising quantity and quality children would raise the number of children and their quality in every period, but the patterns of changes in these absolute magnitudes

Consider members of generation $t$ who are endowed with $h_t$ efficiency units of labor at time $t$. Define potential income, $z_t$, as the amount that they would earn if they devoted their entire time endowment to labor force participation: $z_t \equiv w_t h_t$. Since individuals do not generate utility from consumption at time $t$, their potential income is divided between expenditure on child rearing (quantity as well as quality), at an opportunity cost of $w_t h_t[\tau^q + \tau^e e_{t+1}]$ per child, and savings for future consumption, $s_t$. Hence, in the second period of life (parenthood), the individual faces the budget constraint:

$$w_t h_t n_t(\tau^q + \tau^e e_{t+1}) + s_t \leq w_t h_t. \tag{8}$$

In the third period of life, a member of generation $t$ consumes the value of savings with accrued interest. Hence,

$$c_{t+1} = s_t(1 + \overline{r}). \tag{9}$$

### 2.2.3 The Production of Human Capital

An individual's level of human capital is determined by his quality (education) as well as by the technological environment. Incorporating the insight of Schultz (1964) discussed above, technological progress is assumed to raise the value of education in producing human capital.[24] The level of human capital of children of members of generation $t$, $h_{t+1}$, is an increasing function of their quality (education), $e_{t+1}$, and a decreasing function of the rate of progress in the state of technology from period $t$ to period $t+1$, $g_{t+1} \equiv (A_{t+1} - A_t)/A_t$. The higher is children's quality, the smaller is the adverse effect of technological progress.

$$h_{t+1} = h(e_{t+1}, g_{t+1}), \tag{10}$$

would not differ qualitatively.

[24]Technological progress changes the nature of occupations, and reduces the adaptability of existing human capital for the new technological environment. That is, in the presence of technological progress, the applicability of level of human capital that can be absorbed from the existing technological environment erodes.

where $\forall(e_{t+1}, g_{t+1}) \geq 0$,

$$(A1) \qquad \begin{aligned} &\bullet \quad h_e(e_{t+1}, g_{t+1}) > 0; \quad h_{ee}(e_{t+1}, g_{t+1}) < 0. \\ &\bullet \quad h_g(e_{t+1}, g_{t+1}) < 0; \quad h_{gg}(e_{t+1}, g_{t+1}) > 0; \quad h_{eg}(e_{t+1}, g_{t+1}) > 0. \\ &\bullet \quad h(e_{t+1}, g_{t+1}) > 0; \quad \lim_{g_{t+1} \to \infty} h(0, g_{t+1}) = 0; \end{aligned}$$

Hence, the individual's level of human capital is an increasing, strictly concave function of the quality (education), and a decreasing strictly convex function of the rate of technological progress.[25] Furthermore, education lessens the adverse effect of technological progress. That is, technology complements skills in the production of human capital. The higher the rate of technological progress the higher the relative return to quality.

Moreover, although the number of efficiency units of labor per- worker is diminished during the transition from one technological state to another - the 'erosion effect' - the *effective* number of the efficiency units of labor per worker, which is the product of the workers' level of human capital and the economy's technological state (reflected in the wage per efficiency unit of labor), is presumably higher as a result of technological progress. That is, the overall effect of technological progress from period $t$ to period $t+1$ on the potential income of members of generation $t+1$ may be positive. Furthermore, once technology returns to a stationary state, the 'erosion effect' is eliminated, whereas the positive 'productivity effect' is still in place.

### 2.2.4 Optimization

Members of generation $t$ choose the number and quality of their children, and therefore their own savings and old-age consumption, so as to maximize their intertemporal utility function. Substituting (8)-(10) into (7), the optimization problem of a member

[25] Strict convexity with respect to $g_{t+1}$ and the boundary condition are not essential. They are designed to assure that the level of human capital will not become zero at high rates of technological progress. Alternative assumptions will not affect the qualitative analysis.

of generation $t$ is:

$$\{n_t, e_{t+1}\} = argmax\{(1+\overline{r})w_t h_t[1 - n_t(\tau^q + \tau^e e_{t+1})]\}^{1-\gamma}\{(w_{t+1} n_t h(e_{t+1}; g_{t+1}))\}^{\gamma} \quad (11)$$

$$\text{subject to:} \quad (1+\overline{r})w_t h_t[1 - n_t(\tau^q + \tau^e e_{t+1})] \geq \tilde{c}.$$

$$(n_t, e_{t+1}) \geq 0.$$

The optimization with respect to $n_t$ implies that, as long as potential income at time $t$ is sufficiently high so as to assure that $c_{t+1} > \tilde{c}$, the time spent by individual $t$ raising children is a fixed fraction $\gamma$, whereas the remaining fraction $1-\gamma$ is devoted for labor force participation. However, for low levels of potential income, the inequality constraint binds. The individual consumes the subsistence level, $\tilde{c}$, and uses the rest of the time endowment for childrearing. That is,

$$n_t[\tau^q + \tau^e e_{t+1}] = \begin{cases} \gamma & \text{if} \quad c_{t+1} > \tilde{c} \\ 1 - [\tilde{c}/(1+\overline{r})w_t h_t] & \text{if} \quad c_{t+1} = \tilde{c}. \end{cases} \quad (12)$$

As follows from the budget constraint - (8) - the saving of individual $t$, $s_t$, is therefore

$$s_t = \begin{cases} (1-\gamma)w_t h_t & \text{if} \quad c_{t+1} > \tilde{c} \\ \tilde{c}/(1+\overline{r}) & \text{if} \quad c_{t+1} = \tilde{c}. \end{cases} \quad (13)$$

Since $c_{t+1} = s_t(1 + r_{t+1})$, it follows from (13) that ,

$$c_{t+1} \begin{cases} > \tilde{c} & \text{if} \quad z_t \equiv w_t h_t > \tilde{z} \\ = \tilde{c} & \text{if} \quad z_t \equiv w_t h_t \leq \tilde{z}, \end{cases} \quad (14)$$

where,

$$\tilde{z} \equiv \tilde{c}/(1-\gamma)(1+\overline{r}), \quad (15)$$

is the critical level of potential income above which the individual chooses to consume more than the subsistence level and below which the individual consumes the subsistence level.

As long as the potential income of a member of generation $t$, $z_t \equiv w_t h_t$, is below $\tilde{z}$, then the fraction of time necessary to assure subsistence consumption, $\tilde{c}$, is larger than $1 - \gamma$ and the fraction of time devoted for child rearing is therefore below $\gamma$. As the wage per efficiency unit of labor increases, the individual can generate the subsistence consumption with smaller labor force participation and the fraction of time devoted to childrearing increases.[26] As will become apparent from Proposition 1, the entire increase in the fraction of time devoted to childrearing is used to raise the quantity of children without a change in quality. As long as potential income of a member of generation $t$, $z_t \equiv w_t h_t$, is higher than $\tilde{z}$, the subsistence consumption constraint is not binding and a constant fraction of the unit time endowment, $\gamma$, is devoted to child rearing regardless of how high wages are. Hence further increases in wages are devoted entirely to increased consumption.

Figure 3 shows the effect of an increase in potential income $z_t$ on the individual's choice of total time spent on children and consumption. As is apparent from the diagram the income expansion path is vertical until the level of income passes the critical level that permits consumption to exceed the subsistence level. Thereafter, the income expansion path becomes horizontal at a level $\gamma$ in terms of time devoted for childrearing.

Regardless of whether potential income is above or below $\tilde{z}$, increases in wages will not change the division of child-rearing time between quality and quantity. What *does* affect the division between time spent on quality and time spent on quantity is the

[26] Durand (1975) and Goldin (1994) report that, looking across a large sample of countries, the relationship between women's labor force participation and income is U-shaped. The model presented here explains the negative effect of income on labor force participation for poor countries, and further predicts that this effect should no longer be operative once potential income has risen sufficiently high. It does not, however, explain the positive effect of income on participation for richer countries. See, however, Galor and Weil (1997) for a model that does explain this phenomenon.

rate of technological progress, which changes the return to education. Specifically, using (12), the optimization with respect to $e_{t+1}$ implies that independently of the subsistence consumption constraint

$$G(e_{t+1},g_{t+1}) \equiv (\tau^q+\tau^e e_{t+1})h_e(e_{t+1},g_{t+1})-\tau^e h(e_{t+1},g_{t+1}) \begin{cases} =0 & \text{if} \quad e_{t+1}>0 \\ \leq 0 & \text{if} \quad e_{t+1}=0, \end{cases} \tag{16}$$

where as follows from (A1), $\forall g_{t+1} \geq 0$, and $\forall e_{t+1} > 0$.

$$\begin{aligned} G_e(e_{t+1},g_{t+1}) &= (\tau^q+\tau^e e_{t+1})h_{ee}(e_{t+1},gt+1) < 0; \\ G_g(e_{t+1},g_{t+1}) &= (\tau^q+\tau^e e_{t+1})h_{eg}(e_{t+1},g_{t+1}) - \tau^e h_g(e_{t+1},g_{t+1}) > 0. \end{aligned} \tag{17}$$

In particular, as will become apparent from the proof of Lemma 1 , to assure the existence of a positive level of $g_{t+1}$ such that the chosen level of education is 0, it is assumed that:

$$(A2) \qquad G(0,0) = \tau^q h_e(0,0) - \tau^e h(0,0) < 0.$$

The functional relationship between $e_{t+1}$ and $g_{t+1}$ as depicted in Figure 4, is derived in Lemma 1.

**Lemma 1.** *If (A1) and (A2) are satisfied, then the level of education chosen by members of generation t for their children is an increasing function of* $g_{t+1}$.

$$e_{t+1} = e(g_{t+1}) \begin{cases} =0 & \text{if} \quad g_{t+1} \leq \hat{g} \\ >0 & \text{if} \quad g_{t+1} > \hat{g} \end{cases}$$

*where,* $\hat{g} > 0$, *and*

$$e'(g_{t+1}) > 0 \qquad \forall g_{t+1} > \hat{g}$$

**Proof.** As follow from (A1) and (16), $G(0, g_{t+1})$ is monotonically increasing in $g_{t+1}$. Furthermore, (A1) implies that $\lim_{g_{t+1}\to\infty} G(0, g_{t+1}) > 0$, whereas (A2) imples that $G(0,0) < 0$. Hence, there exists $\hat{g} > 0$ such that $G(0,\hat{g}) = 0$, and therefore, as follows from (16) $e_{t+1} = 0$ for $g_{t+1} \le \hat{g}$. Furthermore, it follows from (17) that $e_{t+1}$ is a single valued function of $g_{t+1}$, where $e'_{t+1}(g_{t+1}) = -G_g(e_{t+1}, g_{t+1})/G_e(e_{t+1}, g_{t+1}) > 0$. □

As is apparent from (17), $e''(g_{t+1})$ depends upon the third derivatives of the production function of human capital. A concave reaction of the level of education to the rate of technological progress appears plausible economically, hence it is assumed that[27]

$$(A3) \qquad e''(g_{t+1}) < 0 \qquad \forall g_{t+1} > \hat{g}.$$

Furthermore, substituting $e_{t+1} = e(g_{t+1})$ into (12), using (14), it follows that $n_t$ is a single-valued function:

$$n_t = \begin{cases} \frac{\gamma}{\tau^q + \tau^e e(g_{t+1})} \equiv n^b(g_{t+1}) & \text{if } z_t \ge \tilde{z} \\ \\ \frac{1-[\tilde{c}/(1+\tilde{r})z_t]}{\tau^q + \tau^e e(g_{t+1})} \equiv n^a(g_{t+1}, z_t) & \text{if } z_t \le \tilde{z}. \end{cases} \tag{18}$$

where as follows from (5), (10), and the definition of $z_t$,

$$z_t = w_t h_t = \alpha(1-\beta)x_t^{(1-\alpha)} h_t^{\alpha} \equiv z(e_t, g_t, x_t), \tag{19}$$

where as follows from (A1) $\forall (e_t, g_t, x_t) >> 0$,[28]

[27] Alternatively, if $e(g_{t+1})$ is strictly convex we may assume that for physiological or other reasons, the maximum amount of education that a child can receive is bounded from above. In the model we ignore integer constraints on the number of children, so that absent a constraint on the quality per child, parents might choose to have an infinitesimally small number of children with infinitely high quality. Thus the existence of integer constraints in the real world may be taken as one justification for an upper bound on level of education.

[28] It should be noted that while the partial derivative of $z_t$ with respect to $g_t$ is negative (holding $x_t$ and thus $A_t$ constant), the total derivative of $z_t$ with respect to $g_t$ (holding $A_{t-1}$ constant) may positive.

$$z_e(e_t, g_t, x_t) > 0,\ z_x(e_t, g_t, x_t) > 0,\ z_g(e_t, g_t, x_t) < 0, \tag{20}$$

The following proposition summarizes the properties of the intertemporal functions $e(g_{t+1})$, $n^a(z_t, g_{t+1})$, and $n^b(g_{t+1})$ and their significance for the evolution in the substitution of quality for quantity in the process of development:

**Proposition 1.** Under (A1)-(A4)

*(a) Technological progress that is expected to occur between the first and second periods of children's lives results in a decline in the parents' chosen number of children and an increase in their quality.*

- $\partial n_t / \partial g_{t+1} \leq 0$
- $\partial e_{t+1} / \partial g_{t+1} \geq 0$;

*(b) If parental potential income is below $\tilde{z}$ (i.e., if the subsistence consumption constraint is binding), an increase in parental potential income raises the number of children, but has no effect on their quality.*

- $\partial n_t / \partial z_t > 0 \;\; if \;\; z_t < \tilde{z}$
- $\partial e_{t+1} / \partial z_t = 0 \;\; if \;\; z_t < \tilde{z}$

*(c) If parental potential income is above $\tilde{z}$, an increase in parental potential income does not change the number of children or their quality.*

- $\partial n_t / \partial z_t = 0 \;\; if \;\; z_t > \tilde{z}$
- $\partial e_{t+1} / \partial z_t = 0 \;\; if \;\; z_t > \tilde{z}$

**Proof.** Follows directly from Lemma 1, (12)-(18), and assumptions (A1)-(A4). □

**Corollary 1.** Under (A1)-(A4)

*(a) If parental potential income is below $\tilde{z}$ (i.e., if the subsistence consumption constraint is binding), an increase in the effective resources per worker raises the number of children, but has no effect on their quality.*

- $\partial n_t / \partial x_t > 0 \;\; if \;\; z_t < \tilde{z}$

- $\partial e_{t+1}/\partial x_t = 0 \;\; if \;\; z_t < \tilde{z}$

*(b) If parental potential income is above $\tilde{z}$, an increase in the effective resources per-worker does not change the number of children or their quality.*

- $\partial n_t/\partial x_t = 0 \;\; if \;\; z_t > \tilde{z}$
- $\partial e_{t+1}/\partial x_t = 0 \;\; if \;\; z_t > \tilde{z}$

**Proof.** In light of parts (b) and (c) of Proposition 1, the corollary follows directly from the fact that $z_t$ increases in $x_t$ as established in (20). □

### 2.3 Technological Progress

Suppose that technological progress, $g_{t+1}$, that takes place between periods $t$ and $t+1$ depends upon the education per capita among the working generation in period $t$, $e_t$, and the population size in period t, $L_t$. [29]

$$g_{t+1} \equiv \frac{A_{t+1} - A_t}{A_t} = g(e_t)f(L_t) \tag{21}$$

where[30]

$$\begin{aligned} &g(0) > 0, \quad g'(e_t) > 0, \; and \; g''(e_t) < 0, \quad \forall e_t \geq 0; \\ &f(L_t) > 0, \quad f'(L_t) > 0, \; and \; f''(L_t) < 0, \quad \forall L_t > 0; \end{aligned} \tag{22}$$

Hence, the rate of technological progress between time $t$ and $t+1$ is a positive, increasing, strictly concave function of the size and level of education of the working generation at time $t$. Furthermore, the rate of technological progress is positive even if labor quality is zero.

As will become apparent, the dynamical system of the described economy is rather complex. Population size does not play a qualitative role in the evolution of the economy,

[29]We consider a modificaiton of equaiton (21) along the lines suggested by Jones (1995) in Section 3.2.2.

[30]It should be noted that since $e_t$ is pre-determined in period $t-1$, the state of technology that will be used for production in period $t+1$ is known prior to the time in which members of generation $t$ choose the education level of their children.

except for its significant role in the takeoff from the Malthusian Regime. Hence, in order to simplify the exposition without affecting the qualitative results, the dynamical system is analyzed initially under the assumption that population size has no effect on technological progress. In particular, let

$$(A4) \qquad f(L_t) = 1 \qquad \forall L_t > 0.$$

In later stages of the analysis the effect of the size of population on the relationship between technological progress and the level of education as specified in (21) is considered.

### 2.4. The Evolution of Population, Technology, and Effective Resources

The size of population at time $t+1$, $L_{t+1}$, is

$$L_{t+1} = n_t L_t, \tag{23}$$

where $L_t$ is the size of population at time $t$, $n_t$ is the number of children per person, and $n_t - 1$ is the rate of population growth. The size of the population at time 0 is historically given at a level $L_0$.

The state of technology at time $t+1$, $A_{t+1}$, as derived from (21), is

$$A_{t+1} = (1 + g_{t+1})A_t, \tag{24}$$

where the state of technology at time 0 is historically given at a level $A_0$.

The evolution of effective resources per worker, $x_t \equiv (A_t X_t)/L_t$, depends on the evolution in the technological level and the rate of population growth:

$$x_{t+1} = \frac{1 + g_{t+1}}{n_t} x_t, \tag{25}$$

where $x_0 \equiv A_0 X/L_0$ is historically given.

Substituting (19),(21) and (A4) into (18), and (18),(A4),(21) into (25), it follows that,

$$x_{t+1} = \begin{cases} \frac{[1+g(e_t)][\tau^q+\tau^e e(g(e_t))]}{\gamma} x_t \equiv \phi^b(e_t)x_t & \text{if} \quad z_t \geq \tilde{z} \\ \\ \frac{[1+g(e_t)][\tau^q+\tau^e e(g(e_t))]}{1-[\bar{c}/(1+\bar{r})z(e_t,g_t,x_t)]} x_t \equiv \phi^a(e_t,g_t,x_t)x_t & \text{if} \quad z_t \leq \tilde{z}, \end{cases} \tag{26}$$

where as follows from Lemma 1, (20), and (22) $\phi^{b\prime}(e_t) > 0$, and $\phi^a_x(e_t, g_t, x_t) < 0$. $\forall e_t \geq 0$.

## 3. The Dynamical System

The development of the economy is characterized by the evolution of output per worker, population, technological level, education per worker, human capital per worker, and effective resources per worker. The evolution of the economy, given (A4) is fully determined by a sequence $\{e_t, g_t, x_t\}_{t=0}^{\infty}$ that satisfies (21), (26), and Lemma 1 in every period $t$.

The dynamical system is characterized by two regimes. In the first regime the subsistence consumption constraint is binding and the evolution of the economy is governed by a *three* dimensional non-linear first-order autonomous system:

$$\begin{cases} x_{t+1} = \phi^a(e_t, g_t, x_t)x_t \\ \\ e_{t+1} = e(g(e_t)) & \text{if} \quad z_t \leq \tilde{z} \\ \\ g_{t+1} = g(e(g_t)), \end{cases} \tag{27}$$

where the initial conditions $e_0, g_0, x_0$ are historically given.

In the second regime the subsistence consumption constraint is not binding and the evolution of the economy is governed by a *two* dimensional non-linear first-order autonomous system:

$$\begin{cases} x_{t+1} = \phi^b(e_t, x_t)x_t \\ \\ e_{t+1} = e(g(e_t)). \end{cases} \qquad \text{if} \quad z_t \geq \tilde{z} \tag{28}$$

In both regimes, however, the analysis of the dynamical system is greatly simplified by the fact that, as follows from Lemma 1, (21), and (A4), the joint evolution of $e_t$ and $g_t$ is determined independently of the $x_t$. Furthermore, the evolution of $e_t$ and $g_t$ is independent of whether the subsistence constraint is binding, and is therefore independent of the regime in which the economy is located. The education level of workers in period $t+1$ depends only on the level of technological progress expected between period $t$ and period $t+1$, while technological progress between periods $t$ and $t+1$ depends only on the level of education of workers in period $t$. Thus we can analyze the dynamics of technology and education independently of the evolution resources per capita.

### 3.1 The Evolution of Technology and Education

The evolution of technology and education, given (A4), is characterized by the sequence $\{g_t, e_t\}_{t=0}^{\infty}$ that satisfies in every period $t$ the equations $g_{t+1} = g(e_t)$, and $e_{t+1} = e(g_{t+1})$. This dynamical sub-system consists in fact of two independent one dimensional, non-linear first-order difference equations that can be written as,

$$e_{t+1} = e(g(e_t)), \tag{29}$$

where the quality of labor in period 0, $e_0$, is historically given, and

$$g_{t+1} = g(e(g_t)), \tag{30}$$

where the rate of technological change from period 0 to period 1, $g_1$, is determined uniquely by $e_0$; $g_1 = g(e_0)$. Hence, the optimal sequence $\{e_t\}_{t=0}^{\infty}$, can be derived directly from (29) and the sequence $\{g_{t+1}\}_{t=0}^{\infty}$, can be generated via (30), or via the static relationship $g_{t+1} = g(e_{t+1})$.

Although the evolution of the sequences $\{e_t\}_{t=0}^{\infty}$, and $\{g_{t+1}\}_{t=0}^{\infty}$, are fundamentally disjoint, and hence can be analyzed in either the plain $(e_{t+1}, e_t)$, or the plain $(g_{t+1}, g_t)$, the structure of this sub-system becomes more apparent in the context of the two dimensional system depicted in the plain $(e_t, g_t)$.

In light of the properties of the functions $e(g_{t+1})$ and $g(e_t)$ given in Lemma 1, (A3)-(A4), and (21)-(22), it follows that in any time period, if population size *does* play a role in technological progress, this dynamical sub-system is characterized by three qualitatively different configurations, which are depicted in Figure 4. The economy shifts endogenously from one configuration to another as population increases and the curve $g(e_t)$ shifts upward to account for the effect of an increase in population.

In Figure 4a, for a range of small population sizes, the dynamical system is characterized by globally stable steady-state equilibria. For a given population size in this range, the steady-state equilibrium is $(\bar{e}, \bar{g}) = (0, g^l)$. As implied by (21), the rate of technological change in a temporary steady state increases monotonically with the size of population, while the level of education remains unchanged.

In Figure 4b, for a range of moderate population sizes, the dynamical system is characterized by three steady-state equilibria. For a given population size in this range, there exist two locally stable steady-state equilibria: $(\bar{e}, \bar{g}) = (0, g^l)$ and $(\bar{e}, \bar{g}) = (e^h, g^h)$, and an interior unstable steady-state $(\bar{e}, \bar{g}) \equiv (e^u, g^u)$. $(e^h, g^h)$ and $g^l$ increase monotonically with the size of population.

Finally, in Figure 4c, for a range of large population sizes, the dynamical system is characterized by globally stable steady-state equilibria. For a given population size in this range, there exists a unique globally stable steady-state equilibrium: $(\bar{e}, \bar{g}) = (e^H, g^H)$. These temporary steady-state levels increase monotonically with the size of population.

### 3.2 Global Dynamics

This section analyzes the evolution of the economy from the Malthusian Regime, through the Post-Malthusian Regime, to the demographic transition and the Modern Growth regime.

#### 3.2.1 Phase Diagrams

The global analysis is based a sequence of phase diagrams that describes the evolution of the system within each regime and in the transition between the different regimes.

The phase diagrams, as depicted in Figure 5, are based on three central elements:

**The Malthusian Frontier**

As was established in (27) and (28) the economy exits from the subsistence consumption regime when potential income, $z_t$, exceeds the critical level $\tilde{z} \equiv \tilde{c}/(1-\gamma)(1+\overline{r})$. This switch of regime changes qualitatively the nature of the dynamical system from a two to a three dimensional system.

Let *The Malthusian Frontier* be the set of all triplets of $(e_t, x_t, g_t)$ for which individuals income equal $\tilde{z}$.[31] Using the definitions of $z_t$ and $\tilde{z}$, it follows from (10), (15) and (19) that the The Malthusian Frontier, $MM$, is

$$MM \equiv \{(e_t, x_t, g_t) : \alpha(1-\beta)x_t^{(1-\alpha)}h(e_t, g_t)^{\alpha} = \tilde{c}/(1-\gamma)(1+\overline{r})\}. \tag{31}$$

Let *The Conditional Malthusian Frontier* be the set of all pairs $(e_t, x_t)$ for which, conditional on a given technological level $g_t$, individuals incomes equal $\tilde{z}$. Following the definitions of $z_t$ and $\tilde{z}$, equations (10), (15) and (19) imply that The Conditional

[31] As was shown in Proposition One, below the Malthusian Frontier, the effect of income on fertility will be positive, while above the frontier there will be no effect of income on fertility. Thus the Malthusian Frontier separates the Malthusian and Post-Malthusian regimes, on the one hand, from the Modern Growth regime, on the other.

Malthusian Frontier, $MM_{|g_t}$, as depicted in each of the panels of Figure 5, is

$$MM_{|g_t} \equiv \{(e_t, x_t) : \alpha(1-\beta)x_t^{(1-\alpha)}h(e_t, g_t)^{\alpha} = \tilde{c}/(1-\gamma)(1+\bar{\tau}) \mid g_t\}. \tag{32}$$

**Lemma 2.** *If (A1) is satisfied, and $(e_t, x_t) \in MM_{|g_t}$ then $x_t$ is a decreasing strictly convex function of $e_t$.*

**Proof.** Since $h_t = h(e_t, g_t)$ is an increasing, strictly concave, function of $e_t$, the lemma follows from (32). □

Hence, the Conditional Malthusian Frontier, as depicted in Figure 5, is a strictly convex, downward sloping, curve in the $(e_t, x_t)$ space. Furthermore, it intersects the $x_t$ axis and approaches asymptotically the $e_t$ axis as $x_t$ approaches infinity. The frontier shifts upward as $g_t$ increases in the transition to a modern growth regime.

### The XX Locus

Let $XX$ be the locus of all triplets $(e_t, g_t, x_t)$ such that the effective resources per worker, $x_t$, is in a steady-state:

$$XX \equiv \{(e_t, x_t, g_t) : \; x_{t+1} = x_t\}. \tag{33}$$

In order to simplify the exposition without affecting the qualitative nature of the dynamical system, the parameters of the model are restricted so as to assure that the XX Locus is non-empty when $z_t \geq \tilde{z}$. That is,

$$(A5) \qquad \hat{g} < (\gamma/\tau^q) - 1 < f(L_0)g(e^H(L_0)).$$

**Lemma 3.** *If (A1)-(A5) are satisfied, then for $z_t \geq \tilde{z}$, there exists a unique value $0 < \hat{e} < e^H$, such that $x_t \in XX$. Furthermore, for $z_t \geq \tilde{z}$*

$$x_{t+1} - x_t \begin{cases} > 0 & if \quad e_t > \hat{e} \\ = 0 & if \quad e_t = \hat{e} \\ < 0 & if \quad e_t < \hat{e} \end{cases}$$

**Proof.** For $z_t \geq \tilde{z}$, it follows from (26) that $x_{t+1} = x_t$ if and only if

$$\phi^b(e_t) \equiv \frac{[1 + g(e_t)][\tau^q + \tau^e e(g(e_t))]}{\gamma} = 1. \tag{34}$$

Since $\phi^b(e_t)$ is strictly monotonic increasing in $e_t$ and since (A5) implies that for all $L_t > 0$, $\phi^b(0) < 1$ and $\phi^b(e^H) > 1$, there exists a unique value $0 < \hat{e} < e^H$, such that $\phi^b(\hat{e}) = 1$ and hence $x_t \in XX$. Furthermore, since $\phi^b(e_t)$ is strictly monotonically increasing in $e_t$, it follows from (26) that $x_{t+1} > x_t$ if and only if $\phi^b(e_t) > 1$ and hence $e_t > \hat{e}$, whereas $x_{t+1} < x_t$ if and only if $\phi^b(e_t) < 1$ and hence $e_t < \hat{e}$. □

Hence, the XX Locus, as depicted in Figure 5 in the space $(e_t, x_t)$, is a vertical line above the Conditional Malthusian Frontier at a level $\hat{e}$.

Lemma 3 holds as long as consumption is above subsistence. In the case where the subsistence constraint is binding, the evolution of $x_t$, as determined by equation (26), is based upon the rate of technological change, $g_t$, the effective resources per-worker, $x_t$ as well as the quality of the labor force, $e_t$.

Let $XX_{|g_t}$ be the locus of all pairs $(e_t, x_t)$ such that $x_{t+1} = x_t$ for a given level of $g_t$. That is,

$$XX_{|g_t} \equiv \{(e_t, x_t) : \ x_{t+1} = x_t \mid g_t\}. \tag{35}$$

**Lemma 4.** *If (A1)-(A5) are satisfied, then for $z_t \leq \tilde{z}$, and for $0 \leq e_t \leq \hat{e}$, there exists a single-valued function $x_t = x(e_t)$ such that $(x(e_t), e_t) \in XX_{|g_t}$. Furthermore, for $z_t \leq \tilde{z}$,*

$$x_{t+1} - x_t \begin{cases} < 0 \quad if \quad (e_t, x_t) > (e_t, x(e_t)) \quad for \quad 0 \le e_t \le \hat{e}, \\ = 0 \quad if \quad x_t = x(e_t) \quad for \quad 0 \le e_t \le \hat{e}, \\ > 0 \quad if \quad [(e_t, x_t) < (e_t, x(e_t)) \quad for \quad 0 \le e_t \le \hat{e},] \quad or \quad [e_t > \hat{e}] \end{cases}$$

**Proof.** As follows from (26), $x_{t+1} = x_t$ if and only if

$$\phi^a(e_t, g_t, x_t) = \frac{[1 + g(e_t)][\tau^q + \tau^e e(g(e_t))]}{1 - [\tilde{c}/(1+\bar{r})z(e_t, g_t, x_t)]} = 1. \tag{36}$$

Since $\phi^a(e_t, g_t, x_t)$ is strictly monotonic *decreasing* in $x_t$, there exists a single valued function, $x_t = x(e_t)$, such that $\phi^a(e_t, x_t | g_t) = 1$ and therefore $(e_t, x(e_t)) \in XX_{|g_t}$. Moreover, since $\phi^a_e(e_t, g_t, x_t)$ is not necessarily monotonic, $x'(e_t)$ is not necessarily monotonic as well. Furthermore, since $\phi^a(e_t, x_t \mid g_t)$ is strictly monotonic decreasing in $x_t$ it follows from (26) that for $0 \le e_t \le \hat{e}$, and for $z_t \le \tilde{z}$ : (a) $x_{t+1} > x_t$ if and only if $x_t < max[x(e_t), x^M_t]$, where $(e_t, x^M_t) \in MM_{|g_t}$, and (b) $x_{t+1} < x_t$ if and only if $x_t > x(e_t)$. □

Hence, without loss of generality, the locus $XX_{|g_t}$ is depicted in Figure 5, as an upward sloping curve in the space $(e_t, x_t)$, defined for $e_t \le \hat{e}$. $XX_{|g_t}$ is strictly below the Conditional Malthusian Frontier for value of $e_t < \hat{e}$, and the two coincides at $\hat{e}$.

**Lemma 5.** *let $(\hat{e}, \hat{x}) \in MM_{|g_t}$. If (A1) and (A5) are satisfied, then $(\hat{e}, \hat{x}) = XX_{|g_t} \cap MM_{|g_t} \cap XX$*

**Proof.** Let $(\hat{e}, \hat{x}) \in MM_{|g_t}$. It follows from the definition of $MM_{|g_t}$ that $z(\hat{e}, \hat{x}|g_t) = \tilde{z}$. Hence, Lemma 2 implies that $(\hat{e}, \hat{x}) \in XX$. Furthermore, since Lemma 2 and 3 are both valid for $z_t = \tilde{z}$, it follows that $x(\hat{e}) = \hat{x}$ and hence $(\hat{e}, \hat{x}) \in XX_{|g_t}$. □

Hence, the Conditional Malthusian Frontier, the XX Locus, and the $XX_{|g_t}$ Locus, as depicted in Figure 5 in the $(e_t, x_t)$ space, coincide at $(\hat{e}, \hat{x})$.

**The $EE$ Locus**

Let $EE$ be the locus of all triplets $(e_t, g_t, x_t)$ such that the quality of labor, $e_t$, is in a steady-state. That is,

$$EE \equiv \{(e_t, x_t, g_t) : \; e_{t+1} = e_t\}. \tag{37}$$

As follows from the analysis in section 3.1, the steady-state values of $e_t$ are independent of the values of $x_t$ and $g_t$. The locus $EE$ evolves through three phases in the process of development, corresponding to the three phases that describe the evolution of education and technology depicted in Figures 4(a), 4(b), and 4(c).

Panel a. In early stages of development, when population size is sufficiently small, the joint evolution of education and technology is characterized by a globally stable temporary steady-state equilibrium, $(\overline{e}, \overline{g}) = (0, g^l)$, as depicted in Figure 4a. The corresponding EE Locus, depicted in the space $(e_t, x_t)$ in Figure 5a, is vertical at the level $e = 0$, for a range of small population sizes. Furthermore, for this range, the global dynamics of $e_t$ in this configuration are given by:

$$e_{t+1} - e_t \begin{cases} = 0 & if \quad e_t = 0 \\ \\ < 0 & if \quad e_t > 0. \end{cases} \tag{38}$$

Panel b. In later stages of development as population size increases sufficiently, the joint evolution of education and technology is characterized by multiple locally stable temporary steady-state equilibria, as depicted in Figure 4b. The corresponding EE Locus, depicted in the space $(e_t, x_t)$ in Figure 5b, consists of 3 vertical lines corresponding the three steady-state equilibria for the value of $e_t$. That is, $e = 0$, $e = e^u$, and $e = e^h$. The vertical lines $e = e^u$, and $e = e^h$ shift rightward as population size increases. Furthermore, the global dynamics of $e_t$ in this configuration are given by:

$$e_{t+1} - e_t \begin{cases} < 0 & \text{if } \; 0 < e_t < e^u \;\; or \;\; e_t > e^h \\ = 0 & \text{if } \; e_t = (0, \; e^u, \; e^h) \\ > 0 & \text{if } \; e^u < e_t < e^h. \end{cases} \tag{39}$$

Panel c. In mature stages of development when population size is sufficiently large, the joint evolution of education and technology is characterized by globally stable steady-state equilibrium at the point $(\overline{e}, \overline{g}) = (e^h, g^h)$, as depicted in Figure 4c. The corresponding EE Locus, as depicted in Figure 5c in the space $(e_t, x_t)$, is vertical at the level $e = e^h$. This vertical line shifts rightward as population size increases. Furthermore, the global dynamics of $e_t$ in this configuration are given by:

$$e_{t+1} - e_t \begin{cases} > 0 & \text{if } \; 0 \leq e_t < e^h \\ = 0 & \text{if } \; e_t = e^h. \\ < 0 & \text{if } \; e_t > e^h. \end{cases} \tag{40}$$

### Conditional Steady-State Equilibria

In early stages of development, when population size is sufficiently small, the dynamical system, as depicted in Figure 5a in the space $(e_t, x_t)$, is characterized by a unique and globally stable conditional steady-state equilibrium.[32] It is given by a point of intersection between the $EE$ Locus and the $XX$ Locus. That is, conditional on a given technological level, $g_t$, the Malthusian steady-state $(0, \overline{x}(g_t))$ is globally stable.[33]

[32]Since the dynamical system is discrete, the trajectories implied by the phase diagrams do not necessarily approximate the actual dynamic path, unless the state variables evolve monotonically over time. As shown in section 3.1 the evolution of $e_t$ is monotonic, whereas the evolution and convergence of $x_t$ may be oscillatory. Non-monotonicity may arise only if $e < \hat{e}$. Non-monotonicity in the evolution of $x_t$ does not affect the qualitative description of the system. Furthermore, if $\phi^a_x(e_t, g_t, x_t)x_t > -1$ the conditional dynamical system is locally non-oscillatory. The phase diagrams in Figure 5a-5c are drawn under the assumptions that assure that there are no oscillations.

[33]The local stability of the steady-state equilibrium $(0, \overline{x}(g_t))$ can be derived formally. The eigenvalues

In later stages of development as population size increases sufficiently, the dynamical system as depicted in Figure 5b is characterized by two conditional steady-state equilibria. The Malthusian conditional steady-state equilibrium is locally stable, whereas the steady-state equilibrium $(e^u, x^u)$ is a saddle point.[34] In addition for education levels above $e^u$ the system converges to a stationary level of education $e^h$ and possibly to a steady-state *growth rate* of $x_t$.

In mature stages of development when population size is sufficiently large, There system convergence globally to an educational level $e^h$ and possibly to a steady-state *growth rate* of $x_t$.

### 3.2.2 Analysis

The transition from the Malthusian Regime through the Post-Malthusian regime to the demographic transition and a Modern Growth regime emerges from Proposition 1, Corollary 1, and Figures 2-5. Consider an economy in early stages of development. Population is low enough that the implied rate of technological change is very small, and parents have no incentive to provide education to their children. As depicted in Figure 4a in the space $(e_t, g_t)$, the economy is characterized by a single temporary steady-state equilibrium in which technological progress is very slow and children's level of education is zero. This temporary steady-state equilibrium corresponds to a globally stable conditional Malthusian steady-state equilibrium, drawn in Figure 5a in the space $(e_t, x_t)$. For a given rate of technological progress, effective resources per capita, as well as the level of education are constant, and hence as follows from (3) and (10) output per-capita is constant as well. Moreover, shocks to population or resources will be undone in a classic Malthusian fashion. Population will be growing slowly, in parallel with technology. As long as the size of the population is sufficiently small, no qualitative changes occurs in the

of the Jacobian matrix of the conditional dynamical system evaluated at the conditional steady-state equilibrium are both smaller than one (in absolute value) under (A1)-(A4).

[34] Convergence to the saddle point takes place only if the level of education is $e^u$. That is, the saddle path is the entire vertical line that corresponds to $e_t = e^u$.

dynamical system described in Figures 4a, and 5a. The temporary steady-state equilibrium depicted in Figure 4a gradually shifts vertically upward reflecting small increments in the rate of technological progress, while the level of education remains constant at zero. Similarly, the conditional Malthusian steady-state equilibrium drawn in Figure 5a for a constant rate of technological progress, shifts upward vertically. However, output per-capita remains constant at the subsistence level.

Over time, the slow growth in population that takes place in the Malthusian regime will raise the rate of technological progress and shift the $g(e_{t+1})$ locus in Figure 4a upward so that it has the configuration shown in Panel B. At this point, the dynamical system of education and technology will be characterized by multiple, history-dependent steady states. One of these steady states will be Malthusian, characterized by constant resources per capita, slow technological progress, and no education. The other will be characterized by a high level of education, rapid technological progress, growing income per capita, and moderate population growth. For the story that we want to tell in this paper, however, the existence of multiple steady states turns out not to be relevant. Since the economy starts out in the Malthusian steady state, it will remain there. If we were to allow for stochastic shocks to education or technological progress, it would be possible for an economy in the Malthusian steady state of panel B to jump to the Modern Growth steady state, but we do not pursue this possibility.

Panel C of figure 4 shows that the increasing size of the population continues to raise the rate of technological progress, reflected in a further upward shift of the $g(e_t)$ function. At a certain level of population, the steady state vanishes, and the economy transitions out of the Malthusian regime. Increases in the rate of technological progress and the level of education feed back on each other until the economy converges to the single, stable steady state shown in the figure.

While the evolution of education and technological progress traced in panel C of

Figure 4 are monotonic once the Malthusian steady state has been left behind, the evolution of population growth and the standard of living, which can be seen in Panel C of figure 5, are more complicated. The reason for this complication is that technological progress has two effects on the evolution of population, as shown in proposition one. First, by inducing parents to give their children more education, technological progress will *ceteris paribus* lower the rate of population growth. But, second, by raising potential income, technological progress will increase the fraction of their time that parents can afford to devote to raising children. Initially, while the economy is in the Malthusian region of Figure 5, the effect of technology on the parent's budget constraint will dominate, and so the growth rate of the population will increase. This is the Post-Malthusian regime.[35]

The positive income effect of technological progress on fertility only functions in the Malthusian region of Figure 5, however. As the figure shows the economy eventually crosses the Malthusian frontier. Once this has happened, further improvements in technology no longer have the effect of changing the amount of time devoted to child-rearing, while faster technological change will continue to raise the quantity of education that parents give each child. Thus once the economy has crossed the Malthusian frontier, population growth will fall as education and technological progress rise.[36]

[35]Taking our model literally, income per capita does not change at all during the Post-Malthusian regime, but rather remains fixed at the subsistence level. But this result follows from the assumption that the only input into child quality is parental time, and that this time input does not produce measured output. A more reasonable description would be that all child-rearing, but especially the production of quality, requires goods or time supplied through a market. The most obvious example of this expenditure is schooling. Thus the shift toward higher child quality that takes place during the post-Malthusian regime would be reflected in higher market expenditures (as opposed to parental time expenditures) on child quality, and thus rising measured income.

[36]Note that in the Post-Malthusian regime, population will be increasing, and so the g(e) curve in Figure 4 will continue to shift upward. In the Modern Growth regime the model gives no concrete prediction about how population size will be changing. It is possible that there will be a steady state with constant population size and education level, and constant growth rates of technology and income per capita. But it is also possible that in the steady state population will be constantly shrinking or growing, with the rate of technological progress also shrinking or growing.

In the modern growth regime, resources per capita will rise, as technological progress outstrips population growth. Figure 4C shows that the levels of education and technological progress will be constant in the steady state, provided that population size is constant (i.e., population growth is zero). This implies that the growth rate of resources per capita, and thus the growth rate of output per capita, will also be constant. However, if population growth is positive in the Modern Growth regime, then education and technological progress will continue to rise, and, similarly, if population growth is negative they will fall. In fact, the model makes no firm prediction about what the growth rate of population will be in the Modern Growth regime, other than that population growth will fall once the economy exits from the Malthusian region. It may be the case that population growth will be zero, in which case the Modern growth regime would constitute a global steady state, in which $e$ and $g$ were constant. Alternatively, population growth could be either positive or negative in the Modern Growth regime, with $e$ and $g$ behaving accordingly.[37]

## 4. Concluding Remarks

This paper develops a unified endogenous growth model in which the evolution of population, technology, and output growth is largely consistent with the process of development in the last millennia. The model generates an endogenous take-off from a Malthusian Regime, through a Post-Malthusian Regime, to a demographic transition and a Modern Growth Regime. In early stages of development - the Malthusian Regime - the economy remains in the proximity of a Malthusian trap, where output per capita is nearly stationary and episodes of technological change bring about proportional increases in output and population. In the intermediate stages of development - the Post-Malthusian

[37]Jones (1995) has argued for a model of technology creation in which the steady state growth rate of technology is related to the growth rate of population, rather than to its level. Under such a specification, our model would have a steady state modern growth regime in which the growth rates of population and technology would be constant. Further, such a steady state would be stable: if population growth fell, the rate of technological progress would also fall, inducing a rise in fertility.

Regime - the intensified pace of technological change that is caused by the increase in the size of population during the Malthusian regime permits the economy to take off. Production takes place under a state of technological disequilibrium in which the relative return to skills rises, inducing the household to shift its spending on children toward quality and away from quantity. Output per capita increases along with an increase in the rate of population growth and human capital accumulation. Eventually, rapid technological progress which results from high human capital accumulation triggers a demographic transition in which fertility rates permanently decline.

The model abstracts from several factors that are relevant for economic growth. Differences between countries in the determination of population growth or in the process of technological change (due to cultural factors, for example) would be reflected in their ability to escape the Malthusian trap and in the speed of their takeoff. Similarly, differences in policies, such as the public provision of education, would change the dynamics of the model. One interesting possibility that the model suggests is that colonialism, by effectively expanding the stock of land available for production, may have played a role in facilitating Europe's emergence from the Malthusian trap.

While our model presents a unified description of the development process followed by Europe and its offshoots, it is clearly not fully applicable to countries that are developing today. For currently developing countries, a large stock of pre-existing technology is available for import, and so the relationship between population size and technology growth, which helped trigger the demographic transition in Europe, is no longer relevant. Similarly, the relationship between income and population growth has changed dramatically, due to the import of health technologies. Countries that are poor, even by the standards of Nineteenth Century Europe, are experiencing growth rates of population far higher than those ever experienced in Europe.

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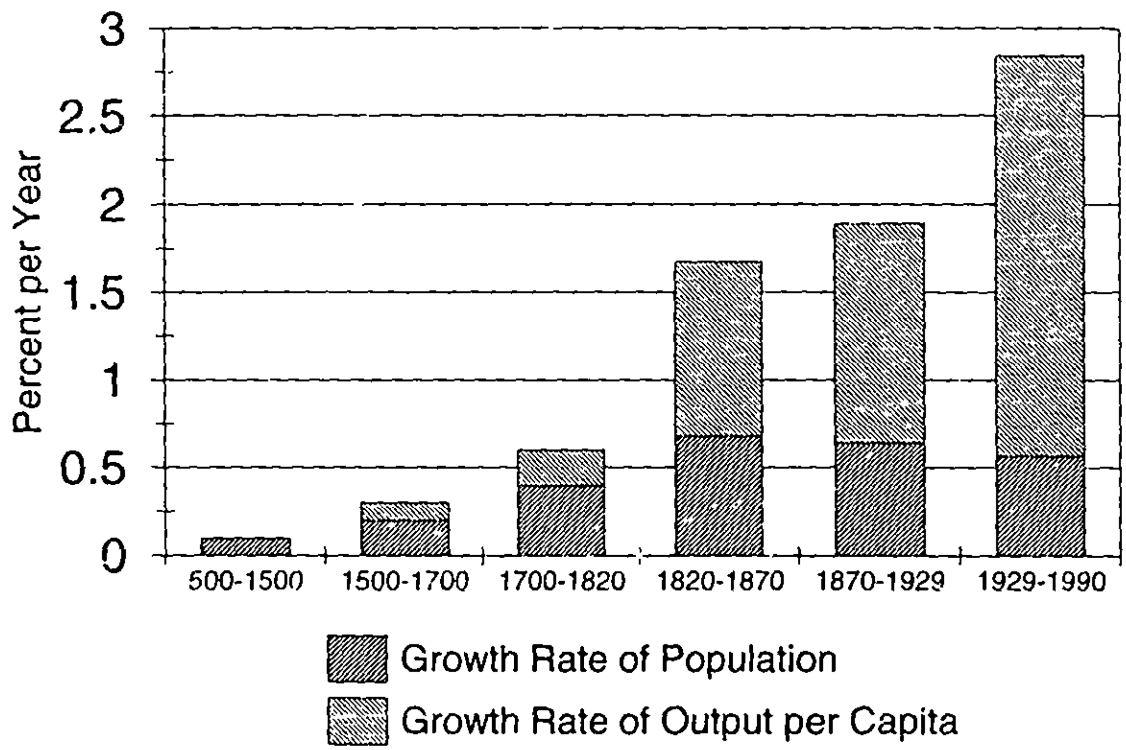
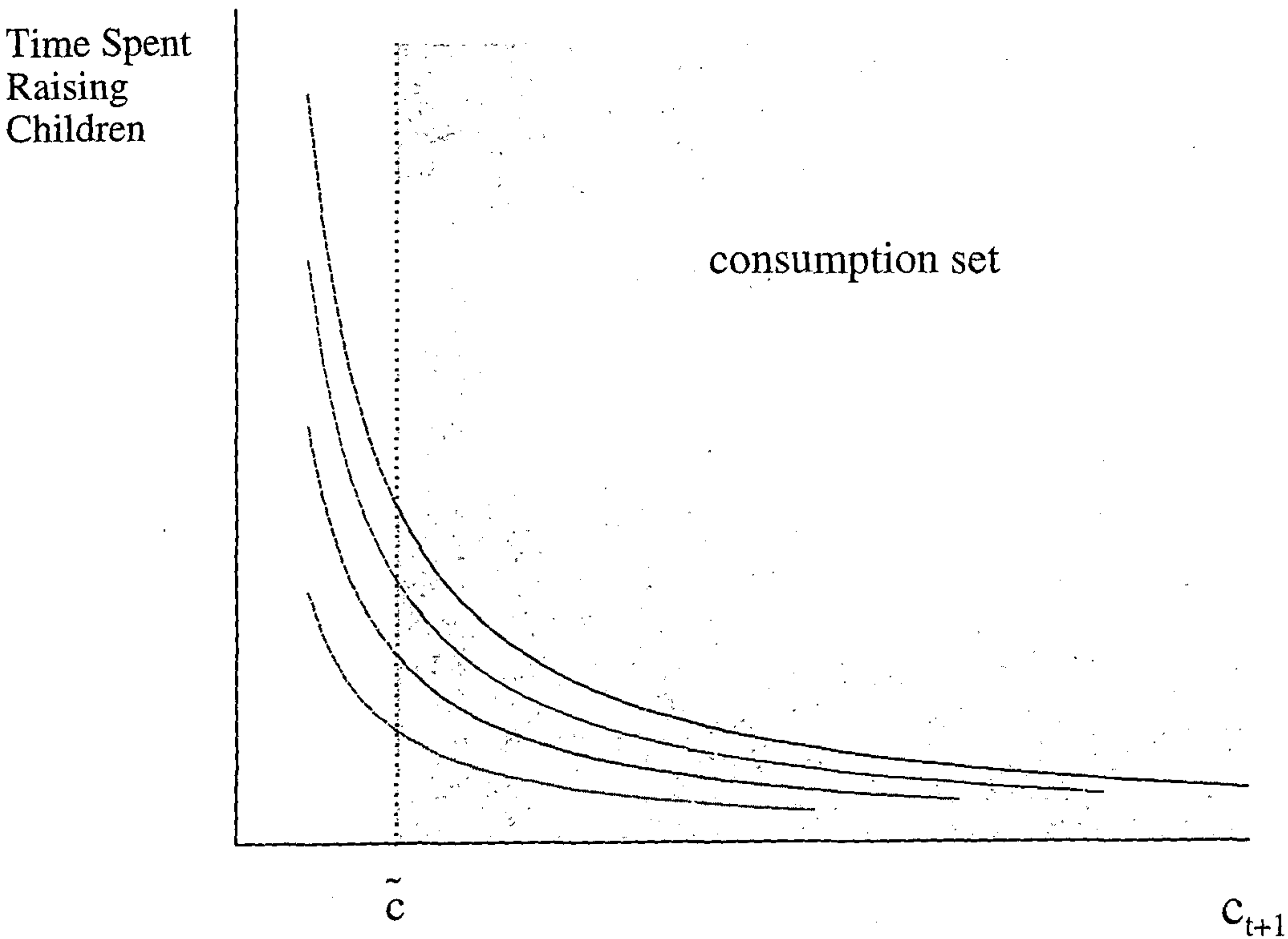
**Figure 1**

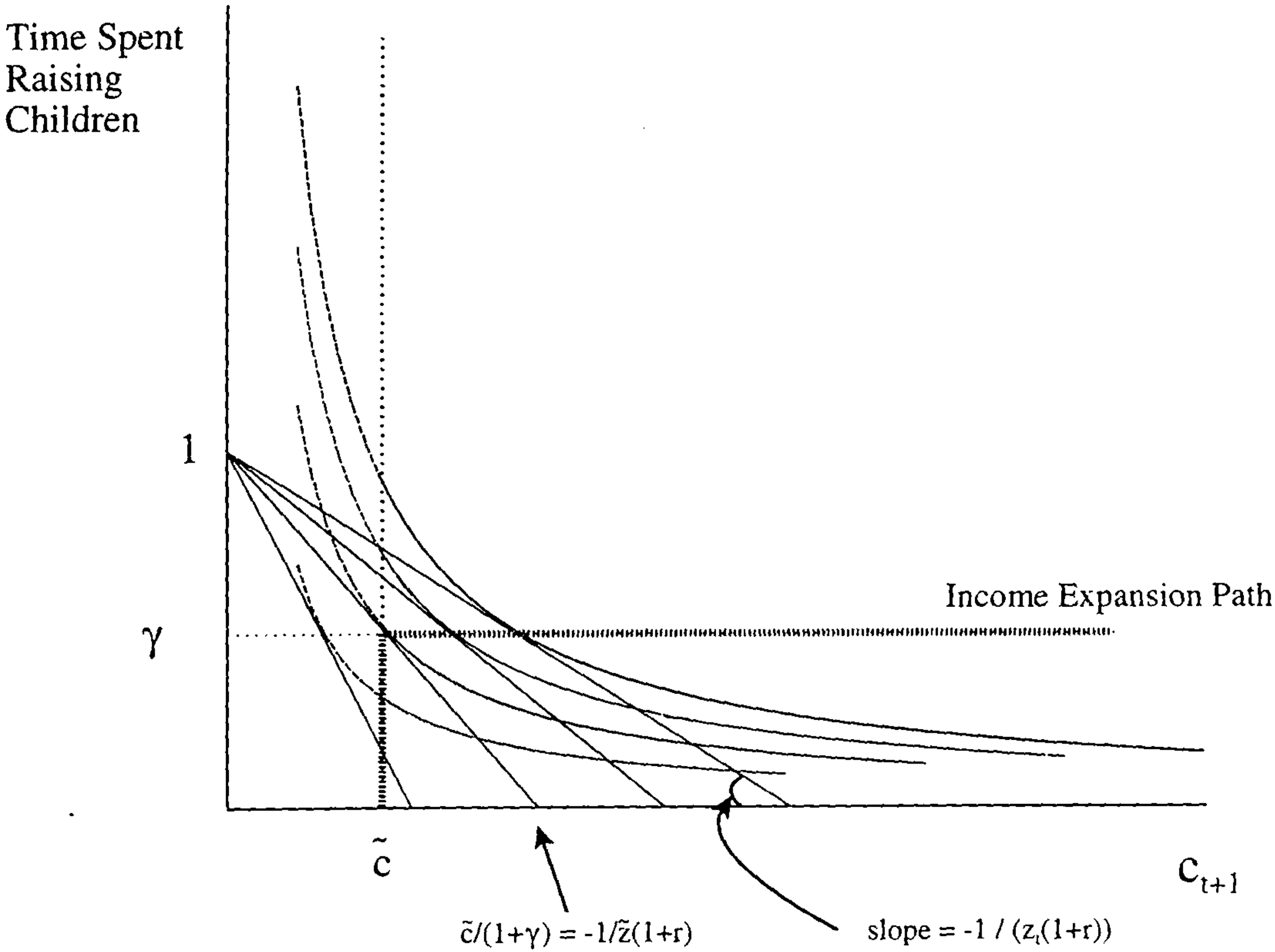
Output Growth in Western Europe



# Figure 2

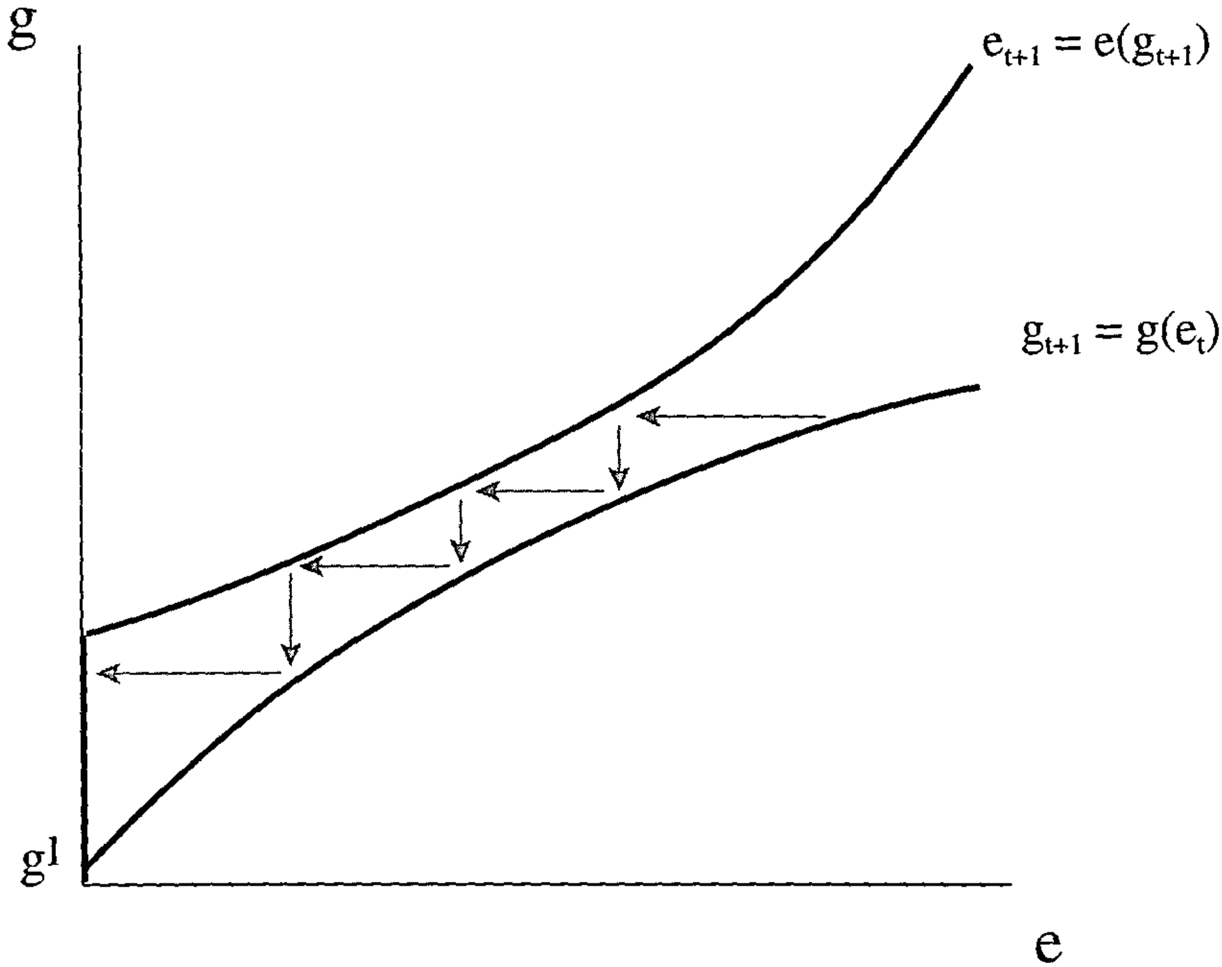


# Figure 3


Time Spent
Raising
Children
1
γ
Income Expansion Path
c̃
$c_{t+1}$
$\tilde{c}/(1+\gamma) = -1/\tilde{z}(1+r)$
slope = $-1 / (z_t(1+r))$

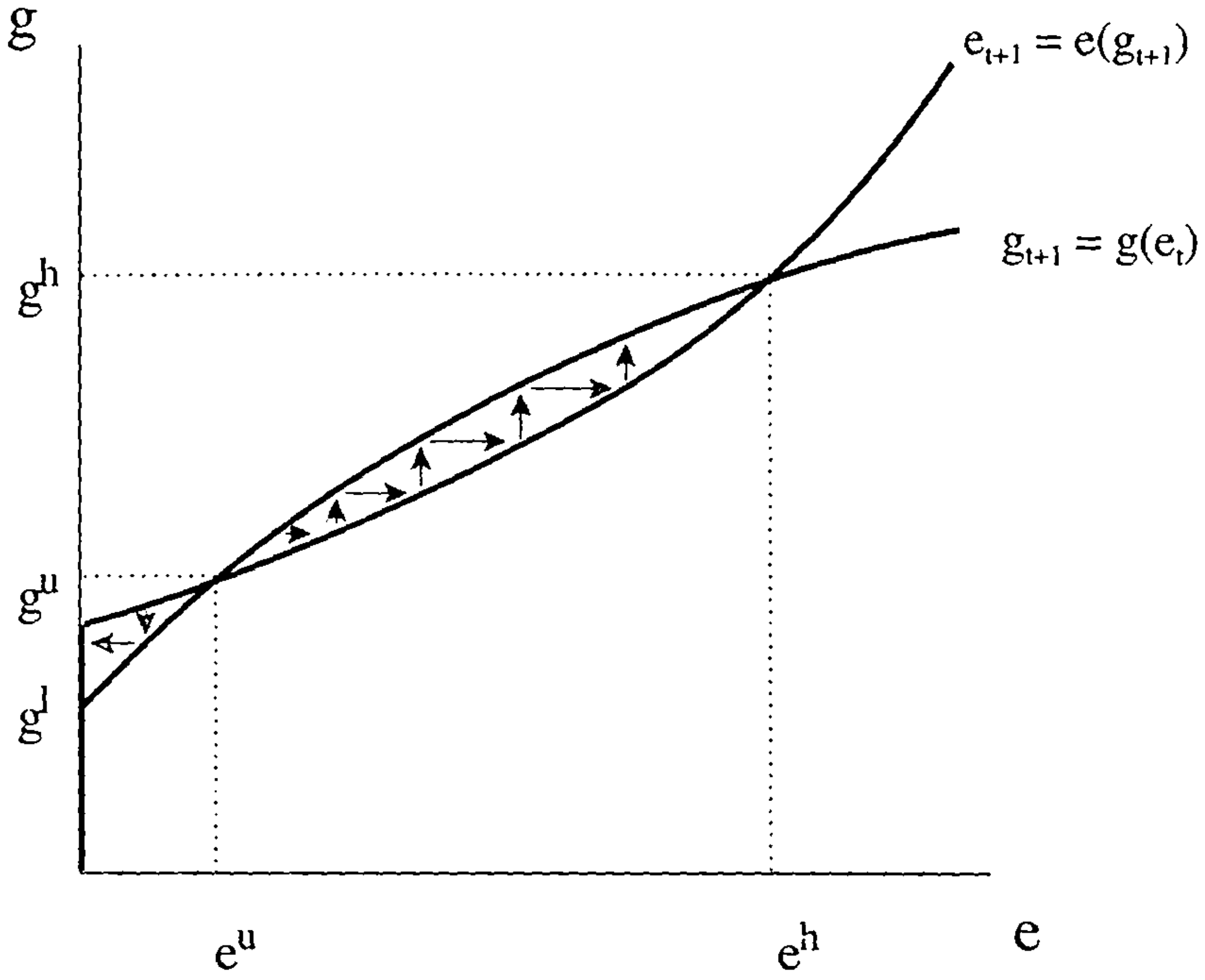
# Figure 4

Panel A



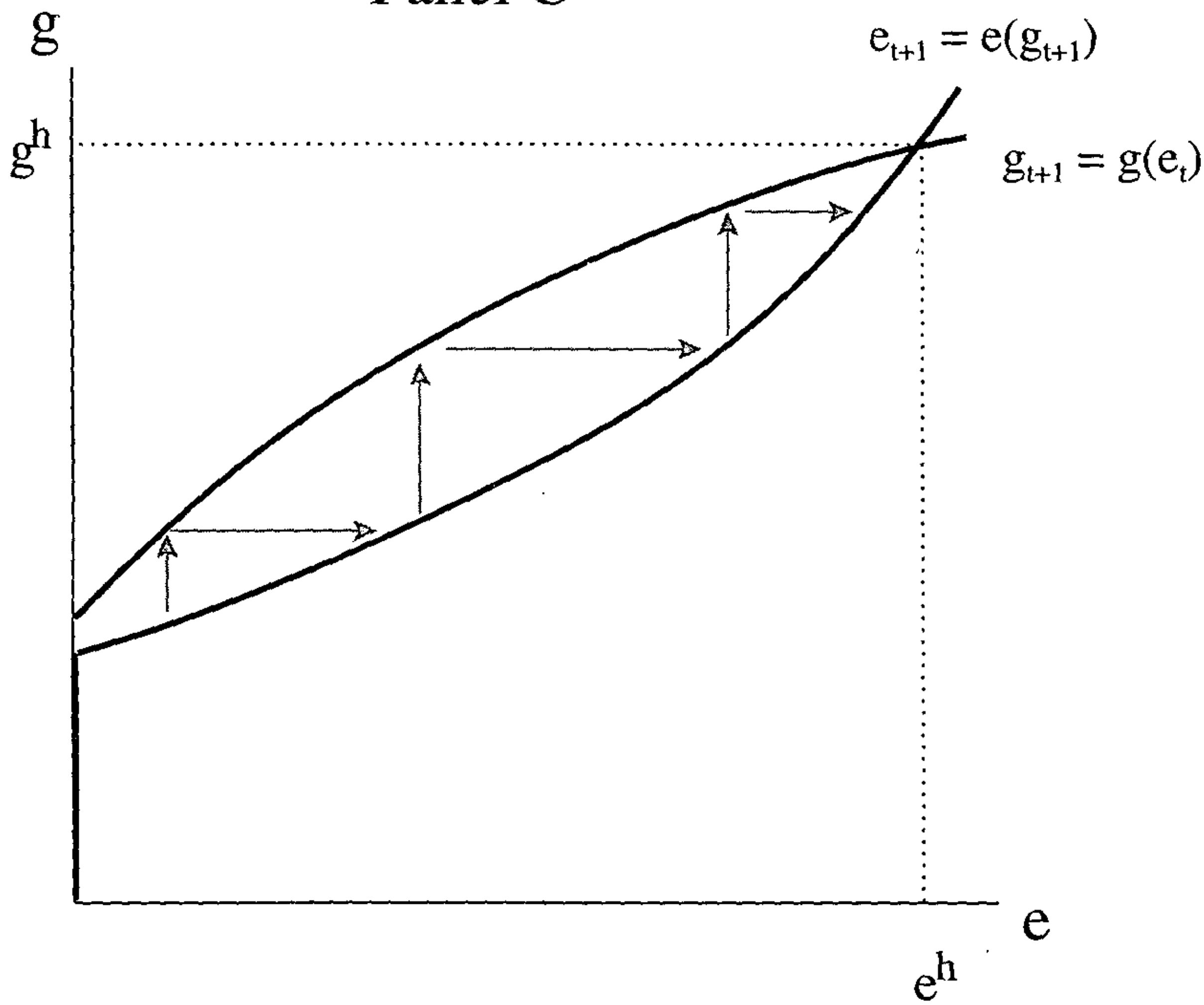
# Figure 4

## Panel B



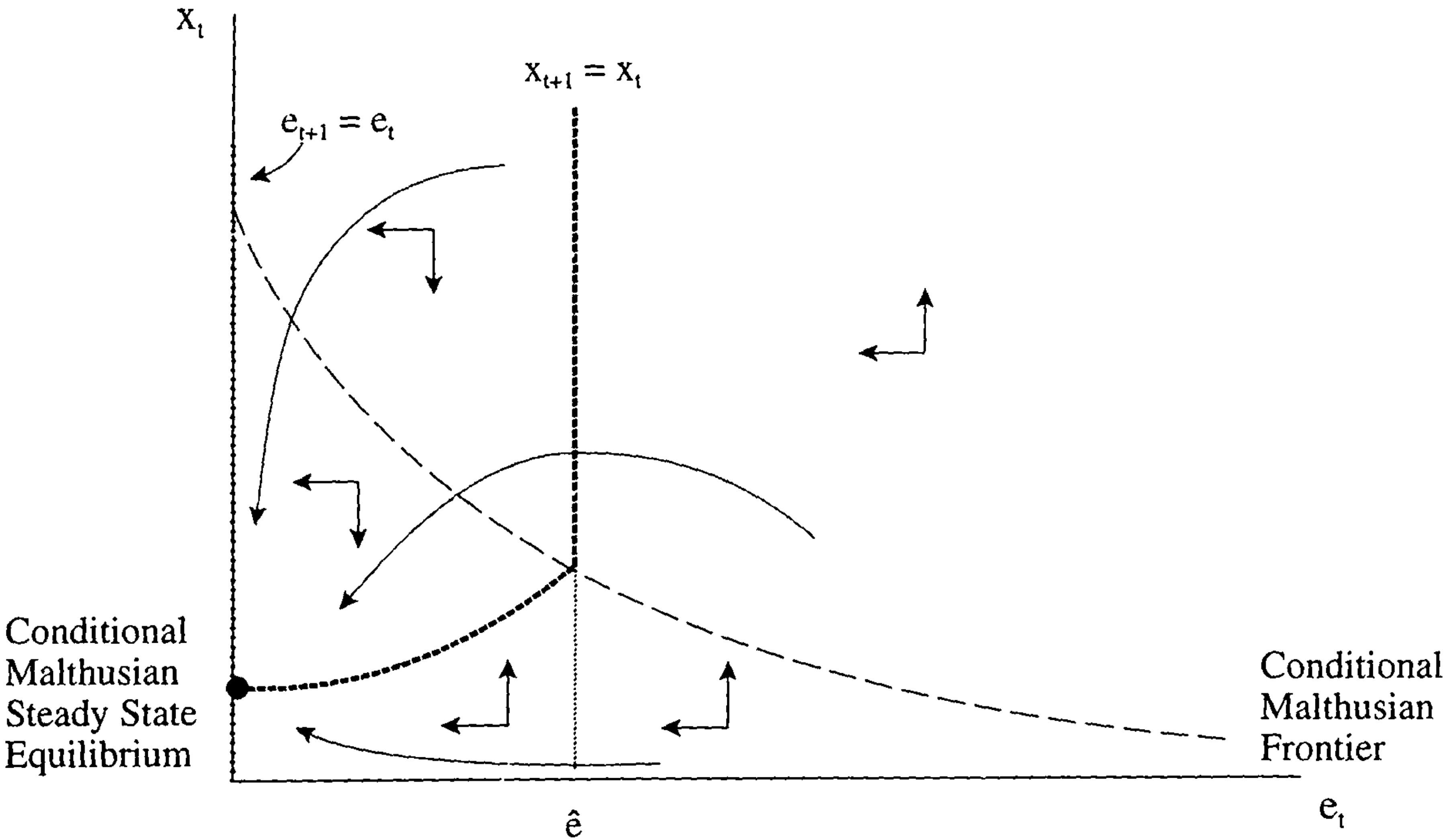
# Figure 4

## Panel C



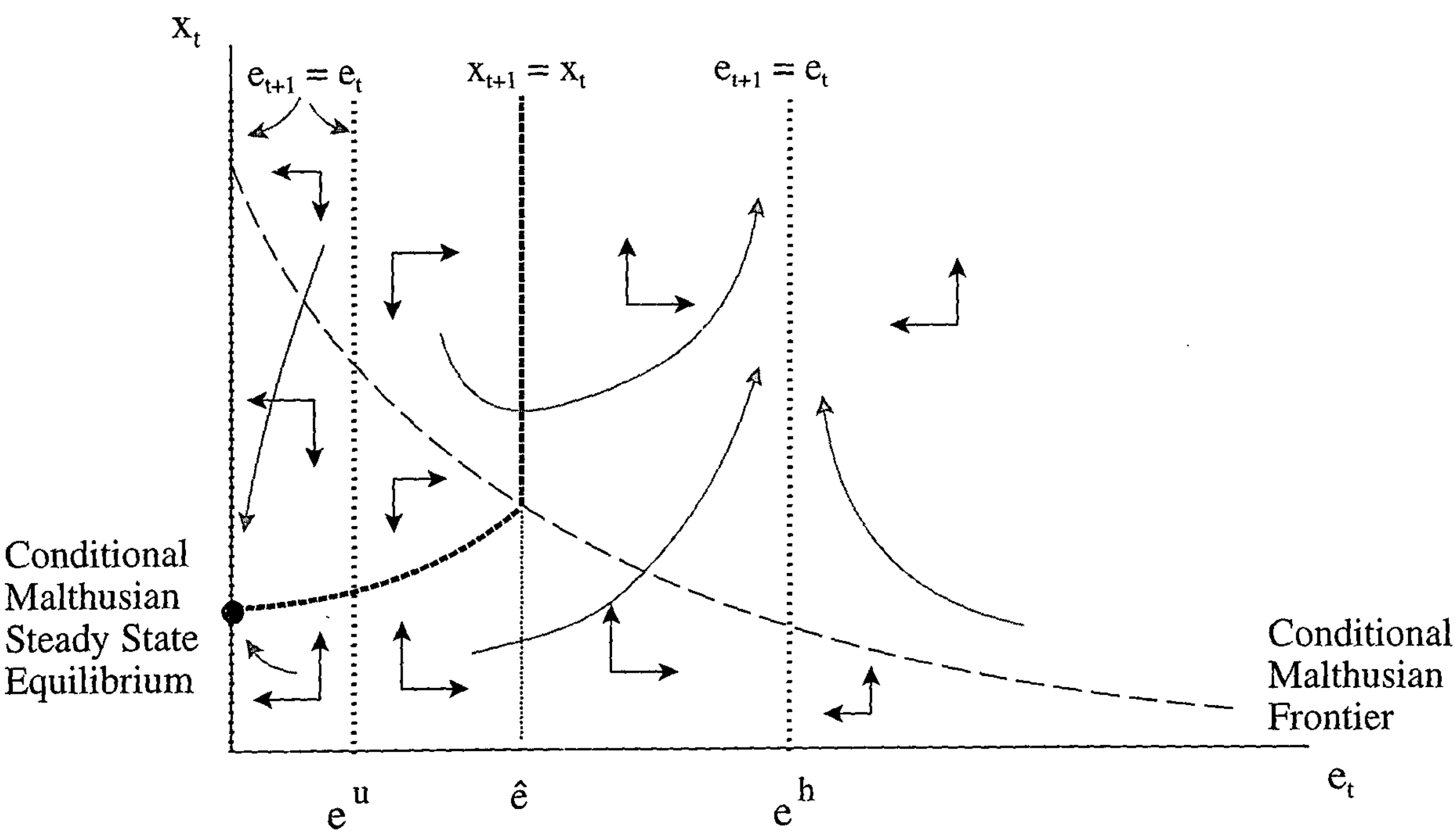
# Figure 5

## Panel A



# Figure 5

## Panel B



# Figure 5

## Panel C