

Structural Change and Economic Growth

Abstract: This paper presents a model in which a country's average propensity to save tends to rise endogenously over time. The paper uses a two-sector neoclassical framework to model the transition from agriculture to manufacturing which typically accompanies economic development. Key assumptions are that only the agricultural sector uses land and a simple version of Engel's law. When a country's income per capita is low, agricultural consumption is important; consequently, land is valuable and capital gains on it may account for most wealth accumulation, making the NIPA APS appear low. If exogenous technological progress raises incomes over time, Engel's law shifts demand to manufactured goods. Then land's importance in portfolios relative to reproducible capital diminishes and the measured average propensity to save can rise.

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This paper presents a model in which a country's average propensity to save rises naturally if and when technological progress sufficiently lifts its income per capita. The effect on the saving propensity follows from structural changes in the economy caused by the operation of Engel's law.

Recent work by Mankiw, Romer, and Weil [1992], Islam [1995], Mankiw [1995], and others emphasizes conditional convergence. Their regressions on international, cross-sectional data assume that each economy has attained, or is moving toward, a steady state growth path. As in Solow's [1956] model, a country's steady-state level of per capita GDP depends on its average propensity to save (APS), its rates of population growth and technological progress, the efficiency of its production sector, etc. Regression outcomes seem to confirm the Solow model's prediction that a higher APS will, *cet. par.*, lead to a higher level of income per capita. Tables of investment rates and income per capita in Dowrick and Gemmell [1991] and Brander and Dowrick [1994] could be interpreted as being consistent with same idea: splitting their samples of countries into rich and poor, reproducible investment as a share of GDP is about twice as high for the rich countries.

An older literature also stresses the importance of international differences in saving propensities but emphasizes that a society may be able to change its behavior over time. For example, Lewis [1954,p.155] wrote, "The central problem in the theory of economic development is to understand the process by which a community which was previously saving and investing 4 or 5 per cent of its national income or less, converts itself into an economy where voluntary saving is running at about 12 to 15 per cent of national income or more."¹ For a second illustration, see Rostow [1962]. According to this view, the government of a low-income country in Dowrick and Gemmell's table might want to encourage its citizens to be thriftier.

Recent work based on less structured regression equations than Mankiw *et. al.* indicates that average propensities to save are not necessarily exogenous. For example, some studies suggest that higher growth rates may cause higher saving rates (e.g., Barro and Lee [1994], Blomstrom, Lipsey, and Zejan [1993], and Barro and Sala-I-Martin [1995, ch.12]). In fact, theoretical models which deduce saving behavior from agents' utility maximization predict that savings propensities are endogenous. In Modigliani's [1986] overlapping generations framework, an economy's APS reflects an average of the saving of young households and the dissaving of retirees. In a faster growing economy, or during a period of faster growth in the course of convergence, young households in effect receive more weight in the average, tending to raise it. On the other hand, faster growth presumably leads to steeper lifetime earnings profiles for households, tending, if it is anticipated, to reduce saving at youthful ages. Fast growth may also raise interest rates, which may raise or lower saving propensities. See, for instance, Carroll and Weil [1994]. The last two phenomena emerge as well in a model with Ramsey-type, dynastic savers (e.g., Barro and Sala-I-Martin [1995,ch.2]).

¹ Gersovitz's [1988] survey of "Saving and Development" begins with the same idea and quotation.

The present paper suggests a different mechanism through which a country's APS may endogenously change over time. The idea is based on Engel's law and its implications for financial variables. Historical figures suggest shifts in the latter can be substantial. For low-income countries, Engel's law predicts a large expenditure share for agricultural products. For these cases, farmland will tend to constitute a significant share of national wealth. If the value of the land increases over time, such societies are "saving" in the sense that households are building up their net worth. The APS used above, however, is a national income and products account measure which excludes capital gains. In characterizing a society's willingness and ability to accumulate wealth, it has the downward bias of overlooking land appreciation. On the other hand, if incomes grow as time passes, Engel's law implies consumer demand will shift toward manufactured goods. Then the bias may disappear. Reproducible capital, presumably more vital to manufacturing than land, will expand in relative importance. Even if the overall rate of wealth accumulation remains the same, the significance of capital gains on land will diminish, and the NIPA APS will tend to rise. This paper suggests that from the positive empirical correlation of saving propensities and standards of living one could mistakenly infer unidirectional causality. Unusual thrift may lead to a high income level, but a high standard of living may lead to a high measured APS as well.

This paper's organization is as follows. Section 1 briefly reviews historical evidence from the United States and United Kingdom on the changing importance of land in national portfolios. Sections 2-3 present a theoretical model illustrating our idea. Section 4 derives results about wealth accumulation and average saving propensities.

1. Historical Background

Historical statistics for the United Kingdom and United States document changes in shares of land and reproducible capital in national portfolios as per capita incomes rose with industrialization.

The reduction in the relative importance of agriculture since the "industrial revolution" is, of course, clear. For the United Kingdom, Deane and Cole [1969, p.137 and Table 30] show that agricultural, forestry, and fishing employment shrinks from 60-80% of total labor in the late seventeenth century, to 30-40% in 1800, 20-25% in 1850, 8-10% in 1900, and 5% by 1950. Meanwhile, employment in manufacturing, mining, transportation, and trade rises from 11-30%, to 40-45%, to 55-60%, to 65-70%, and, by 1950, above 70%. *Historical Statistics of the U.S.* [1975, p.139] show U.S. agricultural employment rising by a factor of 3 between 1840 and 1900, whereas manufacturing rose by 11-12; agriculture shrank by about half between 1900 and 1960, while manufacturing rose by a factor of almost 3.

Investment-to-output ratios for the same period are available for the U.K.: Deane and Cole [p.260 and 266] find the ratio of net physical investment-to-GNP rising from 3-6% in 1688 to 10-12% in the period from 1850 to WWI. Feinstein's [1981] numbers for gross total investment as a fraction of GNP show 8% for 1761-70, 14% for 1791-1800, and 13% for 1851-60. Kuznets [1971] thinks this pattern applied more generally in Europe and Japan, though not necessarily the U.S.²

² *Historical Statistics of the U.S.* [1975] have U.S. investment figures only back to 1869,

Data for both the U.S. and U.K. points to a large decline in the relative share of land in total wealth. Deane and Cole [p.270 and Table 70] put the share of "land" in total national wealth at 64% in 1688 for the U.K., 55% in 1798, 18.1% by 1885, and 4.0% in 1927.³ In Feinstein [1981, Table 7.1], for 1760 private land makes up 51% of "total assets" in Great Britain, and farm land alone 47%; for 1860, land is 30% and farm land 21%. For the U.S., looking at reproducible total assets as a share of total wealth (which includes land) Goldsmith [1952, p.278] finds 40–45% for 1805, 60% by 1850, and 70% by 1900. Indeed, he writes [p.278], "The movement of the share of reproducible tangible wealth in total wealth — essentially the mirror image of the share of land — constitutes one of the most consistent trends observable in the field." *Historical Statistics of the U.S.* [p.255] data show agricultural land (and private forest land) constituting 18% of total wealth in 1900, but only 6% by 1958; in 1900 all private land accounted for 31% of aggregate wealth, and 16% in 1958.

Finally, Kuznets [1971, p.66] thinks of the following as the general scenario for Europe and the United States: wealth-to-GDP ratios ranged from 6–7 prior to industrialization, about half the wealth being land; after industrialization, wealth ratios fell to 4–5.

2. The Model

This paper's model shows that even in the absence of changes in household saving behavior, an economy's APS may well tend to rise naturally during the course of development as farmland's share in national wealth declines. The model has two consumption goods, agricultural goods and manufactured goods. It assumes production of the former requires land while production of the latter uses reproducible capital. There is exogenous technological progress so that standards of living can rise over time. Consumer preferences manifest Engel's law: the fraction of income spent on agricultural goods falls as income rises.⁴

The model's specific structure is as follows. There are overlapping generations of households. Every household lives two periods, is identical to all others born at the same date, and takes prices as given and beyond its control. There are no inheritances or bequests. The number of young households at each t is

$$(1 + n)^t, \quad n > 0.$$

A household inelastically supplies 1 natural unit of labor in its youth and 0 units in old age. Labor-augmenting technological progress occurs at rate g so that the "effective labor supply" of a time- t household is

and ratios from that date show little trend.

³ "Land" in this case should be taken to mean agricultural land and housing for farm labor. See Campion [1939,p.41].

⁴ Kelley and Williamson's [1974] has several of the same elements — though the analytical steps taken and implications drawn differ. See also, for example, the three sector model of Echevarria [1997], the two-sector model of Matsuyama [1991], and the migration model of Glomm [1992].

$$(1 + g)^t, \quad g > 0.$$

A household consumes only in old age. Let the old-age consumption of agricultural and manufactured goods of a household born at t be $c_{A,t+1}$ and $c_{M,t+1}$, respectively; let the prices of these goods be $p_{A,t+1}$ and $p_{M,t+1}$; let the wage per effective labor unit be w_t ; and, let the interest rate on savings carried from time t to $t+1$ be r_{t+1} . Then a household born at t solves

$$\max_{c_{A,t+1}, c_{M,t+1}} u(c_{A,t+1}, c_{M,t+1}) \quad (1)$$

$$\text{subject to: } p_{A,t+1} \cdot c_{A,t+1} + p_{M,t+1} \cdot c_{M,t+1} \leq (1 + r_{t+1}) \cdot w_t.$$

The utility function embodies our version of Engel's law:

$$u(c_A, c_M) = \begin{cases} c_A, & \text{if } c_A < \bar{c}, \\ c_M + \bar{c}, & \text{if } c_A \geq \bar{c}. \end{cases} \quad (2)$$

According to (2), a household whose standard of living is low cares only about agricultural consumption. If, on the other hand, its living standard is high, a household becomes satiated with agricultural products at $c_A = \bar{c}$ and devotes its remaining expenditures exclusively to manufactured goods.⁵

The savings behavior stemming from (1) will be extremely simple: each young household will save all of its labor earnings; each retired household will deplete all of its wealth. The pattern will not shift over time even if incomes change.

On the production side of the economy, the aggregate effective labor supply E_t depends on the number of young households and the current technology:

$$E_t = [(1 + n) \cdot (1 + g)]^t. \quad (3)$$

Production of agricultural goods uses labor E_{At} and land. Letting Q_{At} be agricultural output and T be the economy's land endowment,

$$Q_{At} = [E_{At}]^{1-\alpha} \cdot [T]^\alpha, \quad \alpha \in (0, 1). \quad (4)$$

Land is fixed. Without loss of generality set

$$T = 1.$$

The economy can also produce reproducible capital. That process only requires labor, with one unit of effective labor producing one unit of physical investment. Reproducible

⁵ This is a streamlined characterization of Engel's law. On the other hand, although Engel's law is usually formulated in terms of expenditure shares, note, for example, that according to Houthakker's [1987] description of it, "There is also evidence that the income elasticity of food, like the budget share, is inversely related to income; the elasticity may be as high as .8 or .9 at very low income levels, and close to zero for high income."

capital constructed at t is useable at $t+1$, but it fully depreciates afterward. Manufactured goods are made from reproducible capital, one unit of capital producing one unit of output. Letting I_t be physical investment and E_{It} labor devoted to its production, K_{t+1} the stock or reproducible capital useable at $t+1$, and $Q_{M,t+1}$ the time- $(t+1)$ output of manufactured goods,

$$I_t = E_{It}, \quad (5)$$

$$K_{t+1} = I_t, \quad (6)$$

$$Q_{Mt} = K_t. \quad (7)$$

The time- t prices of land, investment, capital, agricultural output, and manufactured goods are, respectively, p_{Tt} , p_{It} , p_{Kt} , p_{At} , p_{Mt} .

The economy is closed and there is no government sector.

In analyzing the model we use effective labor as the numeraire so that w_t is 1. We also require $Q_{At} > 0$ all t : otherwise workers would have no reason to save at $t-1$. This paper's definition of equilibrium is

Definition: *A nonnegative sequence*

$$\{E_t, E_{At}, E_{It}, Q_{At}, Q_{Mt}, I_t, K_t, c_{At}, c_{Mt}, r_{t+1}, p_{At}, p_{Mt}, p_{It}, p_{Tt}, w_t\}_{t \geq 0}$$

constitutes an "equilibrium" for our model if it satisfies K_0 given and $w_t = 1$ and $Q_{At} > 0$ all t ; if it satisfies (3)-(4) and (5)-(7); if it is consistent with household maximization in (1); and, if

$$E_{At} + E_{Mt} \leq E_t, \quad (i)$$

$$w_t \geq p_{At} \cdot \frac{(1-\alpha) \cdot Q_{At}}{E_{At}}, \text{ equality if } E_{At} > 0, \quad (ii)$$

$$w_t = p_{It} \text{ if } I_t > 0, \quad p_{It} = 0 \text{ otherwise,} \quad (iii)$$

$$p_{Tt} = \frac{\alpha \cdot p_{A,t+1} \cdot Q_{A,t+1}}{1+r_{t+1}} + \frac{p_{T,t+1}}{1+r_{t+1}}, \quad (iv)$$

$$p_{M,t+1} = 1+r_{t+1} \text{ if } Q_{M,t+1} > 0, \quad p_{M,t+1} = 0 \text{ otherwise,} \quad (v)$$

$$p_{Kt} = p_{Mt}, \quad (vi)$$

$$w_t \cdot E_t = p_{Tt} + p_{It} \cdot I_t. \quad (vii)$$

Requiring $w_t = 1$ excludes trivial outcomes with zero wages in some periods. Line (i) is a feasibility constraint. Condition (ii) requires competitive, profit-maximizing behavior in agricultural production; (iii) requires the same in production of physical investment goods. A household which buys land at time t collects rents at $t + 1$ and then can resell the land; hence, p_{Tt} obeys (iv). If households are to be persuaded to finance physical investment at time t , the rate of return must be at least r_{t+1} . Conversely, households will not simultaneously own reproducible capital and the economy's land if the return on reproducible capital is higher. This gives (v). Reproducible capital in place at t is useful only for current production of manufactured goods — implying (vi). Young households save all of their earnings, $w_t \cdot E_t$ in aggregate at t . As young households are the only ones surviving to $t + 1$, their saving must finance the entire stock of land and all current physical investment. That implies (vii) — recall that $T = 1$. All of our analysis assumes perfect foresight.

3. Equilibrium

This paper focuses on economies which start out poor. Accordingly, we assume that the initial reproducible capital stock is 0 and that technological proficiency is too modest to allow satiation with agricultural goods for consumers either at time 0 or 1. Formally,

$$K_0 = 0, \quad [E_0]^{1-\alpha} < \frac{\bar{c}}{1+n}, \quad \text{and} \quad [E_1]^{1-\alpha} < \bar{c}. \quad (8)$$

At the start of period 0, we assume old households own all of the economy's land. Depending on parameter values, the analysis divides into two cases.

Case 1. In the first case, technological change is so feeble that agricultural consumption per household can never reach \bar{c} . In view of (3)–(4), the parameter restriction leading to this outcome is

$$[(1+n) \cdot (1+g)]^{1-\alpha} \leq 1+n. \quad (9)$$

In this case, with diminishing returns in agricultural production from the fixity of land, technological change cannot counterbalance population increase to raise per capita incomes. A Malthusian outcome follows: households never reach the stage of demanding manufactured goods; living standards are stationary or falling over time. For a given g , a larger factor share parameter α for land makes this more likely. Faster population increase does too — e.g., we can rearrange (9) as

$$(1+g)^{1-\alpha} \leq (1+n)^\alpha.$$

Case 1 is a reminder that industrialization is not inevitable, and it serves as an introduction to the early stage of growth for Case 2.⁶

⁶ Note that the stagnation of Case 1 arises from parameter magnitudes rather than from initial conditions for K — which contrasts to the “development traps” of, for example, Azariadis and Drazen [1990] or Baland and Francois [1996].

With (9), there is only one equilibrium: since agricultural consumption per household can never reach \bar{c} , household preferences imply that the price of manufactured goods will always be 0; consequently, physical investment will never be profitable and equilibrium requires $E_{At} = E_t$ and $I_t = E_{It} = 0$ all $t \geq 0$. Let $w_t = 1$ all $t \geq 0$. The first-order condition for labor in agriculture and $w_t = 1$ yield

$$p_{A,t} = \frac{[E_{A,t}]^\alpha}{1-\alpha} \quad \text{all } t \geq 0. \quad (10)$$

The value of agricultural output is $E_t/(1-\alpha)$. Part (vii) from the definition of equilibrium then requires

$$p_{Tt} = E_t \quad \text{all } t \geq 0; \quad (11)$$

part (iv) of the definition requires

$$\begin{aligned} 1 + r_{t+1} &= \frac{\alpha \cdot p_{A,t+1} \cdot Q_{A,t+1} + p_{T,t+1}}{p_{Tt}} = \frac{1}{1-\alpha} \cdot \frac{E_{A,t+1}}{E_t} \\ &= \frac{1}{1-\alpha} \cdot \frac{E_{t+1}}{E_t} = \frac{(1+n) \cdot (1+g)}{1-\alpha}. \end{aligned} \quad (12)$$

Formally,

Proposition 1: *Suppose (8)–(9). Then our model has a unique equilibrium. In the equilibrium, all labor is allocated to agriculture at every date; Q_{At} follows from (4); manufacturing output, physical investment, and reproducible capital are always 0; c_{Mt} , p_{It} , and p_{Mt} are always 0; p_{At} , p_{Tt} , and r_{t+1} are as in (10)–(12); and, $c_{At} = Q_{At}/(1+n)^{t-1}$.*

In the equilibrium, young households spend all of their earnings on land. In retirement, the same households collect the rents on the land and then sell the land to the next generation for the latter's wage bill. The rents plus the sales revenues from land enable retired households to purchase all agricultural production.

Case 2. The second case is our basic one. Condition (8) holds but

$$[(1+n) \cdot (1+g)]^{1-\alpha} > 1+n \quad (9')$$

replaces (9). The latter ensures that feasible agricultural production eventually surpasses \bar{c} per retired household. Let the earliest date on and after which feasible agricultural production can more than satiate consumers be

$$t_1 \equiv \min \{t : [E_t]^{1-\alpha} > \bar{c} \cdot (1+n)^{t-1}\}. \quad (13)$$

Define

$$t_0 \equiv t_1 - 1. \quad (14)$$

Note that (8) implies $t_0 > 0$.

Two families of equilibria exist. It is convenient to think about them in terms of the allocation of labor. The allocation is identical across equilibria except at $t = t_0$ and t_1 .

Prior to time t_0 , young households will refuse to commit their savings to physical investment goods, knowing that next-period demand for agricultural products cannot be satiated, hence, given the nature of household preferences, that there will be no next-period demand for manufactured goods. Thus, as in Case 1,

$$E_{At} = E_t \quad \text{all } t < t_0. \quad (15)$$

For any t , if more than enough labor were allocated to agriculture to satiate consumers, the price of agricultural goods would drop to 0, as would the wage. The latter would not be consistent with equilibrium. Similarly, if at any $t > t_1$ too little labor were allocated to agriculture to satiate consumers, the price of manufactured goods would fall to 0. That would cause physical investment the period before to be 0. Thus at $t - 1$ there would be unemployed labor or more labor allocated to agriculture than necessary to satiate consumers. In either event, $w_{t-1} = 0$ — which again is inconsistent with equilibrium. The time- t agricultural labor input leading to demand satiation is

$$[(1+n)^{t-1} \cdot \bar{c}]^{\frac{1}{1-\alpha}}.$$

The preceding logic shows that for any equilibrium,

$$E_{At} = [(1+n)^{t-1} \cdot \bar{c}]^{\frac{1}{1-\alpha}} \quad \text{all } t > t_1. \quad (16)$$

For a positive wage, we need

$$E_{It} = E_t - E_{At} \quad \text{all } t. \quad (17)$$

Labor allocations to agriculture for t_0 and t_1 remain to be determined. There are only two possibilities for equilibria:

$$E_{A,t_0} = E_{t_0} \quad \text{and any } E_{A,t_1} \in (0, [(1+n)^{t_0} \cdot \bar{c}]^{\frac{1}{1-\alpha}}); \quad (18)$$

$$E_{A,t_1} = [(1+n)^{t_0} \cdot \bar{c}]^{\frac{1}{1-\alpha}} \quad \text{and any } E_{A,t_0} \in (0, E_{t_0}). \quad (19)$$

The logic of the preceding paragraph shows that E_{A,t_1} cannot exceed the bound for agricultural satiation. In (18), it falls short of the bound. Then there will be no demand for manufactured goods at t_1 ; hence, all labor at t_0 must find employment in agriculture. In (19), the economy reaches agricultural satiation at t_1 . Then there must be physical investment at time t_0 for the returns on land and reproducible capital to be the same.

Our reasoning shows that all possible equilibrium labor allocations fit (15)–(19). We now show that any such allocation can be an equilibrium.

For any labor allocation fitting (15)–(19), we can deduce corresponding output quantities from (4) and (5)–(7). Consider prices. In view of (5) and part (vii) of the definition of equilibrium, for any labor allocation from (15)–(19) set

$$P_{Tt} = E_t - E_{It} \quad \text{all } t. \quad (20)$$

A labor allocation and (10) determine the price per unit and total value of agricultural output at all times. Thus, using the price of land and part (iv) of the definition of equilibrium, set

$$1 + r_{t+1} = \frac{\alpha \cdot p_{A,t+1} \cdot Q_{A,t+1} + p_{T,t+1}}{p_{Tt}} \quad \text{all } t. \quad (21)$$

Referring to parts (v)–(vi) of the definition of equilibrium, and noting that any labor allocation determines $I_t = E_{It}$, set

$$p_{Kt} = p_{Mt} = \begin{cases} 1 + r_t, & \text{if } t \geq 1 \text{ and } E_{It} > 0, \\ 0, & \text{otherwise.} \end{cases} \quad (22)$$

Finally, set

$$p_{It} = \begin{cases} 1, & \text{if } E_{It} > 0, \\ 0, & \text{otherwise.} \end{cases} \quad (23)$$

Formally

Proposition 2: *Suppose (8) and (9'). The labor allocation implicit in any equilibrium must satisfy (15)–(19). Conversely, any labor allocation satisfying (15)–(19) uniquely determines an equilibrium: for any such allocation, set output quantities from (4) and (5)–(7); distribute all Q_A and Q_M equally among retired household; set $w_t = 1$; and, set all other prices from (10) and (20)–(23).*

Proof: It remains to show that the price and quantities determined above from any labor allocation satisfying (15)–(19) constitute an equilibrium. By construction all conditions for an equilibrium are satisfied except possibly consumer maximization. But, (vii) shows young households at t purchase all land and physical investment. Next period these are worth

$$\begin{aligned} \alpha \cdot p_{A,t+1} \cdot Q_{A,t+1} + p_{T,t+1} + p_{K,t+1} \cdot K_{t+1} &= \\ \alpha \cdot p_{A,t+1} \cdot Q_{A,t+1} + E_{t+1} - E_{I,t+1} + p_{M,t+1} \cdot Q_{M,t+1} &= \\ \alpha \cdot p_{A,t+1} \cdot Q_{A,t+1} + E_{A,t+1} + p_{M,t+1} \cdot Q_{M,t+1} &= \\ p_{A,t+1} \cdot Q_{A,t+1} + p_{M,t+1} \cdot Q_{M,t+1}. & \end{aligned}$$

Thus, retired households can purchase all of the agricultural and manufactured output of the economy. The construction of the labor allocation shows that retirees are never asked to purchase manufactured goods unless they are satiated with agricultural goods. ■

The rigid nature of preferences and production functions yield a set of equilibria. Elements of the set, however, differ only at dates t_0 and t_1 , and these differences do not matter in the next section, where we consider long-run averages. All of the equilibria in Proposition 2 are clearly Pareto efficient; in terms of welfare, they differ solely in the intergenerational distribution of utility at t_0 and t_1 .

4. Results

Given (9'), the economy at first specializes in agriculture but eventually devotes more and more of its labor to manufacturing. This section compares key variables and ratios in the early stage to their long-run averages after the date at which manufacturing commences. Case-1 variables and ratios are identical to the early-stage outcomes with (9').

We begin with the rate of interest. Line (12) shows

$$1 + r_{t+1} = \frac{(1+n) \cdot (1+g)}{1-\alpha} \quad \text{all } t < t_0. \quad (24)$$

Lines (17) and (20) show $p_{Tt} = E_{At}$. Hence the logic of (12) — together with (16) — yields

$$1 + r_{t+1} = \frac{\frac{\alpha \cdot E_{A,t+1}}{1-\alpha} + E_{A,t+1}}{E_{At}} = \frac{1}{1-\alpha} \cdot \frac{E_{A,t+1}}{E_{A,t}} = \frac{(1+n)^{\frac{1}{1-\alpha}}}{1-\alpha} \quad \text{for } t > t_1. \quad (25)$$

Condition (9') shows expression (25) is smaller than (24).

Thus, the average rate of return for savers is higher in the agricultural stage than afterward. Early on, population growth and technological progress raise the wage bill through time. The wage bill finances spending on agricultural products; hence, p_{At} climbs. Somewhat surprisingly, rents and the value of land end up rising just as fast as E_t .⁷ The interest rate reflects the sum of capital gains on land and rents as a fraction of land's price. After t_1 , although satiation causes agricultural output and prices to grow more slowly, formula (iv) still determines r_{t+1} . Hence, r_{t+1} ends up being lower after t_1 .

Turn next to the ratio of wealth to GDP. The model's ratio of national wealth to GDP is

$$\frac{p_{Tt} + p_{Mt} \cdot K_t}{p_{At} \cdot Q_{At} + p_{Mt} \cdot Q_{Mt} + I_t}$$

The preceding section's analysis shows that at early dates $K_t = I_t = Q_{Mt} = 0$. Accordingly,

$$\frac{p_{Tt}}{p_{At} \cdot Q_{At}} = \frac{E_{At}}{E_{At}/(1-\alpha)} = 1 - \alpha \quad \text{for } t < t_0. \quad (26)$$

Consider times after t_1 . Lines (9') and (16) imply

$$\lim_{t \rightarrow \infty} \frac{E_{A,t}}{E_t} = 0. \quad (27)$$

Hence, $p_{T,t}/E_t \rightarrow 0$, $I_t/E_t \rightarrow 1$, and $p_{At} \cdot Q_{At}/E_t = E_{At}/[(1-\alpha) \cdot E_t] \rightarrow 0$ — recall (10). Thus,

⁷ This is true at all times in Case 1 as well. Rapid population growth, for instance, could then lead to large capital gains on land despite stationary or falling standards of living.

$$\begin{aligned}
\lim_{t \rightarrow \infty} \frac{p_{Tt} + p_{Mt} \cdot K_t}{p_{At} \cdot Q_{At} + p_{Mt} \cdot Q_{Mt} + I_t} &= \lim_{t \rightarrow \infty} \frac{[p_{Tt} + p_{Mt} \cdot K_t]/E_t}{[p_{At} \cdot Q_{At} + p_{Mt} \cdot Q_{Mt} + I_t]/E_t} \\
&= \lim_{t \rightarrow \infty} \frac{p_{M,t} \cdot K_t/E_t}{(p_{M,t} \cdot K_t/E_t) + (I_t/E_t)} = \frac{1+r}{(1+r) + (1+n) \cdot (1+g)} \\
&= \frac{1}{1 + \frac{(1+n) \cdot (1+g)}{1+r}}, \tag{28}
\end{aligned}$$

with r as in (25). Since (24) exceeds (25), the last term of (28) is bounded above by $1/(1+1-\alpha)$. Comparing this with (26),

$$\begin{aligned}
\frac{1}{1+1-\alpha} < 1-\alpha &\iff 0 < (1-\alpha)^2 + (1-\alpha) - 1 \iff \\
1-\alpha > \frac{-1+\sqrt{5}}{2} &\iff \alpha < \frac{3-\sqrt{5}}{2} \approx .38.
\end{aligned}$$

Therefore, unless the share parameter of land in (4) is very large, the model's long-run average wealth ratio will be smaller after t_1 than before.

As Section 1 notes, Kuznets thought that historically wealth ratios typically fell after industrialization — though he reasoned the decline occurred because of accelerated growth. In the present paper, the value of land relative to E and GDP eventually falls — see (17), (20) and (27) — because of the operation of Engel's law, allowing the economy's wealth ratio to decline over time even when g and n are constant. The same process causes the composition of the national portfolio to change from all land to, asymptotically, all reproducible capital.

In measuring the aggregate average propensity to save (APS), we could think about either the increase in aggregate wealth in the economy divided by GDP, or the increase in reproducible capital divided by GDP. Since our households are indifferent between owning land and reproducible capital, the first ratio assesses the increase in how wealthy households feel. The second is the national income and products account (NIPA) investment-to-output ratio and is the most conventional way of actually measuring an economy's saving.

For $t < t_0$, wealth is land. Since the price of agricultural goods rises constantly (see (10)), the value of land grows faster than real output — the latter being Q_{At} at that point: from (20), the price of land is $p_{Tt} = E_{At} = E_t$, which appreciates with factor $(1+n) \cdot (1+g)$. Thus, early on the change in wealth over GDP is $[(1+n) \cdot (1+g) - 1]$ times expression (26). For $t > t_1$, $p_{Mt} \cdot Q_{Mt} = (1+r) \cdot E_{I,t-1}$, with r as in (25). Since $(1+r) \cdot E_{I,t-1}$ is $(1+n) \cdot (1+g)$ times larger after one period, (28) shows that asymptotically the increase-in-wealth to GDP ratio is $[(1+n) \cdot (1+g) - 1]$ times the industrial-stage wealth ratio. In other words, the APS measured in terms of wealth increases is proportional to the wealth-to-GDP ratio — with the same constant of proportionality in both stages. If the wealth ratio is higher before t_0 , the APS is too.

Turning to the national income and product accounts APS, since reproducible capital is not an input for agriculture in our model,

$$APS_t = 0 \quad \text{for } t < t_0. \quad (29)$$

For $t > t_1$, using gross investment

$$APS_t = \frac{I_t}{p_{At} \cdot Q_{At} + p_{Mt} \cdot Q_{Mt} + I_t}. \quad (30)$$

Dividing the numerator and denominator by E_t ,

$$\lim_{t \rightarrow \infty} APS_t = \lim_{t \rightarrow \infty} \frac{(I_t/E_t)}{(p_{Mt} \cdot Q_{Mt}/E_t) + (I_t/E_t)} = \frac{1}{\frac{(1+r)}{(1+n) \cdot (1+g)} + 1}, \quad (31)$$

with r as in (25). The long-run average APS for $t > t_1$ will equal the right-hand side of (31), which clearly is larger (29). For net investment, we replace the numerator of (30) with $I_t - I_{t-1}$, and the result in (31) should be multiplied by $1 - 1/[(1+n) \cdot (1+g)]$.

The idea is as follows. The aggregate amount of wealth which young households desire rises through time because of technological progress and population growth. At first, wealth is land, and increases in life-cycle wealth holdings must equal capital gains on land. Over time, the operation of Engel's law causes land to play a less and less important role in the economy relative to total output. Conversely, reproducible capital becomes increasingly significant. Reproducible capital is a polar opposite to land in terms of supply: land is fixed, but the economy can build new reproducible capital from labor with constant returns to scale. There are no capital gains on K therefore. Under (9'), rising wealth, which is typical of all time periods, leads after t_1 to an increasing stock of reproducible capital, requiring persistent gross and net physical investment, and generating a positive NIPA APS.

5. Conclusion

Although studies often infer that empirical correlations between countries' savings propensities and incomes mean that high rates of saving lead to high incomes, this paper presents a model in which causality can run the other direction as well. The new analysis is based on the changing composition of assets in household portfolios. When incomes are low, agriculture tends to be relatively important and capital gains on land camouflage household wealth accumulation from national income and products accounting. If incomes rise and agriculture's share of total expenditures declines, reproducible capital replaces land in prominence. Almost by definition "reproducible capital" has elastic supply and thus avoids long-term capital gains resembling those on land. A country's NIPA average propensity to save will then rise naturally, and this is true even if household wealth acquisition proceeds at the same rate as before. A low-income country that manages to improve its productive efficiency or gain access to better production technologies may find that its measured APS rises naturally with its standard of living thereafter, so that the economy can finance industrialization without a change in private wealth-accumulation habits.

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