

Devaluation Beliefs and Debt Crisis: The Argentinian Case[†]

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ABSTRACT

In this paper we will consider a simple small open economy with three assets - domestic capital, foreign securities and public debt - to study the government's incentives to devalue and to repay or default the debt. We show that the announcement of a devaluation is anticipated by domestic agents who reduce domestic investments and increase foreign holdings. Once a government devalues, the expectations vanish and the economy recovers its past levels of investment and GDP. However, in a country with international debt denominated in US dollars if a government devalues it requires a higher fraction of GDP to repay its external debt. In consequence, there exists a trade-off between recovering the economy and increasing the future cost of repaying the debt. Our main result is to show that, as devaluation beliefs exists, a devaluation increase government incentives to default and devalue. We calibrate our model to match the decrease in investment of domestic capital, the reduction in production, the increase in trade balance surplus, and the increase in debt levels observed throughout 2001 in Argentina. We show that for a probability of devaluation consistent with the risk premium of the Argentinian Government bonds nominated in dollars issued on April 2001 the external debt of Argentina was in a crisis zone were the government find optimal to default and to devalue.

Keywords: Debt crisis, Devaluation, Argentina.

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1 Introduction

During 2001 the Argentinian GDP fell by more than 20 percent and investment decreased by more than 5 percent of GDP. At the same time, the trade balance yielded a surplus in 2001, and foreign reserves fell dramatically. The ratio of external debt to the GDP increased so much that it forced the Argentinian government to default on December 2001. Afterwards, in January 2002, the government devalued the peso by 40 percent. Figure 1 document these facts.

[Figure 1 about here.]

What happened during 2001 in Argentina? On March 16, President De La Rúa rejected the plan presented by Economics Minister López Murphy to reduce the fiscal deficit. The new minister, Domingo Cavallo presented a new economic plan in the lower house of Argentinian congress. On March 28, the congress refused to allow Cavallo to cut government salary and pension expenditure, and the government sold debt to cover the deficit. Between April and August, several announcements on changes in the exchange rate policy were made. First, on April 12, Cavallo announced that the peso would be peg to the euro (and maybe to the yen). In May, the government announced economic plans that included currency changes, and, on June 18, the Argentine government announced a complex set of new economic policies, including the installation of multiple exchange rates to help the country's exporters. In July, the Province of Buenos Aires announced the issue of a new currency to pay bills, the patacón, and in August, the Banco de la Nación limited sales of dollars at a one-to-one rate with the peso.

[Figure 2 about here.]

Figure 2 reports the daily series of Argentinian reserves. As the announcements on changes were made, the reserves fell. An IMF aid package in September led to a recovery of reserves. But, on October 30, the government could not sell new debt and started to restructure its debt, finally forcing pension funds to buy government bonds. On December 23, the government defaulted, and, on January 11, 2002, the government devalued the peso, after a week of suspended convertibility.

In this paper we will consider a simple small open economy with three assets - domestic capital, foreign securities and public debt - to study the government's incentives to devalue and to repay or default the debt. We will show that expectations on devaluation account for the default on debt.

Our theory is simple. As the expectations of devaluation increase, domestic agents modify their portfolio by reducing their investment in domestic capital and increasing their foreign asset holdings. This reduces GDP and tax revenues. We assume that once a government devalues, the expectations vanish and the economy recovers its past levels of investment and GDP. A government has an incentive to devalue so as to increase the future levels of output, consumption, and capital stock. However, if a government devalues, in the future it requires a higher fraction of GDP to repay its external debt (which is denominated in US dollars). In consequence, the government policy of devaluation faces a trade-off between recovering the economy, and increasing the future cost of repaying the debt.

Our main result shows that under a speculative attack the optimal government policy depends on its level of debt. If the level of debt is low, the government devalues to increase capital but does not default. For higher levels of debt, the government does not devalue and repays its debt because the cost of a default is higher than the benefits of a devaluation. Finally, for sufficiently high levels of debt, the government defaults, because repaying the debt is too costly, and devalues, once the default eliminates the future cost of repaying the debt. Our theory explains why we sometimes observe "good" devaluations, where the economy recovers or "bad" experiences where devaluations took place only after government default, and as a result the economy pays a severe productivity cost that reduces investment and output (as in Argentina in 2002).

We calibrate our model to match the decrease in investment of domestic capital, the reduction in production, the increase in trade balance surplus, and the increase in debt levels observed throughout 2001 in Argentina. We show that for a probability of devaluation consistent with the risk premium of the Argentinian Government bonds nominated in dollars issued on April 2001 the external debt of Argentina was in a crisis zone where the government find optimal to default and to devalue.

The paper proceeds as follows. In the next section we present the economic environment. Section 3 presents the definition of the equilibrium. Section 4 characterizes the optimal behavior of private agents. Section 5 characterizes the levels of debt for which the government always devalues and never defaults and in section 6 we show that self expectation devaluation with default can exist. Section 7 characterizes the government behavior in self-fulfilling crisis and section 8 provides a numerical exercise for the Argentinian case. Finally, in the last section we conclude.

2 The economic environment

There are three agents in the economy –domestic consumers, international bankers, and government– and three assets –domestic capital, K , foreign security, A , and public debt, B . Both A and B are denominated in US dollars and can be exchanged for domestic goods at the real exchange rate e .

The international bankers have perfectly elastic demand for government debt at price q and perfectly elastic supply of the foreign security at price $1/r^*$. The government provides an amount of public good g , obtains revenue from income taxes, τ , and issues public debt, B' . Finally, domestic consumers are the owners of the capital, k , and foreign holdings, a , they inelastically supply a unit of labor, and derive utility from private consumption and the government good.

The consumers

There is a continuum with measure one of identical, infinitely lived consumers who consume, invest, and pay taxes. The individual's utility function is

$$E \sum_{t=0}^{\infty} \beta^t (c_t + v(g_t))$$

where c_t is private consumption and g_t is government consumption. The assumption of risk neutrality of consumers greatly simplifies the modeling of consumer behavior as in Cole and Kehoe (1996). We assume that $0 < \beta < 1$ and that v is continuously differentiable, strictly concave, and monotonically increasing. We also assume that $v(0) = -\infty$. The households'

income, from foreign security holdings from previous period a and factor payment (labor and capital), is devoted paying taxes, consuming domestically produced goods c , and investing in domestic capital k' and in foreign securities a' . The consumer's budget constraint is

$$c_t + k_{t+1} + e_t[a_{t+1} + \Phi(a_{t+1})] = (1 - \tau)\alpha(z_t)\theta(e_t, e_{t-1})f(k_t) + e_t r^* a_t$$

where $e \in \{\underline{e}, \bar{e}\}$ is the real exchange rate (pesos per dollar), z is an indicatrix dealing with default, and r^* is the international interest rate in US dollars. We normalize $\underline{e} = 1$. Here k_t is the consumer's individual capital stock; α is a multiplicative productivity factor that depends on whether or not the government has ever defaulted and $0 < \theta < 1$ is another productivity factor that depends on whether the government devalues or not; τ , with $0 < \tau < 1$, is the constant proportional tax on domestic income; and f is a continuously differentiable, concave, and monotonically increasing production function that satisfies $f(0) = 0$, $f'(0) = \infty$, and $f'(\infty) = 0$. The consumer is endowed with K_0 units of capital and a_0 units of foreign security at period 0. There is also an investment cost on international securities, represented by an increasing, convex function $\Phi(a_{t+1})$, with $\Phi'(0) = 0$. The existence of this function allows us to find the optimal allocations of the foreign security.¹

There are three important assumptions. First, we are assuming that there is a technology that transforms Argentinian goods into foreign goods. The rate of transformation is the real exchange rate, e_t , and, in order to simplify the model, we assume that no changes in nominal prices of Argentinian goods are expected or reported, so a nominal devaluation is also a real devaluation: the government, in choosing e_t , also changes the real terms of trade between the domestic good and the foreign good.² Second, if the government decides to default, there is a permanent negative productivity shock, as in Cole and Kehoe (2000) and other literature on financial economics. Finally, in order to determine the optimal level of a devaluation we assume that the period in which the devaluation happens, the economy is affected by a transitory negative shock in productivity

$$\theta(e_t, e_{t-1}) = \begin{cases} \theta & \text{if government devalues} & e_t > e_{t-1} \\ 1 & \text{if government not devalues} & e_t = e_{t-1} \end{cases}$$

¹This trick is a common one in the small open economy literature. Otherwise, under arbitrage, it turns out to be difficult to compute the amount of resources devoted to the foreign assets.

²This is a reduced form of a model where consumption and investment are composed of tradeable and non-tradeable goods.

There are two formulations that would rationalize our assumption that productivity falls after the government devalues. One story is that firms must renegotiate contracts and, in the short term, firms cannot substitute foreign inputs. We could assume, for example, that there is a foreign produced intermediate good, which cannot be substituted, whose price increases after a devaluation.³ Another story that can rationalize our assumption is that after a devaluation the government increases trade taxes, set different exchange rates for exports and imports, or establishes quotas on trade. In summary, government increases distortions in the economy and reduces output.

The international bankers

There is a continuum with measure one of identical, infinitely lived international bankers. The individual banker is risk neutral and has the utility function

$$E \sum_{t=0}^{\infty} \beta^t x_t$$

where x_t is the banker's private consumption. Analogous to Cole and Kehoe (1996) the assumption of risk neutrality of bankers captures the idea that the domestic economy is small compared to world financial markets. Each banker is endowed with \bar{x} units of the consumption good in each period and faces the budget constraint

$$x_t + q_t B_{t+1} + r^* a_t \leq \bar{x} + z_t B_t + a_{t+1}$$

where q_t is the price of one-period government bonds that pay B_{t+1} in period $t+1$ if $z_{t+1} = 1$, that is, the government decides to repay its debts, and 0 if the government decides not to repay, i.e., $z_{t+1} = 0$.

The government

There is a single government, which is benevolent in the sense that its objective is to maximize the welfare of the consumers. In every period, the government makes three decisions: (i) it chooses the level of government consumption, g_t , financed with household income taxes

³To see how we can incorporate this story into our model, denote by $F(n, k)$ the production function for output as a function the price of the intermediate good n and capital k , and by p the price of the intermediate good. Then, if we require that $f(k) = \max_n F(n, k) - pn$, $\theta(e_t, e_{t-1})f(k) = F(n^*, k) - p(e_t/e_{t-1})n^*$, where $n^* = \arg \max F(n, k) - pn$.

and with some of the dollars obtained from the US dollar denominated bonds issued new borrowing level B_{t+1} ; (ii) it decides whether or not to default on its old debt, $z_t \in \{0, 1\}$; (iii) it chooses the real exchange rate, e_t . Its budget constraint is

$$g_t = \tau\alpha(z_t)\theta(e_t, e_{t-1})f(k_t) + e_t[q_t B_{t+1} - z_t B_t]$$

The government decides to pay, $z_t = 1$, or to default public debt, $z_t = 0$, and whether devalue $e_t > e_{t-1}$ or not $e_t = e_{t-1}$. As in Cole and Kehoe (1996, 2000), national productivity is affected by a default (i.e., $\alpha(z = 0) < \alpha(z = 1)$) and the government loses access to international borrowing and lending after default. Finally, the market clearing condition for the government debt is $b_{t+1} = B_{t+1}$, and we also assume that $k_0 = K_0$ and $b_0 = B_0$.

In each period, the value of an exogenous variable ξ_t is realized. We show that we can construct equilibria where, if the level of government debt B_t is above some crucial level and ξ_t is above another crucial level, then consumers anticipate a devaluation and reduce domestic investment. This creates a self-fulfilling debt crisis in the sense that, since the reduction in domestic investment changes the government incentive to honor its debt. The government chooses to default and then to devalue.

The timing

We assume that the timing of actions within each period is the following:

1. The government sells debt.
2. The international bankers, taking the price of debt as given, choose to buy or not to buy the debt.
3. The government decides to default or not, and chooses the exchange rate and government consumption.
4. The exogenous variable, ξ , is realized.
5. Consumers choose consumption and investment on the domestic and the foreign securities.

One crucial feature of our model is the timing of the consumers' decisions. Given that they observe the sunspot after the government decision making, the government is unable to preclude the effects of sunspot on the consumers' decisions.

3 Equilibrium

As in Cole and Kehoe (2000), the government cannot commit itself either to honoring its debt obligations or to following a fixed borrowing and spending path. It also cannot commit to modify or not the real exchange rate, e . We follow closely Cole and Kehoe's recursive equilibrium definition in which there is no commitment and the agents choose their actions sequentially.

When an individual consumer acts, he knows the following: his individual capital k , and foreign assets holding a , the aggregate state $s = (B, K, A, \alpha_1, e_{-1})$; the government's supply of new debt B' ; the price that bankers are willing to pay for this debt q ; the government's spending, g , and default and devaluation decisions, z and e , respectively; and the sunspot ξ . We define the state of the individual consumer as $(k, a, s, B', g, z, e, \xi)$. We denote the government's policy functions by $B'(s)$, $g(s, B', q)$, $z(s, B', q)$ and $e(s, B', q)$; the price function by $q(s, B')$; and by $K'(s, B', g, z, e, \xi)$, $A'(s, B', g, z, e, \xi)$ the functions that describe the evolution of the aggregate capital and foreign asset stocks, all yet to be defined. The representative consumer's value function is defined by the functional equation

$$\begin{aligned}
V_c(k, a, s, B', g, z, e, \xi) &= \max_{c, k', a'} c + v(g) + \beta E[V_c(k', a', s', B'(s'), g', z', e', \xi')] \\
s.t. \quad c + k' + e[a' + \Phi(a')] &\leq (1 - \tau)\alpha(s, z)\theta(s, e)f(k) + er^*a \\
c, k' > 0 \quad a' &\geq 0 \\
s' &= (B', K'(s, B', g, z, e, \xi), A'(s, B', g, z, e, \xi), \alpha(s, z), e) \\
g' &= g(s', B'(s'), q(s', B'(s'))) \\
z' &= z(s', B'(s'), q(s', B'(s'))) \\
e' &= e(s', B'(s'), q(s', B'(s')))
\end{aligned}$$

The three policy functions of the consumers are $c(k, a, s, B', g, z, e, \xi)$, $k'(k, a, s, B', g, z, e, \xi)$ and $a'(k, a, s, B', g, z, e, \xi)$. Because consumers are also competitive, we need to distinguish between the individual decisions, k_{t+1} and a_{t+1} , and the aggregate values, K_{t+1} and A_{t+1} . In equilibrium, given that all consumers are identical, $k_{t+1} = K_{t+1}$ and $a_{t+1} = A_{t+1}$.

As explained, the production parameters satisfy $\alpha(s, z) = 1$ if the government has not defaulted in the past and has not defaulted this period (otherwise, $\alpha(s, z) = \alpha$), and $\theta(s, e) = 1$ if the government has not devalued in this period (otherwise $\theta(s, e) < 1$).⁴

When an individual banker chooses his new debt level, he knows his individual holdings of government debt b , the aggregate state s , and the government's offering of new debt B' . The state of an individual banker is defined as (b, s, B') . The representative banker's value function is defined by the functional equation

$$\begin{aligned} V_b(b, s, A, B') &= \max_b x + \beta E [V_b(b', s', A', B'(s'))] \\ \text{s.t. } x + q(s, B')b' + r^*A &\leq \bar{x} + z(s, B', q(s, B'))b + A' \\ x > 0, \quad q(s, B')b' &\leq \bar{x} \\ s' &= (B', K'(s, B', g, z, e, \xi), A'(s, B', g, z, e, \xi), \alpha(s, z), e) \end{aligned}$$

Bankers are relatively passive: if \bar{x} is sufficiently large, they purchase the amount of bonds offered by the government as long as the price of these bonds satisfies

$$q(s, B') = \beta E z(s', B'(s'), q(s', B'(s'))),$$

and the assumption that they behave competitively guarantees that they sell the amount of foreign assets demanded by consumers if $r^* = 1/\beta$.

The only strategic agent in the model is the government. Its makes decisions at two points in time. At the beginning of the period, when the government chooses B' , the government's state is simply the initial state s . Later, after it has observed the actions of the

⁴Note that to define the equilibrium we write α and θ as functions of the state. In the next section, we return to our original notation.

bankers, which are summarized in the price q , it will choose whether or not to devalue, e , and default, z , which in turn determines the level of government spending, g , and the levels of productivity, α and θ . This choice is given by the policy functions $g(s, B', q)$, $e(s, B', q)$ and $z(s, B', q)$. In consequence, at the beginning of the period the government knows how the price that its debt will bring, $q(s, B')$, depends on this state and on the level of new borrowing. The government also knows what its own optimizing choices $g(s, B', q(s, B'))$, $e(s, B', q(s, B'))$ and $z(s, B', q(s, B'))$ will be later. The government also realizes that it can affect consumption, c , domestic investment K' , foreign holdings, A' , and the production parameters, α and θ , through its choices. The government's value function is defined by the functional equation

$$\begin{aligned}
V_g(s) &= \max_{B'} E \{c(K, A, s, B', g, z, e, \xi) + v(g) + \beta V_g(s')\} \\
s.t \quad &g = g(s, B', q(s, B')) \\
&z = z(s, B', q(s, B')) \\
&e = e(s, B', q(s, B')) \\
&s = (B', K'(s, B', g, z, e, \xi), A'(s, B', g, z, e, \xi), \alpha(s, z), e)
\end{aligned}$$

We denote by $B'(s)$ the government's debt policy. At a later moment, the government makes its decisions on default, z , and devaluation, e , which determines the level of α and θ and the level of government spending, g . Given $V_g(s)$, we define the policy functions $g(s, B', q)$, $e(s, B', q)$ and $z(s, B', q)$ as the solutions to the following problem

$$\begin{aligned}
&\max_{g, z, e} E \{c(K, A, s, B', g, z, e, \xi) + v(g) + \beta V_g(s')\} \\
s.t \quad &g - \tau \alpha(s, z) \theta(s, e) f(K) \leq e[qB' - zB] \\
&z = 0 \text{ or } z = 1 \\
&g \geq 0 \\
&\theta(s, e) = \begin{cases} \theta(\underline{e}, \bar{e}) & \text{if the government devalues} \\ 1 & \text{if government does not devalue} \end{cases} \\
&s' = (B', K'(s, B', g, z, e, \xi), A'(s, B', g, z, e, \xi), \alpha(s, z), e)
\end{aligned}$$

Definition of an equilibrium.

An equilibrium is a list of value functions V_c , for the representative consumer, V_b for the representative banker, and V_g for the government; policy functions c , k' and a' for the consumer, b' and a' for the banker, and B' , g , z and e for the government; a price function q and an interest rate r^* ; and equations of motion for the aggregate capital stock K' and foreign asset stock A' such that the following conditions hold,

1. Given B' , g , z , e and ξ , V_c is the value function for the solution to the representative consumer's problem, and c , k' and a' are the maximizing choices.
2. Given B' , A' , q , and z , V_b is the value function for the solution to the representative banker's problem, and the value of B' chosen by the government solves the problem when $b = B$.
3. Given q , c , K' , A' , g , z and e , V_g is the value function for the solution to the government's problem first problem, and B' is the maximizing choice. Furthermore, given c , K' , A' , V_g , and B' , then g , z and e maximize the consumer welfare subject to the government's budget constraint
4. $q(s, B') = \beta E z(s', B'(s'), q(s', B'(s')))$, and $r^* = 1/\beta$.
5. $K'(s, B', g, z, e, \xi) = k'(K, A, s, B', g, z, e, \xi)$, $A'(s, B', g, z, e, \xi) = a'(K, A, s, B', g, z, e, \xi)$ and $B'(s) = b'(B, s, B')$.

Finally, consumers and bankers know that the government solves its problem each period, and therefore understand that, under some circumstances, the government will choose to default and/or to devalue.

4 The optimal behavior of private agents

The bankers' optimal behavior depends upon their expectations that they have about the government's future repayment decision z' . If bankers expect that $z' = 0$, then they are not willing to buy any debt unless the price is 0. If bankers expect that $z' = 1$, then they are willing to buy any amount of the government debt up to \bar{x} at price β . If bankers expect

default to occur with probability π they are willing to purchase whatever amount of bonds the government offers up to \bar{x} at price $q = \beta(1 - \pi)$.

The consumers' optimal policy depends solely on what they expect the values of the productivity parameters α and θ will be next period. There are several cases.

No expectations of devaluation. We start first with the cases where consumers have no expectations of devaluation. Consumers believe that the government will not devalue in the next period ($\pi = 0$). Then the first-order conditions are

$$\begin{aligned}\beta(1 - \tau)\alpha(z')f'(k') &= 1 \\ \Phi(a') &= 0 \\ c + k' &= (1 - \tau)\alpha(z')\theta(\underline{e}, \bar{e})f(k) + er^*a\end{aligned}$$

If devaluation has occurred in period t and the government has already defaulted it is optimal for them to set the capital stock for next period to a level k^d that satisfies

$$\beta(1 - \tau)\alpha(0)f'(k^d) = 1$$

to set the level of foreign holdings $a' = 0$, and to eat whatever output is left over

$$c^{dd}(K, a) = (1 - \tau)\alpha(0)\theta(\underline{e}, \bar{e})f(K) + \bar{e}r^*a - k^d$$

their consumption after devaluation and default occur is

$$c^{nd}(k^d, 0) = (1 - \tau)\alpha(0)f(k^d) - k^d$$

If devaluation has occurred in period t and if the government has not defaulted it is optimal for them to set the capital stock for next period to a level k^n that satisfies

$$\beta(1 - \tau)\alpha(1)f'(k^n) = 1$$

to set the level of foreign holdings $a' = 0$, and to eat whatever output is left over

$$c^{dn}(K, a) = (1 - \tau)\alpha(1)\theta(\underline{e}, \bar{e})f(K) + \bar{e}r^*a - k^n$$

their consumption after devaluation and not default occur is

$$c_0^{nn}(k^n, 0) = (1 - \tau)\alpha(1)f(k^n) - k^n$$

If the government does not devalue and has not defaulted it is optimal for consumers to set the capital stock for the next period to the level k^n , to set the level of foreign holdings $a' = 0$ and eat whatever is left over

$$c_0^{nn}(K, a) = (1 - \tau)\alpha(1)f(K) + r^*a\underline{e} - k^n$$

and their consumption thereafter $c_0^{nn}(k^n, 0)$.

If the government does not devalue but has defaulted it is optimal for consumers to set the capital stock for the next period to the level k^d , to set the level of foreign holdings $a' = 0$ and eat whatever is left over

$$c^{nd}(K, a) = (1 - \tau)\alpha(0)f(K) + r^*a\underline{e} - k^d$$

and their consumption thereafter $c^{nd}(k^d, 0)$.

Expectations of devaluation. We are interested in studying the cases in which consumers believe that the productivity parameter θ will be equal to $\theta(\underline{e}, \bar{e})$ for the next period because the government has not previously devaluated, but consumers believe that the government will devalue during the next period ($\pi = 1$). Then the first-order conditions are

$$\begin{aligned}\beta(1 - \tau)\alpha(z')\theta(\underline{e}, \bar{e})f'(k') &= 1 \\ 1 + \Phi(a') &= \frac{1}{\theta(\underline{e}, \bar{e})} \\ c + k' &= (1 - \tau)\alpha(z')f(k) + \left[r^*a - a' - \Phi(a')\right]\underline{e}\end{aligned}$$

If the government does not devalue and has not defaulted it is optimal for consumers to set the capital stock for the next period to a level k^{dn} that satisfies

$$\beta(1 - \tau)\alpha(1)\theta(\underline{e}, \bar{e})f'(k^{dn}) = 1$$

to set the level of foreign holdings a^{dn} that satisfies

$$1 + \Phi'(a^{dn}) = \frac{\bar{e}}{\underline{e}}$$

and eat whatever is left over

$$c_1^{nn}(K, a) = (1 - \tau)\alpha(1)f(K) + \left[r^*a - a^{dn} - \Phi(a^{dn})\right]\underline{e} - k^{dn}$$

If consumers believe that the government will devalue the next period ($\pi = 1$) and the government does not devalue and has defaulted, it is optimal for them to set the capital stock for the next period to a level k^{nd} that satisfies

$$\beta(1 - \tau)\alpha(0)\theta(\underline{e}, \bar{e})f'(k^{nd}) = 1$$

to set the level of foreign holdings a^{dn} and eat whatever is left over

$$c_1^{nd}(K, a) = (1 - \tau)\alpha(0)f(K) + [r^*a - a^{dn} - \Phi(a^{dn})] \underline{e} - k^{nd}$$

5 Devaluation without Default

In this section we will show that an equilibrium exists in which the government brakes a speculative attack imposing a cost to the economy in the current period and maintaining in the future a higher level of debt. For low levels of debt a threshold on government debt will exist under which the government prefers to bear the cost of a devaluation, so that the subsequent recovery of the productivity will permit it to distribute this cost along the future periods. We will denote this level by \underline{b} .

We suppose initially that the government always pays its debt⁵ and that the consumers believe that the government will devalue in the following period, i.e. $\pi = 1$. The bankers do not experience panic and always buy all the debt issued to the level \bar{x} at the price $q = \beta$. We will compare the payments that the government obtains by devaluing and not devaluing to find the level of debt \underline{b} . The payment the government obtains after devaluing and not defaulting is

$$V^{dn}(s, B_0, B_1) = c^{dn}(K_0, a_0) + v\left(\tau\alpha(1)\theta(\underline{e}, \bar{e})f(K_0) + \bar{e}(\beta B_1 - B_0)\right) + \frac{\beta}{1 - \beta} \left\{ c_0^{nn}(k^n, 0) + v\left(\tau\alpha(1)f(k^n) - \bar{e}(1 - \beta)B_1\right) \right\} \quad (1)$$

while if not devaluing and not defaulting

$$V_1^{nn}(s, B_0, B_1) = c_1^{nn}(K_0, a_0) + v\left(\tau\alpha(1)f(K_0) + (\beta B_1 - B_0)\right) + \frac{\beta}{1 - \beta} \left\{ c_1^{nn}(k^{dn}, a^{dn}) + v\left(\tau\alpha(1)f(k^{dn}) - (1 - \beta)B_1\right) \right\} \quad (2)$$

The threshold \underline{b} will be the higher level of debt B_0 that verifies

$$V^{dn}(s, B_0, B_1) \geq V_1^{nn}(s, B_0, B_1) \quad (3)$$

⁵Later on we will show that this will be so in equilibrium.

That is to say, for greater levels of debt, despite consumer expectation on devaluation, the government does not devalue and repays its debt.

To determine the level of debt \underline{b} , however, it is necessary to characterize the conduct of the government relating to the new debt. It is optimal for the government to maintain a constant level of spending $g_{t+1} = g_t$ and, hence, of its debt. Both depend on initial conditions (K_0, B_0) .

If the government has chosen to devalue, given that it is constant, government consumption is given by

$$g^d(B_0, K_0) = \tau\alpha(1) \left[\beta f(k^n) + \theta(\underline{e}, \bar{e})(1 - \beta)f(K_0) \right] - \bar{e}(1 - \beta)B_0$$

while government debt stays constant at

$$B^d(B_0, K_0) = B_0 + \frac{\tau\alpha(1)}{\bar{e}} \left[f(k^n) - \theta(\underline{e}, \bar{e})f(K_0) \right].$$

In the case that the government does not devalue, the constant government consumption will be given by

$$g^n(B_0, K_0) = \tau\alpha(1) \left[\beta f(k^{dn}) + (1 - \beta)f(K_0) \right] - (1 - \beta)B_0$$

while government debt stays constant at

$$B^n(B_0, K_0) = B_0 + \tau\alpha(1) \left[f(k^{dn}) - f(K_0) \right]$$

Given initial conditions (K_0, B_0) , when government consumption is constant, the government's payoff from devaluing and not devaluing (1) and (2) is given, respectively, by

$$V^{dn}(s, B_0, B^d(B_0, K_0))$$

and

$$V_1^{nn}(B_0, B^n(B_0, K_0)).$$

We now argue that when government expenditure is constant, and for β sufficiently high, there is a unique $b^* > 0$ such that

$$V^{dn}(s, b^*, B^d(b^*, K_0)) = V_1^{nn}(s, b^*, B^n(b^*, K_0))$$

When the constraint $V^{dn} \geq V_1^{nn}$ is violated, i.e. $B_0 > b^*$, in the proposed equilibrium described above, there are two possibilities: the government may choose not to devalue, or

it may choose to devalue with a non-stationary expenditure by issuing a new debt level B_1 , to be different from $B^d(B_0, K_0)$, and then maintain this level thereafter. Let $B_1(B_0, K_0, a_0)$ be the value of B_1 that satisfies $V^{dn}(B_0, B_1) = V_1^{nn}(B_0, B_1)$, if such value exists. If no such B_1 exists, then it is optimal for the government not to devalue. We now present a characterization of the equilibrium.

Proposition 1. *For $\beta < 1$ sufficiently close to 1 and \bar{e}_t sufficiently high, there exists a continuous (and increasing) function $\underline{b}(K, a)$ and a positive debt level b^* , such that the following outcomes occur.*

(i) *If $0 \leq B_0 \leq b^*$, then the economy converges to a stationary equilibrium with devaluation, no default and constant government expenditure*

$$g_1 = g_2 = g^d = \tau\alpha(1) \left[\beta f(k^n) + \theta(1 - \beta)f(K_0) \right] - \bar{e}(1 - \beta)B_0$$

and constant government bonds $B_1 = B_2 = B^d = B_0 + \frac{\tau\alpha(1)}{\bar{e}} \left[f(k^n) - \theta f(K_0) \right]$.

(ii) *If $b^* \leq B_0 \leq \underline{b}(K_0, a_0)$, then the economy converges to a stationary equilibrium with devaluation, no default and the dynamics for the government expenditure is $g_1^d < g^d < g_2^d$ and constant at this level thereafter, and for the government bonds $B_0 < B_1$ and constant at this level thereafter.*

(iii) *If $B_0 > \underline{b}(K_0, a_0)$, then the outcome is not in the devaluation no default equilibrium.*

[Figure 3 about here.]

The most interesting case is $K_0 \leq k^n$, where the government issues new debt before devaluing (Figure 3). The reason is that it tries to distribute the cost of the devaluation among every period and to smooth its expenditure. If B_0 is small the government does not have any limit to issue new debt to maintain the public expenditure constant. After the recovery it can face the future higher payment of the debt with higher tax revenue. The highest level of debt for which it is possible to transfer the cost of the devaluation to the new debt and completely smooth the public expenditure is b^* .

If $B_0 > b^*$ is not possible to distribute all the cost of the devaluation over time by issuing new debt and therefore the government must transfer part of the cost to a reduction of the public expenditure of the current period. If it tried to maintain the public expenditure

constant the cost of repaying the debt in the future would be so high that the government would prefer not to devalue.

[Figure 4 about here.]

The proposition 1 establishes that there exist a level of debt that equalize the benefits of the devaluation with the cost that this devaluation causes.⁶ That is to say, on the one hand a devaluation carries to have to confront with a greater debt service in the future, that is because as the devaluation recover the economy the government issue more debt today to smooth the public expenditure. Moreover, an devaluation also means a increase in the future cost of repaying the debt. This two effects are collected by the term $\bar{e}(1 - \beta)B_1$ of (1). On the other hand, the devaluation causes the elimination of the expectation of the consumers what will increase the investment of k^{dn} to k^n , with the consequent increase in consumption and income. Note that the benefits are independent of the level of debt while the costs are increasing in it. The figure 4 represents this intuition.

6 Devaluation with Default

When the level of debt is very high the government has no incentive to repay its debt. Opposite to that which occurred when the level of debt was low, now the cost of productivity that provokes a default is not high enough to oblige the government to repay its debt. Besides, in this case, if the government decides not to repay its debt it will also decide to devalue, since a future cost of repaying the debt does not exists. Thus, beliefs are eliminated and domestic investment recovers.

This section characterizes two critical levels of debt. The first one, denoted by \bar{b} , determines a zone where the government always repays and therefore never devalues. The second, denoted by \bar{B} determines another zone where the government never pays and therefore always devalues.

The levels of debt between \bar{b} and \bar{B} constitute a crisis zone in which, if the consumers expect a devaluation, this is always carried out and the government does not repay its debt. On the contrary, if consumers do not panic the government does not devalue and repays its

⁶Note that if $\beta \rightarrow 1$, the benefits and costs relevant in this case are the ones that are produced in the future.

debt. Besides, we will show that the optimal response of the government consists of reducing its debt to escape from the crisis zone.

In order to show that a crisis zone can exist, we will follow Cole and Kehoe (2000) and show that: first, if the level of debt is lower than the critical level \bar{b} , even if consumers expected a devaluation and bankers decide not to buy the new debt (observing the lower level of domestic investment), the government does not devalue and repay its debt ; second, if the level of debt is higher than the other critical level \bar{B} , even if consumers do not expect a devaluation and bankers buy all the debt issues, the government defaults and devalues. These two levels of debt determine three zones: (1) the no crisis zone, where the government does not recover the economy and repay its debt; (2) the crisis zone without expectations of devaluation, where the government always defaults and devalues; (3) the crisis zone, where if consumers believe that a devaluation will happen the government defaults and then devalues.

No Crisis Zone

To see how the government does not devalue and repay its debt, we study the case where the government repays even if bankers do not buy government bonds and consumers expect a devaluation.

In order to show that this equilibrium exists we must show that the level of debt satisfies two conditions. First the government prefers not to default and not to devalue than to default and devalue. Second, the government prefers not to default and not to devalue than to devalue and repay. Finally, we must show that the upper bound found in proposition 1 is lower than the threshold found in this case.

With the purpose of characterizing the maximum level of debt for which the government prefers to repay and not devalue (\bar{b}) is assumed, as in Cole and Kehoe (2000), that the government offers new debt B' to international bankers, but, given that consumers expect a devaluation, $\pi = 1$, the bankers know that the government devalue only after default on its old debt B . Given this conjecture, the price of the new government bonds falls to $q = 0$.

The payoff to the government if it devalues and defaults is given by

$$V^{dd}(s, B_0, B_1) = c^{dd}(K_0, a_0) + v\left(\tau\alpha(0)\theta(\underline{e}, \bar{e})f(K_0) + \bar{e}\beta B_1\right) + \frac{\beta}{1-\beta} \left[c^{nd}(k^d, 0) + v\left(\tau\alpha(0)f(k^d)\right) \right] \quad (4)$$

with $B_1 = 0$ because bankers do not buy any government bonds, i.e., $V^{dd}(s, B_0, 0)$.

The two constraints on government debt must be satisfied simultaneously in any equilibrium with no default and no devaluation are

$$\begin{aligned} V^{dn}(s, B_0, 0) &\leq V_1^{nn}(s, B_0, 0) \\ V^{dd}(s, B_0, 0) &\leq V_1^{nn}(s, B_0, 0) \end{aligned}$$

Let us define

$$\begin{aligned} (V^{dd} - V_1^{nn})(B_0, B_1) &= H_1^{dd-nn} + v\left(\tau\alpha(0)\theta(\underline{e}, \bar{e})f(K_0) + \bar{e}\beta B_1\right) - \\ &v\left(\tau\alpha(1)f(K_0) + \beta B_1 - B_0\right) + \frac{\beta}{1-\beta} \left\{ v\left(\tau\alpha(0)f(k^d)\right) - v\left(\tau\alpha(1)f(k^{dn}) - (1-\beta)B_1\right) \right\} \end{aligned}$$

where $H_1^{dd-nn} \equiv c^{dd}(K_0, a_0) - c_1^{nn}(K_0, a_0) + \beta/(1-\beta) [c^{nd}(k^d, 0) - c_1^{nn}(k^{dn}, a^{dn})]$.

We let $\bar{b}(K, a)$ be the largest value of B_0 for which the government weakly prefers to repay its debt, even if it cannot sell new bonds at a positive price, i.e., $(V^{dd} - V_1^{nn})(\bar{b}(K_0, a_0), 0) = 0$. We refer to the range of debt value for which both constraints are satisfied as the no crisis zone, $B \in (\underline{b}(K), \bar{b}(K)]$. The following proposition establishes when we can have a non-empty zone and shows that in the equilibrium characterized in the proposition 1 the government always pays its debt.

Proposition 2. *For $\beta < 1$ sufficiently close to 1 and \bar{e} sufficiently high, there exists a continuous (and increasing) function $\bar{b}(K, a) > \underline{b}(K, a)$, such that there exists a non-empty interval of levels of government debt B , $\underline{b}(K_0, a_0) < B < \bar{b}(K_0, a_0)$ where the government will not devalue and repays its debt.*

Crisis without expectations

In order to determine the zone where crisis with default can be possible, we show that there exists a level of debt for which, even in the case that consumers have no expectations on devaluation ($\pi = 0$) and the bankers buy government debt, the government will default and then devalue. This level of debt determines the zone where crisis can only occur if consumers believe that a devaluation will take place in the next period.

In this case the government's payoff of not devaluing is

$$\begin{aligned} V_0^{nn}(s, B_0, B_1) &= c_0^{nn}(K_0, a_0) + v(\tau\alpha(1)f(K_0) + (\beta B_1 - B_0)) + \\ &\frac{\beta}{1-\beta} \{c_0^n(k^n, 0) + v(\tau\alpha(1)f(k^n) - (1-\beta)B_1)\} \end{aligned} \quad (5)$$

Let be $\bar{B}(K_0, a_0)$ the lower level of debt for which the payoff to the government if it devalues, given by (4), is greater than the payoff given by (5). Formally

$$V^{dd}(s, B_0, B_1) \geq V_0^{nn}(s, B_0, B_1)$$

In consequence, for an intermediate level of government bonds, the government will devalue and default if consumers expect a devaluation with one-probability ($\pi = 1$), and repay and does not devalue if consumers believe that no devaluation will occur $\pi = 0$.

Proposition 3. *For $\beta < 1$ sufficiently close to 1 and \bar{e} sufficiently high, there is a non-empty interval of levels of government debt B , $\bar{b} < B \leq \bar{B}$, for which one-probability devaluation crises are possible.*

[Figure 5 about here.]

In summary, there exist four zones (see Figure 5): (1) If $B \leq \underline{b}$, the government devalues but repays its debt; (2) If $\underline{b} < B \leq \bar{b}$, the government does not devalue and repays its debt; (3) If $\bar{b} < B \leq \bar{B}$, if consumers reduce investments, the government devalues and does not repay its debt; (4) If $\bar{B} < B$, the government always defaults and devalues.

[Figure 6 about here.]

The cost of the default is a fraction $\alpha(0)$ of the gross national product and therefore independent of the level of debt, while the benefits of the default are increasing in the level of debt issued today. Figure 6 represents this intuition. The level of debt \bar{B} is the level of debt such that benefits and costs of a default are equals. It is important to remark that for levels of debt lowers than \bar{B} the government always repay its debt if the consumers does not expect a devaluation ($\pi = 0$), since, as opposed to Cole and Kehoe (2000), the panic is not in the international bankers.

[Figure 7 about here.]

Figure 7 show us the conditions under which the four zones exists. A default eliminate the future cost of repaying the debt and always increase the benefits of a devaluation. The level of debt characterized by proposition 2, \bar{b} , is the level of debt for which the benefits of a devaluation with default are equal to the net cost of a default. For levels of debt higher

than \underline{b} and lower than \bar{b} the government never defaults because the cost of a default is always higher than the benefits of an devaluation with default. Then the government always repay its debt and not devalue. Moreover, for levels of debt higher than \bar{b} and lower than \bar{B} the benefits of a devaluation with default are greater than the net cost of the default. Then the government default and devalue, because the net benefits of the devaluation compensate the net cost of the default. Finally, note that the benefits of the devaluation depends of the consumers beliefs. If the consumers have not expectations on devaluation ($\pi = 0$), the benefits of the devaluation are zero and the government never default and devalue for levels of debt lower than \bar{B} . In summary, for levels of debt between \bar{b} and \bar{B} the equilibrium depends on the devaluation beliefs of the consumers.

7 Self-Fulfilling Devaluation Crises

We can now characterize the optimal government behaviour in equilibria in which devaluation can occur with a positive probability $0 < \pi < 1$ depending on realizations of the sunspot variable.

A self-fulfilling devaluation crisis arises when there are two possible equilibrium outcomes, one in which the government does not devalue and chooses to repay the old debt and another in which the government devaluates and defaults on the existing debt. Self-fulfilling crises are possible in these equilibria for certain values of the fundamentals (K, B) ; the realization of the sunspot variable determines which of these two outcomes ensues.

In equilibrium, if $\zeta \leq \pi$ and B is greater than the crucial level $\bar{b}(K, a)$, then consumers predict that the government will devalue. Consumers reduce their investment in domestic capital and increase their foreign asset holdings. This reduces output and tax revenues in the next period and bankers are therefore not willing to pay a positive price for the new debt offered and thus provoke a default. If, however, $\zeta > \pi$, then consumers predict that the government will not devalue. If B is less than or equal to the crucial level $\bar{b}(K, a)$, however, then no crisis can occur, no matter what the realization of ζ . Because ζ is uniformly distributed on the unit interval, π is both the crucial value of ζ , and the probability that $\zeta \leq \pi$. If $\zeta \leq \pi$, a crisis takes place if the debt is above $\bar{b}(K, a)$ and below the upper bound,

which we now denote $\bar{B}(K, a, \pi)$ since this bound also will vary with π . In the previous sections we have analyzed the limiting cases where $\pi = 0$ and $\pi = 1$.

Before characterizing the government's behavior in this equilibrium, we need to know for what regions of (B, K) values a self-fulfilling devaluation crisis are possible and for what regions devaluation and default are the only outcome.

The lower bound $\bar{b}(K, a)$ does not change. No crisis equilibrium is possible if the government weakly prefers to repay its debt, even if it cannot sell new bonds and consumers predict a devaluation. Explicitly characterizing the upper bound on debt $\bar{B}(K, a, \pi)$ is more difficult here because, as we shall see, optimal government policy will not, in general, be stationary in the crisis zone. We can explicitly characterize the upper bound on debt under a stationary debt policy where the capital stock is equal to k^n . Let $B^s(\pi)$ be the largest value of B for which

$$\begin{aligned} & c_0^{nn}(k^n, a) + v \left[\tau\alpha(1)f(k^n) - (1 - \hat{\beta})B \right] + \frac{\hat{\beta}}{1 - \hat{\beta}} \left\{ c_0^{nn}(k^n, 0) + v \left[\tau\alpha(1)f(k^n) - (1 - \hat{\beta})B \right] \right\} \\ & + \frac{\beta\pi \left(c_1^{nn}(k^n, 0) + v \left[\tau\alpha(1)f(k^n) - (1 - \hat{\beta})B \right] \right)}{1 - \hat{\beta}} + \frac{\beta^2\pi \left(c^{dd}(k^{dn}, a^{dn}) + v \left[\tau\alpha(0)f(k^{dn}) \right] \right)}{(1 - \hat{\beta})} \\ & + \frac{\beta^3\pi \left(c_0^{nd}(k^d, 0) + v \left[\tau\alpha(0)f(k^d) \right] \right)}{(1 - \hat{\beta})(1 - \beta)} \geq c^{dd}(k^n, a) + v \left[\tau\alpha(0)\theta(\underline{e}, \bar{e})f(k^n) + \hat{\beta}\bar{e}B \right] + \\ & + \frac{\beta}{1 - \beta} \left(c_0^{nd}(k^d, 0) + v \left[\tau\alpha(0)f(k^d) \right] \right) \end{aligned}$$

[Figure 8 about here.]

where we have denoted $\hat{\beta} = \beta(1 - \pi)$. As π tends to 0 this constraint tends to $V^{dd} - V_0^{nn}(B, B) \leq 0$ in lemma 4; hence $B^s(0) = B^s$.

Lemma 1. *If the economy is such that, in the $\pi = 1$ bad crisis equilibrium, the stationary upper bound on debt implied by lemma 4 satisfies $B^s(0) > \bar{b}(k^n)$ then for any probability π and for $K_0 = k^n$, there is a non-empty region of debt levels $\bar{b}(k^n, a) < B < \bar{B}(k^n, a, \pi)$.*

We now construct an equilibrium in which devaluation and default occur with positive probability. Suppose that $K_0 = k^n$ and $B_0 > \bar{b}(k^n, a)$, and the government is faced with

the following choices in period 0: devalue and default now; plan to run the debt down to $\bar{b}(k^n, a)$ or less in T periods if no devaluation occurs; or never run the debt down. For each of these choices, we can calculate the expected payoff. The equilibrium is determined by the choice that yields the maximum expected payoff. Assuming that $B_0 \leq B^s(\pi)$, the government maintains a constant level of government spending if a devaluation does not occur but is possible. If the government plans to run its debt down to $\bar{b}(k^n, a)$ in T periods, we can use the government's budget constraints to calculate that level of government spending:

$$g^T(B_0) = \tau\alpha(1)f(k^n) + \frac{\hat{\beta}^{T-1}\beta(1-\hat{\beta})}{1-\hat{\beta}^T}\bar{b}(k^n, a) - \frac{1-\hat{\beta}}{1-\hat{\beta}^T}B_0$$

If the government chooses to never run its debt down to $\bar{b}(k^n, a)$, then government spending is

$$g^\infty(B_0) = \tau\alpha(1)f(k^n) - (1-\hat{\beta})B_0$$

We can now calculate the expected payoff of running the debt down to $\bar{b}(k^n, a)$ in T periods

$$\begin{aligned} V^T(B_0) = & c_0^{nn}(k^n, a) + v[g^T(B_0)] + \frac{\hat{\beta} - \hat{\beta}^T}{1 - \hat{\beta}} \{ (1 - \pi) [c_0^{nn}(k^n, 0) + v(g^T(B_0))] + \\ & + \pi c_1^{nn}(k^n, 0) + \pi v[g^T(B_0)] + \pi V_\pi^{dd} \} + \frac{\hat{\beta}^{T-1}\beta}{1 - \beta} [c_0^{nn}(k^n, 0) + v(\tau\alpha(1)f(k^n) - (1 - \beta)\bar{b}(k^n, a_0))] \end{aligned}$$

where $\hat{\beta} = \beta(1 - \pi)$ and

$$V_\pi^{dd} = \beta (c^{dd}(k^{dn}, a^{dn}) + v[\tau\alpha(0)f(k^{dn})]) + \beta^2 (c_0^{nd}(k^d, 0) + v[\tau\alpha(0)f(k^d)]) / (1 - \beta)$$

To determine T , we merely choose the maximum of

$$V^1(B_0), V^2(B_0), \dots, V^\infty(B_0)$$

where

$$\begin{aligned} V^\infty(B_0) = & c_0^{nn}(k^n, a) + v[g^T(B_0)] + \frac{\hat{\beta}}{1 - \hat{\beta}} \{ (1 - \pi) [c_0^{nn}(k^n, 0) + v(g^\infty(B_0))] + \\ & + \pi c_1^{nn}(k^n, 0) + \pi v[g^T(B_0)] + \pi V_\pi^{dd} \} \end{aligned}$$

Lemma 2. For any K_0 and $B_0 \leq B^s(\pi) - \tau\alpha(1)(f(K_0) - f(k^n))$, if we denote by V^T the government's payoff when its policy is to lower its debt to $\bar{b}(k^n)$ in T periods while keeping

g constant, then a $T \in \{1, 2, \dots, \infty\}$ that maximizes $\{V^1(B_0), V^2(B_0), \dots, V^\infty(B_0)\}$ exists, and the following are true:

- (i) If $K_0 \geq k^n$, as B_0 increases, $T(B_0)$ passes through critical points where it increases by one period. Furthermore, for π close enough to 0, there necessarily are regions of $B_0 \leq B^s(\pi)$ with the full range of possibilities $T(B_0) = 1, 2, \dots, \infty$;
- (ii) If $K_0 < k^n$, then the debt may increase in the first period, but afterwards follows the same characterization as in (i) since $K_1 = k^n$ and $B_1 \leq B^s(\pi)$.

Lemma 3. For any $\pi > 0$ for which there exists a non-empty crisis zone $\bar{b}(k^n, a) < B \leq \bar{B}(k^n, a, \pi)$, there can exist a crisis equilibrium in which the transition function for capital and the price function on government debt are given by

$$K(B') = \begin{cases} k^n & \text{if } B' \leq \bar{B}(k^n, a, \pi) \text{ and } \alpha = \alpha(1) \xi > \pi \\ k^{dn} & \text{if } B' \leq \bar{B}(k^n, a, \pi) \text{ and } \alpha = \alpha(1) \xi < \pi \\ k^d & \text{otherwise} \end{cases}$$

$$q(B') = \begin{cases} \beta & \text{if } B' \leq \bar{b}(k^n, a) \text{ and } z(s, B', \beta) = 1 \\ \hat{\beta} & \text{if } \bar{b}(k^n) < B' \leq \bar{B}(k^n, a, \pi) \text{ and } z(s, B', \hat{\beta}) = 1 \\ 0 & \text{otherwise} \end{cases}$$

and, depending on B_0 , the following outcomes occur

- (i) If $K_0 \geq k^n$ and $B_0 \leq \bar{b}(k^n, a)$ then $c_0 = c^{nn}(K_0, a_0)$ and all other equilibrium variables are stationary: $K = k^n$, $a = 0$, $c_t = c^{nn}(k^n, 0)$ for $t \geq 1$, $B = B_0 - \tau\alpha(1)(f(K_0) - f(k^n))$, $g = \tau\alpha(1)f(k^n) - (1 - \beta)B$ $q = \beta$ and $e = \underline{e}$. In this case no devaluation occurs;
- (ii) If $\bar{b}(k^n, a) < B_0 \leq \bar{B}(k^n, a, \pi)$, then a devaluation and default occurs with probability π in the first period and every subsequent period in which $B > \bar{b}(k^n, a)$. If $B_0 \leq B^s(\pi) - \tau\alpha(1)(f(K_0) - f(k^n))$, optimal government policy involves running down the debt to $\bar{b}(k^n, a)$ in $T(B_0)$ periods, while smoothing government expenditures as described in Proposition 6. If $T(B_0)$ is finite and a crisis does not occur, then following period $T(B_0)$, the equilibrium outcomes are those in (i). For $B_0 > B^s(\pi) - \tau\alpha(1)(f(K_0) - f(k^n))$, the equilibrium converges to the outcome described in Lemma 2 in at most two periods.
- (iii) If $K_0 < k^n$ and $B_0 \leq \bar{b}(k^n, a)$, then there is no possibility of a devaluation in period 0, and from period 1 onward, the outcomes correspond to those described in (i) if under the government's optimal policy, $B_1 \leq \bar{b}(k^n, a)$ or in (ii) if not.

(iv) If $B_0 > \bar{B}(K_0, a, \pi)$, then the only outcome is the devaluation default outcome in which $c_0 = c^{dd}(K_0, a_0)$, $g_0 = \tau\alpha(0)\theta(\underline{e}, \bar{e})f(K_0)$, and all other equilibrium variables are stationary: $K = k^d$, $c = c^{dd}(k^d, 0)$, $B = 0$, $a = 0$, $g = \tau\alpha(0)f(k^d)$, $q = 0$, and $e = \bar{e}$.

8 A Numerical Exercise

This section presents a numerical exercise whose parameters have been chosen so that the initial period matches the situation of Argentina in 2000. We use the model to help us interpret events in Argentina in 2001. We show that the crisis zone for our stylized model of Argentina is fairly large, and that the evolution of the variables of the model matches the evolution of the aggregate variables of Argentina's economy during 2001.

The utility function for the consumers and the government is

$$E \sum_{t=0}^{\infty} \beta^t (c_t + \log(g_t))$$

The technology and the feasibility constraint are given by

$$\begin{aligned} f(K) &= AK^s \\ c + g + k' - (1 - \delta)k &\leq AK^s + [qB' - B - a' - \Phi(a') + r^*a]e \end{aligned}$$

and the adjustment cost function is given by

$$\Phi(a) = \phi_1 + \frac{(\phi_2 a)^2}{2}$$

The capital share in GDP was taken from Kydland and Zarazaga (2002), $s = 0.4$. The discount factor $\beta = 0.963$ corresponds to an international interest rate of 3.84% that was taken from the interest rate in 2001 of US. 1 year Government Securities Treasury bills. The permanent drop of the productivity associated with a default is taken from Cole and Kehoe (2000) and implies a fall in productivity of 5%, $\alpha(0) = 0.95$. The temporary drop of the productivity related to a devaluation is established to reproduce the reduction in the investment rate observed between the year 2000 and 2001, $\theta(\underline{e}, \bar{e}) = 0.9892$ that represents a fall in productivity of 1.92%.

Setting the probability of devaluation $\pi = 0.0473$ the yield of the Argentinian Government bonds nominated in dollars with a year of maturity is $0.09 = [\beta(1 - \pi)]^{-1} - 1$ that corresponds with the government bonds issued with those characteristics on April 19, 2001. This means a risk premium of a 5.16% upon the argentinian government bonds. The previous exchange rate to the crisis is fixed in $\underline{e} = 1$ and the exchange rate after the devaluation in $\bar{e} = 1.4$ that corresponds to the exchange rate set by the Argentine government on January 11, 2002. Table I shows the values of the parameters calibrated without solving the model.

The next five parameters $\phi_1, \phi_2, \tau, A, \delta$ and a_0 are calibrated solving the model. The parameters ϕ_1 and ϕ_2 of the adjustment cost is fixed to reproduce the the investment rate in the Argentinian GDP 2000, $i/y = 0.18$, and the reduction in international reserves of the Central Bank that during the year 2001 reached, 9200 million of dollars 3.42% of the 2001 output. The tax rate and the TFP, A , are calibrated from the steady state budget constraint of the government to reproduce the shares of government spending and public debt in the Argentinian GDP 2000: , $g/y = 0.19$ and $B/y = 0.45$ respectively. We obtain a depreciation rate of $\delta = 0.0815$ for a capita-output ratio of $K/Y = 3$. Finally, the initial value of the foreign assets a_0 is chosen to reproduce the share in GDP of the trade balance in 2000, that is to say, a surplus of 0.41%. Table II shows the values of the parameters.

[Table 1 about here.]

[Table 2 about here.]

With these values of the parameters and with $K_0 = k^n$ the levels of debt that determine the different zones of the model are presented in Table III. Consequently, the initial values of debt for which a self-fulfilling devaluation crisis can occur in the first period are between 19,59% and 236,78% of the output. The level of Argentinian debt over GDP in 2000 reached 45% of the output, which means that it was in the crisis zone.

[Table 3 about here.]

The numerical exercise show that during 2001 Argentina was in a equilibrium in which devaluation and default occur with positive probability. As happened during 2001 the government plans to run its debt down before devalue and default in January 2002. We can use

the calibrated parameters to calculate the optimal policy of the government debt before the government default (see Figure 9).

[Figure 9 about here.]

Finally, Figure 10 presents the evolution of the variables of the economy calibrated when the variable sunspot generates the expectation of devaluation at the moment t , i.e. $\zeta \leq \pi$ in t .

[Figure 10 about here.]

9 Conclusions

In this paper a theory is built to study the relationship between the politics of exchange rate and the debt defaults of governments when this debt is nominated in foreign currency. The most interesting result that is obtained is that there exist low levels of debt for which devaluation permits the recovery of the economy and thus an increase in levels of public expenditure, while for higher levels of debt the possibility of a devaluation generates the default of the government. Besides, the model is calibrated for the Argentine economy just before the crisis of 2001 and shows that the debt of the government was in the crisis zone. In future research, we will model how changes in real terms of trade induce debt crises in an economy with tradeable and non-tradeable goods.

References

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A APPENDIX

A.1 Proof of Proposition 1

Proof. We will define the following difference as

$$(V^{dn} - V_1^{nn})(B_0, B_1) = H_1^{dn-nn} + v\left(\tau\alpha(1)\theta(\underline{e}, \bar{e})f(K_0) + \bar{e}(\beta B_1 - B_0)\right) - v\left(\tau\alpha(1)f(K_0) + (\beta B_1 - B_0)\right) \\ + \frac{\beta}{1-\beta} \left\{ v\left(\tau\alpha(1)f(k^n) - (1-\beta)\bar{e}B_1\right) - v\left(\tau\alpha(1)f(k^{nn}) - (1-\beta)B_1\right) \right\}$$

where $H_1^{dn-nn} \equiv c^{dn}(K_0, a_0) - c_1^{nn}(K_0, a_0) + \beta/(1-\beta) [c^{nd}(k^d, 0) - c_1^{nn}(k^{nn}, a^{nn})]$.

Notice that $(V^{dn} - V_1^{nn})(0, B_1) > 0$ as $\beta \rightarrow 1$, which requires $c_0^{nn}(k^n, 0) - c_1^{nn}(k^{dn}, a^{dn}) + v\left(\tau\alpha(1)f(k^n) - (1-\beta)\bar{e}B_1\right) - v\left(\tau\alpha(1)f(k^{dn}) - (1-\beta)B_1\right) > 0$.⁷ In addition, observe that $(V^{dn} - V_1^{nn})(B_0, B_1) \rightarrow -\infty$ as $B_0 \rightarrow (\tau\alpha(1)\theta(\underline{e}, \bar{e})/\bar{e})f(K_0) + \beta B_1 \equiv B_0^{dn-nn}(B_1, \beta)$.

We need to characterize optimal government behavior with respect to B_1 . It is optimal for the government to maintain a constant level of spending $g_{t+1} = g_t$ and, hence, of its debt. Both depend on initial conditions (K_0, B_0) .

If the government has chosen to devalue, given that it is constant, the government consumption is given by

$$g^d(B_0, K_0) = \tau\alpha(1) \left[\beta f(k^n) + \theta(\underline{e}, \bar{e})(1-\beta)f(K_0) \right] - \bar{e}(1-\beta)B_0$$

while government debt stays constant at

$$B^d(B_0, K_0) = B_0 + \frac{\tau\alpha(1)}{\bar{e}} \left[f(k^n) - \theta(\underline{e}, \bar{e})f(K_0) \right]$$

If the government does not devalue, the constant government consumption will be given by

$$g^n(B_0, K_0) = \tau\alpha(1) \left[\beta f(k^{dn}) + (1-\beta)f(K_0) \right] - (1-\beta)B_0$$

while government debt stays constant at

$$B^n(B_0, K_0) = B_0 + \tau\alpha(1) \left[f(k^{dn}) - f(K_0) \right]$$

let be k^* such that $f(k^n) = \theta(\underline{e}, \bar{e})f(k^*)$, for all $K_0 < k^*$ then $B^d = B_1 > B_0$; for all $K_0 = k^*$ then $B_1 = B_0$; and, for all $K_0 > k^*$ then $B_1 < B_0$. Notice that $k^n < k^*$.

Given initial conditions (K_0, B_0) , when the government consumption is constant, the government's payoff from devaluing and no devaluing (1) and (2) is given, respectively, by $V^{dn}(s, B_0, B^d(B_0, K_0))$ and $V_1^{nn}(B_0, B^n(B_0, K_0))$. If these payoffs satisfy the constraint $V^{dn} \geq V_1^{nn}$, then this is an equilibrium outcome.

We argue that when the government expenditure is constant, and for β sufficiently close to one, there is a unique $b^* > 0$ such that

$$V^{dn}(s, b^*, B^d(b^*, K_0)) = V_1^{nn}(s, b^*, B^n(b^*, K_0))$$

⁷This condition is guaranteed if the adjustment cost is sufficiently high so that $k^{dn} - (r^* - 1)a^{dn} + \Phi(a^{dn}) - (1-\tau)\alpha(1)[f(k^n) - f(k^{dn})] > k^n$ which implies $c_0^{nn}(k^n, 0) > c_1^{nn}(k^{dn}, a^{dn})$, and also $g_1^d > g_1^n$.

let us write this constraint as $(V^{dn} - V_1^{nn})(b^*) = 0$ where

$$\begin{aligned} (V^{dn} - V_1^{nn})(b^*) &= H_1^{dn-nn} + v\left(\tau\alpha(1)\theta(\underline{e}, \bar{e})f(K_0) + \bar{e}(\beta B^d(b^*, K_0) - b^*)\right) - \\ &- v\left(\tau\alpha(1)f(K_0) + (\beta B^n(b^*, K_0) - b^*)\right) + \frac{\beta}{1-\beta}\left\{v\left(\tau\alpha(1)f(k^n) - (1-\beta)\bar{e}B^d(b^*, K_0)\right) - \right. \\ &\left. - v\left(\tau\alpha(1)f(k^{dn}) - (1-\beta)B^n(b^*, K_0)\right)\right\} \end{aligned}$$

where $H_1^{dn-nn} \equiv c^{dn}(K_0, a_0) - c_1^{nn}(K_0, a_0) + \beta [c_0^{nn}(k^n, 0) - c_1^{nn}(k^{dn}, a^{dn})] / (1 - \beta)$.

Notice that $(V^{dn} - V_1^{nn})(0) > 0$ as $\beta \rightarrow 1$, and that $(V^{dn} - V_1^{nn})(b) \rightarrow -\infty$ as $b \rightarrow \tau\alpha(1)/\bar{e}[\theta(\underline{e}, \bar{e})f(K_0) + \beta f(k^n)/(1 - \beta)] \equiv B_0^{dn-nn}(B^d(b, K_0), \beta)$, i.e., $g^d(b^*, K_0)$ goes to zero. Finally differentiating $V^{dn} - V_1^{nn}$ yields

$$\frac{d(V^{dn} - V_1^{nn})(b)}{db} < 0$$

when \bar{e} is sufficiently high.

Consequently, since $(V^{dn} - V_1^{nn})$ is continuous in b , there is a unique b^* such that $0 < b < B_1^{dn-nn}(B^n(b, K_0), \beta)$. That is, $(V^{dn} - V_1^{nn})(b^*) = 0$ and $(V^{dn} - V_1^{nn})(b) > 0$ for all $B < b^*$, while $(V^{dn} - V_1^{nn})(b) < 0$ for all $B > b^*$.

Whenever the constraint $V^{dn} \geq V_1^{nn}$ is violated, i.e. $B_0 > b^*$, in the proposed equilibrium described above, there are two possibilities: the government may choose not to devalue, or it may choose to devalue with a non-stationary expenditure by issuing a new debt level B_1 , to be different from $B^d(B_0, K_0)$, and then maintain this level thereafter.

Let $B_1(B_0, K_0, a_0)$ be the value of B_1 that satisfies $V^{dn}(B_0, B_1) = V_1^{nn}(B_0, B_1)$, if such value exists. If no such B_1 exists, then it is optimal for the government not to devalue.

We now argue that there is a continuous increasing function $\underline{b}(K, a)$ such that for all $b^* \leq B_0 \leq \underline{b}(K_0, a_0)$ it is optimal for the government to devalue in period 0 and maintain a constant level of government expenditure different from period 1 on. In this case the government maintains a level of debt that differs from B_0 . For all $B_0 > \underline{b}(K_0, a_0)$, it is optimal for the government not to devalue. We then let

$$\begin{aligned} \underline{b}(K_0, a_0) &= \max B_0(B_1, K_0, a_0) \\ &\text{subject to} \\ &0 \leq B_1 \leq B_1(B_0, K_0, a_0) \end{aligned}$$

the constraint $B_1 \leq B_1(B_0, K_0, a_0)$ binds if and only if the constraint $V^{dn} \geq V_1^{nn}$ binds in period 0 when $B_0 = \underline{b}(K_0, a_0)$, i.e. $V^{dn}(B_0, B_1) = V_1^{nn}(B_0, B_1)$. Differentiating $(V^{dn} - V_1^{nn})(B_0, B_1)$, we obtain

$$\frac{\partial (V^{dn} - V_1^{nn})(B_0, B_1)}{\partial B_0} < 0$$

when \bar{e} is sufficiently high. Furthermore, since $(V^{dn} - V_1^{nn})(0, B_1) > 0$ as $\beta \rightarrow 1$ and $(V^{dn} - V_1^{nn})(B_0, B_1) \rightarrow -\infty$ as $B_0 \rightarrow B_0^{dn-nn}$, then there is a unique $B_0(B_1)$ for which the constraint holds with equality; due to $\partial(V^{dn} - V_1^{nn})(B_0, B_1)/\partial B_0 \neq 0$ the implicit function theorem implies that $B_0(B_1, K_0, a_0)$ is continuous. Since $B_0(B_1, K_0, a_0)$ is continuous in B_1 , it achieves a maximum on the compact constraint set.

The dynamics of the government expenditure and government bonds are the following,

$$\begin{aligned} g_1^d &= \tau\alpha(1)\theta f(K_0) + \bar{e}\beta B_1 - \bar{e}B_0 \\ g_2^d &= \tau\alpha(1)f(k^n) - \bar{e}(1 - \beta)B_1 \end{aligned}$$

In order to prove part (ii) recall first that for \bar{e} sufficiently high $\frac{\partial(V^{dn}-V^{nn})(B_0, B_1)}{\partial B_0} < 0$. Also observe that for \bar{e} sufficiently high, if $g_2^d - g_1^d > 0$ then $\frac{\partial(V^{dn}-V^{nn})(B_0, B_1)}{\partial B_1} > 0$, and if $g_2^d - g_1^d < 0$ then $\frac{\partial(V^{dn}-V^{nn})(B_0, B_1)}{\partial B_1} < 0$. In consequence, the implicit function theorem implies $\frac{dB_1}{dB_0}$ a positive sign for the former, and a negative for the latter.

Second, observe that as B_0 increases up to b^* , a positive sign is resulting in $(V^{dn}-V^{nn})(B_0, B_1(B_0))$ with constant public expenditure as indicated in part (i). However, as B_0 sets beyond b^* this positivity does not hold any longer, so an equilibrium with non constant public expenditure may exist. Having reached the threshold $B_0 = b^*$ and then $B^d = B^{d*} = b^* + \frac{\tau\alpha(1)}{\bar{e}}[f(k^n) - \theta f(K_0)]$, the dynamics of the public expenditure is, then, given by

$$g_2^d - g_1^d = \tau\alpha(1)\left(f(k^n) - \theta f(K_0)\right) + \bar{e}(B_0 - B_1) = \bar{e}\left[(B_0 - b^*) - (B_1 - B^{d*})\right] \quad (6)$$

Now, beyond b^* the first increment is always positive. So the public expenditure can only be increased in this case by increasing government issue B_1 , and lower than the difference $B_0 - b^*$. Note that no other case is possible. Think of a decrease in public expenditure $g_2^d - g_1^d < 0$. As we indicated above, the implicit function theorem implies that $\frac{dB_1}{dB_0} < 0$. If initially $B_0 = b^*$ and afterwards it were increased, then B_1 would be consequently increased $B_1 > B^{d*}$. This is a contradiction since (6) implies a positive increase in the public expenditure.

Finally it is easy to show that $g_1^d < g^d < g_2^d$, since $g_1^d - g^d = -\bar{e}\beta\left[(B_0 - b^*) - (B_1 - B^{d*})\right]$ and $g_2^d - g^d = \bar{e}(1 - \beta)\left[(B_0 - b^*) - (B_1 - B^{d*})\right]$. \square

A.2 Proof of Proposition 2

Proof. The proof follows the following steps.

First, we define the difference $(V^{dn} - V_1^{nn})(B_0, B_1)$ and $(V^{dd} - V_1^{nn})(B_0, B_1)$, as well as some of their properties in the case where consumers have no expectations on devaluation. Two thresholds spring from these properties. Firstly, from proposition 1, the government, after deciding not to devalue, will devalue if the initial government bond level B_0 is lower than $\underline{b}(K_0, a_0)$, beyond which there will be no devaluation. Next, after the bankers decide not to buy government bonds, the government will default and devalue if the initial government bond level B_0 is higher than $\bar{b}(K_0, a_0)$, below which there will be no devaluation and no default.

Second, in order that the one-probability zone exists, we will show that it will be required that the threshold $\underline{b}(K_0, a_0)$ is lower than $\bar{b}(K_0, a_0)$, and then, a sufficiently high $\beta < 1$ can be found such that the zone exists.

Consider the definition and properties of $(V^{dd} - V_1^{nn})(B_0, B_1)$ stated in proposition 1. That is, in proposition 1, for some \bar{e} high enough, it was proved that given any B_1 , the difference $(V^{dn} - V_1^{nn})(B_0, B_1)$ is a decreasing function in B_0 , i.e. $\partial(V^{dn} - V_1^{nn})(B_0, B_1)/\partial B_0 < 0$, and that there exists a threshold $\underline{b}(K_0, a_0)$ such that $(V^{dn} - V_1^{nn})(\underline{b}(K_0, a_0), B_1(\underline{b}(K_0, a_0))) = 0$ and for $B_0 > \underline{b}(K_0, a_0)$, $V^{dn}(B_0, B_1) < V_1^{nn}(B_0, B_1)$ for all B_1 .

In addition, let us define now

$$(V^{dd} - V_1^{nn})(B_0, 0) = H_1^{dd-nn} + v\left(\tau\alpha(0)\theta(\underline{e}, \bar{e})f(K_0)\right) - v\left(\tau\alpha(1)f(K_0) - B_0\right)$$

$$+\frac{\beta}{1-\beta} \left\{ v\left(\tau\alpha(0)f(k^d)\right) - v\left(\tau\alpha(1)f(k^{nn})\right) \right\}$$

where $H_1^{dd-nn} \equiv c^{dd}(K_0, a_0) - c_1^{nn}(K_0, a_0) + \beta/(1-\beta) [c^{nd}(k^d, 0) - c_1^{nn}(k^{nn}, a^{nn})]$.

Observe that $(V^{dd} - V_1^{nn})(0, 0) < 0$ as $\beta \rightarrow 1$, which implies that $c^{nd}(k^d, 0) - c_1^{nn}(k^{nn}, a^{nn}) + v\left(\tau\alpha(0)f(k^d)\right) - v\left(\tau\alpha(1)f(k^{nn})\right) < 0$; and that $(V^{dd} - V_1^{nn})(B_0, 0) \rightarrow +\infty$ as $B_0 \rightarrow \tau\alpha(1)f(K_0) \equiv B_0^{dd-nn}(0, \beta)$. Here, it is easy to prove that $\partial(V^{dd} - V_1^{nn})(B_0, 0)/\partial B_0 = v'(\tau\alpha(1)f(K_0) - B_0) > 0$ so that there exists a threshold $(V^{dd} - V_1^{nn})(\underline{b}(K_0, a_0, \beta), 0) = 0$.

Now, in order the no crisis no devaluation zone exists, we must require that

$$\underline{b}(K_0, a_0) < \bar{b}(K_0, a_0, \beta). \quad (7)$$

See Figure 1. First, considering $B_1 = 0$ this condition holds since $B_0^{dn-nn}(0, \beta) < B_0^{dd-nn}(\beta)$, due to $\theta(\underline{e}, \bar{e})/\bar{e} < 1$. Now increasing B_1 , observe that $B_0^{dn-nn}(B_1, \beta)$ increases (and eventually exceeds $B_0^{dd-nn}(0, \beta)$). However, in order that the condition (7) holds, we will require that⁸ $(V^{dn} - V_1^{nn})(B_0^{dd-nn}, B_1(\underline{b}(K_0, a_0))) < 0$.

Under this assumption, for a $\beta < 1$ sufficiently high, there exists some level B such that $\underline{b}(K_0, a_0) < B < B_0^{dd-nn}$. In addition, $\bar{b}(K_0, a_0, \beta)$ is strictly increasing in β and we can set $\bar{b}(K_0, a_0, \beta)$ as close as B_0^{dd-nn} as wished. This means that the no crises zone without devaluation exists.

[Figure 11 about here.]

□

A.3 Proof of Proposition 3

Proof. The proof consists on showing that the region exists for all B_1 .

In the region of crises, $\pi = 1$, whatever the bankers do. It can be proved now that the difference $(V^{dd} - V_1^{nn})(B_0, B_1)$ is increasing in B_1 for a sufficiently high \bar{e} .

In proposition 2 it was proved that in the crises region $(V^{dn} - V_1^{nn})(B_0, B_1) < 0$ when $(V^{dd} - V_1^{nn})(B_0, B_1) > 0$.

Hence gains of default and devalue for the government increase with the bonds bought by the bankers.

In addition, in the crises region the government prefers to devalue and default than to default and no-devalue for all B_1 , i.e., $(V^{dd} - V_1^{nd})(B_0, B_1) > 0$, with

$$(V^{dd} - V_1^{nd})(B_0, B_1) = H_1^{dd-nd} + v\left(\tau\alpha(0)f(K_0) + \beta B_1\right) - v\left(\tau\alpha(0)\theta(\underline{e}, \bar{e})f(K_0) + \beta\bar{e}B_1\right)$$

where $H_1^{dd-nd} \equiv c^{dd}(K_0, a_0) - c_1^{nd}(K_0, a_0) + \beta/(1-\beta) [c^{nd}(k^d, 0) - c_1^{nd}(k^{nd}, a^{nn})]$, $c_1^{nd}(K, a) = (1-\tau)\alpha(0)f(K_0) + [r^*a - a^{nn} - \phi(a^{nn})]\underline{e} - k^{nd}$ and k^{nd} satisfies $\beta(1-\tau)\alpha(0)\theta(\underline{e}, \bar{e})f'(k^{nd}) = 1$. □

A.4 Lemma 1

Proof. If the government prefers not to devalue and not default to devalue and to default, conditional on keeping a constant debt level, then it certainly does so under the optimal debt policy; hence, $B^s(\pi) \leq \bar{B}(k^n, a, \pi)$. As π increases, we can use the implicit function theorem to show that

⁸Observe that this is the same as to require $(V^{dd} - V_1^{nn})(\underline{b}(K_0, a_0), 0) < 0$.

$B^s(\pi)$ decreases, making it more difficult for a nonempty interval $\bar{b}(k^n, a) < B \leq B^s(\pi)$ to exist. Notice that $B^s(0) > \bar{b}(k^n, a)$ implies that, if $K_0 = k^n$ and $B_0 = B_1 = \bar{b}(k^n, a)$, then the constraint $V^{dd} - V_0^{nn}(B, B) \leq 0$ with $q = \beta$ and $K_1 = k^n$ is strictly satisfied, and hence it is also satisfied by B_0 slightly larger than $\bar{b}(k^n, a)$. Since this holds for any π , $\bar{B}(k^n, a, \pi) > \bar{b}(k^n, a)$. \square

A.5 Lemma 2

Proof. Consider first the case where $K_0 = k^n$. Either the maximum of $V^T(B_0)$ is achieved for some finite T or it is not. If it is not, then

$$V^T(B_0) = \lim_{T \rightarrow \infty} V^T(B_0) > V^T(B_0)$$

for all finite T , and never running down the debt is optimal. We now argue that as B_0 increases above $\bar{b}(k^n, a)$, it can pass through critical points where the optimal T increases by one period. For $\bar{b}(k^n, a) < B_0 \leq \bar{b}(k^n, a)/(1 - \pi)$, it is optimal to set $T = 1$, because the yield from selling $\bar{b}(k^n, a)$, $\beta \bar{b}(k^n, a)$, is greater than $\hat{\beta} B_1$ for any $\bar{b}(k^n, a) < B_1 < B_0$. As we increase B_0 , we can pass through a critical point where the optimal T increases to $T = 2$. It cannot increase by more, because the optimal government policy is to steadily decrease B if it is to decrease at all. Therefore, there must be a region of values of B_0 where it is optimal to set $T = 2$ in between the regions where it is optimal to set $T = 1$ and where it is optimal to set $T = 3$. As we increase B_0 even further, we can increase T , but we can never decrease it. To see why the optimal T can never decrease, observe that for $\bar{b}(k^n, a) < B_0 < \bar{b}(k^n, a)/(1 - \pi)$,

$$V^1(B_0) > V^2(B_0) > \dots > V^\infty(B_0)$$

By setting $T = 1$ for B_0 in this region, the government can avoid crises without sacrificing government spending, and every period that it delays is costly. Differentiating our formula for $V^T(B_0)$, we obtain

$$\frac{\partial^2 V^T(B_0)}{\partial B_0 \partial T} = -v''(g^T(B_0)) \frac{\partial g^T(B_0)}{\partial T} = -v''(g^T(B_0)) \frac{\hat{\beta}^{T-1} \ln \hat{\beta}(1 - \hat{\beta})}{(1 - \hat{\beta}^T)^2} (\beta \bar{b}(k^n, a) - \hat{\beta} B_0) < 0$$

because $B_0 > \bar{b}(k^n, a)$ and $0 < \hat{\beta} \leq \beta < 1$. Hence

$$0 > \frac{d}{dB_0} V^\infty(B_0) > \dots > \frac{d}{dB_0} V^2(B_0) > \frac{d}{dB_0} V^1(B_0)$$

Therefore, if $V^T(B) \leq V^{T^*}(B)$ for $T^* > T$, then for any $B^* > B$, $V^T(B^*) \leq V^{T^*}(B^*)$. Hence, if it is optimal to reduce the debt from B_0 to $\bar{b}(k^n, a)$ in T periods, then for higher B_0 , it cannot be optimal to reduce the debt in $T - 1$ periods.

As π tends to 0, the equilibria tend to those of the zero-probability crisis equilibria. For π close enough to 0, it is easy to show that there are necessarily regions of B_0 with the full range of possibilities $T = 1, 2, \dots \infty$. Let $V(B_0, T) = V^T(B_0)$ and $g(B_0, T) = g^T(B_0)$ where we think of T as a continuous variable. We can differentiate our formula for V^T with respect to T . If we can find that for fixed $B_0 > \bar{b}(k^n, a)$ and for π small enough, $\partial V(B_0, T)/\partial T > 0$ for all T , then we know that even for discrete T , increasing T yields a higher expected payoff for the government.

Differentiating $V(B_0, T)$, we obtain

$$\begin{aligned} \frac{\partial V(B_0, T)}{\partial T} = & \hat{\beta}^{T-1} \left\{ \frac{\ln \hat{\beta} (\beta \bar{b}(k^n, a) - \hat{\beta} B_0) v'(g^T(B_0))}{1 - \hat{\beta}^T} + \right. \\ & + \frac{\beta \ln \hat{\beta} [c_0^{nn}(k^n, 0) + v(\tau \alpha(1) f(k^n) - (1 - \beta) \bar{b}(k^n, a))]}{1 - \beta} \\ & \left. - \frac{\hat{\beta} \ln \hat{\beta} [(1 - \pi) [c_0^{nn}(k^n, 0) + v(g^T(B_0))] + \pi c_1^{nn}(k^n, 0) + \pi v[g^T(B_0)] + \pi V_\pi^{dd}]}{1 - \hat{\beta}} \right\} \end{aligned}$$

Fortunately, this formula is easy to interpret. As T tends to ∞ , $\partial V(B_0, T)/\partial T$ tends to 0, because $V(B_0, T)$ tends to $V^\infty(B_0)$. Even so, we are concerned with the sign of $\partial V(B_0, T)/\partial T$. The benefit of increasing T is that the government can maintain a higher level of government spending, and this benefit is captured by the first term in the formula above. Notice that as π tends to 0, this benefit remains positive once we factor out $\hat{\beta}^{T-1}$. The cost of increasing T is that the government risks crises for more periods, and this cost is captured by the last two terms in the formula above. Notice that as π tends to 0, this cost goes to zero, even after we factor out $\hat{\beta}^{T-1}$.

Now fix a $B_0 > \bar{b}(k^n, a)$ and a π for which $\partial V(B_0, T)/\partial T > 0$ for all T . The optimal government policy is to set spending equal to $g^\infty(B_0)$ and maintain debt at B_0 . Our previous arguments now imply that for any T , there exists some initial B , $\bar{b}(k^n, a) < B < B_0$, for which the optimal government policy is to run its debt down to $\bar{b}(k^n, a)$ in T periods. We know that for $\bar{b}(k^n, a) < B < \bar{b}(k^n, a)/(1 - \pi)$, it is optimal to run down the debt in one period. We also know that for $B = B_0 > \bar{b}(k^n, a)/(1 - \pi)$, it is optimal to never run down the debt. Somewhere between $\bar{b}(k^n, a)/(1 - \pi)$ and B_0 , all other intermediate possibilities must exist.

To rule out the possibility of there being a sudden jump from a finite T being optimal to it being optimal to maintain the debt level constant, suppose to the contrary that such a jump does occur. Then, at the debt level B where this jump occurs, we know that $V^T(B) = V^\infty(B)$, but $V^{T^*}(B) < V^T(B)$ for all $T^* > T$. Furthermore, $V^\infty(B^*) > V^T(B^*)$ for all $B^* > B$. The continuity of V^T and V^{T+1} implies that we can choose $\hat{B} > B$ so that $V^T(\hat{B}) > V^{T+1}(\hat{B})$. Since

$$0 > \frac{d}{dB} V^{T+1}(B) > \frac{d}{dB} V^T(B)$$

$V^T(B) > V^{T+1}(B)$ for all $B \leq \hat{B}$. Since $V^T(\hat{B}) \rightarrow V^\infty(\hat{B})$ as $T \rightarrow \infty$, however, we know that there exists a $\hat{T} > T + 1$ sufficiently large so that $V^{\hat{T}}(\hat{B}) > V^T(\hat{B})$. Consequently, if we restrict the government's choices to the set $1, 2, \dots, \hat{T}$ we know that at $B_0 = \hat{B}$ it would choose to run down its debt in \hat{T} periods. Our previous arguments now imply that there has to be a region where $B < \hat{B}$ and where it is optimal to run down the debt in $T + 1$ periods, in particular where $V^{T+1}(\hat{B}) > V^T(B)$. This contradiction rules out the possibility of a sudden jump.

For the case when $K_0 \neq k^n$, a similar variational argument implies that under the optimal policy, g is constant during the transition to $\bar{b}(k^n, a)$. Furthermore, if instead of (K_0, B_0) as its state, the government has $(k^n, B_0 + \tau \alpha(1)(f(k^n) - f(K_0)))$, where $B_0 + \tau \alpha(1)(f(k^n) - f(K_0)) < B^s(\pi)$, the government's problem is unchanged, except that private consumption is different in period 0. Hence, the solution is unchanged. \square

A.6 Proof Lemma 3

Proof. The characterization of the crisis equilibrium works similarly to that of the no devaluation equilibrium in Proposition 3. In the no devaluation equilibrium, the stationary debt policy charac-

terizes optimal government behaviour and, implicitly, equilibrium outcomes when the participation constraint does not bind. In the crisis equilibrium, $T(B_0)$ and $V_g^T(B_0)$ characterize optimal government behaviour and, implicitly, equilibrium outcomes when the participation constraint does not bind.

When the participation constraint does bind, we can use the identical logic as that in the proof of Lemma 4 to argue that, if $K = k^n$ then the equilibrium adjusts to that characterized by $T(B)$ and $V_g^T(B)$ in at most one period; in particular, if $B_1 > B^s(\pi)$, then $B_2 < B^s(\pi)$ and the government runs down its debt in $T(B_2)$ periods starting in the period after $K = k^n$. If $K_0 = k^n$, this is period 1, but if $K_0 \neq k^n$ and if the participation constraint binds in period 1, it is period 2. We need to also allow for the possibility that $K = k^n$ if the government needs to lower either B_1 or B_2 so much as to satisfy the participation constraints in period 0 or period 1 so that B_1 or B_2 is less than or equal to $\bar{b}(k^n, a)$. Otherwise, the proof follows the identical logic as that of Lemma 4. The notation involved in writing out the expressions for $V_g^{dn} - V_g^{nn}$ analogous to those found in the proofs in Lemma 4 and Proposition 3 is straightforward, but tedious. We omit it here. \square

A.7 Lemma 4

Lemma 4. *For $\beta < 1$ sufficiently close to 1 and \bar{e} sufficiently high, there exists a continuous and increasing function $\bar{B}(K_0, a_0)$, and a positive debt level B^s , such that $\bar{B}(K_0, a_0) > B^s$ and $\bar{B}(K_0, a_0) > \bar{b}(K_0, a_0)$ for all K_0, a_0 , such that the following outcomes occur:*

- i) If $K_0 = k^n$, and $B_0 \leq B^s$, then the economy will be in the stationary no devaluation equilibrium in which government debt stays constant at its initial debt B_0 .*
- ii) If $B_0 \leq \bar{B}(K_0, a_0)$, then the economy converges to the stationary no-default continuation equilibrium after at most two periods.*
- iii) If $B_0 > \bar{B}(K_0, a_0)$, then the outcome is the devalue and default equilibrium outcome.*

Proof. Observe that we are going to compare the constraints on the government's debt that must be satisfied simultaneously in any equilibrium with no default and no devaluation, in the case that consumers have no expectations on devaluation and bankers buy government bonds

$$\begin{aligned} V^{dd}(s, B_0, B_1) &\geq V_0^{nd}(s, B_0, B_1) \\ V^{dn}(s, B_0, B_1) &\leq V_0^{nn}(s, B_0, B_1) \\ V^{dd}(s, B_0, B_1) &\leq V_0^{nn}(s, B_0, B_1) \end{aligned}$$

It is easy to show, for β close to one and \bar{e} sufficiently high, the two first constraints always hold. First, the government always devalue after a default even in the case that consumers do not expect a devaluation: given that,

$$(V^{dd} - V_0^{nd})(B_0, B_1) = c^{dd}(K_0, a_0) + v\left(\tau\alpha(0)\theta(\underline{e}, \bar{e})f(K_0) + \beta\bar{e}B_1\right) - c_0^{nd}(K_0, a_0) - v\left(\tau\alpha(0)f(K_0) + \beta B_1\right)$$

this condition holds if \bar{e} is sufficiently high. Second, if the government does not default it never devalues: for β close to one and \bar{e} sufficiently high, the following difference is always negative

$$\begin{aligned} (V^{dn} - V_0^{nn})(B_0, B_1) &= H_0^{dn-nn} + v\left(\tau\alpha(1)\theta(\underline{e}, \bar{e})f(K_0) + \bar{e}(\beta B_1 - B_0)\right) - v\left(\tau\alpha(1)f(K_0) + (\beta B_1 - B_0)\right) \\ &\quad + \frac{\beta}{1-\beta} \left\{ v\left(\tau\alpha(1)f(k^n) - (1-\beta)\bar{e}B_1\right) - v\left(\tau\alpha(1)f(k^n) - (1-\beta)B_1\right) \right\} \end{aligned}$$

where $H_0^{dn-nn} \equiv c^{dn}(K_0, a_0) - c_0^{nn}(K_0, a_0)$.

The previous environment is so close to Cole and Kehoe (1999, Proposition 1) that i)-iii) can be followed in their proof. We have pointed out the slight difference shown in our model.

Finally, it remains to prove that the crises zone exists, i.e., $\bar{B}(K_0, a_0) > \bar{b}(K_0, a_0)$. First, we define the following difference

$$(V^{dd} - V_0^{nn})(B_0, B_1) = H_0^{dd-nn} + v\left(\tau\alpha(0)\theta(\underline{e}, \bar{e})f(K_0) + \beta\bar{e}B_1\right) - v\left(\tau\alpha(1)f(K_0) + (\beta B_1 - B_0)\right) \\ + \frac{\beta}{1-\beta} \left\{ v\left(\tau\alpha(0)f(k^d)\right) - v\left(\tau\alpha(1)f(k^n) - (1-\beta)B_1\right) \right\}$$

where $H_0^{dd-nn} \equiv c^{dd}(K_0, a_0) - c_0^{nn}(K_0, a_0) + \beta/(1-\beta) [c^d(k^d) - c^n(k^n)]$.

Next, we will compare the constraints that set both levels $(V^{dd} - V_1^{nn})(B_0, 0)$ and $(V^{dd} - V_0^{nn})(B_0, B_1)$. We will show that for $\beta < 1$ sufficiently close to one $(V^{dd} - V_1^{nn})(B_0, 0) > (V^{dd} - V_0^{nn})(B_0, B_1)$ for all B_1 . Given any $B_1 > 0$, it is verified $(V^{dd} - V_0^{nn})(B_0, B_1) \rightarrow +\infty$ as $B_0 = \tau\alpha(1)f(K_0) + \beta B_1$ is greater than B^{dd-nn} stated in proposition 2, and in addition it is easy to show that $(V^{dd} - V_1^{nn})(B_0, 0) > (V^{dd} - V_0^{nn})(B_0, 0)$ given that subtracting both we find

$$(V^{dd} - V_0^{nn})(B_0, 0) - (V^{dd} - V_1^{nn})(B_0, 0) = c_1^{nn}(K_0, a_0) - c_0^{nn}(K_0, a_0) + \\ \beta/(1-\beta) \left[c_1^{nn}(k^{nn}, a^{nn}) - c_0^{nn}(k^n, 0) + v\left(\tau\alpha(1)f(k^{nn})\right) - v\left(\tau\alpha(1)f(k^n)\right) \right]$$

which is negative for $\beta < 1$ sufficiently close to one. Then, by continuity of $(V^{dd} - V_0^{nn})(B_0, B_1)$ this means that the one-probability crises zone exists. \square

Table I: Parameters Calibrated without Solving the Model

Parameter value	Parameter meaning	Source
$s = 0.4$	Capital Share.	Kydland and Zarazaga (2000)
$\beta = 0.963$	Discount Factor.	1 year Gov. Securities Treasury bills
$\alpha(0) = 0.95$	Permanent drop in productivity.	Cole and Kehoe (2000)
$\theta(\underline{e}, \bar{e}) = 0.9808$	Temporal drop in productivity	Investment rate reduction in 2001
$\pi = 0.0473$	Devaluation Probability	Risk premium of Argentinian Debt
$\underline{e} = 1$	Exchange rate pegged to US dollar	
$\bar{e} = 1.4$	Exchange rate set on January 11, 2002	

Table II: Parameters Calibrated by Solving the Model

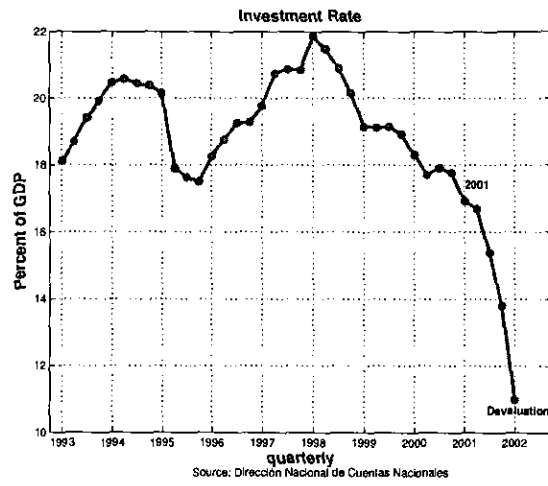
Parameters		Calibration Targets
$\phi_1 = 71.54$	Adjustment cost	Investment rate in 2000 ($i/y = 0.18$)
$\phi_2 = 4.3478 \times 10^{-5}$	Adjustment cost	Reduction in reserves over GDP (3.42%)
$\tau = 0.2593$	Tax Rate	Government spending over GDP in 2000 ($g/y = 0.19$)
$A = 1206$	Scale factor	External Debt over GDP in 2000 ($B/y = 0.45$)
$\delta = 0.0815$	Depreciation rate	Capital-Output ratio ($k/y = 3$)
$a_0 = 3436.9$	Initial foreign assets	Trade balance surplus (0.41%)

	Level	% of Output
$\underline{b}(k^n, a_0)$	24623	0.0866
$\bar{b}(k^n, a_0)$	55672	0.1959
$B^s(\pi)$	672930	2.3678

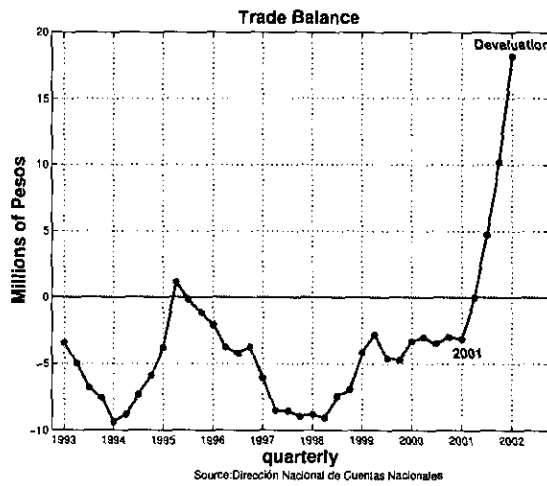
Table III: Debt levels and percentage of output.



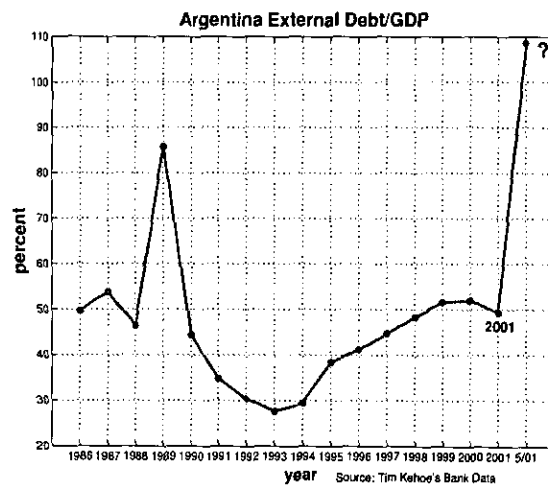
1.1: Output.



1.2: Investment Rate.



1.3: Trade Balance over output.



1.4: Debt over output.

Figure 1: Argentinian facts I

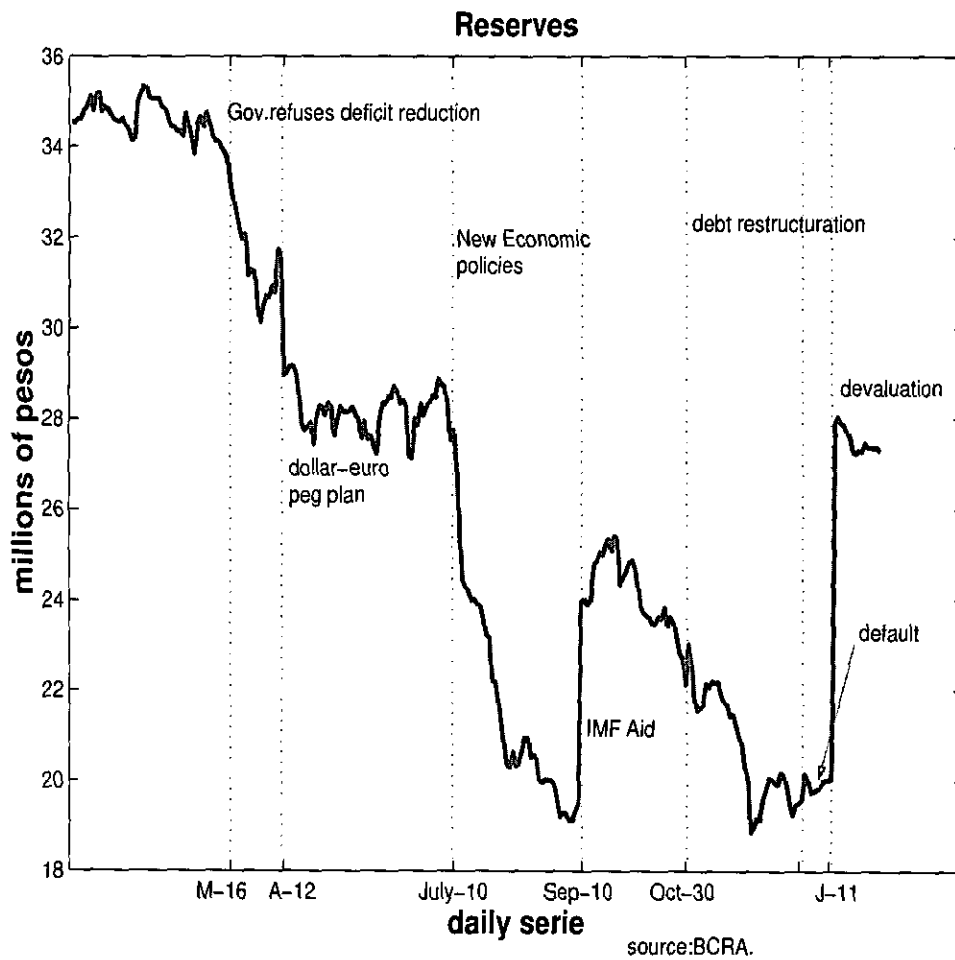


Figure 2: Argentinian Facts II

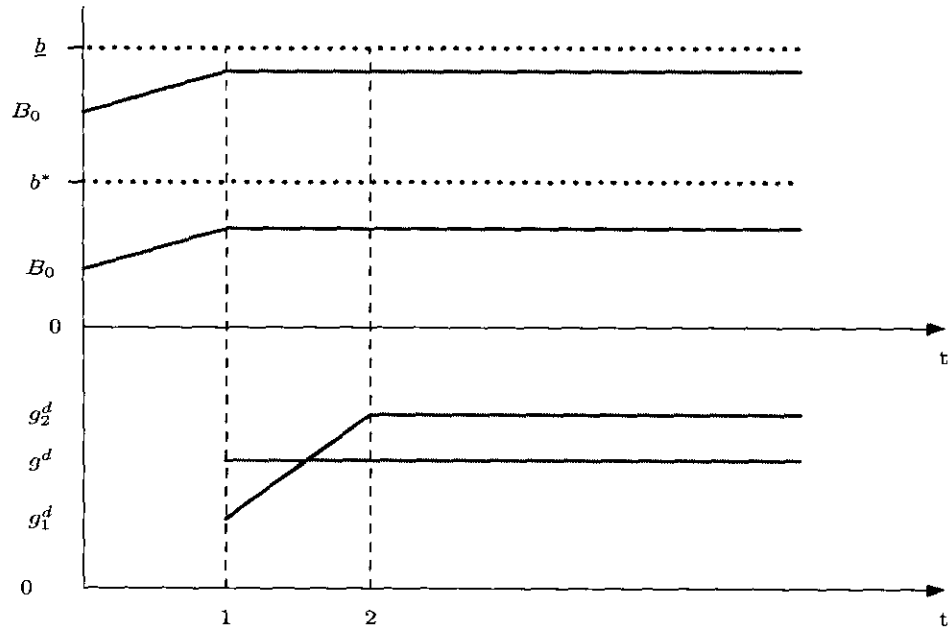


Figure 3: Bonds and government expenditure paths described at proposition 1 when $K_0 \leq k^n$.

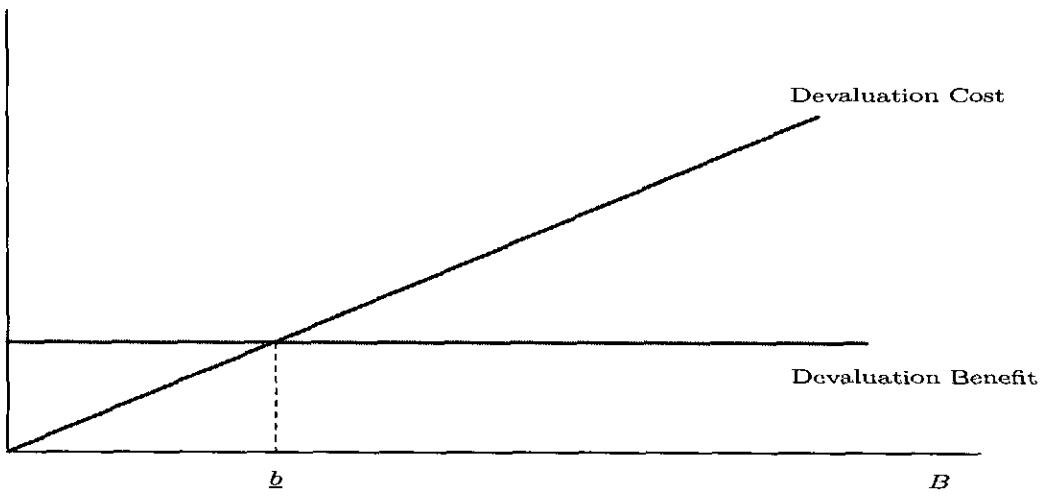


Figure 4: Devaluation without default.

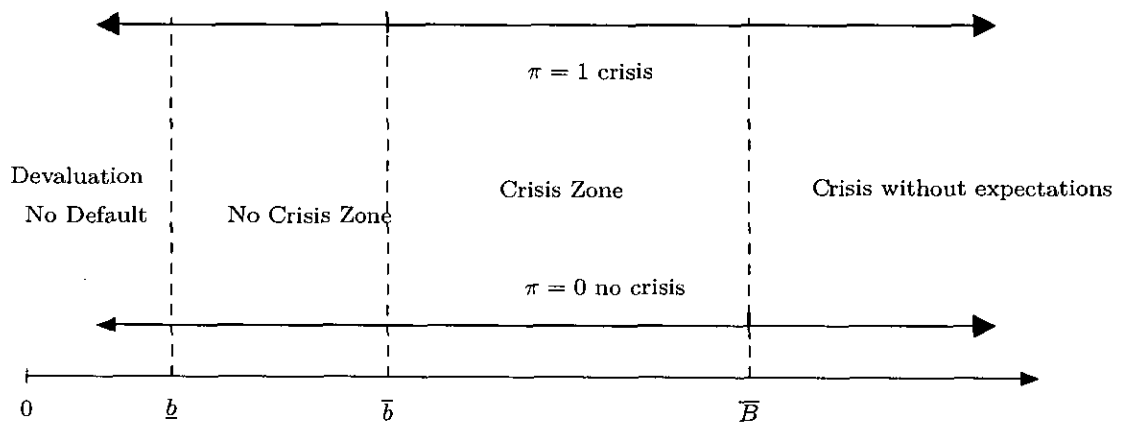


Figure 5: Existing zones I.

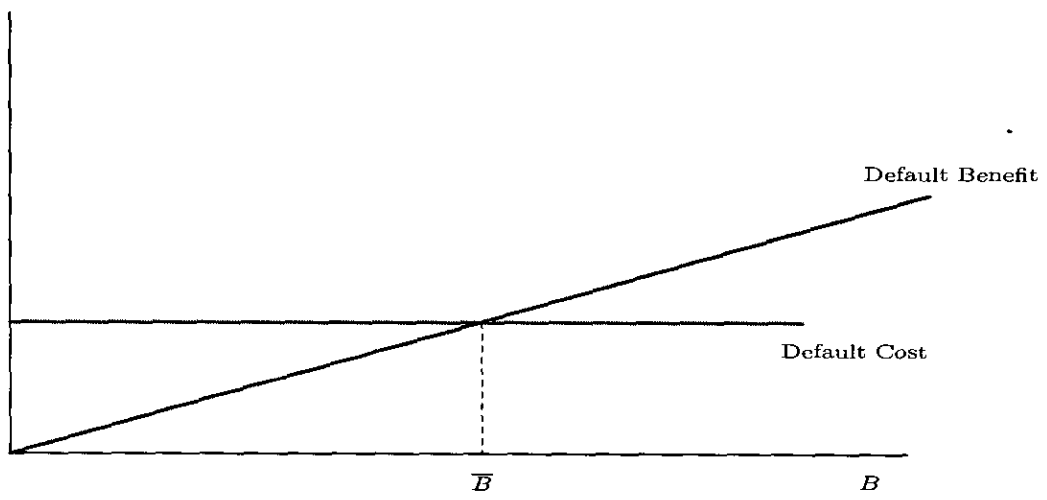


Figure 6: Default without expectations.

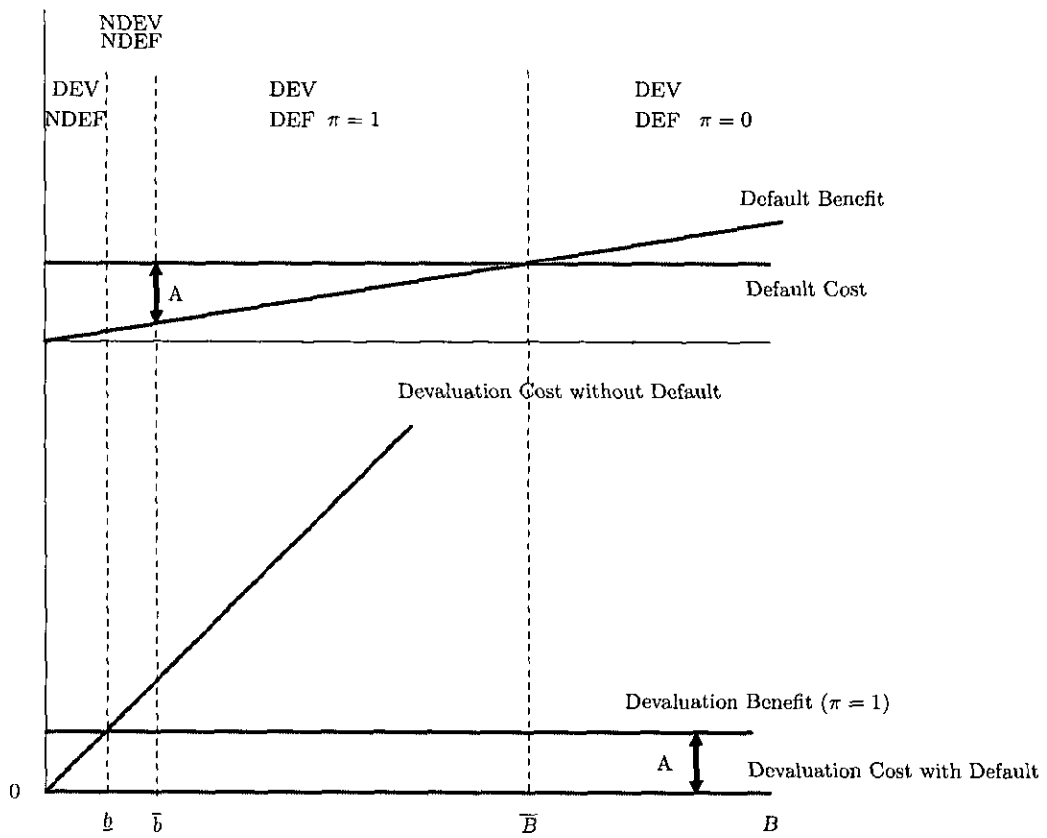


Figure 7: Existing zones II

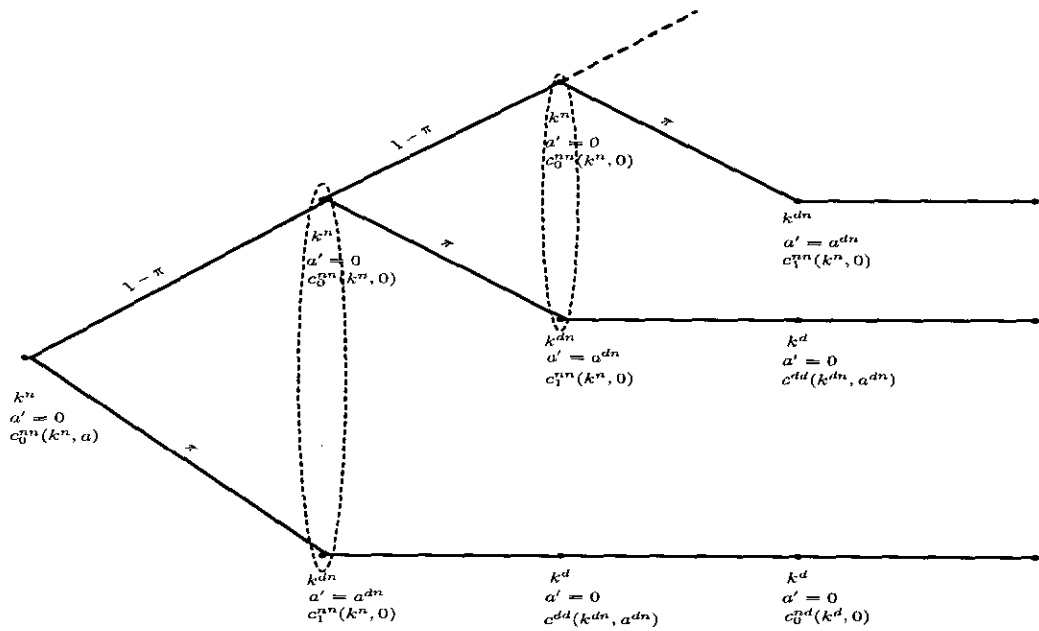


Figure 8: Consumer decisions.

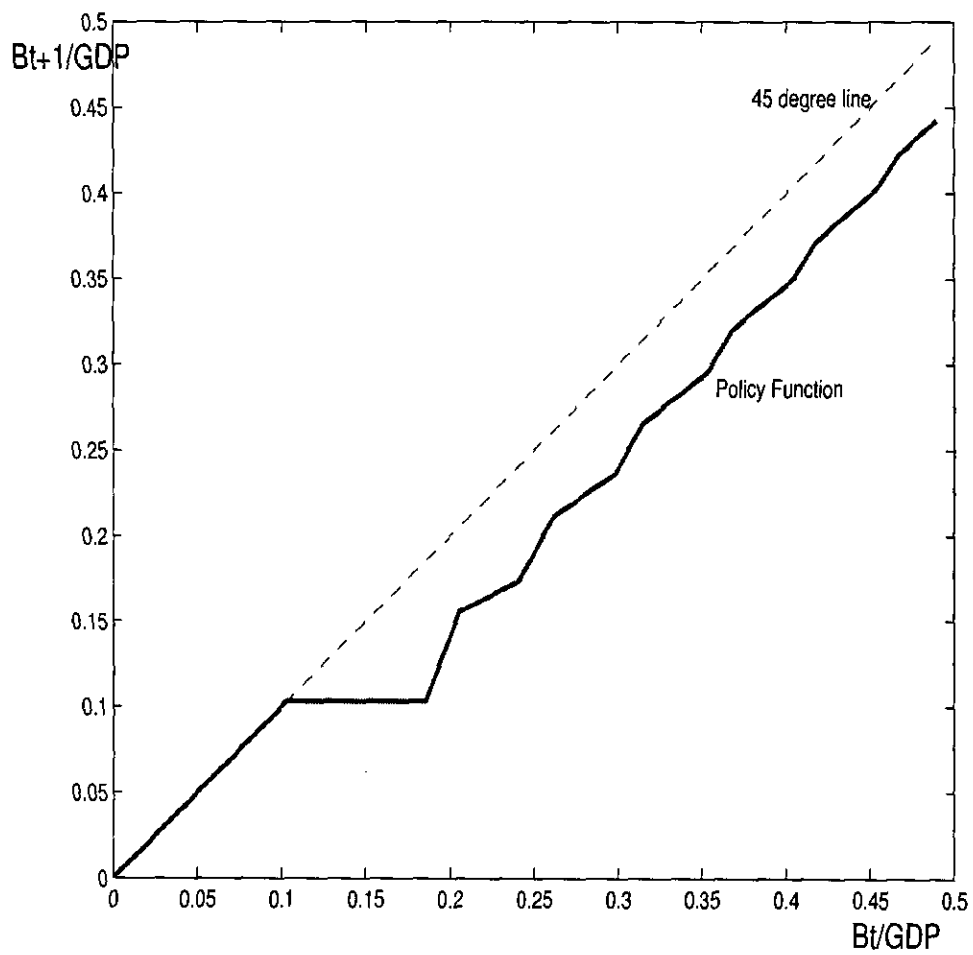
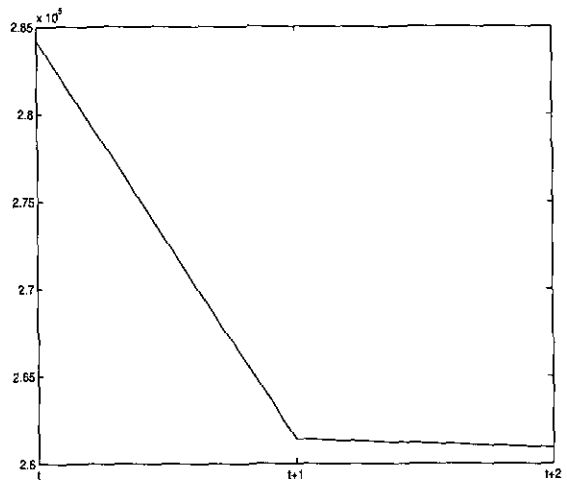
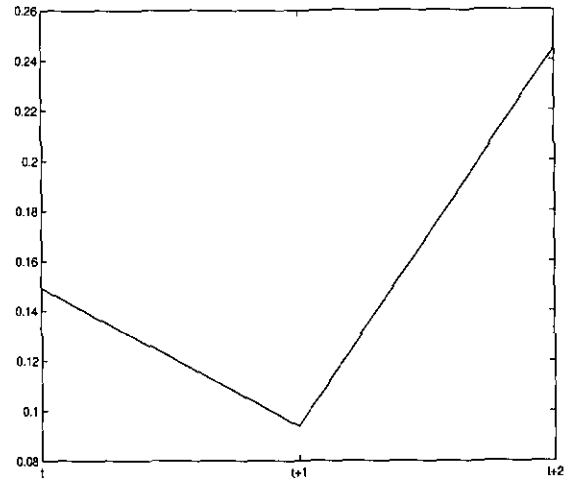


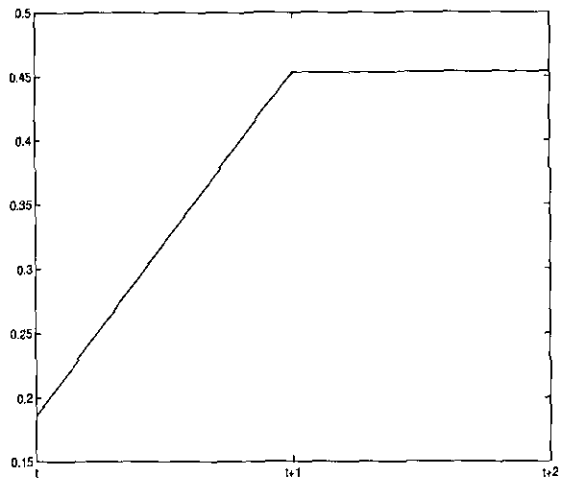
Figure 9: Optimal policy for the parameters calibrated.



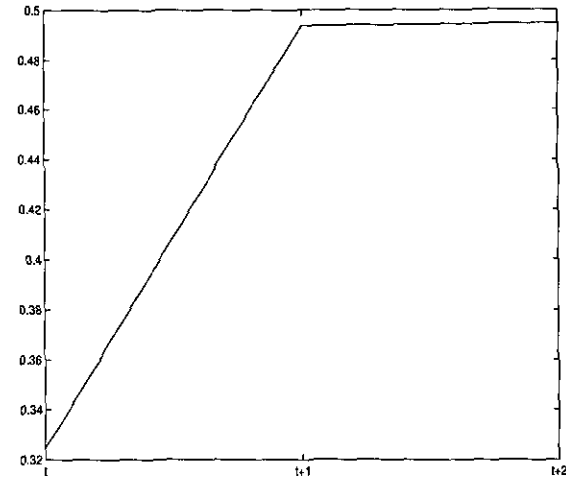
10.1: Output.



10.2: Investment Rate.



10.3: Trade Balance over output.



10.4: Debt over output.

Figure 10: Variables of the economy calibrated.

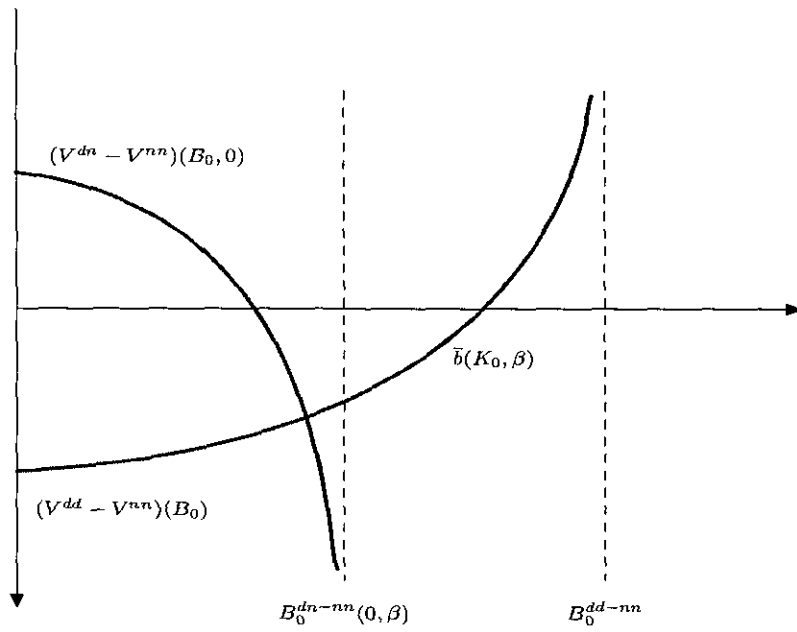


Figure 11: The no crises without devaluation zone upper bound $\bar{b}(K_0, \beta)$, and the $(V^{dn} - V_1^{nn})(B_0, B_1)$ function evaluated at $B_1 = 0$.