Discussion of Sophisticated Monetary Policies By A. Atkeson, V. Chari and P. Kehoe

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- Apply to implementation of interest rate rules. Comparisons with monetary supply rules.

Outline

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- Substantive message of first model is very simple.
  Simple static deterministic set-up is rich enough to illustrate it.

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- *Our approach, in contrast, insists that policies be specified so that a competitive equilibrium can exist following a deviation.*
- *Best responses be controllable, in the sense that policies can be found which ensures that, following any deviation, the best response of any individual private agent is different from the average choice of the private agents.*

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- Objective function: $U(x(i), x, y, \delta)$
  - x, y, δ determines prices, taxes, position of demand curve, etc.

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- Multiple Eqm. for policy outcome δ :
  - $a = (x, y, \delta)$ is CEO,
  - $a' = (x', y', \delta)$ is CEO.

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- Competitive Eqm. Outcome (CEO): fixed point of best response $x = b(x, \delta)$.
- Advantage: simplicity.
  Disadvantage: does not cover case gov. purchases positive, taxes zero.
  Which gov. policies are restricted is encoded in the definition of u.

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- Consider a CEO a^*.
  We can now talk about in and out of equilibrium policy actions implementing it.

- Unique Implementation of CEO a^* $(x^* = B(a^*), 0 = E(a^*))$ for all (x', y') such that

$$x' \neq x \text{ and that } 0 = E(x', y', \sigma_g(x'))$$

it is not optimal for the private sector to choose x':

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- Non-Trivial Implementation: Restrict sophisticated policies σ_g:
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- If there is a function $y = e(\cdot)$ such that $0 = E(x, e(x, \delta), \delta)$:
  - All sophisticated policies are non-trivial.
  - Can eliminate y from problem, use u, b (easier).
  - Makes less transparent the reason why an equilibrium does not exist.

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- Notice that for this sophisticated policy σ_g to be *non trivial* we require that for all $x' \neq x^*$ there is a y' such that

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- Using a Controllable best response if private agents deviate from x^* they are faced with a switch to gov. policy outcome $\tilde{\delta}$.
- The new policy $\tilde{\delta}$ was chosen so that it has a unique equilibrium associated with it.
- This is the sense in which *uniqueness somewhere leads to uniqueness everywhere.*
- Notice that implementation in the paper is a bit different:
  The regime to which the sophisticated policy switches depends on the private agent deviations (there is no unique regime to revert to $\tilde{\delta}$)

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- Authors use more complicated model because it is (more similar to) the one used more frequently in the literature.

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- Conclusion: Central Banks do use interest rates as instrument. But, according to the two models in this paper, it is not clear that theoretically the reason is due to out of equilibrium concerns.

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- Instead, money supply rules are used upon a deviation.
- This brings the obvious point, why not used money supply rules directly?
- Conclusion: Central Banks do use interest rates as instrument. But, according to the two models in this paper, it is not clear that theoretically the reason is due to out of equilibrium concerns.
- (Two) 'big picture' comment(s).
  Positive Analysis: what are reasonable ways to model the off equilibrium conjectures ?
  Are extrapolation from behaviour in Eq. path reasonable? (Taylor rules)
  Normative Analysis: How should government announce and conduct policy to influence these conjectures?