Sophisticated Monetary Policies

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Barro, Lucas-Stokey Approach to Policy

- Solve Ramsey problem
  - choose $a = \{\text{policies, prices, allocations}\}$ to
    \[
    \text{max } \text{Utility} \quad \text{s.t. } a \in \text{Competitive Equilibrium}
    \]

  Answer: Ramsey outcome $a^*$ function of exogenous shocks
Barro, Lucas-Stokey Approach to Policy

- Solve Ramsey problem
  - choose $a = \{\text{policies, prices, allocations}\}$ to
    $$\text{max } Utility$$
    $$\text{s.t. } a \in \text{Competitive Equilibrium}$$

  Answer: Ramsey outcome $a^*$ function of exogenous shocks

- Want to get
  - How to get there uniquely

- Left open: Implementation
  - Designing policies so Ramsey outcome is unique equilibrium
Our Solution to Implementation Problem

- *Sophisticated policies* can
  - Depend on histories of agents’ actions
  - Differ on and off equilibrium path
  - Equilibrium concept that specifies outcomes for all histories
Our Solution to Implementation Problem

- *Sophisticated policies* can
  - Depend on histories of agents’ actions
  - Differ on and off equilibrium path

- Example
  - If $\pi_t \in Acceptable\ Region$, follow Ramsey policy
  - If not, switch to alternative policy (*reversion*)
Our Solution to Implementation Problem __________________________

• *Sophisticated policies* can
  
  ○ Depend on histories of agents’ actions
  
  ○ Differ on and off equilibrium path

• Main Result
  
  ○ Can uniquely implement any desired competitive outcome
Our Solution to Implementation Problem

• *Sophisticated policies* can
  
  ○ Depend on histories of agents’ actions
  
  ○ Differ on and off equilibrium path

• Main Result
  
  ○ Can uniquely implement any desired competitive outcome

• Note:
  
  ○ Differs from “implementation via nonexistence”
  
  ○ Here continuation equilibria exist after all deviations
Follow Barro, Lucas-Stokey Approach to Policy
Follow Barro, Lucas-Stokey Approach to Policy Implementation Problem Now Solved
Implementation: A Nontrivial Problem in Monetary Models

- Sargent-Wallace result
  - Indeterminacy if interest rates depend only on exogenous events
- Indeterminacy risky
Contrasts with Literature

- Our approach: Implementation by discouraging deviations
- Literature: Implementation via nonexistence
Contrasts with Literature

- Our approach: Implementation by discouraging deviations
- Literature: Implementation via nonexistence

Our Approach: Discourage Deviations

Outcome path

Equilibrium exists

Equilibrium exists
Contrasts with Literature

- Our approach: Implementation by discouraging deviations
- Literature: Implementation via nonexistence

**Literature: Nonexistence after Deviations**
Contrast Concerning Taylor Principle

- Literature: Taylor principle needed for uniqueness
  - Taylor principle
    \[ i_t = \bar{i} + \phi(\pi_t - \bar{\pi}), \quad \phi > 1 \]
    raise interest rates more than 1 for 1 with inflation
Main Results

- Simple Sticky Price model
  - Implement with sophisticated policies
  - Indeterminacy with linear feedback rules

- Extend to
  - New Keynesian model
  - Imperfect Information
Simple Sticky Price Model
Outline of Section

- Model Setup and 4 Equilibrium Conditions
- Implement with Sophisticated Policies
- Cannot implement with linear feedback rules
Setup and 4 Equilibrium Conditions
Setup: Technology and Preferences

- Final good technology

\[ Y_t = \left[ \int Y_t(j)^\theta \, dj \right]^{\frac{1}{\theta}} \]

- Intermediate good technology

\[ Y_t(j) = L_t(j) \]

- Preferences

\[ E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \]

where \( L_t = \int L_t(j) \)

- Cash-in-advance
Setup: Technology and Preferences

• Final good technology

\[ Y_t = \left[ \int Y_t(j)^\theta dj \right]^{1/\theta} \]

• Intermediate good technology (some producers sticky \( p \) some flexible \( p \))

\[ Y_t(j) = L_t(j) \]

• Preferences

\[ E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \]

where \( L_t = \int L_t(j) \)

• Cash-in-advance
One-Period Stickiness

- Let $h_{t-1}$ be history of past actions and shocks at start of period $t$

\[ h_{t-1}, h_{gt} = (h_{t-1}, x_t), h_{yt} = (h_{gt}, \text{policy}, \eta_t) \]

- Strategies of agents and central bank depend on relevant history
- Sticky price producers only interesting strategic players
One-Period Stickiness

- Let $h_{t-1}$ be history of past actions and shocks at start of period $t$

\[
\begin{align*}
    h_{t-1} & \quad h_{gt} = (h_{t-1}, x_t) \\
    h_{yt} & = (h_{gt}, \text{policy}, \eta_t) \\
    h_t & \\
\end{align*}
\]

Sticky price Regime Shock Flexible prices set

\[
\begin{align*}
    x_t(j, h_{t-1}) & \\
    \delta(h_{gt}) & \\
    \eta_t & \\
    \text{Policy} & \text{Consumers move} \\
\end{align*}
\]

\[
\begin{align*}
    x_t = p_{st} - p_{t-1} & \\
    i(h_{gt}) & \text{or } \mu(h_{gt})
\end{align*}
\]

- Strategies of agents and central bank depend on relevant history
- Sticky price producers only interesting strategic players
- Next, 4 equations of this “New Classical” sticky price model
Derive 4 equations of New Classical System

- Sticky price producer’s best response
  - price set as markup over “expected” marginal cost

\[ P_{st}(j) = \frac{1}{\theta} \frac{E_{t-1}\left[ Q_t \frac{1}{P_t^{1-\theta}} W_t y_t \right]}{E_{t-1}\left[ Q_t \frac{1}{P_t^{1-\theta}} y_t \right]} \]

when log linearize and use \( W / p = -u_t / u_c \) get

\[ p_{st}(j) = E_{t-1}[p_t + \gamma y_t] \]

letting \( x_t(j) \equiv p_{st}(j) - p_{t-1} \) and \( \pi_t \equiv p_t - p_{t-1} \)

\[ x_t(j) = E_{t-1}[\gamma y_t + \pi_t] \]
Derive 4 equations of New Classical System

- New Classical Phillips Curve
  - use flexible price producers’ problem and aggregate price index to get
  \[ \pi_t = \kappa y_t + x_t \] \hspace{2cm} (2)
  - Log Linearized Euler equation (\( \eta_t \) is flight-to-quality shock)
  \[ y_t = E_t y_{t+1} - \sigma (i_t - E_t \pi_{t+1}) + \eta_t \] \hspace{2cm} (3)
  - Log Linearized quantity equation (cash-in-advance)
  \[ \pi_t = \mu_t - (y_t - y_{t-1}) \] \hspace{2cm} (4)
New Classical System with Strategies and Interest Rate Regime

1. Sticky producers’ best response

\[ x(j, h_{t-1}, x_t) = E \left[ \gamma y(h_{yt}) + \pi(h_{yt}) \ \bigg| \ h_{t-1}, x_t \right] \]

2. New Classical Phillips curve

\[ \pi(h_{yt}) = \kappa y(h_{yt}) + x(h_{t-1}) \]

3. Euler equation

\[ y(h_{yt}) = E \left[ y(h_{yt+1}) \ \bigg| \ h_{yt} \right] - \sigma (i(h_{gt}) - x(h_t)) + \eta_t \]

4. Quantity theory

\[ \pi(h_{yt}) = \mu(h_{yt}) - \left( y(h_{yt}) - y_{t-1} \right) \]
Outcome Path versus Strategies

- Strategies say what to do at all possible histories
- Outcome path describes what actually happens:
Strategies say what to do at all possible histories

Outcome path describes what actually happens:

\[ \{ x_t(\eta^{t-1}), i_t(\eta^{t-1}), y_t(\eta^t), \pi_t(\eta^t) \} \equiv \{ a_t(\eta^t) \} \]
How Strategies Induce Future Histories

- Fix strategies for all players \( \{x_t(\cdot)\}, \{\delta_t(\cdot), i_t(\cdot), \mu_t(\cdot)\}, \{y_t(\cdot), \pi_t(\cdot)\} \)

- Strategies recursively define future histories

  - Given \( h_{t-1} \), history \( h_t \) generated from strategies and realization of \( \eta_t \)

    \[
    h_{t-1} \rightarrow x_t = x_t(h_{t-1})
    \]

    \[
    \Rightarrow h_{gt} = (h_{t-1}, x_t) \rightarrow \delta_t = \delta_t(h_{gt}), \ i_t = i_t(h_{gt})
    \]

    \[
    \Rightarrow h_{yt} = (h_{gt}, \delta_t, i_t, \eta_t) \rightarrow y_t(h_{yt}), \ \pi_t(h_{yt})
    \]

    and so on
Outcome Path \( \{a_t(\eta^t)\} \) is a Competitive Equilibrium

1. Sticky producers’ equilibrium response

\[
x_t(\eta^{t-1}) = E\left[ \gamma y_t(\eta^t) + \pi_t(\eta^t) \mid \eta^{t-1} \right]
\]

2. New Classical Phillips curve

\[
\pi_t(\eta^t) = \kappa y_t(\eta^t) + x_t(\eta^{t-1})
\]

3. Euler equation

\[
y_t(\eta^t) = E\left[ y_{t+1}(\eta^{t+1}) - \sigma(i_t(\eta^t) - x_{t+1}(\eta^t)) \right] + \eta_t
\]

4. Quantity theory

\[
\pi_t(\eta^t) = \mu_t(\eta^t) - (y_t(\eta^t) - y_{t-1}(\eta^{t-1}))
\]
Implementation with Sophisticated Policies
Implementation with Sophisticated Policies

Policies can differ on and off the equilibrium path
Implementation Theorem

- Suppose some given outcome path

\[ a_t^* (\eta^t) \]

is a competitive equilibrium. There are sophisticated policies with a unique equilibrium which generate given outcome path.
Sketch of Proof

- To implement specific outcome path \( \{ x_t^* (\eta^{t-1}), i_t^* (\eta^{t-1}), y_t^* (\eta^t), \pi_t^* (\eta^t) \} \)
  
  o If \( \tilde{x}_t = x_t^* \) stay with original policy \( i_t^* \)
  
  o If \( \tilde{x}_t \neq x_t^* \) switch to money for one period and
    
    choose money to generate original inflation \( \pi_t^* \)
Sketch of Proof

- Best response of sticky producer

\[ x_t(j) = E_{t-1}[\gamma y_t + \pi_t] \]

- Want to discourage deviations, that is make

\[ x_t(j) \neq \tilde{x}_t \]
Sketch of Proof

- Can make $E_{t-1}[\gamma y_t + \pi_t]$ to be whatever we want
- Because money regime $\pi_t$ and $y_t$ determined by
  - Flexible producers’ decisions
    $$\pi_t = \kappa y_t + x_t$$
  - Cash-in-advance in first differences
    $$\pi_t = \mu_t - (y_t - y_{t-1})$$
- So for any sticky producer choice $x_t$ (with $y_{t-1}$ given)
  - $\pi_t$ and $y_t$ uniquely determined by $\mu_t$
  - $E_{t-1}[\gamma y_t + \pi_t]$ is monotone in $\mu_t$
Sketch of Proof

• Proof exploits *controllability with money* i.e.
  
  ○ Central bank can induce any best response by individual sticky price producer following aggregate deviation $\tilde{x}_t$
Recap

- Unique Implementation with Sophisticated Policies

- Next, show why regime switching is necessary
Necessity of Regime Switching
• Linear feedback rule

\[ i_t = \bar{i}_t + \sum_{s=0}^{\infty} \phi_{xs} x_{t-s} + \sum_{s=1}^{\infty} \phi_{ys} y_{t-s} + \sum_{s=1}^{\infty} \phi_{\pi s} \pi_{t-s} \]

• Necessarily yields indeterminacy
  
  ○ Under this rule continuum of competitive equilibria

\[ x_{t+1} = i_t + c \eta_t, \quad \pi_t = x_t + \kappa (1 + \psi c) \eta_t, \quad y_t = (1 + \psi c) \eta_t \]

indexed by \( c \) and \( x_0 \).
• King rule (from King 2000 and Svensson and Woodford 2005)

\[ i_t = i_t^* + \phi(x_t - x_t^*) \]

where \( i_t^* \), \( x_t^* \) are desired outcomes.

• Our approach: King rule yields indeterminacy
Recap

- Simple sticky price model
  - Unique implementation with sophisticated policies
  - Necessity of regime switching

- Next, extend to standard New Keynesian model with
  - Sticky price producers use Calvo-pricing
  - No flexible producers
Standard New Keynesian Model

Model Setup
Standard New Keynesian Model

- Timing (w/o shocks)

Fraction $1 - \alpha$
reset prices to $p_{st}(j,h_{t-1})$

Regime $\delta(h_{gt})$
Policy $i_t(h_{gt})$ or $\mu(h_{gt})$

Consumers

Only 1 equation changes from simple sticky price model

- Sticky price producers’ best response
Implementation in Standard New Keynesian Models

- Works with reversion to money

- Works with reversion to interest rates
  
  ○ Along equilibrium path
  
  \[
  i_t(h_{gt}) = i_t^* \left( \eta^{t-1} \right)
  \]

  ○ Any deviation at time \( t \) switch to new regime

  \[
  i_t(h_{gt}) = \bar{i} \text{ at } t \text{ with } \bar{i} \neq 0
  \]

  and for \( \hat{\phi} \) with unique continuation equilibrium set

  \[
  i_s(h_{gs}) = \hat{\phi} x_s \text{ all } s \geq t + 1
  \]
Can we get Implementation with Linear Feedback Rules?
King Rule Works Here but Differently from Literature

• King rule

\[ i_t = i_t^* + \phi(\pi_t - \pi_t^*), \quad \phi > 1 \]

• Our approach
  ○ Implements bounded outcomes
  ○ After deviation, returns to desired outcomes

• Literature
  ○ After deviation, leads to nonexistence (\( \pi \) explodes)
Standard New Keynesian Model
Robust to Imperfect Information
Imperfect Information

- Imperfect monitoring
  - See agents’ actions every period with probability $q$
  - Get exact implementation

- Measurement error
  - See agents’ actions with measurement error
  - Get approximate implementation
Conclusion

• Follow Barro, Lucas-Stokey
  ○ Check controllability of best responses
  ○ If controllable, move on to next paper
  ○ If not?...

• Extend to financial crises, fiscal policy, and so on