Liquidity and Trading Dynamics

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Motivation

Broad question: what are the effects of financial frictions on macroeconomic volatility?

Specific frictions: limited access to credit and limited supply of liquid assets

Result: if no credit, scarce liquidity can amplify aggregate shocks by introducing counter-cyclical “self-insurance” motive
Motivation

Broad question: what are the effects of financial frictions on macroeconomic volatility?

Specific frictions: limited access to credit and limited supply of liquid assets

Result: if no credit, scarce liquidity can amplify aggregate shocks by introducing counter-cyclical “self-insurance” motive

3 main ingredients:

- idiosyncratic income risk
- decentralized model of production and exchange
- public supply of liquid assets
Application

**Facts:** after mid 1980s US has experienced

1. decline in aggregate volatility (Great Moderation)
2. decline in sectoral comovement
Application

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2. decline in sectoral comovement

At the same time:

- expansion of credit market
- high inflation in the 1970s

Simple calibration to study the quantitative contribution of our mechanism to explain facts 1 and 2
Coordination in Trade

- amplification (and comovement) related to coordination element

- two meanings here:
  1. you want to buy more goods if others buy more
  2. you want to sell more goods if others sell more

- coordination element arises endogenously only when liquidity supply is low
• Money search, Kiyotaki and Wright (1989), Shi (1997), Lagos and Wright (2006)

• In Diamond (1982) coordination element with increasing returns built in matching function

• Aggregate shocks in money-search models, Berentsen, Camera, and Waller (2003)

• Aggregate effects of uninsurable idiosyncratic risk, Krusell and Smith (1998)
Environment

- Continuum of infinitely-lived Producer/Consumer households
- Discrete time, each date $t$ divided in $s = 1, 2, 3$
- Agents produce, trade and consume a perishable good
- Households start with an initial endowment $M_0$ of money
- We consider two extremes:
  1. anonymous markets $\rightarrow$ fiat money
  2. perfect credit markets $\rightarrow$ money is useless
Monetary Economy

- Continuum of islands with representative sample of P and C and competitive markets *a la* Lucas and Prescott (1974)

- At $s = 1$: P and C travel to different islands $k$ and $k'$ → no communication

- Island $k$ characterized by productivity shock $\theta^k_t \sim F(\cdot | \zeta)$

$$y_{1,t} = \theta^k_t n_t$$

- $\zeta = $ aggregate shock (fix it for now)
Monetary Economy (continued)

- At $s = 2$: C and P travel to different islands
  $\rightarrow$ no communication

- Fixed endowment $y_{2,t} = e_2$

- At $s = 3$: C and P in same island
  $\rightarrow$ centralized market

- Fixed endowment $y_{3,t} = e_3$
Geography

C/P k C/P

C/P k' C/P

C/P C/P k k'}
Geography
Geography
Preferences

- Quasi-linear utility (LW):

\[ \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t (u(c_{1,t}) - v(n_t) + U(c_{2,t}) + c_{3,t}) \right], \]

\( u, U \) strictly concave

\( v \) convex
Government

- \( \gamma = \) constant money growth rate

- at the end of period 3 government injects \((\gamma - 1)M_t\) units of money by lump-sum transfer/tax

- we take the monetary policy \( \gamma \) as given and compare economies with different policies
Stationarity

- focus on equilibria where nominal variables grow at rate $\gamma$

- recursive representation of household problem with one state variable:
  
  $m = \text{normalized money balances at beginning period 1}$

- in equilibrium:
  1. stationary distribution of $m$
  2. stationary normalized prices

  $$\{p_1(\theta), p_2, p_3\}$$

- relevant shocks: $(\theta, \tilde{\theta})$ where
  
  $\theta = \text{producer island shock}$
  $\tilde{\theta} = \text{consumer island shock}$
Individual Optimization

Bellman equation

\[
V(m) = \max_{\{c_s, m_s\}, n} \int_0^{\theta} \int_0^{\tilde{\theta}} [u(c_1(\tilde{\theta})) - v(n(\theta)) + U(c_2(\theta, \tilde{\theta})) + c_3(\theta, \tilde{\theta}) + \beta V(\gamma^{-1} m_3(\theta, \tilde{\theta}))]dF(\theta)dF(\tilde{\theta}),
\]

s.t.

\[
y_1(\theta) = \theta n(\theta)
\]

\[
y_2 = e_2, \quad y_3 = e_3
\]

and ...
Budget and Liquidity Constraints

Period 1:

\[ m_1(\tilde{\theta}) + p_1(\tilde{\theta})c_1(\tilde{\theta}) \leq m \]
\[ m_1(\tilde{\theta}) \geq 0 \]
Budget and Liquidity Constraints

Period 1:

\[ m_1(\tilde{\theta}) + p_1(\tilde{\theta})c_1(\tilde{\theta}) \leq m \]
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Period 2:

\[ m_2(\theta, \tilde{\theta}) + p_2c_2(\theta, \tilde{\theta}) \leq m_1(\tilde{\theta}) + p_1(\theta)y_1(\theta) \]
\[ m_2(\theta, \tilde{\theta}) \geq 0 \]
Budget and Liquidity Constraints

Period 1:
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\[ m_2(\theta, \tilde{\theta}) \geq 0 \]

Period 3:
\[ m_3(\theta, \tilde{\theta}) + p_3c_3(\theta, \tilde{\theta}) \leq m_2(\theta, \tilde{\theta}) + p_2y_2 + p_3y_3 + (\gamma - 1) \]
\[ m_3(\theta, \tilde{\theta}) \geq 0 \]
Equilibrium Characterization

Labor supply:

$\nu'(n(\theta)) = \theta \frac{p_1(\theta)}{p_2} \int_0^\theta U'(c_2(\theta, \tilde{\theta}))dF(\tilde{\theta})$

3 Euler equations (with complementary slackness):

$u'(c_1(\tilde{\theta})) \geq \frac{p_1(\tilde{\theta})}{p_2} \int_0^{\tilde{\theta}} U'(c_2(\theta, \tilde{\theta}))dF(\theta) \quad (m_1(\tilde{\theta}) \geq 0)$

$U'(c_2(\theta, \tilde{\theta})) \geq \frac{p_2}{p_3} \quad (m_2(\theta, \tilde{\theta}) \geq 0)$

$1 \geq p_3 \beta RV'(Rm_3(\theta, \tilde{\theta})) \quad (m_3(\theta, \tilde{\theta}) \geq 0)$
Equilibrium Characterization

Labor supply:

\[ v'(n(\theta)) = \theta \frac{p_1(\theta)}{p_2} \int_0^{\theta} U'(c_2(\theta, \tilde{\theta})) dF(\tilde{\theta}) \]

3 Euler equations (with complementary slackness):

\[ u'(c_1(\tilde{\theta})) = \frac{p_1(\tilde{\theta})}{p_2} \int_0^{\tilde{\theta}} U'(c_2(\theta, \tilde{\theta})) dF(\theta) \quad (m_1(\tilde{\theta}) > 0) \]

\[ U'(c_2(\theta, \tilde{\theta})) \geq \frac{p_2}{p_3} \quad (m_2(\theta, \tilde{\theta}) \geq 0) \]

\[ 1 = p_3 \beta RV' \left( Rm_3(\theta, \tilde{\theta}) \right) \quad (m_3(\theta, \tilde{\theta}) > 0) \]
Two polar regimes

1. Unconstrained economy
   - $m_2(\theta, \tilde{\theta}) > 0$ for all $\theta$ and $\tilde{\theta}$
   - $\gamma = \beta$

2. Fully constrained economy
   - $m_2(\theta, \tilde{\theta}) = 0$ for all $\theta$ and $\tilde{\theta}$
   - $\gamma \geq \hat{\gamma} > \beta$
Two polar regimes

\[ \frac{M_t}{P_{t,2}} \]

Friedman rule

Fully constrained

Motivation  Model  Equilibrium  U and C economies  Aggregate shocks  Great Moderation  Conclusions

\[ \gamma \]

\[ \beta \]

\[ \hat{\gamma} \]
Equilibrium on island $\theta$ (Period 1)

Euler equation

$$u'(y_1(\theta)) = \frac{p_1(\theta)}{p_2} \int U'(c_2(\tilde{\theta}, \theta)) \, dF(\tilde{\theta})$$

Labor supply

$$v'(n(\theta)) = \theta \frac{p_1(\theta)}{p_2} \int U'(c_2(\theta, \tilde{\theta})) \, dF(\tilde{\theta})$$

Market clearing

$$y_1(\theta) = \theta n(\theta)$$
Unconstrained economy

- Euler equation in period 2
  \[ U'(c_2(\theta, \tilde{\theta})) = \frac{p_2}{p_3} \]

- From budget constraints
  \[ c_2(\theta, \tilde{\theta}) = \frac{1}{p_2} \left( m - p_1(\tilde{\theta})y_1(\tilde{\theta}) + p_1(\theta)y_1(\theta) \right) - \frac{m_2(\theta, \tilde{\theta})}{p_2} \]

- \( m_2(\theta, \tilde{\theta}) \) adjusts to keep \( U' \) constant
Unconstrained economy

- Euler equation in period 2

\[ U'(c_2(\theta, \tilde{\theta})) = \frac{p_2}{p_3} \]

- From budget constraints

\[ c_2(\theta, \tilde{\theta}) = \frac{1}{p_2} \left( m - p_1(\tilde{\theta})y_1(\tilde{\theta}) + p_1(\theta)y_1(\theta) \right) - \frac{m_2(\theta, \tilde{\theta})}{p_2} \]

- \( m_2(\theta, \tilde{\theta}) \) adjusts to keep \( U' \) constant

- full insurance
Unconstrained economy (continued)

Proposition

(i) An unconstrained equilibrium exists if and only if $\gamma = \beta$.
(ii) The equilibrium implements the first-best allocation.

- Equilibrium boils down to

$$u'(\theta n(\theta)) = \theta v'(n(\theta))$$

- Equilibrium in island $\theta$ independent of what’s going on in other islands
Constrained economy

- Euler equation in period 2

\[ U'(c_2(\theta, \tilde{\theta})) \geq \frac{p_2}{p_3} \]

- From budget constraints

\[ c_2(\theta, \tilde{\theta}) = \frac{1}{p_2} \left( m - p_1(\tilde{\theta})y_1(\tilde{\theta}) + p_1(\theta)y_1(\theta) \right) - \frac{m_2(\theta, \tilde{\theta})}{p_2} \]

- \( m_2(\theta, \tilde{\theta}) = 0 \) cannot adjusts
Constrained economy

- Euler equation in period 2
  \[ U'(c_2(\theta, \tilde{\theta})) \geq \frac{p_2}{p_3} \]

- From budget constraints
  \[ c_2(\theta, \tilde{\theta}) = \frac{1}{p_2} \left( m - p_1(\tilde{\theta})y_1(\tilde{\theta}) + p_1(\theta)y_1(\theta) \right) - 0 \]

- \( m_2(\theta, \tilde{\theta}) = 0 \) cannot adjusts

- uninsurable income risk
Proposition

There is a cutoff $\hat{\gamma} \in (\beta, \infty)$ such that a fully constrained equilibrium exists if and only if $\gamma \geq \hat{\gamma}$.

Solve functional equations for $p_1(\cdot)$ and $y_1(\cdot)$:

(normalize $p_2 = 1$)

\[
\begin{align*}
    u'(y_1(\theta)) &= p_1(\theta) \int_0^{\theta} U'(M - p_1(\theta)y_1(\theta) + p_1(\tilde{\theta})y_1(\tilde{\theta})) \, dF(\tilde{\theta}) \\
    v'(y_1(\theta)/\theta) &= \theta p_1(\theta) \int_0^{\theta} U'(M - p_1(\tilde{\theta})y_1(\tilde{\theta}) + p_1(\theta)y_1(\theta)) \, dF(\tilde{\theta})
\end{align*}
\]
Proof Idea

- define $x(\theta) \equiv p_1(\theta) y_1(\theta)$
- define a mapping
  $$T : B([0, \theta]) \rightarrow B([0, \theta])$$
- find a fixed point $x(.)$ and then go back to $(p_1(\cdot), y_1(\cdot))$
- check Euler equation in period 2 to obtain $\hat{\gamma}$
- sufficient conditions for $T$ to be a contraction:
  $$-u''(c)c/u'(c) \in [\rho, 1)$$
  for some $\rho > 0$
Credit Economy

- assume all households are not anonymous and have access to perfect credit

- consider real non-state-contingent bonds in periods 1 and 2 which pay off in 3

Proposition

The economy with perfect credit market has a stationary equilibrium which achieves the same allocation than the monetary economy under Friedman rule.
Aggregate shocks

- $\zeta \sim G(\zeta)$ with support $[\underline{\zeta}, \overline{\zeta}]$, i.i.d.

- $F(\theta | \zeta^{II}) \leq F(\theta | \zeta^I)$ if $\zeta^{II} \geq \zeta^I$ (FOSD)

- Aggregate output

$$Y_1(\zeta) \equiv \int y_1(\theta, \zeta) dF(\theta | \zeta)$$

**Questions:** How does output respond to aggregate shocks? To what extent do outputs in different islands comove?
Output response to $\zeta$

A decomposition

$$\frac{d \ln Y_1}{d \zeta} = \int y_1(\theta, \zeta) \frac{\partial f(\theta|\zeta)}{\partial \zeta} d\theta \underbrace{Y}_{\text{composition}} + \int \frac{\partial y_1(\theta, \zeta)}{\partial \zeta} dF(\theta|\zeta) \underbrace{Y}_{\text{coordination}}$$

**Effects:**

<table>
<thead>
<tr>
<th>Unconstrained</th>
<th>Composition</th>
<th>Coordination</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>Fully Constrained</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>
Consumption response to $\zeta$
Unconstrained economy

$y_1(\theta)$
Constrained economy

\[ y_1(\theta) \]

Figure 3:
Coordination effect

Proposition

When $\gamma \geq \hat{\gamma}$ then $y_1(\theta, \zeta)$ is increasing in $\zeta$ for all $\theta$

Partial equilibrium exercise

- imagine $\zeta'' > \zeta'$
- look at market $\theta$
- keep fixed $p_1(\tilde{\theta}, \zeta')$ and $y_1(\tilde{\theta}, \zeta')$ for $\tilde{\theta} \neq \theta$

- **Result:** $p_1(\tilde{\theta}, \zeta)y_1(\tilde{\theta}, \zeta)$ increasing in $\tilde{\theta}$ for any $\zeta$
Coordination effect (continued)

- In Euler equation: RHS falls

\[ u'(y_1) = p_1 \int_{0}^{\tilde{\theta}} U'(e_2 - p_1 y_1 + p_1(\tilde{\theta}, \zeta^l) y_1(\tilde{\theta}, \zeta^l))dF(\tilde{\theta}|\zeta^{II}) \]

- In labor supply: RHS increases

\[ v'(y_1/\theta) = \theta p_1 \int_{0}^{\tilde{\theta}} U'(e_2 - p_1(\tilde{\theta}, \zeta^l) y_1(\tilde{\theta}, \zeta^l) + p_1 y_1)dF(\tilde{\theta}|\zeta^{II}) \]

\[ \Rightarrow y_1 > y_1(\theta, \zeta^l) \text{ and } y_1 p_1 > y_1(\theta, \zeta^l) p_1(\theta, \zeta^l) \]
Complete Proof (sketch)

Let $T_I$ and $T_{II}$ be the maps associated to $\zeta^I$ and $\zeta^{II}$.

Let $x^I$ and $x^{II}$ be the fixed points of $T_I$ and $T_{II}$.

**Step 1.** “Partial equilibrium”

$$x^0 = T_{II}x^I \Rightarrow x^0 \geq x^I$$

**Step 2.** From $T_{II}$ monotone and contraction (here it is key!!)

$$x^{II} = T_{II}...T_{II}x^I \Rightarrow x^{II} \geq x^I$$

**Step 3.** Output response

$$x^{II} \geq x^I \Rightarrow y^{II} \geq y^I$$
Extended Model

- fraction $\phi$ of households travel to subset of islands with credit access $\rightarrow$ credit economy

- fraction $1 - \phi$ travel to subset of islands with anonymity $\rightarrow$ monetary economy

- two extremes are now:
  1. unconstrained economy: either $\phi = 1$ or $\gamma = \beta$
  2. fully constrained economy: $\phi = 0$ and $\gamma \geq \hat{\gamma}$
Calibration

- functional forms: \( \theta \) lognormal \((\mu, \sigma)\) and

\[
\begin{align*}
    u(c) &= c^{1-\rho_1} / (1 - \rho_1) \\
    U(c) &= c^{1-\rho_2} / (1 - \rho_2) \\
    v(n) &= n
\end{align*}
\]

- data on revolving consumer credit: \( \phi = .05 \) before 1984 and \( \phi = 4 \) after 1984

- match temporary income volatility to .17 (Hubbard et al)

- match frequency of recessions (NBER) and money demand in 1948 – 1984

**Exercise:** match average output volatility and sectoral comovement pre-1984 and evaluate how much we explain of changes post-1984 due to changes in \( \phi \) and \( i \)
Amplification

aggregate output volatility vs. Nominal interest rate (percent)

- before 1985
- after 1985
- Great Moderation

Nominal interest rate (percent)
Conclusions

• When liquidity is scarce and there is no access to credit, there is a coordination element in trade

• This tends to magnify the response of the economy to aggregate shocks and the sectoral comovement

• When liquidity is abundant and/or more agents have access to credit, this coordination element vanishes (Great Moderation)

• Current crisis: low credit access $\rightarrow$ bigger response to confidence shocks