Liquidity and Trading Dynamics

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Motivation

Broad question: what are the effects of financial frictions on macroeconomic volatility?

Specific frictions: limited access to credit and limited supply of liquid assets

Result: if no credit, scarce liquidity can amplify aggregate shocks by introducing counter-cyclical “self-insurance” motive

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Broad question: what are the effects of financial frictions on macroeconomic volatility?

Specific frictions: limited access to credit and limited supply of liquid assets

Result: if no credit, scarce liquidity can amplify aggregate shocks by introducing counter-cyclical "self-insurance" motive

3 main ingredients:

- idiosyncratic income risk
- decentralized model of production and exchange
- public supply of liquid assets

Application

Facts: after mid 1980s US has experienced

1. decline in aggregate volatility (Great Moderation)
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At the same time:

- expansion of credit market
- high inflation in the 1970s

Simple calibration to study the quantitative contribution of our mechanism to explain facts 1 and 2

Coordination in Trade

- amplification (and comovement) related to coordination element
- two meanings here:
  1. you want to buy more goods if others buy more
  2. you want to sell more goods if others sell more
- coordination element arises endogenously only when liquidity supply is low

Literature

- Money search, Kiyotaki and Wright (1989), Shi (1997), Lagos and Wright (2006)
- In Diamond (1982) coordination element with increasing returns built in matching function
- Aggregate shocks in money-search models, Berentsen, Camera, and Waller(2003)
- Aggregate effects of uninsurable idiosyncratic risk, Krusell and Smith (1998)

Environment

- Continuum of infinitely-lived Producer/Consumer households
- Discrete time, each date t divided in $s = 1, 2, 3$
- Agents produce, trade and consume a perishable good
- Households start with an initial endowment M_0 of money
- We consider two extremes:
  1. anonymous markets → fiat money
  2. perfect credit markets → money is useless

Monetary Economy

- Continuum of islands with representative sample of P and C and competitive markets *a la* Lucas and Prescott (1974)
- At $s = 1$: P and C travel to different islands k and k'
  $\rightarrow$ no communication
- Island k characterized by productivity shock $\theta_t^k \sim F(\cdot|\zeta)$

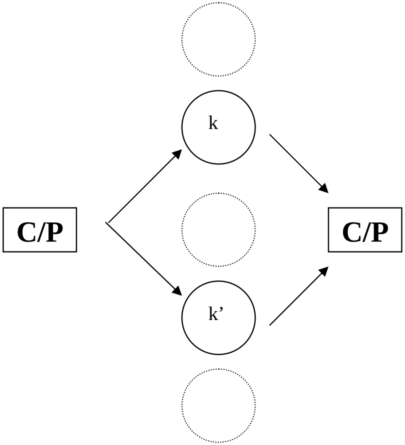
$$y_{1,t} = \theta_t^k n_t$$

- ζ = aggregate shock (fix it for now)

Monetary Economy (continued)

- At $s = 2$: C and P travel to different islands
  $\longrightarrow$ no communication
- Fixed endowment $y_{2,t} = e_2$
- At $s = 3$: C and P in same island
  $\longrightarrow$ centralized market
- Fixed endowment $y_{3,t} = e_3$

Geography



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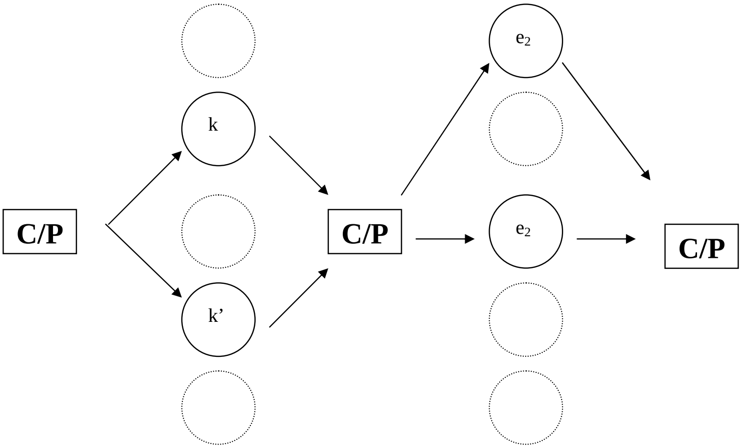

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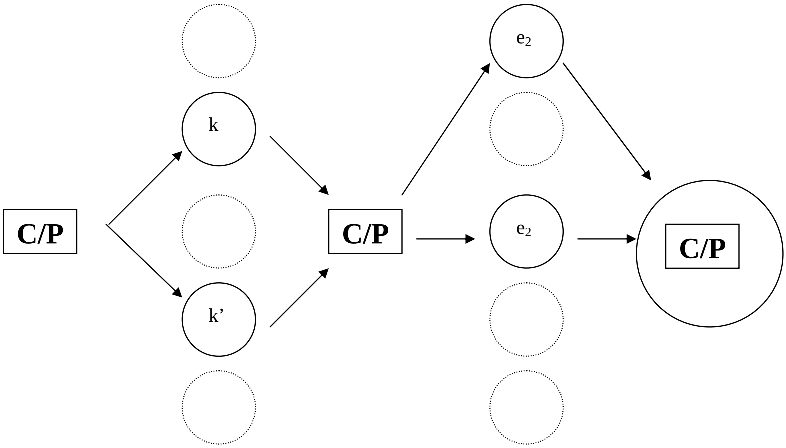

Preferences

- Quasi-linear utility (LW):

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^{t}\left(u(c_{1,t})-v(n_t)+U(c_{2,t})+c_{3,t}\right)\right],$$

u, U strictly concave

v convex

Government

- $\gamma =$ constant money growth rate
- at the end of period 3 government injects $(\gamma - 1)M_t$ units of money by lump-sum transfer/tax
- we take the monetary policy γ as given and compare economies with different policies

Stationarity

- focus on equilibria where nominal variables grow at rate γ
- recursive representation of household problem with one state variable:

  m = normalized money balances at beginning period 1

- in equilibrium:
  1. stationary distribution of m
  2. stationary normalized prices

$$\{p_1(\theta)\}_\theta, p_2, p_3$$

- relevant shocks: $(\theta, \tilde{\theta})$ where

$$\theta = \text{producer island shock}$$
$$\tilde{\theta} = \text{consumer island shock}$$

Individual Optimization

Bellman equation

$$V(m) = \max_{\{c_s\},\{m_s\},n} \int_0^{\overline{\theta}} \int_0^{\overline{\theta}} [u(c_1(\tilde{\theta})) - v(n(\theta)) + U(c_2(\theta,\tilde{\theta})) + \\ + c_3(\theta,\tilde{\theta}) + \beta V(\gamma^{-1} m_3(\theta,\tilde{\theta}))] dF(\theta) dF(\tilde{\theta}),$$

s.t.

$$y_1(\theta) = \theta n(\theta)$$

$$y_2 = e_2 \; , \; y_3 = e_3$$

and ...

Budget and Liquidity Constraints

Period 1:

$$m_1(\tilde{\theta}) + p_1(\tilde{\theta})c_1(\tilde{\theta}) \leq m$$
$$m_1(\tilde{\theta}) \geq 0$$

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$$m_2(\theta, \tilde{\theta}) + p_2 c_2(\theta, \tilde{\theta}) \leq m_1(\tilde{\theta}) + p_1(\theta)\, y_1(\theta)$$
$$m_2(\theta, \tilde{\theta}) \geq 0$$

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Period 3:

$$m_3(\theta,\tilde{\theta}) + p_3 c_3(\theta,\tilde{\theta}) \leq m_2(\theta,\tilde{\theta}) + p_2 y_2 + p_3 y_3 + (\gamma - 1)$$
$$m_3(\theta,\tilde{\theta}) \geq 0$$

Equilibrium Characterization

Labor supply:

$$v'(n(\theta)) = \theta \frac{p_1(\theta)}{p_2} \int_0^{\overline{\theta}} U'(c_2(\theta,\tilde{\theta}))dF(\tilde{\theta})$$

3 Euler equations (with complementary slackness):

$$\begin{aligned} u'(c_1(\tilde{\theta})) &\geq \frac{p_1(\tilde{\theta})}{p_2} \int_0^{\overline{\theta}} U'(c_2(\theta,\tilde{\theta}))dF(\theta) & (m_1(\tilde{\theta}) \geq 0) \\ U'(c_2(\theta,\tilde{\theta})) &\geq \frac{p_2}{p_3} & (m_2(\theta,\tilde{\theta}) \geq 0) \\ 1 &\geq p_3 \beta R V'\left(R m_3(\theta,\tilde{\theta})\right) & (m_3(\theta,\tilde{\theta}) \geq 0) \end{aligned}$$

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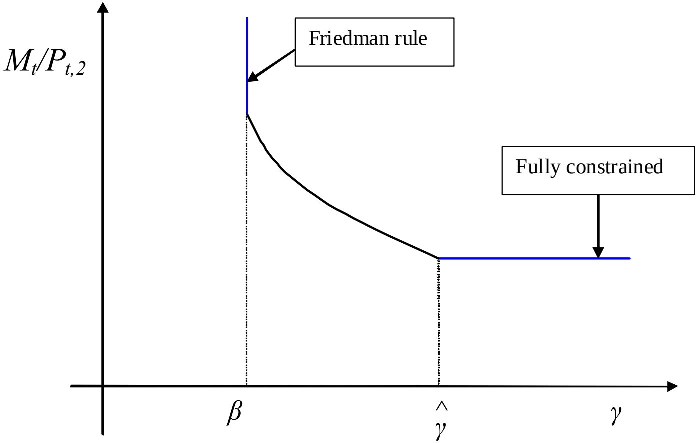
Two polar regimes

1. Unconstrained economy
   - $m_2(\theta, \tilde{\theta}) > 0$ for all θ and $\tilde{\theta}$
   - $\gamma = \beta$
2. Fully constrained economy
   - $m_2(\theta, \tilde{\theta}) = 0$ for all θ and $\tilde{\theta}$
   - $\gamma \geq \hat{\gamma} > \beta$

Two polar regimes



Equilibrium on island θ (Period 1)

Euler equation

$$u'(y_1(\theta)) = \frac{p_1(\theta)}{p_2} \int U'\left(c_2(\tilde{\theta}, \theta)\right) dF(\tilde{\theta})$$

Labor supply

$$v'(n(\theta)) = \theta \frac{p_1(\theta)}{p_2} \int U'\left(c_2(\theta, \tilde{\theta})\right) dF(\tilde{\theta})$$

Market clearing

$$y_1(\theta) = \theta n(\theta)$$

Unconstrained economy

- Euler equation in period 2

$$U'(c_2(\theta,\tilde{\theta})) = \frac{p_2}{p_3}$$

- From budget constraints

$$c_2(\theta,\tilde{\theta}) = \frac{1}{p_2}\left(m - p_1(\tilde{\theta})y_1(\tilde{\theta}) + p_1(\theta)y_1(\theta)\right) - \frac{m_2(\theta,\tilde{\theta})}{p_2}$$

- $m_2(\theta,\tilde{\theta})$ adjusts to keep U' constant

Unconstrained economy

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- $m_2(\theta,\tilde{\theta})$ adjusts to keep U' constant
- full insurance

Unconstrained economy (continued)

Proposition

(i) *An unconstrained equilibrium exists if and only if $\gamma = \beta$.*

(ii) *The equilibrium implements the first-best allocation.*

- Equilibrium boils down to

$$u'(\theta n(\theta)) = \theta v'(n(\theta))$$

- $\rightarrow$ Equilibrium in island θ independent of what's going on in other islands

Constrained economy

- Euler equation in period 2

$$U'(c_2(\theta,\tilde{\theta})) \geq \frac{p_2}{p_3}$$

- From budget constraints

$$c_2(\theta,\tilde{\theta}) = \frac{1}{p_2}\left(m - p_1(\tilde{\theta})y_1(\tilde{\theta}) + p_1(\theta)y_1(\theta)\right) - \frac{m_2(\theta,\tilde{\theta})}{p_2}$$

- $m_2(\theta,\tilde{\theta}) = 0$ cannot adjusts

Constrained economy

- Euler equation in period 2

$$U'(c_2(\theta,\tilde{\theta})) \geq \frac{p_2}{p_3}$$

- From budget constraints

$$c_2(\theta,\tilde{\theta}) = \frac{1}{p_2}\left(m - p_1(\tilde{\theta})y_1(\tilde{\theta}) + p_1(\theta)y_1(\theta)\right) - 0$$

- $m_2(\theta,\tilde{\theta}) = 0$ cannot adjusts

- uninsurable income risk

Constrained economy (continued)

Proposition

There is a cutoff $\hat{\gamma} \in (\beta, \infty)$ such that a fully constrained equilibrium exists if and only if $\gamma \geq \hat{\gamma}$.

Solve functional equations for $p_1(\cdot)$ and $y_1(\cdot)$:

(normalize $p_2 = 1$)

$$u'(y_1(\theta)) = p_1(\theta) \int_0^{\overline{\theta}} U'\left(M - p_1(\theta) y_1(\theta) + p_1(\tilde{\theta}) y_1(\tilde{\theta})\right) dF(\tilde{\theta})$$

$$v'(y_1(\theta)/\theta) = \theta p_1(\theta) \int_0^{\overline{\theta}} U'\left(M - p_1(\tilde{\theta}) y_1(\tilde{\theta}) + p_1(\theta) y_1(\theta)\right) dF(\tilde{\theta})$$

Proof Idea

- define $x(\theta) \equiv p_1(\theta) y_1(\theta)$
- define a mapping

$$T : B([0, \overline{\theta}]) \to B([0, \overline{\theta}])$$

- find a fixed point $x(.)$ and then go back to $(p_1(\cdot), y_1(\cdot))$
- check Euler equation in period 2 to obtain $\hat{\gamma}$
- sufficient conditions for T to be a contraction:

$$-u''(c)c/u'(c) \in [\underline{\rho}, 1)$$

for some $\underline{\rho} > 0$

Credit Economy

- assume all households are not anonymous and have access to perfect credit
- consider real non-state-contingent bonds in periods 1 and 2 which pay off in 3

Proposition

The economy with perfect credit market has a stationary equilibrium which achieves the same allocation than the monetary economy under Friedman rule.

Aggregate shocks

- $\zeta \sim G(\zeta)$ with support $[\underline{\zeta}, \overline{\zeta}]$, i.i.d.
- $F(\theta|\zeta^{II}) \leq F(\theta|\zeta^{I})$ if $\zeta^{II} \geq \zeta^{I}$ (FOSD)
- Aggregate output

$$Y_1(\zeta) \equiv \int y_1(\theta, \zeta)\, dF(\theta|\zeta)$$

Questions: How does output respond to aggregate shocks? To what extent do outputs in different islands comove?

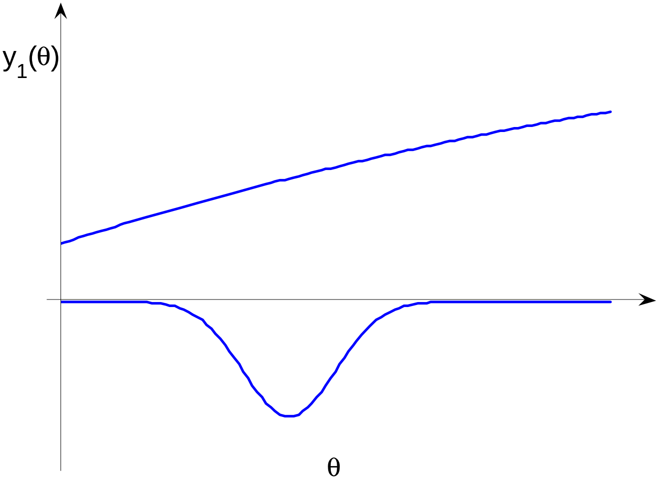
Output response to ζ

A decomposition

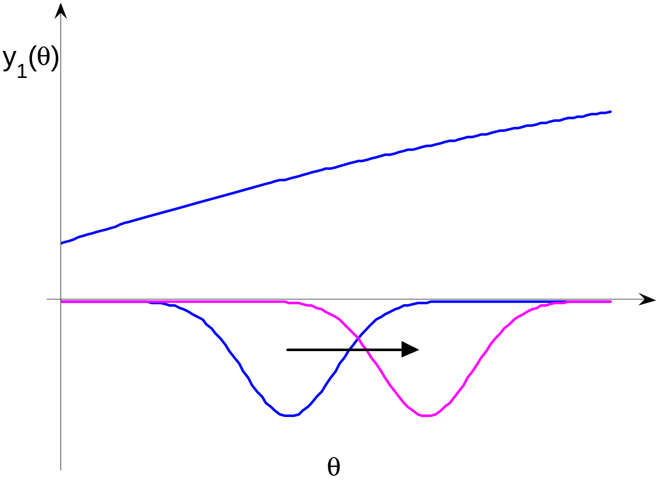
$$\frac{d \ln Y_1}{d\zeta} = \underbrace{\frac{\int y_1(\theta,\zeta) \frac{\partial f(\theta|\zeta)}{\partial \zeta} d\theta}{Y}}_{\text{composition}} + \underbrace{\frac{\int \frac{\partial y_1(\theta,\zeta)}{\partial \zeta} dF(\theta|\zeta)}{Y}}_{\text{coordination}}$$

Effects:	Composition	Coordination
Unconstrained	$+$	0
Fully Constrained	$+$	$+$

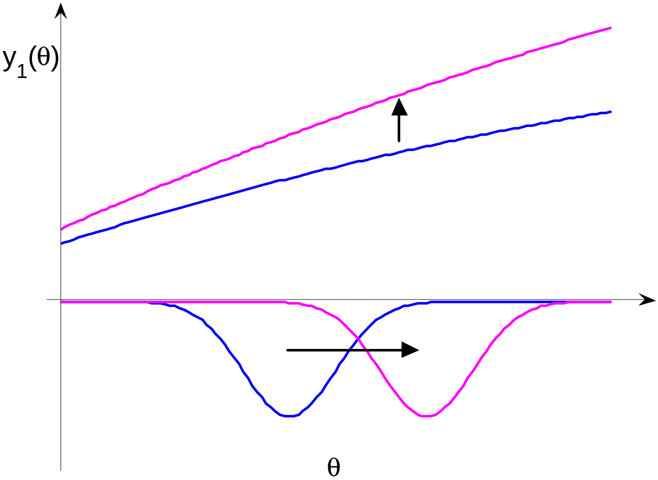
Consumption response to ζ



Unconstrained economy



Constrained economy



Coordination effect

Proposition

When $\gamma \geq \hat{\gamma}$ then $y_1(\theta, \zeta)$ is increasing in ζ for all θ

Partial equilibrium exercise

- imagine $\zeta^{II} > \zeta^{I}$
- look at market θ
- keep fixed $p_1(\tilde{\theta}, \zeta^{I})$ and $y_1(\tilde{\theta}, \zeta^{I})$ for $\tilde{\theta} \neq \theta$
- **Result:** $p_1(\tilde{\theta}, \zeta) y_1(\tilde{\theta}, \zeta)$ increasing in $\tilde{\theta}$ for any ζ

Coordination effect (continued)

- In Euler equation: RHS falls

$$u'(y_1) = p_1 \int_0^{\overline{\theta}} U'(e_2 - p_1 y_1 + p_1(\tilde{\theta}, \zeta^I) y_1(\tilde{\theta}, \zeta^I)) dF(\tilde{\theta}|\zeta^{II})$$

- In labor supply: RHS increases

$$v'(y_1/\theta) = \theta p_1 \int_0^{\overline{\theta}} U'(e_2 - p_1(\tilde{\theta}, \zeta^I) y_1(\tilde{\theta}, \zeta^I) + p_1 y_1) dF(\tilde{\theta}|\zeta^{II})$$

- $\Rightarrow y_1 > y_1(\theta, \zeta^I)$ and $y_1 p_1 > y_1(\theta, \zeta^I) p_1(\theta, \zeta^I)$

Complete Proof (sketch)

Let T_I and T_{II} be the maps associated to ζ^I and ζ^{II}.

Let x^I and x^{II} be the fixed points of T_I and T_{II}.

Step 1. "Partial equilibrium"

$$x^0 = T_{II} x^I \Rightarrow x^0 \geq x^I$$

Step 2. From T_{II} monotone and contraction (here it is key!!)

$$x^{II} = T_{II} \ldots T_{II} x^I \Rightarrow x^{II} \geq x^I$$

Step 3. Output response

$$x^{II} \geq x^I \Rightarrow y^{II} \geq y^I$$

Extended Model

- fraction ϕ of households travel to subset of islands with credit access $\rightarrow$ credit economy
- fraction $1-\phi$ travel to subset of islands with anonymity $\rightarrow$ monetary economy
- two extremes are now:

  1. unconstrained economy: either $\phi = 1$ or $\gamma = \beta$
  2. fully constrained economy: $\phi = 0$ and $\gamma \geq \hat{\gamma}$

Calibration

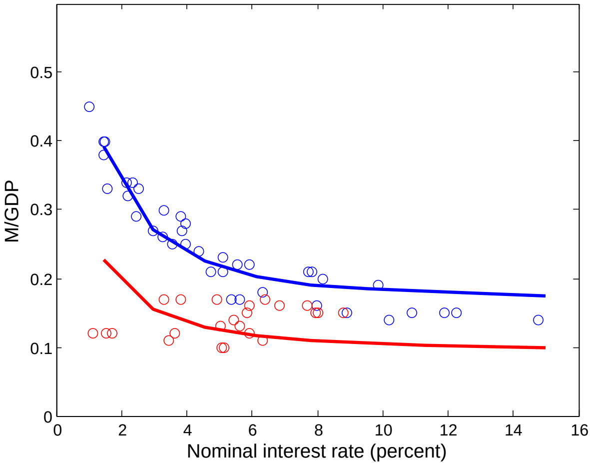
- functional forms: θ lognormal (μ, σ) and

$$u(c) = c^{1-\rho_1}/(1-\rho_1) \quad U(c) = c^{1-\rho_2}/(1-\rho_2) \quad v(n) = n$$

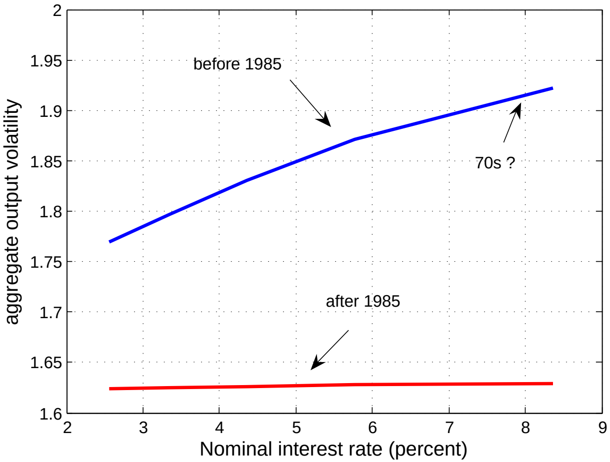
- data on revolving consumer credit: $\phi = .05$ before 1984 and $\phi = 4$ after 1984
- match temporary income volatility to .17 (Hubbard et al)
- match frequency of recessions (NBER) and money demand in $1948 - 1984$

Exercise: match average output volatility and sectoral comovement pre-1984 and evaluate how much we explain of changes post-1984 due to changes in ϕ and i

Money demand



Amplification



Conclusions

- When liquidity is scarce and there is no access to credit, there is a coordination element in trade
- This tends to magnify the response of the economy to aggregate shocks and the sectoral comovement
- When liquidity is abundant and/or more agents have access to credit, this coordination element vanishes (Great Moderation)
- Current crisis: low credit access $\rightarrow$ bigger response to confidence shocks