Real Effects of Price Stability with Endogenous Nominal Indexation

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THE REAL EFFECTS OF INFLATION

- The transmission of inflation shocks to the real sector of the economy requires some form of nominal rigidity.

- We explore the role played by ‘nominal financial contracts’ in a model with incomplete markets.

- We then study how different monetary regimes affect the propagation of inflation shocks:
  - Across different types of borrowers (firms);
  - for the whole economy.
Why are contracts nominally denominated?

• We need a theory of nominal rigidities (endogenous nominal indexation).

• We adopt the idea of Jovanovic-Ueda (1997): The general price is observed with delay.

• We embed this idea in an industry dynamics model with repeated moral hazard:
  – Clementi-Hopenhyan (2006);
  – Gertler (1992);
1. Optimal and renegotiation-proof contracts not fully indexed.  
   Therefore, ‘unexpected’ inflation shocks have real effects.

2. Contracts are less indexed for smaller and more constrained firms.  
   Therefore, the impact of an inflation shock is bigger for small firms.

3. The degree of nominal indexation increases with price uncertainty.  
   Therefore, a given inflation shock has a bigger impact in economies with lower price uncertainty.
MODEL

• Continuum of risk-neutral investors with discounting $\delta$.

• Continuum of risk neutral entrepreneurs with discounting $\beta \leq \delta$.

• Entrepreneurs generate cash revenues:

$$s = pzF(k)$$

$k =$ input of capital.
$F(.) =$ strictly increasing and concave; $F(0) = 0$.
$z =$ idiosyncratic shock; $z \sim LN(\mu_z, \sigma_z^2)$.
$p =$ aggregate nominal price; $p \sim LN(\mu_p, \sigma_p^2)$. 
Information and timing

- Investment $k$ is publicly observable. It is chosen before knowing $z$ and $p$.

- The idiosyncratic shock $z$ is NOT publicly observable but it can be inferred once we know $s$ and $p$.

- After observing $s = p z F(k)$, the entrepreneur can divert the revenues without being detected:
  
  $\hat{s} = p \hat{z} F(k) = \text{reported revenues.}$
  
  $\hat{z} = \text{shock inferred from } \hat{s}, \text{once we know } p.$

- **ASSUMPTION:** The price is observed with delay.
The entrepreneur observes $s = pzF(k)$.

Chooses reporting, $\hat{s}$ (and $\hat{z}$).

Diverted revenue, $s - \hat{s}$

Public revenue, $\hat{s}$

Price $p$ is observed

\[
\frac{s - \hat{s}}{p} = (z - \hat{z})F(k)
\]

\[
\frac{\hat{s}}{p} = \hat{z}F(k)
\]
VALUE OF DIVERSION

EX-POST

\[ \frac{s - \hat{s}}{p} = (z - \hat{z}) F(k) \]

EX-ANTE (when choosing diversion)

\[ E\left( \frac{s - \hat{s}}{p} \middle| s \right) = E\left( z - \hat{z} \middle| s \right) F(k) \]
LONG-TERM CONTRACT
Max investor’s value subject to entrepreneur’s value

\[ V(q) = \max_{k, u(z, p)} \left\{ -k + \delta E \left[ z k^\theta + W(u(z, p)) \right] \right\} \]

subject to
\[ E \left[ u(z, p) \mid s \right] \geq E \left[ \phi z k^\theta + u(0, p) \mid s \right] \]

\[ q = \beta E \left[ u(z, p) \right] \]

\[ u(z, p) \geq 0. \]
Proposition. The optimal policy for the entrepreneur’s value depends only on \( z \), not \( p \).

Therefore, \( u' = u(z) \).
VALUE FUNCTION

\[ V(q) \]

\[ \bar{q} \]
TIMING

The entrepreneur observes $s = pzF(k)$

Chooses reporting, $\hat{s}$ (and $\hat{z}$)

Diverted revenue, $s - \hat{s}$

Public revenue, $\hat{s}$

RENEGOTIATION

Price $p$ is observed
VALUE FUNCTION

\[ W(u(z)) \]

\[ u(z) \]

\[ \bar{q} \]
RENEGOTIATION-PROOF CONTRACT

\[ V(q) = \max_{k, u(s)} \left\{ -k + \delta E \left[ z k^\theta + W(u(s)) \right] \right\} \]

subject to

\[ u(s) \geq \phi E \left[ z k^\theta \mid s \right] + u(0), \quad \forall s \]

\[ q = \beta E u(s) \]

\[ u(s) \geq u \]
VALUE FUNCTION

$V(q)$

$q$

$u$  $\bar{q}$
Dynamics of individual net worth

\[ u' = \phi \left[ E(z \mid s) - \bar{z} \right] k^\theta + \frac{q}{\beta} \]

- Case I: \( \sigma_p = 0 \) \( \Rightarrow \) \( E(z \mid s) = z \)

- Case I: \( \sigma_p = \infty \) \( \Rightarrow \) \( E(z \mid s) = \bar{z} \)
RESPONSE TO A NOMINAL PRICE SHOCK

Proposition. Consider a one-time unexpected increase in price $\Delta p$. The impact of the shock on the next period net worth strictly decreases in $\sigma_p$. 
Investment Decision and Distribution of Firms

(a) Investment Decision
Entrepreneur’s Value (q)

(b) Invariant Distribution
Firm Size (Capital)
FIRM SIZE AND INDEXATION
(Elasticity of net worth to price shock)
AVERAGE INDEXATION
(Elaticity of net worth to price shock)

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<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Low Price Uncertainty</td>
<td>0.667</td>
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<tr>
<td>(small $\sigma_p$)</td>
<td></td>
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<tr>
<td>High Price Uncertainty</td>
<td>0.011</td>
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<tr>
<td>(large $\sigma_p$)</td>
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FIRM SIZE AND PRICE SHOCK RESPONSE

- Small firms;  - - - Large firms

(a) Low Uncertainty, Price Change = 0.25
(b) Low Uncertainty, Price Change = −0.25
(c) High Uncertainty, Price Change = 0.25
(d) High Uncertainty, Price Change = −0.25
### AGGREGATE VOLATILITY OF CAPITAL

<table>
<thead>
<tr>
<th>$\sigma_p$</th>
<th>Value 1</th>
<th>Value 2</th>
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<tbody>
<tr>
<td>0.02</td>
<td>0.008</td>
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<tr>
<td>0.20</td>
<td>0.073</td>
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<tr>
<td>1.50</td>
<td>0.134</td>
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<tr>
<td>1.70</td>
<td>0.120</td>
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## DEVELOPMENT AND PRICE VOLATILITY

<table>
<thead>
<tr>
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<th>More developed financial system ($\phi = 0.50$)</th>
<th>Less developed financial system ($\phi = 1.00$)</th>
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</thead>
<tbody>
<tr>
<td><strong>Low Price Uncertainty ($\sigma_p = 0.02$)</strong></td>
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</tr>
<tr>
<td>Aggregate Capital</td>
<td>0.803</td>
<td>0.644</td>
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<tr>
<td>Standard Deviation Capital</td>
<td>0.006</td>
<td>0.008</td>
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<tr>
<td><strong>High Price Uncertainty ($\sigma_p = 1.5$)</strong></td>
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<tr>
<td>Aggregate Capital</td>
<td>0.984</td>
<td>0.963</td>
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<tr>
<td>Standard Deviation Capital</td>
<td>0.092</td>
<td>0.134</td>
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<tr>
<td><strong>Extreme Price Uncertainty ($\sigma_p = 1.70$)</strong></td>
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<tr>
<td>Aggregate Capital</td>
<td>0.986</td>
<td>0.955</td>
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<tr>
<td>Standard Deviation Capital</td>
<td>0.085</td>
<td>0.130</td>
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CONCLUSION

- We have shown that inflation shocks can have real economic effects because of limited indexation of financial contracts.

- We asked whether an increase in nominal price uncertainty increases the volatility of the real economy.

- We find that this is not necessarily the case because the degree of nominal indexation is ‘endogenous’ and increases with nominal price uncertainty.

- The analysis also points out that inflation shocks have a different impact on firms of different types.