Trend and Cycle in Bond Premia

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Motivation

• stylized fact: excess returns on long bonds are predictable
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• definitions

  - log excess return on zero coupon bond of maturity \( n \) years, held for 1 year

    \[
    = \log \text{price next year} - \log \text{price today} - 1 \text{ year interest rate}
    \]

    \[
    = p_{t+1}^{(n-1)} - p_t^{(n)} - i_t^{(1)} := r x_{t+1}^{(n)}
    \]

  - predictability: premium \( \hat{E}_t r x_{t+1}^{(n)} \) moves around

    \( \hat{E}_t \) computed from statistical model \( \rightarrow \) "statistical premium"

    e.g., fitted value from regressing \( r x_{t+1}^{(n)} \) on time-\( t \) variables
Motivation

• stylized fact: excess returns on long bonds are predictable

\[ \hat{E}_t r x_{t+1}^{(n)} \] moves around; e.g.

– higher after recessions, lower at end of booms
– higher in 1980s, lower in 1970s

\[ \Rightarrow \] puzzle: why did investors not exploit predictability? Two candidate answers:

1. historical expectations \( \neq \hat{E}_t r x_{t+1}^{(n)} \)

2. historical expectations \( = \hat{E}_t r x_{t+1}^{(n)} = (\text{compensation for risk})_t \)

• most models: only 2.

• This paper: model of both; evidence of 1. from surveys
This paper

- investors learn about dynamics of interest rates, inflation, real activity

- learning process determines
  1. subjective forecasts $E_t r x^{(n)}_{t+1}$ – compare to survey data
  2. subjective risk

- Euler equation: $E_t r x^{(n)}_{t+1} = \text{(compensation for subjective risk)}_t$

- decompose statistical premium
  $\hat{E}_t r x^{(n)}_{t+1} = \left(\hat{E}_t r x^{(n)}_{t+1} - E_t r x^{(n)}_{t+1}\right) + \text{(compensation for subjective risk)}_t$
  forecast difference
Message

• decompose statistical premium

\[
\hat{E}_t r x_{t+1}^{(n)} = \left( \hat{E}_t r x_{t+1}^{(n)} - E_t r x_{t+1}^{(n)} \right) + \text{(compensation for subjective risk)}_t
\]

forecast difference

• Movements in forecast difference

  – forecasts lag behind trends, do not react much to sudden changes

  \[\implies\text{forecast differences low in 70s, high in 80s, countercyclical}.\]

• Movements in compensation for subjective risk

  – with recursive utility, relevant risk is Cov (bond returns, news about growth)

  – risk increases over 1970s, drops later

  \[\implies\text{subjective premium slow-moving, high in 80s}\]
1. Stylized facts about survey forecasts

   (a) pictures for one bond, one forecast horizon

   (b) compress info from forecasts for many maturities, horizons
       (using factor model)

\[ \Rightarrow \text{forecasts made as if level & slope of yield curve more persistent} \]

\[ \Rightarrow \text{subjective premium much less volatile, cyclical, especially for long maturities} \]

2. consumption based asset pricing model with learning:

   learning explains forecast differences and subjective risk premia
Related Literature

- predictability regressions
  
  Fama & Bliss 1987, Campbell & Shiller 1991, etc

- statistical analysis of interest rate survey data
  

- role of survey expectations in other markets
  

- asset pricing with recursive utility
  
Properties of Survey Forecasts


- each quarter, 40 market participants are asked about their interest-rate expectations

- look at median, max horizons: 2 quarters for GN, 1 year for Bluechip

- decomposition for bond of maturity $n$ years, held for horizon $h$ years

\[
\hat{E}_{t+r} x_{t+h}^{(n)} = \left( \hat{E}_{t+r} x_{t+h}^{(n)} - E_{t+r} x_{t+h}^{(n)} \right) + \text{(compensation for risk)}_t
\]

- measure $\hat{E}_{t+r} x_{t+h}^{(n)}$ with regressions

- measure $E_{t+r} x_{t+h}^{(n)} = E_{t+r} p_{t+h}^{(n-h)} - p_t^{(n)} - i_t^{(h)}$

  with interest-rate surveys $E_{t+r} p_{t+1}^{(n-1)} = -(n - 1) E_{t+r} i_{t+1}^{(n-1)}$

example: $n = 11$ years, $h = 1$ year for Bluechip
Premia, maturity = 11 years, horizon = 1 year

statistical (regression)
Premia, maturity = 11 years, horizon = 1 year

statistical (regression)
Premia, maturity = 11 years, horizon = 1 year

statistical (regression) - subjective (survey data)
statistical (regression)
Outline

1. Stylized facts about survey forecasts
   
   (a) pictures for one bond, one forecast horizon
   
   (b) compress info from forecasts for many maturities, horizons
       (using factor model)

   $\Rightarrow$ forecasts made as if level & slope of yield curve more persistent

   $\Rightarrow$ subjective premium much less volatile & cyclical, especially for long maturities

2. consumption based asset pricing model with learning:

   learning explains forecast differences and subjective risk premia
Compressing info

- goal: combine info in forecasts for all dates, bond maturities, forecast horizons

- quarterly state space system:
  observables are interest rates, inflation, consumption growth

  \[
  \text{observables}_t = \mu_z + \phi_z \text{factors}_{t-1} + \sigma_z e_t \\
  \text{factors}_t = \phi_f \text{factors}_{t-1} + \sigma_f e_t
  \]

- statistical forecasts

  \[
  \hat{E}_t [\text{observables}_{t+h}] = \mu_z + \phi_z (\phi_f)^h \text{factors}_t
  \]

- assume same functional form for survey forecasts

  \[
  E_t [\text{observables}_{t+h}] = \mu_z + \phi^*_z (\phi^*_f)^h \text{factors}_t
  \]

- estimate \( \phi^*_z, \phi^*_f \) from survey forecast data

\[\implies\] survey forecasts look like from system with more persistent level, slope
Comparison of subjective & statistical premia

maturity 10 years

subjective premium

<table>
<thead>
<tr>
<th>volatility</th>
<th>% trend</th>
<th>% cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.96</td>
<td>81</td>
<td>15</td>
</tr>
</tbody>
</table>

statistical premium

| 6.44       | 94      | 31      |
Premia, maturity = 11 years, horizon = 1 year

- statistical
- subjective
Outline

1. Stylized facts about survey forecasts
   (a) pictures for one bond, one forecast horizon
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2. consumption based asset pricing model with learning:

   learning explains forecast differences and subjective risk premia
Consumption based asset pricing

• Beliefs: adaptive learning
  
  – cond. distribution of cons. growth, inflation, interest rates for every date $t$
  
  – estimate linear state space system
    (geometric downweighting of past data with forget factor $\nu$)
  
  – use price variables to capture conditioning info, use in Euler equation checks

• Preferences: recursive utility (Epstein-Zin 1989)
  
  – special case: intertemporal elasticity of substitution $= 1$.

$$\ln V_t = (1 - \beta) \ln C_t + \beta \ln CE_t (V_{t+1})$$

$$CE_t (V_{t+1}) = E_t \left( V_{t+1}^{1-\gamma} \right)^{1/(1-\gamma)}$$

  – aversion against persistence $\gamma > 1$
Nominal pricing kernel (in logs)

- Euler equation: for a nominal return $R_{t+1}$, must have $E [\exp (m_{t+1}) R_{t+1}] = 1$

- With normal homoskedastic shocks, linear state space system:
  
  $$m_{t+1} = \text{const.} \quad - \quad \Delta c_{t+1} - (\gamma - 1) (E_{t+1} - E_t) \sum_{\tau=1}^{\infty} \beta^\tau \Delta c_{t+1+\tau} - \pi_{t+1}$$

- Agents want more consumption if
  - low consumption growth
  - bad news about future consumption growth

- Agents dislike assets that pay off little if
  - lo growth, hi inflation or bad news about growth

- Compensation for risk = Cov(return, $m$)
  - expected return higher if higher Cov (return, news about growth)
  - nom. $n$-period interest rate higher if higher Cov (inflation, news about growth) over lifetime of bond.
Euler equations for bonds

For a nominal return $R_{t,t+h}$, must have $E \left[ \exp \left( \sum_{i=1}^{h} m_{t+i} \right) R_{t,t+h} \right] = 1$.

1. Holding period = maturity: log return = interest rate satisfies

$$i_t^{(h)} = -\frac{1}{h} E_t \left[ \sum_{i=1}^{h} m_{t+i} \right] - \frac{1}{2h} Var_t \left[ \sum_{i=1}^{h} m_{t+i} \right]$$

$i_t^{(h)}$ higher if higher cov (inflation, news about growth) between $t$ and $t + h$

2. Holding period < maturity: log excess return

$$E_t \left[ p_t^{(n-h)} \right] - p_t^{(n)} - h i_t^{(h)} + \frac{1}{2} Var_t \left[ p_t^{(n-h)} \right] = -cov_t \left( \sum_{i=1}^{h} m_{t+i}, p_t^{(n-h)} \right)$$

$cov_t$ higher if higher cov ($p_t^{(n-h)}$, news about growth)

Can evaluate cond. moments under agent’s subjective belief for every $t$ & check errors
Results

Decompose statistical premium

\[ \hat{E}_t r x_{t+1}^{(n)} = \left( \hat{E}_t r x_{t+1}^{(n)} - E_t r x_{t+1}^{(n)} \right) + cov_t \left( m_{t+1}, r x_{t+1}^{(n)} \right) \]

- forecast difference
- compensation for subjective risk

• Subjective beliefs implied by learning \( \rightarrow \) movements in forecast difference, subjective risk

• Pick preference parameters \( \beta, \gamma \) to fit Euler equations

• Time variation in subjective, statistical risk premia
Results: subjective beliefs from learning

• Forecasts
  – fit survey forecasts better than statistical model
    (MAE drops for most forecast horizons, maturities)
  – forecast differences:
    (statistical minus surveys) versus (statistical minus learning):
    learning tracks change in sign around 1980, many cyclical movements

• Subjective Risk
  – Cov$_t$ (bond price, news about growth) high & positive after 70s, negative later
  – Cov$_t$ (inflation, news about growth) high after 70s, lower later
Forecast differences

4 qtr statistical minus subjective risk premium; 40 qtr bond

-25 -20 -15 -10 -5 0 5 10 15


red: surveys
green: learning
Preference parameters

- Horizon $h = 4$ quarters.

- Consider three Euler equations
  - 4 and 40 quarter interest rate
  - excess return for holding 40 quarter bond over 4 quarters
    (express in terms of forward rates)

- Select $\beta, \gamma$ to match equally weighted sum of squared errors

- For comparison, use statistical model as belief, and log utility
## Euler equation errors

### Table 8: Euler Equations Errors

<table>
<thead>
<tr>
<th>model</th>
<th>Preferences</th>
<th>Euler equation errors</th>
<th>Mean absolute error (% p.a.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\beta$</td>
<td>$\gamma$</td>
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<tr>
<td>statistical belief</td>
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</tr>
<tr>
<td>log utility</td>
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<td>1.96</td>
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<tr>
<td>Epstein-Zin</td>
<td>.9966</td>
<td>80</td>
<td>1.76</td>
</tr>
<tr>
<td>learning $\nu = .95$</td>
<td></td>
<td></td>
<td></td>
</tr>
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</table>
Forecast differences

4 qtr statistical minus subjective risk premium (maturity 40 quarters, percent)
Subjective and Statistical Risk Premia

4 quarter subjective risk premium (maturity 40 quarters, percent)

4 statistical risk premium (maturity 40 quarters, percent)
Message

• decompose statistical premium

\[
\hat{E}_{t+1}rx^{(n)}_{t+1} = (\hat{E}_{t+1}rx^{(n)}_{t+1} - E_{t+1}rx^{(n)}_{t+1}) + cov_t \left( m_{t+1}, rx^{(n)}_{t+1} \right)
\]

forecast difference

subjective risk premium

• Movements in forecast difference

  – forecasts lag behind trends, do not react much to sudden changes

\[\Rightarrow\] forecast differences low in 70s, high in 80s, countercyclical.

• Movements in (subjective) risk premium

  – with recursive utility, Cov (returns, news about growth) drives premia

  – high interest rates, inflation bad news for growth, more so after 1970s

\[\Rightarrow\] subjective premium slow-moving, high in 80s