Liquidity and Trading Dynamics*

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Abstract

In this paper, we build a model where the presence of liquidity constraints tend to magnify the economy's response to aggregate productivity shocks. We consider a decentralized model of trade, where agents may use credit or money to buy goods. When agents do not have access to credit and the real value of money balances is low, agents are more likely to be liquidity constrained. This makes them more concerned about their short-term earning prospects when making their consumption decisions and more concerned about their short-term spending opportunities when making their production decisions. This generates a coordination element in spending and production, which leads to greater aggregate volatility and to greater comovement across different producers. We use the model to tell a story about the decline in aggregate volatility in the U.S. after the mid 1980s, the so-called Great Moderation. We show that our mechanism can explain a sizeable fraction of the observed decline in volatility and it also captures the reduction in sectoral comovement experienced in the U.S. during the same period.

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1 Introduction

Over the course of the postwar period, the U.S. has experienced a marked decline in aggregate volatility, the so-called Great Moderation. Real output volatility has been consistently lower in the period following the mid 1980s, after peaking in the high inflation period of the 1970s and early 1980s. A growing body of research, starting with Kim and Nelson (1999), McConell and Pérez-Quirós (2000), Blanchard and Simon (2001), and Stock and Watson (2002), documents this phenomenon and proposes a number of possible explanations. A second stylized fact, pointed out by Comin and Philippon (2005), is that the degree of comovement among different sectors in the economy has also declined over the same period.[1]

In this paper, we explore formally the idea that structural changes which increase access to liquidity in a broad sense, can lead both to lower aggregate volatility and to less comovement. To this end, we build a model which focuses on the role of liquidity constraints and precautionary behavior in the transmission of aggregate shocks. We identify a novel mechanism by which the relaxation of liquidity constraints can affect aggregate outcomes. If agents are less likely to be liquidity constrained, they are less concerned about their short-term earnings prospects when making their spending decisions and about their short-term consumption opportunities when making their production decisions. This breaks the link between individual trading decisions and aggregate cyclical conditions, dampening the effect of aggregate shocks on aggregate output and reducing the degree of comovement between different sectors.

We use this model to tell a story about the Great Moderation which emphasizes two factors: the steady expansion of credit markets and the effect of lower inflation on the real value of money balances. A number of researchers have suggested that financial innovation and improved access to credit, for both households and businesses, are important structural changes which may help explain the decline in volatility (Campbell and Hercowitz, 2005, Cecchetti, Flores-Lagunes and Krause, 2005, and Dynan, Elmendorf and Sichel, 2006). Many have also pointed to the reduction in inflation that the U.S. has experienced in the same period. We argue that both factors may have concurred to relax liquidity constraints in the period following the mid 1980s, contributing to the

[1] In Section 4.1, we provide some summary evidence on both stylized facts.

observed decline in volatility and comovement.

We consider a decentralized model of production and exchange in the tradition of search models of money, where credit frictions arise from the limited ability to verify the agents' identity. There is a large number of households, each with one consumer and one producer. Consumers and producers from different households meet and trade in spatially separated markets, or islands. In each island, the gains from trade are determined by a local productivity shock. An exogenous aggregate shock determines the distribution of local shocks across islands. A good aggregate shock reduces the proportion of low productivity islands and increases that of high productivity islands, that is, it leads to a first-order stochastic shift in the distribution of local productivities. Due to limited credit access, households accumulate precautionary money balances to bridge the gap between current spending and current income. Money is supplied by the government and grows at a constant rate, which, in equilibrium, is equal to the rate of inflation. Higher inflation reduces the equilibrium real value of the money stock in the hands of the consumers.

In the model, we distinguish different regimes along two dimensions: credit access and inflation. In regimes with less credit access and higher inflation agents are more likely to face binding liquidity constraints. In such regimes, we show that there is a coordination element in spending and production decisions: agents are less willing to trade when they expect others to trade less. This leads both to greater comovement between different sectors of the economy and to greater aggregate volatility.

We first obtain analytical results in two polar cases which we call "unconstrained" and "fully constrained" regimes. An unconstrained regime arises when either households have full access to credit or inflation is sufficiently low. In this case, households are never liquidity constrained in equilibrium. Our first result is that in an unconstrained regime the quantity traded in each island is independent of what happens in other islands. The result follows from the fact that households are essentially fully insured against idiosyncratic shocks. This makes their expected marginal value of money constant, allowing the consumer and the producer from the same household to make their trading decisions independently. At the opposite end of the spectrum, a fully constrained regime arises when households have no credit access and inflation is sufficiently high that they expect to be liquidity constrained for all realizations of the idiosyncratic shocks. In this case,

the decisions of the consumer and the producer are tightly linked. The consumer needs to forecast the producer's earnings and the producer needs to forecast the consumer's spending in order to evaluate the household's marginal value of money.

Next, we look at the aggregate implications of these linkages. In all regimes, a good aggregate shock has a positive compositional effect: as more islands have high productivity, aggregate output increases. However, in an unconstrained regime there is no feedback from this aggregate increase in output to the level of trading in an island with a given local shock. In a fully constrained regime, instead, the linkage between the trading decisions in different islands generates an additional effect on trading and output. A good aggregate shock raises the probability of high earnings for the producer, inducing the consumer to increase spending. At the same time, the producer expects his partner to spend more, increasing his incentive to produce. These two effects imply that a higher level of aggregate activity induces higher levels of activity in each island, conditional on the local shock, leading to an amplified response of aggregate activity. Numerical results show that our mechanism is also at work in intermediate regimes, where the liquidity constraint is occasionally binding, and that in this intermediate region increased credit access and lower inflation lead to lower volatility and lower comovement.

We then perform a simple quantitative exercise in order to evaluate how much our mechanism can explain of the reduction in both output volatility and sectoral comovement experienced in the U.S. during the Great Moderation. We look at U.S. data for the period 1947-2007 and split the sample in two, pre-1984 and post-1984. These two sub-samples are characterized by two different regimes in terms of average inflation and credit access, with lower inflation and increased credit access in the second sub-sample. We first choose the model's parameters to match some relevant features of the U.S. economy in the first sub-sample. We then compute the effect of the regime change on aggregate volatility and sectoral comovement and compare it to the changes observed in the data. Our model can account for about 1/4 of the observed reduction in volatility and for about 3/4 of the observed change in comovement.

This paper is related to the literature on search models of decentralized trading, going back to Diamond (1982, 1984) and Kiyotaki and Wright (1989). In particular, Diamond (1982) puts forth the idea that "the difficulty of coordination of trade" may contribute to cyclical volatility. The contribution of our paper is to show that the pres-

ence of this coordination problem depends crucially on credit market conditions and on the monetary regime. This allows us to identify a novel connection between financial development, liquidity supply, and aggregate dynamics. Our model allows for divisible money and uses the Lagos and Wright (2005) approach to simplify the analysis. In Lagos and Wright (2005) agents alternate trading in a decentralized market to trading in a centralized competitive market. The combination of quasi-linear preferences and periodic access to a centralized market ensures that the distribution of money holdings across agents is degenerate when they enter the decentralized market. Here we use these same two ingredients, with a modified periodic structure. In our model, agents have access to a centralized market every three periods. The extra period of decentralized trading is necessary to make the precautionary motive matter for trading decisions in the decentralized market of the previous period. This is at the core of our amplification mechanism. A three-period structure is also used by Berentsen, Camera and Waller (2005) to study the short-run neutrality of money. They show that, away from the Friedman rule, random monetary injections can be non-neutral, since they have a differential effect on agents with heterogeneous money holdings. Although different in its objectives, their analysis also relies on the lack of consumption insurance. Our work is also related to a large number of papers who have explored the implications of different monetary regimes for risk sharing, in environments with idiosyncratic risk (e.g. Aiyagari and Williamson, 2000, Reed and Waller, 2006) and is related to Rocheteau and Wright (2005) for the use of competitive pricing *à la* Lucas and Prescott (1974) in a money search model.

More broadly, the paper is related to the literature exploring the relation between financial frictions and aggregate volatility, including Bernanke and Gertler (1989), Bencivenga and Smith (1991), Acemoglu and Zilibotti (1997), and Kiyotaki and Moore (1997). In particular, Kiyotaki and Moore (2001) also emphasize the effect of a limited supply of liquid assets (money) on aggregate dynamics. Their paper studies a different channel by which limited liquidity can affect the transmission of aggregate shocks, focusing on the effects on investment and capital accumulation. Campbell and Hercowitz (2005) also explore the question whether financial deepening can account for the reduction in aggregate volatility in the U.S. after the mid 1980s. In particular, they consider a model with collateral constraints and study the effects of relaxing collateral constraints

on the cyclical behavior of labor supply. Their argument is that when the collateral constraint is tighter, an increase in durable goods spending in a boom raises the need for funds for borrowing households in the short-run, which translates into a positive shift of the labor supply schedule. Although developed in a very different environment, their mechanism bears some relation to the one at work on the producer's side of our model.

Our paper is also related to the vast literature on the effect of liquidity constraints on consumption decisions. In particular, our argument relies on the idea that when liquidity constraints are binding less often, consumption becomes less sensitive to short-term income expectations. Some evidence consistent with this idea is in Jappelli and Pagano (1989), who show that the "excess sensitivity" of consumption to current income is less pronounced in countries with more developed credit markets, and in Bacchetta and Gerlach (1997), who show that excess sensitivity has declined in the United States as a consequence of financial deregulation. Dynan, Elmendorf and Sichel (2006) argue informally that a relaxation of credit constraints may lead to a reduction in the marginal propensity to spend out of income in a standard keynesian model, and use this insight as the basis for their empirical analysis of the Great Moderation.

Finally, numerous papers have argued that the conduct of monetary policy is in part responsible for the higher volatility of the 1970s. However, these papers have focused on the destabilizing effects of poorly designed interest rate rules in environments with nominal rigidities (e.g., Clarida, Gali, and Gertler, 2000). In this paper, we completely abstract from these effects, by considering a flexible price environment and only focusing on the effects of monetary policy on the supply of liquid monetary balances.

The rest of the paper is organized as follows. In Section 2, we introduce our model, characterize the competitive equilibrium, and derive our main analytical results. Section 3 presents an extended version of the model which is used for the numerical analysis. Section 4 presents our calibration exercise. Section 5 discusses an extension with imperfect information and public signals. Section 6 concludes. The appendix contains all the proofs not in the text.

2 The Model

2.1 Setup

The economy is populated by a unit mass of infinitely-lived households, composed of two agents, a consumer and a producer. Time is discrete and each period agents produce and consume a single, perishable consumption good. The economy has a simple periodic structure: each time period t is divided into three subperiods, $s = 1, 2, 3$. We will call them "periods" whenever there is no risk of confusion.

In periods 1 and 2, the consumer and the producer from each household travel to spatially separated markets, or islands, where they interact with consumers and producers from other households. There is a continuum of islands and each island receives the same mass of consumers and producers in both periods 1 and 2. The assignment of agents to islands is random and satisfies a law of large numbers, so that each island receives a representative sample of consumers and producers. In each island there is a competitive goods market, as in Lucas and Prescott (1974). The consumer and the producer from the same household do not communicate while traveling in periods 1 and 2, but get together at the end of each period. In period 3, all consumers and producers trade in a single centralized market.[2]

In period 1 of time t, a producer located in island k, has access to the linear technology

$$y_{t,1} = \theta_t^k n_t,$$

where $y_{t,1}$ is output, n_t is labor effort, and θ_t^k is the local level of productivity, which is random and can take two values: 0 and $\overline{\theta} > 0$. At time t, a fraction ζ_t of islands is randomly assigned the high productivity level $\overline{\theta}$, while a fraction $1 - \zeta_t$ is unproductive. The aggregate shock ζ_t is independently drawn and publicly revealed at the beginning of period 1, and takes two values, ζ_H and ζ_L, in $(0, 1)$, with probabilities α and $1 - \alpha$. The island-specific productivity θ_t^k is only observed by the consumers and producers located in island k. In Section 3, we will generalize the distributions of local and aggregate shocks.

In periods 2 and 3, each producer has a fixed endowment of consumption goods,

[2]The use of a household made of two agents, buyer and seller, who cannot communicate during a trading period, goes back to Lucas (1990) and Fuerst (1992).

$y_{t,2} = e_2$ and $y_{t,3} = e_3$. We assume that the value of e_3 is large, so as to ensure that equilibrium consumption in period 3 is strictly positive for all households.

The household's preferences are represented by the utility function

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t (u(c_{t,1}) - v(n_t) + U(c_{t,2}) + c_{t,3})\right],$$

where $c_{t,s}$ is consumption in subperiod (t,s) and $\beta \in (0,1)$ is the discount factor. Both u and U are increasing, strictly concave, with continuous first and second derivatives on $(0,\infty)$. The function u is bounded below, with $u(0) = 0$, has finite right-derivative at 0 and satisfies the Inada condition $\lim_{c\to\infty} u'(c) = 0$. The function U satisfies the Inada conditions $\lim_{c\to 0} U'(c) = \infty$ and $\lim_{c\to\infty} U'(c) = 0$. The function v represents the disutility of effort, is increasing and convex, has continuous first and second derivatives on $[0,\bar{n})$ and satisfies $v'(0) = 0$ and $\lim_{n\to\bar{n}} v'(n) = \infty$.

In this section, we analyze two versions of the model. First, we consider a monetary version of the model where the consumers' identity cannot be verified in the islands they visit in periods 1 and 2, so credit contracts are not feasible. There is an exogenous supply of perfectly divisible notes issued by the government, money, and the only feasible trades in periods 1 and 2 are trades of money for goods. Each household is endowed with a stock of money M_0 at date 0. At the end of each subperiod 3, the government injects $(\gamma - 1) M_t$ units of money by a lump-sum transfer to each household (a lump-sum tax if $\gamma < 1$). Therefore, the stock of money M_t grows at the constant gross rate γ. In this paper we make no attempt to explain the government's choice of the monetary regime, but simply explore the effect of different regimes on equilibrium behavior. Second, we consider a version of the model with perfect credit markets, where the consumers' identity can be costlessly verified in each period and credit contracts are perfectly enforced.

Let us comment briefly on two of the assumptions made. First, the fact that in subperiod 3 consumers and producers trade in a centralized market and have linear utility is essential for tractability, as it allows us to derive an equilibrium with a degenerate distribution of money balances at the beginning of $(t,1)$, as in Lagos and Wright (2005).[3] Second, we assume that the household is split in a consumer and a producer who make separate decisions in period 1, without observing the shock of the partner. This assump-

[3]See Shi (1997) for a different approach to obtain a degenerate distribution of money holdings.

tion allows us to capture in a simple way the effects of short-term income uncertainty on consumption and production decisions.

2.2 First-best

The first-best allocation provides a useful benchmark for the rest of the analysis. Consider a social planner with perfect information who can choose the consumption and labor effort of the households. Given that there is no capital, there is no real intertemporal link between time t and $t+1$. Therefore, we can look at a three-period planner's problem.

Each household is characterized by a pair $(\theta, \tilde{\theta})$, where the first element represents the shock in the producer's island and the second represents the one in the consumer's island. An allocation is given by consumption functions $\{c_s(\theta, \tilde{\theta}, \zeta)\}_{s\in\{1,2,3\}}$ and an effort function $n(\theta, \tilde{\theta}, \zeta)$. The planner chooses an allocation that maximizes the ex ante utility of the representative household

$$\mathbb{E}[u(c_1(\theta, \tilde{\theta}, \zeta)) - v(n(\theta, \tilde{\theta}, \zeta)) + U(c_2(\theta, \tilde{\theta}, \zeta)) + c_3(\theta, \tilde{\theta}, \zeta)],$$

subject to the economy's resource constraints. Given an aggregate shock ζ, in period 1 there is one resource constraint for each island θ[4]

$$\mathbb{E}[c_1(\tilde{\theta}, \theta, \zeta)|\theta, \zeta] \leq \mathbb{E}[y_1(\theta, \tilde{\theta}, \zeta)|\theta, \zeta],$$

where $y_1(\theta, \tilde{\theta}, \zeta) = \theta n(\theta, \tilde{\theta}, \zeta)$. In period $s = 2, 3$, the resource constraint is

$$\mathbb{E}[c_s(\theta, \tilde{\theta}, \zeta)|\zeta] \leq e_s.$$

The resource constraints for periods 1 and 2 reflect the assumption that each island receives a representative sample of consumers and producers.

The following proposition characterizes the optimal allocation.

Proposition 1 *The optimal output level in period* 1 *is* $y_1(\theta, \tilde{\theta}, \zeta) = y_1^*(\theta)$ *for all* $\tilde{\theta}$ *and* ζ*, where* $y_1^*(0) = 0$ *and* $y_1^*(\overline{\theta})$ *satisfies*

$$\overline{\theta} u'(y_1^*(\overline{\theta})) = v'(y_1^*(\overline{\theta})/\overline{\theta}). \tag{1}$$

Optimal consumption in period 2 *is* $c_2(\theta, \tilde{\theta}, \zeta) = e_2$ *for all* $\theta, \tilde{\theta}$ *and* ζ*.*

[4]From now on, "island θ" is short for "an island with productivity shock θ."

Due to the separability of the utility function, the optimal output level in a given productive island is not affected by the fraction ζ of productive islands in the economy. Moreover, at the optimum, c_2 is constant across households, that is, households are fully insured against the shocks θ and $\tilde{\theta}$. Finally, given linearity, the consumption levels in period 3 are not pinned down, as consumers are ex ante indifferent among any profile $c_3(\theta, \tilde{\theta}, \zeta)$ such that $\mathbb{E}[c_3(\theta, \tilde{\theta}, \zeta)] = e_3$.

2.3 Monetary Equilibrium

Let us first concentrate on the monetary economy, where no credit contracts are feasible in periods 1 and 2. We focus on stationary equilibria where all nominal variables grow at rate γ. We can then normalize nominal prices and money holdings in period t, dividing them by the aggregate money stock M_t, and study stationary equilibria where quantities, normalized prices and normalized money holdings only depend on the current shocks. Therefore, from now on, we drop the time index t.

We begin by characterizing optimal individual behavior. Let $p_1(\theta, \zeta)$ denote the normalized price of goods in period 1 in island θ, and $p_2(\zeta)$ and $p_3(\zeta)$ denote the normalized prices in periods 2 and 3. Consider a household with an initial stock of money m (normalized), at the beginning of period 1 after the realization of ζ. The consumer travels to island $\tilde{\theta}$ and consumes $c_1(\tilde{\theta}, \zeta)$. Since money holdings are non-negative, the budget constraint and the liquidity constraint in period 1 are

$$\begin{aligned} m_1(\tilde{\theta}, \zeta) + p_1(\tilde{\theta}, \zeta) c_1(\tilde{\theta}, \zeta) &\leq m, \\ m_1(\tilde{\theta}, \zeta) &\geq 0, \end{aligned}$$

where $m_1(\tilde{\theta}, \zeta)$ denotes the consumer's normalized money holdings at the end of period 1. In the meantime, the producer, located in island θ, produces and sells $y_1(\theta, \zeta) = \theta n(\theta, \zeta)$. At the end of period 1, the consumer and the producer get together and pool their money holdings. Therefore, in period 2 the budget and liquidity constraints are

$$\begin{aligned} m_2(\theta, \tilde{\theta}, \zeta) + p_2(\zeta) c_2(\theta, \tilde{\theta}, \zeta) &\leq m_1(\tilde{\theta}, \zeta) + p_1(\theta, \zeta) \theta n(\theta, \zeta), \\ m_2(\theta, \tilde{\theta}, \zeta) &\geq 0, \end{aligned}$$

where consumption, $c_2(\theta, \tilde{\theta}, \zeta)$, and end-of-period normalized money holdings, $m_2(\theta, \tilde{\theta}, \zeta)$, are now contingent on both shocks θ and $\tilde{\theta}$. Finally, in period 3, the consumer and the

producer are located in the same island and the revenues $p_3(\zeta) e_3$ are immediately available. Moreover, the household receives a lump-sum transfer equal to $\gamma - 1$, in normalized terms. The constraints in period 3 are then

$$\begin{aligned} m_3(\theta, \tilde{\theta}, \zeta) + p_3 c_3(\theta, \tilde{\theta}, \zeta) &\leq m_2(\theta, \tilde{\theta}, \zeta) + p_2(\zeta) e_2 + p_3(\zeta) e_3 + \gamma - 1, \\ m_3(\theta, \tilde{\theta}, \zeta) &\geq 0. \end{aligned}$$

A household with normalized money balances $m_3(\theta, \tilde{\theta}, \zeta)$ at the end of subperiod 3, will have normalized balances $\gamma^{-1} m_3(\theta, \tilde{\theta}, \zeta)$ at the beginning of the following subperiod 1, as the real rate of return on money between $(t, 3)$ and $(t+1, 1)$ is equal to the inverse of the inflation rate, γ^{-1}. Let $V(m)$ denote the expected utility of a household with normalized money balances m at the beginning of period 1, before the realization of the aggregate shock ζ. The household's problem is then characterized by the Bellman equation

$$V(m) = \max_{\{c_s\}, n, \{m_s\}} \mathbb{E}[u(c_1(\tilde{\theta}, \zeta)) - v(n(\theta, \zeta)) + U(c_2(\theta, \tilde{\theta}, \zeta)) + c_3(\theta, \tilde{\theta}, \zeta) + \beta V(\gamma^{-1} m_3(\theta, \tilde{\theta}, \zeta))], \tag{2}$$

subject to the budget and liquidity constraints specified above. The solution to this problem gives us the optimal household's choices as functions of the shocks and of the initial money balances m, which we denote by $c_1(\theta, \zeta, m)$, $c_2(\theta, \tilde{\theta}, \zeta, m)$, etc.

We are now in a position to define a stationary competitive equilibrium.

Definition 1 *A stationary competitive equilibrium of the economy with no credit is given by prices* $\{p_1(\theta, \zeta), p_2(\zeta), p_3(\zeta)\}$, *a distribution of money holdings with c.d.f.* $H(\cdot)$ *and support* $\mathcal{M}$, *and an allocation* $\{n(\theta, \zeta, m)$, $c_1(\theta, \zeta, m)$, $c_2(\theta, \tilde{\theta}, \zeta, m)$, $c_3(\theta, \tilde{\theta}, \zeta, m)$, $m_1(\theta, \zeta, m)$, $m_2(\theta, \tilde{\theta}, \zeta, m)$, $m_3(\theta, \tilde{\theta}, \zeta, m)\}$ *such that:*

(i) the allocation solves problem (2) for each $m \in \mathcal{M}$;

(ii) goods markets clear

$$\int_{\mathcal{M}} \mathbb{E}[c_1(\theta, \zeta, m) | \theta, \zeta] dH(m) = \theta \int_{\mathcal{M}} \mathbb{E}[n(\theta, \zeta, m) | \theta, \zeta] dH(m) \text{ for all } \theta, \zeta,$$

$$\int_{\mathcal{M}} \mathbb{E}[c_s(\theta, \tilde{\theta}, \zeta, m) | \zeta] dH(m) = e_s \text{ for } s = 2, 3 \text{ and all } \zeta;$$

(iii) the distribution $H(\cdot)$ satisfies $\int_{\mathcal{M}} m dH(m) = 1$ and

$$H(m) = \Pr[(\theta, \tilde{\theta}, \tilde{m}) : \gamma^{-1} m_3(\theta, \tilde{\theta}, \zeta, \tilde{m}) \leq m | \zeta] \text{ for all } m \text{ and } \zeta.$$

The last condition in (iii) ensures that the distribution $H(\cdot)$ is stationary. As we will see below, we can focus on equilibria where the distribution of money balances is degenerate at $m = 1$. Therefore, from now on, we drop the argument m from the equilibrium allocations.

In order to characterize the equilibrium, it is useful to derive the household's first order conditions. From problem (2) we obtain three Euler equations, with respective complementary slackness conditions,

$$u'(c_1(\tilde{\theta}, \zeta)) \geq \frac{p_1(\tilde{\theta}, \zeta)}{p_2(\zeta)} \mathbb{E}[U'(c_2(\theta, \tilde{\theta}, \zeta)) | \tilde{\theta}, \zeta] \quad (m_1(\tilde{\theta}, \zeta) \geq 0) \quad \text{for all } \tilde{\theta}, \zeta, \tag{3}$$

$$U'(c_2(\theta, \tilde{\theta}, \zeta)) \geq \frac{p_2(\zeta)}{p_3(\zeta)} \quad (m_2(\theta, \tilde{\theta}, \zeta) \geq 0) \quad \text{for all } \theta, \tilde{\theta}, \zeta, \tag{4}$$

$$1 \geq p_3(\zeta) \beta \gamma^{-1} V'(\gamma^{-1} m_3(\theta, \tilde{\theta}, \zeta)) \quad (m_3(\theta, \tilde{\theta}, \zeta) \geq 0) \quad \text{for all } \theta, \tilde{\theta}, \zeta, \tag{5}$$

the optimality condition for labor supply

$$v'(n(\theta, \zeta)) = \theta \frac{p_1(\theta, \zeta)}{p_2(\zeta)} \mathbb{E}[U'(c_2(\theta, \tilde{\theta}, \zeta)) | \theta, \zeta] \quad \text{for all } \theta, \zeta, \tag{6}$$

and the envelope condition

$$V'(m) = \mathbb{E}\left[\frac{u'(c_1(\tilde{\theta}, \zeta))}{p_1(\tilde{\theta}, \zeta)}\right]. \tag{7}$$

Our assumptions allow us to simplify the equilibrium characterization as follows. Since $\theta = 0$ with probability $\zeta > 0$, the Inada condition for U implies that $m_1(\tilde{\theta}, \zeta)$ and $m_3(\theta, \tilde{\theta}, \zeta)$ are strictly positive for all θ, $\tilde{\theta}$, and ζ. To insure against the risk of entering period 2 with zero money balances, households always carry positive balances into periods 1 and 2. This implies that (3) and (5) always hold as equalities.

Condition (5), holding with equality, shows why we obtain equilibria with a degenerate distribution of money balances, as in Lagos and Wright (2005). Given that the normalized supply of money is always equal to 1, a stationary equilibrium with a degenerate distribution $H(\cdot)$ must satisfy

$$\gamma^{-1} m_3(\theta, \tilde{\theta}, \zeta) = 1 \text{ for all } \theta, \tilde{\theta}, \zeta.$$

In equilibrium, all agents adjust their consumption in period 3, so as to reach the same level of m_3, irrespective of their current shocks. The assumptions that utility is linear in period 3 and that e_3 is large enough imply that the marginal utility of consumption in period 3 is constant, ensuring that this behavior is optimal.[5] Moreover, equation (5), as an equality, implies that in all stationary equilibria $p_3(\zeta)$ is independent of the aggregate shock ζ and equal to $\gamma/(\beta V'(1))$. From now on, we just denote it as p_3.

This leaves us with condition (4). In general, this condition can be either binding or slack for different pairs $(\theta, \tilde{\theta})$, depending on the parameters of the model. However, we are able to give a full characterization of the equilibrium by looking at specific monetary regimes, namely, by making assumptions about γ. First, we look at equilibria where the liquidity constraint $m_2(\theta, \tilde{\theta}, \zeta) \geq 0$ is never binding. We will show that this case arises if and only if $\gamma = \beta$, that is, in a monetary regime that follows the Friedman rule. Second, we look at equilibria where the constraint $m_2(\theta, \tilde{\theta}, \zeta) \geq 0$ is binding for all pairs $(\theta, \tilde{\theta})$ and for all ζ. We will show that this case arises if and only if the rate of money growth is sufficiently high, that is, when $\gamma \geq \hat{\gamma}$ for a given cutoff $\hat{\gamma} > \beta$.

These two polar cases provide analytically tractable benchmarks which illustrate the mechanism at the core of our model. The quantitative analysis in Section 4 considers the case of economies with $\gamma \in (\beta, \hat{\gamma})$, where the liquidity constraint in period 2 is binding for a subset of agents.

2.3.1 Unconstrained equilibrium

We begin by considering "unconstrained equilibria," that is, equilibria where the liquidity constraint in period 2 is never binding. In this case, condition (4) always holds as an equality. Combining conditions (3)-(5), all as equalities, and (7) gives

$$\frac{u'(c_1(\tilde{\theta}, \zeta))}{p_1(\tilde{\theta}, \zeta)} = \beta\gamma^{-1}\mathbb{E}\left[\frac{u'(c_1(\tilde{\theta}', \zeta'))}{p_1(\tilde{\theta}', \zeta')}\right], \tag{8}$$

where $\tilde{\theta}'$ and ζ' represent variables in the next time period. Taking expectations with respect to $\tilde{\theta}$ and ζ on both sides shows that a necessary condition for an unconstrained

[5] When $\gamma > \beta$, all stationary equilibria are characterized by a degenerate distribution of money holdings. One can show that, in this case, the value function V is strictly concave. This, together with (5) implies that m_3 is constant across households.

equilibrium is $\gamma = \beta$. The following proposition shows that this condition is also sufficient. Moreover, under this monetary regime, the equilibrium achieves an efficient allocation.[6]

Proposition 2 *In the economy with no credit, an unconstrained equilibrium exists if and only if $\gamma = \beta$ and achieves a first-best allocation.*

For our purposes, it is especially interesting to understand how the level of activity is determined in a productive island in period 1. Let $\overline{p}_1(\zeta)$ and $\overline{y}_1(\zeta)$ denote $p_1(\overline{\theta}, \zeta)$ and $y_1(\overline{\theta}, \zeta)$. Substituting (4) into (3) (both as equalities), we can rewrite the consumer's optimality condition in period 1 as

$$u'(\overline{y}_1(\zeta)) = \frac{\overline{p}_1(\zeta)}{p_3}. \tag{9}$$

Similarly, the producer's optimality condition (6) can be rewritten as

$$v'(\overline{y}_1(\zeta)/\overline{\theta}) = \overline{\theta}\frac{\overline{p}_1(\zeta)}{p_3}. \tag{10}$$

These two equations describe, respectively, the demand and the supply of consumption goods in island $\overline{\theta}$, as a function of the price $\overline{p}_1(\zeta)$. Jointly, they determine the equilibrium values of $\overline{p}_1(\zeta)$ and $\overline{y}_1(\zeta)$ for each ζ. These equations highlight that, in an unconstrained equilibrium, consumers and producers do not need to forecast the income/spending of their partners when making their optimal choices, given that their marginal value of money is constant and equal to $1/p_3$. This implies that trading decisions in a given island are independent of trading decisions in all other islands. We will see that this is no longer true when we move to a constrained equilibrium. Conditions (9) and (10) can be easily manipulated to obtain the planner's first order condition (1), showing that in an unconstrained equilibrium $\overline{y}_1(\zeta)$ is independent of ζ and equal to its first-best level.

2.3.2 Fully constrained equilibrium

We now turn to the case where the liquidity constraint is always binding in period 2, that is, $m_2(\theta, \tilde{\theta}, \zeta) = 0$ for all $\theta, \tilde{\theta}$ and ζ. We refer to it as a "fully constrained equilibrium." We will show that such an equilibrium arises when inflation γ is sufficiently high.

[6] See Rocheteau and Wright (2005) for a general discussion of the efficiency of the Friedman rule in a wide class of search models of money.

Again, our main objective is to characterize how output is determined in period 1. First, however, we need to derive the equilibrium value of $p_2(\zeta)$. At the beginning of each period the entire money supply is in the hands of the consumers. Since in a fully constrained equilibrium consumers spend all their money in period 2, market clearing gives us a simple "quantity theory" equation

$$p_2(\zeta)e_2 = m = 1, \tag{11}$$

which pins down $p_2(\zeta)$. To simplify notation, we choose units in period 2 such that $e_2 = 1$, so as to have $p_2(\zeta) = 1$.

Consider now a consumer and a producer in a productive island in period 1. Given that the consumer will be liquidity constrained in period 2, his consumption in that period will be fully determined by his money balances. In period 1, the consumer is spending $\overline{p}_1(\zeta)\overline{y}_1(\zeta)$ and expects his partner's income to be $\overline{p}_1(\zeta)\overline{y}_1(\zeta)$ with probability ζ, and zero otherwise. Therefore, he expects total money balances at the beginning of period 2 to be 1 in the first case and $1 - \overline{p}_1(\zeta)\overline{y}_1(\zeta)$ in the second. Using $p_2(\zeta) = 1$, we can then rewrite the Euler equation (3) as

$$u'(\overline{y}_1(\zeta)) = \overline{p}_1(\zeta)\left[\zeta U'(1) + (1-\zeta)\,U'\left(1 - \overline{p}_1(\zeta)\overline{y}_1(\zeta)\right)\right]. \tag{12}$$

A symmetric argument on the producer's side shows that the optimality condition (6) can be written as

$$v'\left(\overline{y}_1(\zeta)/\overline{\theta}\right) = \overline{\theta}\overline{p}_1(\zeta)\left[\zeta U'(1) + (1-\zeta)\,U'\left(1 + \overline{p}_1(\zeta)\overline{y}_1(\zeta)\right)\right]. \tag{13}$$

These two equations correspond to (9) and (10) in the unconstrained case and jointly determine $\overline{p}_1(\zeta)$ and $\overline{y}_1(\zeta)$ for each ζ. The crucial difference with the unconstrained case is that now ζ, the fraction of productive islands in the economy, enters the optimal decisions of consumers and producers in a given productive island, since it affects their expected income and consumption in the following period. We will see in a moment how this affects aggregate volatility and comovement.

Notice that (12) and (13) implicitly define a "demand curve" and a "supply curve," $\overline{y}^D(\overline{p}_1, \zeta)$ and $\overline{y}^S(\overline{p}_1, \zeta)$.[7] It is easy to show that, for any ζ, there exists a price where

[7] These are not standard partial-equilibrium demand and supply functions, as they represent the relation between the price $\overline{p}_1$ and the demand/supply of goods in a symmetric equilibrium where prices and quantities are identical in all productive islands.

the two curves intersect. For comparative statics, it is useful to impose an additional restriction, ensuring that the supply curve is positively sloped at the equilibrium. We then make the assumption

$$-(1-\zeta_H)cU''(c)/U'(c) \leq 1 \text{ for all } c, \tag{A1}$$

which ensures that the income effect on labor supply is not too strong and that the supply curve is positively sloped everywhere.

Lemma 1 *The function $\overline{y}^D(\overline{p}_1, \zeta)$ is decreasing in $\overline{p}_1$. Under assumption A1, the function $\overline{y}^S(\overline{p}_1, \zeta)$ is increasing in $\overline{p}_1$ and, for given ζ, there is a unique pair $(\overline{p}_1(\zeta), \overline{y}_1(\zeta))$ which solves (12)-(13).*

To complete the equilibrium characterization, it remains to find p_3 and check that consumers are indeed constrained in period 2, that is, that (4) holds. In the next proposition, we show that this condition is satisfied as long as γ is above some cutoff $\hat{\gamma}$.

Proposition 3 *There is a $\hat{\gamma} > \beta$ such that, in the economy with no credit, a fully constrained equilibrium exists if and only if $\gamma \geq \hat{\gamma}$.*

It is useful to clarify the role of the inflation rate γ in determining whether we are in a constrained or unconstrained equilibrium. Notice that in an unconstrained equilibrium the household's money balances at the beginning of period 1 must be sufficient to purchase both $\overline{p}_1(\zeta)\overline{y}_1(\zeta)$ and $p_2(\zeta)e_2$, in case the consumer is assigned to a productive island and the producer to an unproductive one. Therefore in an unconstrained equilibrium the following inequality holds for all ζ

$$\frac{1}{p_2(\zeta)} \geq e_2 + \frac{\overline{p}_1(\zeta)}{p_2(\zeta)}\overline{y}_1(\zeta).$$

On the other hand, (11) shows that $1/p_2(\zeta)$ is constant and equal to e_2 in a fully constrained equilibrium. That is, the real value of money balances in terms of period 2 consumption is uniformly lower in a fully constrained equilibrium. This lower real value of money balances is sustained by the fact that inflation is high and the real rate of return on money is low. This reduces the agents' willingness to hold money, reducing the equilibrium real value of money balances. Through this channel high inflation reduces the households' ability to self-insure.

2.4 Perfect credit markets

The economy with perfect credit markets is formally equivalent to the monetary economy under the Friedman rule and achieves a first-best allocation. To prove this claim it is sufficient to consider the case where households trade real non-state-contingent bonds in periods 1 and 2, which pay off in period 3. Let $b_1(\tilde{\theta},\zeta)$ and $b_2(\theta,\tilde{\theta},\zeta)$ denote the bond holdings in the hands of consumers at the end of periods 1 and 2 and $p_1^C(\theta,\zeta)$ and $p_2^C(\zeta)$ denote goods' prices in terms of period 3 consumption.[8] The household's budget constraints are then

$$\begin{aligned} b_1(\tilde{\theta},\zeta)+p_1^C(\theta,\zeta)c_1(\tilde{\theta},\zeta) &\leq 0, \\ b_2(\theta,\tilde{\theta},\zeta)+p_2^C(\zeta)\,c_2(\theta,\tilde{\theta},\zeta) &\leq b_1(\tilde{\theta},\zeta)+p_1^C(\theta,\zeta)\theta n(\theta,\zeta), \\ c_3(\theta,\tilde{\theta},\zeta) &\leq b_2(\theta,\tilde{\theta},\zeta)+p_2^C(\zeta)\,e_2+e_3. \end{aligned}$$

The analysis of the household's problem can then be developed as in the monetary economy, where $b_1(\tilde{\theta},\zeta)$ and $b_2(\theta,\tilde{\theta},\zeta)$ take the place of $m_1(\tilde{\theta},\zeta)-m$ and $m_2(\theta,\tilde{\theta},\zeta)-m$. The crucial difference is that here there are no non-negativity constraint on bond holdings, which implies that the Euler equation (4) will always hold as an equality. This observation is behind the following result.[9]

Proposition 4 *The economy with perfect credit markets has a stationary equilibrium, which achieves the same consumption allocation as the economy with no credit under the Friedman rule.*

Allowing for intertemporal trade between periods $(t,3)$ and $(t+1,1)$ or, more generally, for any set of state-contingent securities, would not change this result.

2.5 Coordination, amplification and comovement

We now turn to the effects of the aggregate shock ζ on the equilibrium allocation in the various regimes considered. Aggregate output in period 1 is given by

$$Y_1(\zeta)=\zeta\overline{y}_1(\zeta). \tag{14}$$

[8] Using period 3 consumption as numeraire, bond prices are always equal to 1.

[9] The proof follows closely that of Proposition 2, and is omitted.

Consider the proportional effect of a small change in ζ on aggregate output,

$$\frac{d\log Y_1(\zeta)}{d\zeta} = \frac{1}{\zeta} + \frac{d\ln \overline{y}_1(\zeta)}{d\zeta}. \tag{15}$$

When ζ increases, there is a larger fraction of productive islands, so aggregate output mechanically increases in proportion to ζ. This "composition effect" corresponds to the first term in (15). The open question is whether a change in ζ also affects the endogenous level of activity in a productive island. This effect is captured by the second term in (15) and will be called "coordination effect."

In an unconstrained equilibrium, we know that $\overline{y}_1(\zeta)$ is independent of ζ. Therefore, if money growth follows the Friedman rule or if there are perfect credit markets the coordination effect is absent. What happens in a fully constrained equilibrium, that is, when credit contracts are not available and inflation is high enough? Consider the demand and supply curves in a productive island, $\overline{y}^D(\overline{p}_1, \zeta)$ and $\overline{y}^S(\overline{p}_1, \zeta)$, derived above. Applying the implicit function theorem to (12) and (13) yields

$$\frac{\partial \overline{y}^D(\overline{p}_1(\zeta), \zeta)}{\partial \zeta} = p_1 \frac{U'(1) - U'(1 - \overline{p}_1(\zeta)\overline{y}_1(\zeta))}{u''(\overline{y}_1(\zeta)) + (\overline{p}_1(\zeta))^2 (1-\zeta) U''(1 - \overline{p}_1(\zeta)\overline{y}_1(\zeta))} > 0,$$

and

$$\frac{\partial \overline{y}^S(\overline{p}_1(\zeta), \zeta)}{\partial \zeta} = p_1 \frac{U'(1) - U'(1 + \overline{p}_1(\zeta)\overline{y}_1(\zeta))}{v''(\overline{y}_1(\zeta)/\overline{\theta})/\overline{\theta} - \overline{\theta}(\overline{p}_1(\zeta))^2 (1-\zeta) U''(1 + \overline{p}_1(\zeta)\overline{y}_1(\zeta))} > 0.$$

Both inequalities follow from risk aversion in period 2, that is, from the concavity of U. On the demand side, the intuition is the following. In period 1, a consumer in a productive island is concerned about receiving a bad income shock. Given that he is liquidity constrained, this shock will directly lower his consumption from 1 to $1 - \overline{p}_1(\zeta)\overline{y}_1(\zeta)$. An increase in ζ lowers the probability of a bad shock, decreasing the expected marginal value of money and increasing the consumer's willingness to spend, for any given price. On the supply side, as ζ increases, a producer in a productive island expects his partner to spend $\overline{p}_1(\zeta)\overline{y}_1(\zeta)$ with higher probability. This generates a negative income effect which induces him to produce more, for any given price. These two effects shift both demand and supply to the right and, under assumption A1, lead to an increase in equilibrium output.[10] This proves the following result.

[10] If is useful to mention what would happen in an environment where the producer and consumer

Proposition 5 *(Coordination) Under assumption A1, in a fully constrained equilibrium, the output in the productive islands,* $\overline{y}_1(\zeta)$*, is increasing in* ζ*.*

This is the central result of our paper and shows that when liquidity constraints are binding there is a positive coordination effect, as consumers and producers try to keep their spending and income decisions aligned. Consumers spend more when they expect their partners to earn more, and producers work more when they expect their partners to spend more. This has two main consequences. First, the impact of an aggregate shock on the aggregate level of activity is magnified, leading to increased volatility. Second, there is a stronger degree of comovement across islands. Let us analyze these two implications formally.

Since $\overline{y}_1(\zeta)$ is independent of ζ in an unconstrained equilibrium and increasing in ζ in a fully constrained one, equation (15) implies immediately that $\partial \log Y_1(\zeta)/\partial\zeta$ is larger in a fully constrained equilibrium than in an unconstrained equilibrium. This leads to the following result.

Proposition 6 *(Amplification) Under assumption A1,* $Var[\log Y_1(\zeta)]$ *is larger in a fully constrained equilibrium than in an unconstrained equilibrium.*

To measure comovement we look at the coefficient of correlation between local output in any given island and aggregate output. In an unconstrained equilibrium, there is some degree of correlation between the two, simply because ζ is an aggregate shock which increases aggregate output and increases the probability of the high productivity shock in any given island. However, in a fully constrained equilibrium the correlation tends to be stronger. Now, even conditionally on the island receiving the high productivity shock, an increase in ζ tends to raise both local and aggregate output, due to the coordination effect. This leads to the following result.

Proposition 7 *(Comovement) Under assumption A1,* $Corr[y_1(\theta,\zeta), Y_1(\zeta)]$ *is larger in a fully constrained equilibrium than in an unconstrained equilibrium.*

from the same household can communicate (but not exchange money) in period 1. In that case, in a productive island there will be two types of consumers and producers, distinguished by the local shock of their partners. Consumers (producers) paired with a low productivity partner, will have lower demand (supply). So also in that case an increase in ζ would lead to a reduction in activity in the productive island. However, that case is less tractable, due to the four types of agents involved, and it fails to capture the effect of uncertainty on the agents' decisions.

An alternative measure of comovement is the correlation between the level of activity in any given pair of islands, that is, $Corr[y_1(\theta,\zeta), y_1(\tilde{\theta},\zeta)]$. In a setup with i.i.d. idiosyncratic shocks the two measures are interchangeable as there is a simple monotone relation between them.[11]

3 The Extended Model

We now enrich the model by generalizing the distributions of local and aggregate shocks and by allowing a fraction of households to have access to credit. We will use this version of the model to extend the analytical results of the previous section and to set the stage for the quantitative analysis in Section 4.

3.1 Setup

The setup is as in Section 2 except for two differences. First, we generalize the distribution of the shocks. The aggregate shock ζ_t is i.i.d. with cumulative distribution function $G(\cdot)$ continuous on the support $[\underline{\zeta}, \overline{\zeta}]$. Conditional on ζ_t, the local productivity, θ_t^k, is randomly drawn from the cumulative distribution function $F(\cdot|\zeta_t)$ and support $\Theta = [0, \overline{\theta}]$. We assume that $F(\cdot|\zeta)$ has an atom at 0, i.e., $F(0|\zeta) > 0$, and is continuous on $(0, \overline{\theta}]$. Moreover, $F(\theta|\zeta)$ is continuous and non-increasing in ζ, for each θ. This implies that a distribution with a higher ζ first-order stochastically dominates a distribution with lower ζ. As before, a law of large numbers applies, so $F(\cdot|\zeta)$ also represents the distribution of productivity shocks across islands for a given ζ.

Second, we assume that a fraction $\phi \in [0,1]$ of the households have access to credit. These households all travel to a given subset of islands, so they behave as described in section 2.4 and always produce and consume the first-best level of output. A fraction $1-\phi$ of the households, instead, are anonymous and travel to a different subset of islands, so they behave as in the monetary equilibrium described in Section 2.3. The parameter ϕ can be interpreted as the degree of financial development of the economy.

We focus on stationary equilibria, defined along the lines of Definition 1. The superscripts C and M refer, respectively, to the households who have access to credit and to the anonymous households who need money to trade goods. In equilibrium credit

[11] In particular, it is possible to prove that $Corr[y(\theta,\zeta), y(\tilde{\theta},\zeta)] = (Corr[y(\theta,\zeta), Y(\zeta)])^2$.

households never hold money and we focus on equilibria where the distribution of money balances of anonymous households is degenerate at $m = 1/(1-\phi)$.

In a stationary equilibrium, anonymous households behave as in Section 2.3 and their behavior is characterized by the optimality conditions (3)-(6). The assumption that $F(\cdot|\zeta)$ has an atom at 0, together with the Inada condition for U, ensures that (3) and (5) always hold as equalities, as in the binary case. In general, (4) can hold with equality or not depending on the shocks θ and $\tilde{\theta}$ and on the monetary regime.

The optimal behavior of credit households is described by the same equations (3)-(6), with the difference that (3) and (4) always hold as equalities, while (5) is always slack, consistently with credit households being at a corner solution $m_3^C(\theta, \tilde{\theta}, \zeta) = 0$. Notice that in this model, credit access makes money completely unnecessary. We make this modelling choice to focus on the simple idea that both increased credit access and a better monetary environment contribute to relax borrowing constraints. However, it would be clearly interesting to investigate environments with a richer interaction between credit and money.

In the rest of this section, we focus on the polar cases of unconstrained and fully constrained equilibria, we generalize the main analytical result of the previous section, the coordination result in Proposition 5, and we discuss its implications for amplification and comovement. In the appendix, we characterize the equilibrium for general monetary and credit regimes, that is, for any $\gamma \in (0, \infty)$ and $\phi \in [0, 1]$, which will be used in the numerical analysis.

3.2 Unconstrained and fully constrained equilibria

The equilibrium characterization is a natural generalization of that in Section 2. First, we look at unconstrained equilibria, which arise when either monetary policy follows the Friedman rule or when all households have access to credit, that is, when either $\gamma = \beta$ or $\phi = 1$. Second, we look at fully constrained equilibria where anonymous households are always liquidity constrained in period 2 and there are no credit households, that is, when $\gamma \geq \hat{\gamma}$ and $\phi = 0$.

Proposition 8 *In the extended model, an unconstrained equilibrium exists if and only if either $\phi = 1$ or $\phi < 1$ and $\gamma = \beta$ and it always achieves a first-best allocation. Output in*

period 1 *is* $y_1^C(\theta,\zeta) = y_1^M(\theta,\zeta) = y_1^*(\theta)$ *where* $y_1^*(\theta)$ *satisfies* $\theta u'(y_1^*(\theta)) = v'(y_1^*(\theta)/\theta)$ *for all* $\theta > 0$*, is increasing in* θ*, and is independent of* ζ*.*

In an unconstrained equilibrium the real allocation is the same for all households, regardless of their access to credit. In particular, they consume and produce the first-best level of output in all islands. As in the binary model, the separability of the utility function implies that equilibrium output in island θ depends only on the local productivity and is not affected by the distribution of productivities in other islands.

The characterization of a fully constrained equilibrium is also analogous to the binary model, but the analysis is richer because of the continuous distribution of θ. In a fully constrained equilibrium, all households are liquidity constrained in period 2. Hence, in that period, their consumption depends on both the consumer's and the producer's shock in period 1. Following similar steps to Section 2.3.2, we can show that $p_2^M(\zeta)$ is constant and equal to 1 (under the normalization $e_2 = 1$). The consumer's budget constraints in periods 1 and 2 then yield

$$c_2^M(\theta,\tilde{\theta},\zeta) = 1 - p_1^M(\tilde{\theta},\zeta)y_1^M(\tilde{\theta},\zeta) + p_1^M(\theta,\zeta)\,y_1^M(\theta,\zeta)\,. \tag{16}$$

Consider a consumer and a producer in island θ. The consumer's Euler equation and the producer's optimality condition can be rewritten as

$$u'(y_1^M(\theta,\zeta)) = p_1^M(\theta,\zeta)\int_0^{\overline{\theta}} U'(c_2^M(\tilde{\theta},\theta,\zeta))dF(\tilde{\theta}|\zeta), \tag{17}$$

$$v'\left(y_1^M(\theta,\zeta)/\theta\right) = \theta p_1^M(\theta,\zeta)\int_0^{\overline{\theta}} U'(c_2^M(\theta,\tilde{\theta},\zeta))dF(\tilde{\theta}|\zeta), \tag{18}$$

where $c_2^M(\theta,\tilde{\theta},\zeta)$ is given by (16). These two equations are analogous to (12) and (13) and represent, for a given ζ, the demand and supply in island θ, taking as given prices and quantities in other islands. They define two functional equations in $p_1^M(\cdot,\zeta)$ and $y_1^M(\cdot,\zeta)$. In the Appendix, we show that this pair of functional equations has a unique solution. To do so, we define nominal income $x(\theta,\zeta) \equiv p_1^M(\theta,\zeta)y_1^M(\theta,\zeta)$ and we solve a fixed point problem in terms of the function $x(\cdot,\zeta)$.

To solve our fixed point problem, we use a contraction mapping argument, making the following assumption:

$$-\frac{cu''(c)}{u'(c)} \in [\underline{\rho},1) \text{ for all } c, \tag{A2}$$

for some $\underline{\rho} > 0$. The upper bound on $-cu''(c)/u'(c)$ is needed to ensure that the demand elasticity in a given island θ is high enough. This guarantees that in islands where productivity is higher prices do not fall too much, so that nominal income is increasing in θ. That is, producers in more productive islands receive higher earnings. This property is both economically appealing and useful on technical grounds, as it allows us to prove the monotonicity of the mapping used in our fixed point argument.[12] The lower bound $\underline{\rho}$ is used to prove the discounting property of the same mapping.[13]

As in the binary model, we can then characterize a fully constrained equilibrium and find a cutoff $\hat{\gamma}$ such that such an equilibrium exists whenever $\gamma \geq \hat{\gamma}$.

Proposition 9 *In the extended model, under assumption A2, there is a cutoff $\hat{\gamma} > \beta$ such that a fully constrained equilibrium exists if and only if $\gamma \geq \hat{\gamma}$. In a fully constrained equilibrium, both output $y_1^M(\theta, \zeta)$ and nominal income $p_1^M(\theta, \zeta) y_1^M(\theta, \zeta)$ are monotone increasing in θ.*

We could prove existence under weaker conditions, using a different fixed point argument. However, the contraction mapping approach helps us derive the coordination result in Proposition 10 below.

3.3 Aggregate implications

We now turn to the analysis of the impact of the aggregate shock ζ on the equilibrium allocation. Summing over credit and money islands, aggregate output in period 1 is given by

$$Y_1(\zeta) \equiv \sum_{i=C,M} \omega^i \int_0^{\overline{\theta}} y_1^i(\theta, \zeta)\, dF(\theta|\zeta), \tag{19}$$

[12] It is useful to mention alternative specifications which can deliver the same result (nominal income increasing in θ) without imposing restrictions on risk aversion in period 1. One possibility is to introduce local shocks as preference shocks. For example, we could assume that the production function is the same in all islands while the utility function takes the form $\theta u(c_1)$ where θ is the local shock. In this case, it is straightforward to show that both $p_1^M(\theta, \zeta)$ and $y_1^M(\theta, \zeta)$ are increasing in θ, irrespective of the curvature of u. This immediately implies that nominal income is increasing in θ. Another possibility is to use more general preferences, which allow to distinguish risk aversion from the elasticity of intertemporal substitution. For example, using a version of Epstein and Zin (1989) preferences, it is possible to show that this result only depends on the elasticity of substitution between c_1 and c_2 and not on risk aversion.

[13] This assumption is minimally restrictive, as $\underline{\rho}$ is only required to be non-zero.

where the weight ω^i is equal to the average share of nominal output produced in islands of type i. The proportional response of output to a small change in ζ, can be decomposed as in the binary case,

$$\frac{d\ln Y_1}{d\zeta} = \frac{1}{Y_1}\sum_{i=C,M}\omega^i \int_0^{\overline{\theta}} y_1^i(\theta,\zeta)\frac{\partial f(\theta|\zeta)}{\partial\zeta}d\theta + \frac{1}{Y_1}\sum_{i=C,M}\omega^i\int_0^{\overline{\theta}}\frac{\partial y_1^i(\theta,\zeta)}{\partial\zeta}dF(\theta|\zeta). \quad (20)$$

The first term is the mechanical composition effect of having a larger fraction of more productive islands. This effect is positive both in an unconstrained and in a fully constrained equilibrium. This follows from the fact that an increase in ζ leads to a first-order stochastic shift in the distribution of θ and that $y_1^i(\theta,\zeta)$ is increasing in θ in both regimes, as shown by Propositions 8 and 9.

The second term in (20) is our coordination effect. As in the binary case, this effect is zero in an unconstrained equilibrium, since, by Proposition 8, output in any island θ is independent of the economy-wide distribution of productivity. Turning to a fully constrained equilibrium, we can generalize Proposition 5 and show that $y_1^M(\theta,\zeta)$ is increasing in ζ, for any realization of the local productivity shock θ. For this result, we make a stronger assumption than the one used in the binary model, that is, we assume that U has a coefficient of relative risk aversion smaller than one

$$-\frac{cU''(c)}{U'(c)} \leq 1 \text{ for all } c. \quad \text{(A1')}$$

This condition is sufficient to prove that the labor supply in each island is positively sloped. Assumption A1' is usually stronger than needed. In the simulations of Section 4, we check numerically that labor supply is positively sloped also for parametrizations with a coefficient of relative risk aversion greater than 1, so our coordination result extends to those cases. In fact, as we will see, the coordination effect can be stronger when agents are more risk averse.[14]

Proposition 10 *(Coordination) Consider the extended model. Under assumption A1' and A2, in a fully constrained equilibrium, for each $\theta > 0$, the output $y_1^M(\theta,\zeta)$ is increasing in ζ.*

[14] A way of relaxing this assumption on risk aversion could be to consider more general preferences that distinguish between risk aversion and the elasticity of substitution between consumption and leisure (see footnote 12).

To understand the mechanism behind this result, it is useful to consider the following partial equilibrium exercise. Let us focus on island θ and take as given $p_1^M(\tilde{\theta}, \zeta)$ and $y_1^M(\tilde{\theta}, \zeta)$ for all $\tilde{\theta} \neq \theta$. Consider the demand and supply equations for island θ, (17) and (18). Proposition 9 shows that $p_1^M(\theta, \zeta) y_1^M(\theta, \zeta)$ is an increasing function of θ. It follows that $U'(c_2^M(\tilde{\theta}, \theta; \zeta))$ is decreasing in $\tilde{\theta}$, while $U'(c_2^M(\theta, \tilde{\theta}; \zeta))$ is increasing in $\tilde{\theta}$. Hence, when ζ increases the integral on the right-hand side of (17) decreases, while the integral on the right-hand side of (18) increases.[15] The intuition is similar to the one for the binary model. On the demand side, when a liquidity constrained consumer expects higher income from his partner, his marginal value of money decreases. Then, he reduces his reserves and increases consumption for any given price $p_1^M(\theta, \zeta)$. On the supply side, when a producer expects higher spending by his partner, he faces a negative income effect and hence produces more for any given $p_1^M(\theta, \zeta)$. The first effect shifts the demand curve to the right, the second shifts the supply curve to the right. The combination of the two implies that equilibrium output in island θ increases.

On top of this partial equilibrium mechanism, there is a general equilibrium feed-back due to the endogenous response of prices and quantities in islands $\tilde{\theta} \neq \theta$. This magnifies the initial effect. As the nominal value of output in all other islands increases, there is a further increase in the marginal value of money for the consumers and a further decrease for the producers, leading to an additional increase in output.

Summing up, the coordination effect identified in Proposition 10 is driven by the agents' expectations regarding nominal income in other islands. This effect tends to magnify the output response to aggregate shocks in a fully constrained economy and to generate more comovement across islands, as in the binary case.

Going back to equation (20), we have established that the coordination effect is zero in the unconstrained case and positive in the fully constrained one. However, this is not sufficient to establish that output volatility is greater in the constrained economy, since we have not compared the relative magnitude of the compositional effects, which are positive in both cases. In the binary model, the comparison was unambiguous, given that this effect was identical in the two regimes. However, except in specific cases, it is difficult to compare the relative size of this effect in the two regimes and to

[15] This follows immediately from the fact that an increase in ζ leads to a shift of the distribution of θ in the sense of first order stochastic dominance.

obtain the analog of Proposition 6. In fact, it is possible to construct examples where this effect is larger in the unconstrained economy and where it is strong enough to dominate our coordination effect. Similar problems limit our ability to generalize the analytical result on comovement in Proposition 7. However, the numerical analysis in the next section shows that, under realistic parametrizations, the coordination mechanism identified in Proposition 10 tends to generate higher volatility and higher comovement in more liquidity constrained regimes.

4 The Great Moderation

In this section, we use a simple calibrated version of the model to evaluate to what extent our mechanism can explain the changes in output volatility and sectoral comovement associated to the Great Moderation. We first choose the model's parameters to match some basic features of the U.S. economy pre-1984. Then we only change the values of ϕ and γ, to reflect observed changes in credit access and average inflation after 1984, and look at their effect on aggregate volatility and comovement.

4.1 Stylized facts

Let us first summarize the stylized facts which motivate our exercise. The first is the decline in aggregate volatility in the U.S. after the mid 1980s, a well-established fact in the literature.[16] The first panel in Figure 1 shows the path of the volatility of real GDP from 1951 to 2003. To measure volatility, we use HP filtered series for real GDP (in logs) from the National Income and Product Account (NIPA) and compute rolling standard deviations over a 9 year window.[17] The second fact is the decline in sectoral comovement over the same time period, a fact documented by Comin and Philippon (2005), using the 35-sector KLEM dataset of Jorgenson and Stiroh (2000). The second panel in Figure 1 shows the path of sectoral comovement from 1951 to 2003. To measure comovement, we look at real value added industry-level data from NIPA and compute rolling coefficients

[16] See references in the introduction.

[17] We begin in 1951 because the first available observation for chain-weighted GDP is 1947. We look at yearly data and use a smoothing parameter of 10 in the HP filter, which is appropriate for capturing volatility at business-cycle frequency, as suggested by Baxter and King (1999) (see also Ravn and Uhlig, 2002).

of correlation with aggregate GDP over a 9 year window. Our measure of comovement is the average coefficient of correlation across sectors.[18]

Figure 1 shows a marked drop in both volatility and comovement occurring in the mid 1980s. Splitting the sample in two, before and after 1984, we find that the standard deviation of real GDP decreased from 1.86% in the first subsample to 0.92% in the second one, while sectoral comovement went from 0.56 to 0.39. These are the numbers we will try to explain. The figure also shows an increase in volatility in correspondence to the high inflation period of the 1970s and first half of the 1980s. Comovement displays a similar, if less dramatic, increase in the same period.[19] We will also explore the ability of our model to capture these patterns.

To explain these facts, we focus on two underlying changes in the U.S. economy. First, the steady expansion of credit markets, documented extensively in Dynan, Elmendorf and Sichel (2006). Here, given our focus on households' liquidity management, we look at the evolution of revolving consumer credit. Panel (a) of Figure 2 shows that the ratio of revolving credit over GDP has risen steadily over time from 1968 to 2007.[20] Second, the high inflation of the 1970s and early 1980s. In panel (b) we plot the behavior of average CPI inflation and of the nominal interest rate over a 9 year window, from 1951 to 2003. The path of both variables clearly shows that in the intermediate part of the sample both realized inflation and inflation expectations (reflected in the nominal interest rate) were higher.

4.2 Calibration

For our numerical analysis, we choose isoelastic functional forms for the instantaneous utility functions, $u(c) = c^{1-\rho_1}/(1-\rho_1)$ and $U(c) = c^{1-\rho_2}/(1-\rho_2)$, and for the disutility of labor effort, $v(n) = n^{1+\eta}/(1+\eta)$. There are two aggregate states, ζ_L and ζ_H, with probabilities α and $1-\alpha$. Conditional on the aggregate state, the island specific shock θ are log-normally distributed with mean μ_H in state ζ_H, μ_L in state ζ_L, and variance

[18] We use 22-industry level data, which go back to 1947, excluding the last two sectors (federal and state government). As for aggregate GDP, we use real value added, chain-weighted series, in logs, HP filtered with a smoothing parameter of 10.

[19] Similar patterns are found in the literature. See, e.g., Figure 1 in Blanchard and Simon (2000) and Figure 9 in Comin and Philippon (2005).

[20] The data are from the Federal Reserve Board, Table G.19.

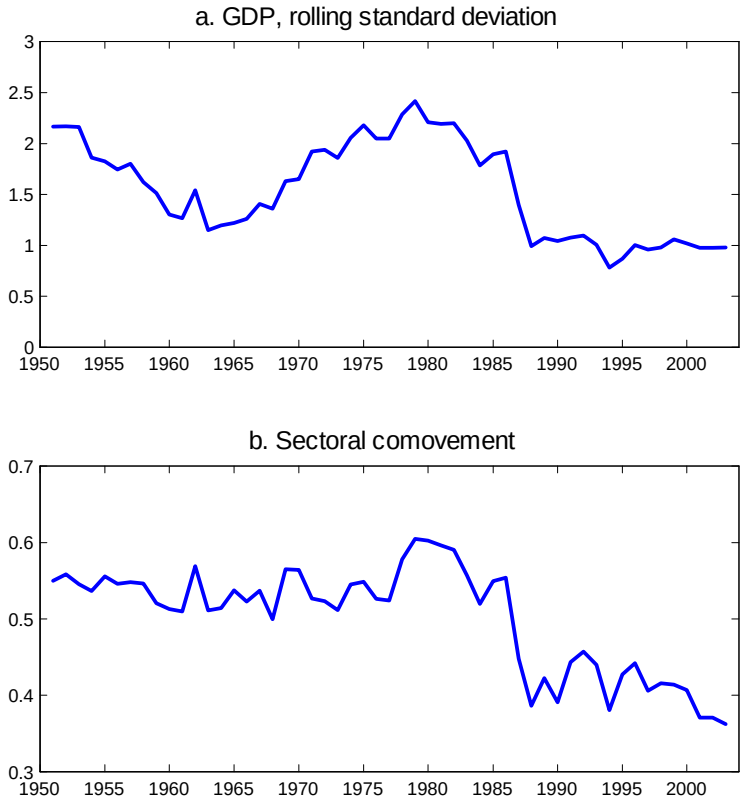

Figure 1: Volatility and comovement.
Panel (a): Rolling standard deviation of real GDP over 9 year window from $t-4$ to $t+4$ (in percentage points). Chain-weighted yearly data, HP filtered with smoothing parameter 10. Panel (b): Rolling average coefficient of correlation between 20 sectors and aggregate GDP over 9 year window from $t-4$ to $t+4$. Chain-weighted yearly data, HP filtered with smoothing parameter 10. Source: BEA.

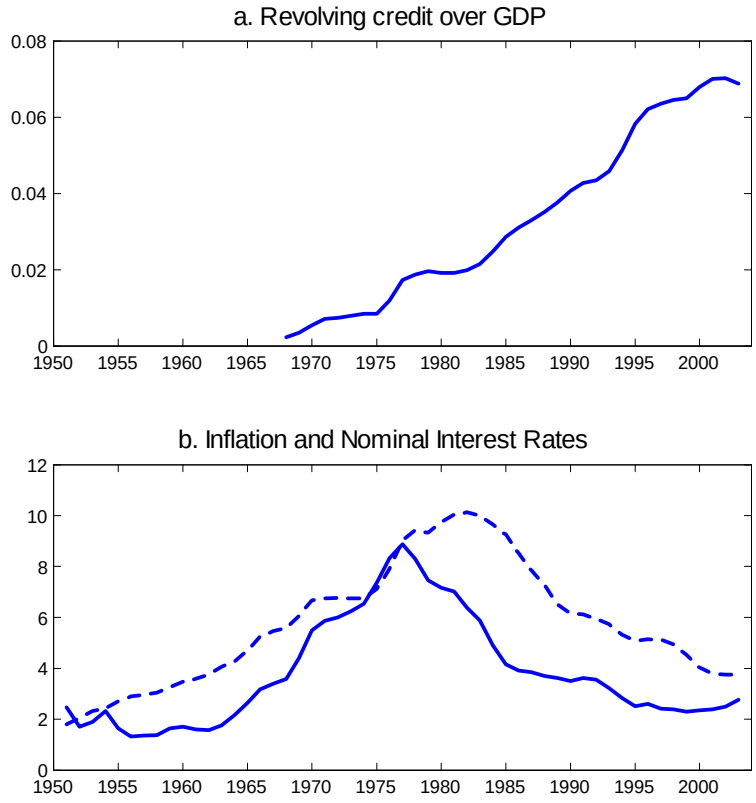

Figure 2: Revolving credit, inflation and nominal interest rates.
Panel (a): Total consumer revolving credit owned and securitized, over GDP. Source: Federal Reserve Board. Panel (b): CPI inflation and 3-month nominal interest rate, rolling averages from $t-4$ to $t+4$ (percentage points). Sources: BLS and Federal Reserve Board.

σ^2.[21]

We interpret each sequence of three subperiods as a year and set the discount factor β equal to 0.97. In our baseline parametrization, we set the coefficient of relative risk aversion ρ_2 equal to 1 and the inverse Frisch elasticity η to 0. Later, we will present various robustness checks with different values for these parameters. Given the assumption of isoelastic preferences, we can normalize $e_2 = 1$. The dispersion of local shocks, σ^2, is chosen to match observed income volatility at the household level. In particular, in the model we identify transitory idiosyncratic income volatility with the average standard deviation of nominal income in the first two periods conditional on ζ. We choose $\sigma^2 = 0.1971$ to obtain a standard deviation of income equal to 0.17, consistently with estimates in Hubbard, Skinner and Zeldes (1994).

The remaining parameters are calibrated to match relevant features of the U.S. economy for the period pre-1984. The values of ρ_1, e_3 and $\mathbb{E}[\mu]$ are chosen to match the shape of the empirical relation between money velocity and the nominal interest rate in the period 1947-1984. This approach follows Lucas (2000), Lagos and Wright (2005) and Craig and Rocheteau (2007). In the data, inverse money velocity is obtained as the ratio of M1 to nominal GDP and the nominal interest rate is measured by the short-term commercial paper rate. In the model, inverse money velocity is measured as $1/Y_t^N$, where Y_t^N is aggregate nominal output in periods 1 to 3 at time t. To derive the nominal interest rate in the model, we assume that the real interest rate is $1/\beta$, so that the nominal interest rate is equal to $1 + i = \gamma/\beta$. Notice that the Friedman rule corresponds, as it should, to $i = 0$.[22] The parameters we obtain are $\rho_1 = 0.2005$, $e_3 = 4.0101$ and $\mathbb{E}[\mu] = 0.5768$. The value of ϕ, the fraction of consumers with credit access, is chosen to match the average ratio of revolving credit to GDP in 1947-1984, which is equal to 0.0073.[23] This gives us a value of $\phi = 0.0465$. We then calibrate the volatility of aggregate shocks to match average volatility in 1947-1984. We interpret

[21] Even though this distribution does not have an atom at 0, our numerical results show that in equilibrium consumers never exhaust their money balances in period 1.

[22] For each yearly observation of i, we compare the empirical value of the inverse money velocity and the one generated by the model. We choose the model parameters to minimize the mean quadratic distance between the two. Given that the distribution of μ has a negligible effect on money velocity, for this estimation we compute money velocity setting μ at its mean value $\mathbb{E}[\mu]$.

[23] Data before 1968 are not available, so we make the conservative assumption that the ratio was at its 1968 value in all previous periods. In the model, the average stock of credit in the economy is equal to $\frac{2}{3}\int p_1^C(\theta)\, y_1^C(\theta)\, dF(\theta) + \frac{1}{3} p_2^C e_2$.

the bad aggregate state ζ_L as a recession and choose $\alpha = 0.8$ to match the frequency of recessions as defined by the National Bureau of Economic Research (NBER), in the subsample 1947-1984. The size of the shock, $\mu_H - \mu_L$, is chosen equal to 0.0157 to match the average volatility of aggregate output in the same subsample, 1.86%, under a constant nominal interest rate equal to the subsample average of 5.53%. Finally, to compare the model's predictions on comovement to evidence from sectoral data, we assume that islands are grouped in sectors made of N islands. We choose $N = 5,050$ to match our evidence on sectoral comovement in 1947-1984, that is, a coefficient of correlation equal to 0.56.

4.3 Results

Having calibrated the model to match volatility and comovement pre-1984, we look at the effects of improved credit access and lower inflation in the second subsample, 1985-2007. Splitting the sample in two, before and after 1984, the ratio of revolving consumer credit over GDP goes from 0.73% to 5.37%. To match this increase, in the model we increase ϕ from 0.0465 to 0.4376. The data on money velocity provide a useful consistency check for our estimated change in ϕ. Figure 3 shows the relation between inverse money velocity and the nominal interest rate in the data (scatter plot) and in the model (solid and dashed lines) for the two subsamples. The increase in ϕ in the second period implies that the model-generated curve shifts downwards because of the lower money demand from credit households. Changing ϕ from 0.0465 to 0.4376 generates a downward shift which is roughly consistent with the shift in the data.

In terms of monetary regimes, given the structure of the model and the way we have calibrated the "money demand" relation in Figure 3, we focus on changes in the nominal interest rate, which reflect underlying changes in expected inflation. The results are similar if we use realized inflation, given the similar path of the two variables (see panel (b) of Figure 2). Splitting the sample in two, before and after 1984, the average nominal interest rate goes from 5.53% to 5.19%. Overall, the high rates of the 1970s and 1980s have a small effect on the period's average in 1947-1984, so this period only displays slightly higher rates than 1985-2007. Therefore, in order to better gauge the effects of a high inflation regime, we also consider three separate regimes, with nominal

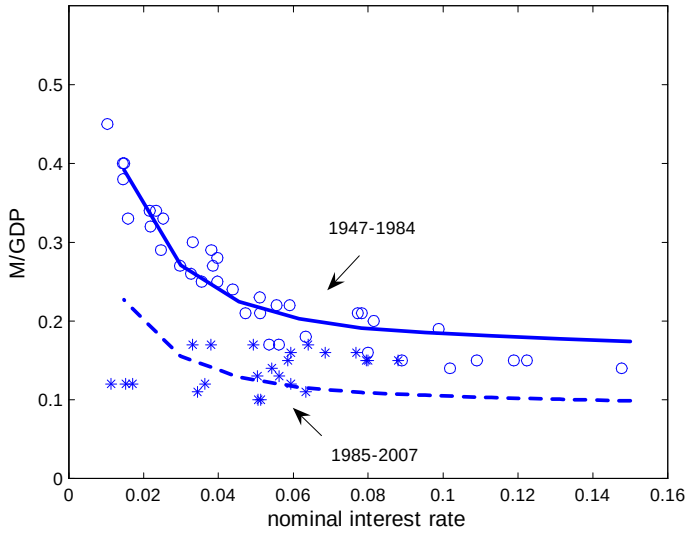

Figure 3: Money demand pre-1984 and post-1984.
The scatter plot represents the data: dots for 1947-1984, stars for 1985-2007. The lines represent the simulated relation between the nominal rate and the inverse velocity in the model: solid line for 1947-1984, dashed line for 1985-2007.

interest rates equal to 3.31%, 8.65%, and 5.19%, corresponding, respectively, to 1947-1969, 1970-1984, and 1985-2007.[24]

We then simulate the model under the different regimes and compute the volatility of aggregate output and our measure of sectoral comovement, equal to the average coefficient of correlation between sectoral output and aggregate output. Given that in subperiods 2 and 3 output is constant by assumption, we focus on volatility and comovement in subperiod 1. In our three-period exercise, we take the middle period as our reference point, and re-calibrate $\mu_H - \mu_L$ and N to match volatility and comovement in this period.

Table 1 shows the results of our benchmark calibration. On the left, we report the observed values of volatility and comovement in the U.S. in the different subsamples, on the right, the values generated by the model. In our two-period exercise, the model is able to explain approximately 1/4 of the observed decline in volatility and more than 3/4

[24]Given that the data on revolving consumer credit are available only from 1968, we only consider two credit regimes, before and after 1984.

of the decline in sectoral comovement. In our three-period exercise, the model is able to explain almost half of the increase in volatility between the first and the second period, due to the substantial increase in inflation, and about 1/4 of the subsequent decline, from the second to the third period. The model-generated changes in comovement are, again, quite close to the observed ones.

Table 1: Benchmark Calibration

	US Data		*Benchmark Calibration* $\rho_2 = 1, \eta = 0$	
	Volatility	Comovement	Volatility	Comovement
1947-1984	1.86	0.56	1.86	0.56
1985-2007	0.92	0.39	1.62	0.43
1947-1969	1.67	0.54	1.93	0.52
1970-1984	2.15	0.58	2.15	0.58
1985-2007	0.92	0.39	1.86	0.43

Note: Data from NIPA. The remaining model parameters are $\beta = 0.97$, $\rho_1 = 0.2005$, $e_2 = 1$, $e_3 = 4.0101$, $\mathbb{E}[\mu] = 0.5768$, $\alpha = 0.8$, $\sigma^2 = 0.1971$. In the two-period exercise we set $\mu_H - \mu_L = 0.0157$ and $N = 5,050$. In the three-period exercise $\mu_H - \mu_L = 0.0166$ and $N = 4,550$. The regimes for credit access are $\phi_{47-84} = 0.0465$ and $\phi_{85-07} = 0.4376$. The monetary regimes are $\gamma_{47-84} = \beta \cdot 1.0553$, $\gamma_{47-69} = \beta \cdot 1.0331$, $\gamma_{70-84} = \beta \cdot 1.0865\%$, and $\gamma_{85-07} = \beta \cdot 1.0519$.

We then consider some alternative parametrizations, both to check the robustness of our results and to investigate how they depend on different features of the model. Table 2 reports the values of volatility and comovement pre- and post-1984 generated by our model when we vary η or ρ_1. For each pair of values of η and ρ_1, we re-calibrate all the remaining parameters following the steps outlined above. Therefore, volatility and comovement in the reference periods are, by construction, equal to their empirical counterpart. The table shows that our results remain qualitatively unchanged and that the quantitative effects remain sizeable.

Increasing η tends to reduce the size of our effect, as it makes the labor supply in each island less elastic, thus reducing the response of labor effort to the demand shifts driven by our coordination motive. However, when we increase η, our calibration requires that we reduce ρ_1 in order to match the money demand relation plotted in Figure 3. This

makes the consumers' demand in each island more elastic, and it magnifies the effects of the supply shift due to our coordination motive on the producer's side. This combination of effects, implies that changing η *and* re-calibrating the model can, in general, have non-monotone effects on the size of our amplification effect. This is illustrated in Table 2, which show that the reduction in volatility in the post-84 period is bigger at $\eta = 0.25$ than at $\eta = 0.1$.[25]

Table 2: Changing the elasticity of labor supply η

	$\rho_2 = 1, \eta = 0.1$		$\rho_2 = 1, \eta = 0.25$	
	Volatility	Comovement	Volatility	Comovement
1947-1984	1.86	0.56	1.86	0.56
1985-2007	1.75	0.43	1.64	0.42
1947-1969	1.90	0.48	2.00	0.53
1970-1984	2.15	0.58	2.15	0.58
1985-2007	1.71	0.39	1.83	0.42

Note: The model's parameters are calibrated for each value of η, as described in the text. The values of $\mu_H - \mu_L$ and N are separately calibrated in the two-period and three-period exercises, so as to match volatility and comovement, respectively, in 1947-1984 and in 1970-1984.

Table 3: Changing risk aversion ρ_2

	$\rho_2 = 0.5, \eta = 0$		$\rho_2 = 2, \eta = 0$	
	Volatility	Comovement	Volatility	Comovement
1947-1984	1.86	0.56	1.86	0.56
1985-2007	1.52	0.42	1.72	0.42
1947-1969	1.97	0.53	1.98	0.48
1970-1984	2.15	0.58	2.15	0.58
1985-2007	1.69	0.41	1.90	0.42

Note: The model's parameters are calibrated for each value of ρ_2, as described in the text. The values of $\mu_H - \mu_L$ and N are separately calibrated in the two-period and three-period exercises, so as to match volatility and comovement, respectively, in 1947-1984 and in 1970-1984.

[25] Notice also that our calibration approach places effectively an upper bound on η. As we increase η, the calibrated value of ρ_1 is smaller. For values of η around 0.35 and higher, the calibrated value of ρ_1 hits a corner at 0, and the fit of the money demand relation gets worse for larger values of η. Furthermore, as usual in real business cycle models, larger values of η tend to worsen the performance of the model in terms of labor supply volatility.

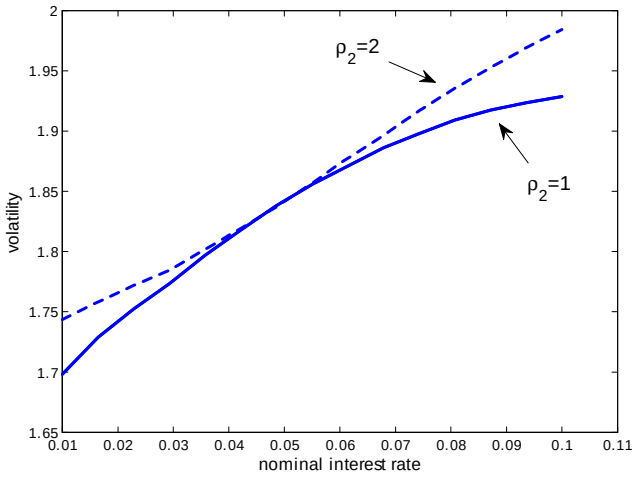

Figure 4: Risk aversion and volatility

Table 3 shows that the effects remain sizeable when we experiment with different values of ρ_2, the coefficient of relative risk aversion in period 2. The effects of changing ρ_2 are also subtle. Let us focus on the columns on the right, with $\rho_2 = 2$. When we do our two-periods exercise, the model with $\rho_2 = 2$, displays a smaller reduction in volatility and comovement from the first to the second period, relative to the baseline case $\rho_2 = 1$. However, in the three-period exercise, the model with $\rho_2 = 2$ displays a larger run up in volatility and comovement from the first to the second period. Therefore, depending on the specific regimes we are comparing, a larger risk aversion can lead to smaller or bigger effects. Figure 4 confirms this observation, by plotting volatility against different values of the nominal interest rate for $\rho_2 = 1$ (solid line) and $\rho_2 = 2$ (dashed line). Each line is derived using the respective values of the calibrated parameters and keeping ϕ at its pre-1984 value. On the one hand, the effect of going from relative low inflation towards the Friedman rule is smaller when $\rho_2 = 2$. On the other hand, the effect of going towards a high inflation regime is larger.

To understand this result, notice that when ρ_2 increases two effects are at work. First, given the value of money balances in period 2, $1/p_2(\zeta)$, an increase in ρ_2 tends to increase our coordination effect. If consumers are more risk averse, an increase in the probability of high income realizations, tend to increase their current demand more. A symmetric

mechanism is at work on the producers' side. So both demand and supply in a given island θ tend to shift more to the right under larger risk aversion. However, an increase in risk aversion also affects the endogenous value of money balances. In particular, as agents have a stronger precautionary motive, $1/p_2(\zeta)$ tends to be larger. This reduces the severity of the liquidity problem in period 2, possibly making the liquidity constraint not binding for some realizations of the local shocks, and it reduces the relevance of our coordination effect. Therefore, the net effect of changing ρ_2, is, in general ambiguous. Figure 4 shows that, under our parametrization, the second effect dominates near the Friedman rule, while the first one dominates for higher inflation rates.[26]

Table 4: Uniform distribution

	Volatility	Comovement
1947-1984	1.86	0.56
1985-2007	1.66	0.46
1947-1969	1.78	0.48
1970-1984	2.15	0.58
1985-2007	1.75	0.45

Note. The model parameters are $\beta = 0.97$, $\rho_1 = 0.1819$, $\rho_2 = 1$, $\eta = 0$, $e_2 = 1$, $e_3 = 3.4482$, $\alpha = 0.8$, $\overline{\theta} = 1.1098$. For the two-periods exercise $\Delta = 0.0331$ and $N = 1,895$, while for the three-periods one $\Delta = 0.0349$ and $N = 1,500$. The regimes for credit access are $\phi_{47-84} = 0.0428$ and $\phi_{85-07} = 0.4426$. The monetary regimes are the same as above.

Finally, we experiment with different assumptions on the distribution of θ. Namely, we assume that in the high aggregate state, ζ_H, the island specific shock θ has a discrete uniform distribution, with 10 equally spaced observations on the interval $[0, \overline{\theta}]$. In the low state, ζ_L, the probability of $\theta = 0$ increases by $\Delta > 0$, and the probability of all positive realizations of θ decrease proportionally. The two parameters $\overline{\theta}$ and Δ are calibrated, respectively, to match transitory idiosyncratic income volatility and aggregate volatility, while the remaining parameters are calibrated to match the money demand

[26] Notice that this discussion of the effects of changing ρ_2 is quite unrelated to our use of the assumption $\rho_2 \leq 1$ in the proof of Proposition 10. That assumption was only made as a sufficient condition for a positively sloped labor supply curve in each island. In our numerical examples, we checked that the labor supply is always positively sloped, even when we experiment with larger values of ρ_2. So the coordination result in Proposition 10 applies in all our examples.

relation, as in the baseline calibration. Table 3 reports the results of both the two-period and the three-period exercises described above. In the two-period exercise, we obtain results similar to the ones obtained with a log-normal distribution. In the three-period exercise, the model with the uniform distribution can actually explain a larger fraction of the observed changes in volatility and comovement.

5 News shocks

Consider now the general model of Section 3, with the only difference that the aggregate shock ζ is not observed by the households in period 1. Instead, they all observe a public signal $\xi \in [\underline{\xi}, \overline{\xi}]$, which is drawn at the beginning of each period, together with the aggregate shock ζ, from a continuous distribution with joint density function $g(\zeta, \xi)$.

Take an agent located in an island with productivity θ, his posterior density regarding ζ can be derived using Bayes' rule:

$$g(\zeta|\xi, \theta) = \frac{f(\theta|\zeta)\, g(\zeta, \xi)}{\int_{\underline{\zeta}}^{\overline{\zeta}} f(\theta|\tilde{\zeta}) g(\tilde{\zeta}, \xi) d\tilde{\zeta}}.$$

The distribution $g(\zeta|\xi, \theta)$ is then used to derive the agent's posterior beliefs regarding $\tilde{\theta}$ in the island where his partner is located

$$F(\tilde{\theta}|\xi, \theta) = \int_{\underline{\zeta}}^{\overline{\zeta}} F(\tilde{\theta}|\zeta) g(\zeta|\xi, \theta)\, d\zeta.$$

We will make the assumption that $F(\tilde{\theta}|\xi, \theta)$ is non-increasing in ξ, for any pair $(\theta, \tilde{\theta})$. This means, that conditional on θ, the signal ξ is "good news" for $\tilde{\theta}$, in the sense of Milgrom (1981). We also make the natural assumption that $F(\tilde{\theta}|\xi, \theta)$ is non-increasing in θ. In period 2, the actual shock ζ is publicly revealed.

In this environment, we study a stationary equilibrium, along the lines of the one described in Section 3, where credit households never hold money, and the distribution of money holdings for anonymous households at the beginning of period 1 is degenerate. Prices and allocations now depend on the local shocks and on the aggregate shocks ξ and ζ. In particular, prices and quantities in period 1 depend only on θ and ξ, given that ζ is not in the information set of the households in that period. Aggregate output

in period 1 becomes

$$Y_1(\zeta,\xi) \equiv \sum_{i=C,M} \omega^i \int_0^{\overline{\theta}} y_1^i(\theta,\xi)\, dF(\theta|\zeta). \tag{21}$$

We can now look separately at the output response to the productivity shock ζ and to the news shock ξ. In particular, next proposition shows that the output response to ζ is positive both in an unconstrained and in a fully constrained equilibrium, while the output response to the signal ξ is positive only in the fully constrained case.

Proposition 11 *Consider an economy with imperfect information regarding the aggregate shock. Under assumptions A1' and A2, in an unconstrained equilibrium* $\partial Y_1(\zeta,\xi)/\partial\zeta > 0$ *and* $\partial Y_1(\zeta,\xi)/\partial\xi = 0$. *In a fully constrained equilibrium* $\partial Y_1(\zeta,\xi)/\partial\zeta > 0$ *and* $\partial Y_1(\zeta,\xi)/\partial\xi > 0$.

This result is not surprising, in light of the analysis in the previous sections. Compare the expression for aggregate output under imperfect information (21) with the correspondent expression in the case of full information (19). By definition, the productivity shock ζ affects the distribution of local shocks $F(\cdot|\zeta)$ in both cases. However, the trading decisions of anonymous households in island θ are affected only by the agents' expectations about that distribution, which, in the case of imperfect information, are driven by the signal ξ. It follows that the effect of ζ is analogous to the mechanical composition effect in the model with full information on ζ, while the effect of ξ is analogous to the coordination effect. The advantage of an environment with imperfect information, is that these two effects can be disentangled. In an unconstrained economy, as we know from Proposition 8, output in island θ is independent of the economy-wide distribution of productivity and thus does not respond to ξ. The result that the output response to ξ is positive in a fully constrained economy is a natural extension of Proposition 10. In island θ, trading is higher the more optimistic agents are about trading in all other islands. The only difference is how expectations are formed. The perceived distribution of productivities for an agent in island θ depends now on the signal ξ, instead that on the actual ζ. A positive signal ξ makes both consumers and producers in island θ more optimistic about trading in other islands, even if the underlying ζ is unchanged. This highlights that expectations are at the core of our amplification result.

6 Concluding Remarks

In this paper, we have analyzed how different liquidity regimes affect the response of an economy to aggregate shocks. A liquidity regime depends both on the households' access to credit and on the value of their money holdings. In regimes where liquidity constraints are binding more often, we show that there is a coordination motive in the agents' trading decisions. This generates both an amplified response to aggregate shocks and a larger degree of comovement.

Our mechanism is driven by the combination of risk aversion, idiosyncratic uncertainty, and decentralized trade. All three ingredients are necessary for the mechanism to operate. Risk aversion and idiosyncratic risk give rise to an insurance problem. Decentralized trade implies that agents with no access to credit can only self-insure using their money holdings.[27] A nice feature of our setup is that simply by changing the credit and monetary regimes, we move from an environment in which idiosyncratic risk is perfectly insurable (unconstrained equilibrium) to an environment in which idiosyncratic risk is completely uninsurable (fully constrained equilibrium). In this sense, the mechanism identified in this paper speaks more broadly about the effect of uninsurable idiosyncratic risk on aggregate behavior.

In our model, money is the only liquid asset available to households. This is clearly a strong simplification. It was motivated by the fact that money balances constitute a major fraction of liquid savings for many households (especially for middle-low income households, who are plausibly more likely to be constrained), and by the fact that there is a well understood channel relating inflation to the real value of money balances. Nonetheless, it would be interesting to expand the model to allow for a richer menu of liquid assets. On the empirical side, the period following the 1980s has seen the development of many highly liquid alternatives to checking accounts (e.g. money market accounts), which arguably have contributed to better liquidity management at the household level. Including these assets into the picture may thus lead to larger quantitative effects than those documented in Section 4.

In this paper, we have focused on liquidity problems at the household level. However,

[27] Reed and Waller (2006) also point out the risk sharing implications of different monetary regimes in a model *à la* Lagos and Wright (2005).

the mechanism described could also be applied to firms' liquidity problems. In particular, one can think of extending our mechanism to an input-output framework, where firms buy intermediate goods from each other and where the spending decisions of liquidity constrained firms affect the rest of the economy through a channel similar to the one in our model.

For analytical tractability, we have developed our argument in a periodic framework *à la* Lagos and Wright (2005). This framework is clearly special in many respects, and, in particular, displays no endogenous source of persistence. Therefore, it would be interesting to investigate, numerically, the quantitative implications of our mechanism in a version of the model that allows for richer dynamics of individual asset positions.[28] A similar extension would also help to clarify the relation between our results and the literature on the aggregate implications of imperfect risk sharing, such as Krusell and Smith (1989).[29]

Finally, in terms of empirical applications, it would be interesting to investigate the behavior of volatility and comovements in countries other than the U.S., who have either experienced periods of fast financial innovation or periods of high inflation. For example, the cross-country evidence in Cecchetti, Flores-Lagunes and Krause (2005) shows a negative effect of financial deepening on aggregate volatility, which is consistent with our story.

Appendix

Proof of Proposition 1

For any θ and $\tilde{\theta}$, the first order conditions for $c_1(\theta, \overline{\theta}, \zeta)$ and $n(\overline{\theta}, \tilde{\theta}, \zeta)$ are

$$\begin{aligned} \zeta u'(c_1(\theta, \overline{\theta}, \zeta)) &= \Lambda(\overline{\theta}, \zeta), \\ \zeta v'(n(\overline{\theta}, \tilde{\theta}, \zeta)) &= \overline{\theta}\Lambda(\overline{\theta}, \zeta), \end{aligned}$$

where $\Lambda(\overline{\theta}, \zeta)$ is the Lagrange multiplier of the resource constraints in island $\overline{\theta}$. Substituting $c_1(\theta, \overline{\theta}, \zeta) = y_1^*(\overline{\theta})$, $n(\overline{\theta}, \theta, \zeta) = y_1^*(\overline{\theta})/\overline{\theta}$, and $\Lambda(\theta, \zeta) = \zeta u'(y_1^*(\overline{\theta}))$ confirms that

[28] See, for example, the computational approach in Molico (2006).

[29] In Krusell and Smith (1989) the entire capital stock of the economy is a liquid asset and the presence of uninsurable idiosyncratic risk has minor effects on aggregate dynamics. To explore our mechanism, it would be interesting to assume that capital is at least partially illiquid.

the proposed allocation in period 1 for productive islands is optimal. Similar derivations show the optimality of the allocation in period 1 for unproductive islands and in period 2 for all island.

Proof of Proposition 2

In the main text we show that $\gamma = \beta$ is a necessary condition for an unconstrained equilibrium and that an unconstrained equilibrium achieves first-best efficiency in period 1. In period 2, if the liquidity constraint is slack, all households' consume the same amount, as $U'(c_2(\theta, \tilde{\theta}, \zeta)) = p_2(\zeta)/p_3$ for all θ and $\tilde{\theta}$. By market clearing $c_2(\theta, \tilde{\theta}, \zeta)$ must then be equal to e_2. Since any stationary allocation $c_3(\theta, \tilde{\theta}, \zeta)$ is consistent with first-best efficiency, this completes the proof of efficiency. It remains to prove that $\gamma = \beta$ is sufficient for an unconstrained equilibrium to exist. To do so, we construct such an equilibrium. Let the prices be

$$\begin{aligned} p_1(\theta) &= p_3 u'(y_1^*(\theta)) \text{ for } \theta \in \left\{0, \overline{\theta}\right\}, \\ p_2 &= p_3 U'(e_2), \end{aligned}$$

and p_3 take any value in $(0, \hat{p}_3]$, where

$$\hat{p}_3 \equiv 1/\left(u'(y_1^*(\overline{\theta}))y_1^*(\overline{\theta}) + U'\left(e_2\right)e_2\right).$$

From the argument above, the consumption levels in periods 1 and 2 must be at their first-best level. Substituting in the budget constraints the prices above and the first-best consumption levels in periods 1 and 2, we obtain

$$c_3(\theta, \tilde{\theta}, \zeta) = e_3 - u'(y_1^*(\tilde{\theta}))y_1^*(\tilde{\theta}) + u'(y_1^*(\theta))y_1^*(\theta).$$

The assumption that e_3 is large ensures that $c_3(\theta, \tilde{\theta}) > 0$ for all θ and $\tilde{\theta}$. Moreover, it is easy to show that money holdings are non-negative, thanks to the assumption $p_3 \leq \hat{p}_3$. It is also easy to check that the allocation is individually optimal and satisfies market clearing, completing the proof.

Proof of Lemma 1

Applying the implicit function theorem, to (12) and (13) we obtain

$$\frac{\partial \overline{y}^D(\overline{p}_1,\zeta)}{\partial \overline{p}_1} = \frac{\zeta U'(e_2)+(1-\zeta)U'(e_2-\overline{p}_1\overline{y}_1)-\overline{p}_1\overline{y}_1(1-\zeta)U''(e_2-\overline{p}_1\overline{y}_1)}{u''(\overline{y}_1)+\overline{p}_1^2(1-\zeta)U''(e_2-\overline{p}_1\overline{y}_1)}, \quad (22)$$

$$\frac{\partial \overline{y}^S(\overline{p}_1,\zeta)}{\partial \overline{p}_1} = \overline{\theta}\frac{\zeta U'(e_2)+(1-\zeta)U'(e_2+\overline{p}_1\overline{y}_1)+\overline{p}_1\overline{y}_1(1-\zeta)U''(e_2+\overline{p}_1\overline{y}_1)}{v''\left(\overline{y}_1/\overline{\theta}\right)/\overline{\theta}-\overline{p}_1^2(1-\zeta)U''(e_2+\overline{p}_1\overline{y}_1)}. \quad (23)$$

The concavity of u and U imply that the numerator of (22) is positive and the numerator is negative, proving that $\partial \overline{y}^D(\overline{p}_1,\zeta)/\partial p_1 < 0$. The concavity of U and the convexity of v show that the denominator of (23) is positive. It remains to show that the numerator is also positive. The following chain of inequalities is sufficient for that:

$$\begin{aligned}\zeta U'(e_2)+(1-\zeta)U'(e_2+\overline{p}_1\overline{y}_1)+(1-\zeta)\overline{p}_1\overline{y}_1U''(e_2+\overline{p}_1\overline{y}_1) &> \\ U'(e_2+\overline{p}_1\overline{y}_1)+(1-\zeta)(e_2+\overline{p}_1\overline{y}_1)U''(e_2+\overline{p}_1\overline{y}_1) &\geq 0.\end{aligned}$$

The first inequality follows because the concavity of U implies $U'(e_2) > U'(e_2+\overline{p}_1\overline{y}_1)$ and $e_2U''(e_2+\overline{p}_1\overline{y}_1) < 0$. The second follows from assumption A1, completing the proof that $\partial \overline{y}^S(\overline{p}_1,\zeta)/\partial p_1 > 0$. Existence can be shown using similar arguments as in the proof of Lemma 2 below. Uniqueness follows immediately.

Proof of Proposition 3

First, we complete the characterization of a fully constrained equilibrium, presenting the steps omitted in the text. Then, we will define $\hat{\gamma}$ and prove that such an equilibrium exists iff $\gamma \geq \hat{\gamma}$. Suppose for the moment that (12) and (13) have a unique solution, $p_1(\overline{\theta},\zeta)$ and $y_1(\overline{\theta},\zeta)$. In unproductive islands, output and nominal output are zero, $y_1(0,\zeta)=0$ and $p_1(0,\zeta)y_1(0,\zeta)=0$. From the consumer's budget constraint in period 2, we obtain

$$c_2(\theta,\tilde{\theta},\zeta) = e_2 - p_1(\tilde{\theta},\zeta)y_1(\tilde{\theta},\zeta) + p_1(\theta,\zeta)y_1(\theta,\zeta).$$

The price level in unproductive islands is obtained from the Euler equation (3),

$$p_1(0,\zeta) = u'(0)\left(\mathbb{E}[U'(c_2(0,\tilde{\theta},\zeta))|\tilde{\theta},\zeta]\right)^{-1}.$$

From the consumer's budget constraint in period 3 we obtain $c_3 = e_3$. Combining the Euler equations (3) and (5) and the envelope condition (7), p_3 is uniquely pinned down

by

$$\frac{1}{p_3} = \beta\gamma^{-1}\mathbb{E}[U'(c_2(\theta,\tilde{\theta},\zeta))]. \tag{24}$$

The only optimality condition that remains to be checked is the Euler equation in period 2, (4). Notice that given our construction of $c_2(\theta,\tilde{\theta},\zeta)$ and the concavity of U, $U'(c_2(\theta,\tilde{\theta},\zeta)) \geq \min_{\tilde{\zeta}} U'(c_2(\overline{\theta},\underline{\theta},\tilde{\zeta}))$ for all $\theta,\tilde{\theta},\zeta$. It follows that a necessary and sufficient condition for (4) to hold for all $\theta,\tilde{\theta},\zeta$ is

$$\min_{\zeta} U'(c_2(\overline{\theta},\underline{\theta},\zeta)) \geq \frac{1}{p_3}. \tag{25}$$

We now define the cutoff

$$\hat{\gamma} \equiv \beta\frac{\mathbb{E}[U'(c_2(\theta,\tilde{\theta},\zeta))]}{\min_{\zeta} U'(c_2(\overline{\theta},\underline{\theta},\zeta))}$$

and prove the statement of the proposition. Using (24) to substitute for p_3, condition (25) is equivalent to $\gamma \geq \hat{\gamma}$. Therefore, if an unconstrained equilibrium exists, (25) implies $\gamma \geq \hat{\gamma}$, proving necessity. If $\gamma \geq \hat{\gamma}$, the previous steps show how to construct a fully constrained equilibrium, proving sufficiency. In the case where (12) and (13) have multiple solutions, one can follow the steps above and find a value of $\hat{\gamma}$ for each solution. The smallest of these values gives us the desired cutoff.

Proof of Proposition 6

The argument in the text shows that $d\log Y_1(\zeta)/d\zeta$ is larger in a fully constrained equilibrium, for all $\zeta \in [\zeta_L,\zeta_H]$, which implies that $\log Y_1(\zeta_H) - \log Y_1(\zeta_L)$ is larger as well. This proves our statement, since $Var[\log Y_1(\zeta)] = \alpha(1-\alpha)[\log Y_1(\zeta_H) - \log Y_1(\zeta_L)]^2$.

Proof of Proposition 7

Let $\mu_y = \mathbb{E}[y_1(\theta,\zeta)]$. Since $Y_1(\zeta) = \mathbb{E}[y_1(\theta,\zeta)|\zeta]$, we have

$$Cov[y_1(\theta,\zeta), Y_1(\zeta)] = \mathbb{E}[\mathbb{E}[\left(y_1(\theta,\zeta) - \mu_y\right)(Y_1(\zeta) - \mu_y)|\zeta]] = Var[Y_1(\zeta)],$$

and hence $Corr[y_1(\theta,\zeta), Y_1(\zeta)] = (Var[Y_1(\zeta)]/Var[y_1(\theta,\zeta)])^{1/2}$. Using the decomposition $Var[y_1(\theta,\zeta)] = Var[Y_1(\zeta)] + \mathbb{E}[Var[y_1(\theta,\zeta)|\zeta]]$, rewrite this correlation as

$$Corr[y_1(\theta,\zeta), Y_1(\zeta)] = \left(1 + \frac{\mathbb{E}[Var[y_1(\theta,\zeta)|\zeta]]}{Var[Y_1(\zeta)]}\right)^{-1/2} =$$

$$= \left(1 + \frac{(1-\alpha)\zeta_L(1-\zeta_L)\left(\overline{y}_1(\zeta_L)\right)^2 + \alpha\zeta_H(1-\zeta_H)\left(\overline{y}_1(\zeta_H)\right)^2}{\alpha(1-\alpha)\left(\zeta_H\overline{y}_1(\zeta_H) - \zeta_L\overline{y}_1(\zeta_L)\right)^2}\right)^{-1/2}. \tag{26}$$

Define

$$f(\xi) \equiv \frac{\alpha(1-\alpha)(\zeta_H \xi - \zeta_L)^2}{(1-\alpha)\zeta_L(1-\zeta_L) + \alpha\zeta_H(1-\zeta_H)\xi^2}.$$

After some algebra, one can see that the expression on the right-hand side of (26) is monotone increasing in $f(\xi)$, where $\xi = \overline{y}_1(\zeta_H)/\overline{y}_1(\zeta_L)$. Therefore, the correlation is lower in the unconstrained economy if and only if $f\left(\xi^U\right) < f\left(\xi^C\right)$, where ξ^U and ξ^C denote, respectively, the ratio $\overline{y}_1(\zeta_H)/\overline{y}_1(\zeta_L)$ in the unconstrained and in the fully constrained regimes. Notice that $f(\xi)$ is continuous and differentiable. Moreover, $\xi^U = 1$ from Proposition 2, and $\xi^C > 1$ from Proposition 5. Therefore, to prove our statement it is sufficient to show that $f'(\xi) > 0$ for $\xi \geq 1$. Differentiating $f(\xi)$ shows that $f'(\xi)$ has the same sign as

$$\zeta_H(\zeta_H\xi - \zeta_L)\left((1-\alpha)\zeta_L(1-\zeta_L) + \alpha\zeta_H(1-\zeta_H)\xi^2\right) - \alpha\zeta_H(1-\zeta_H)(\zeta_H\xi - \zeta_L)^2\xi.$$

Since $\zeta_H > \zeta_L$, if $\xi \geq 1$ then $\zeta_H\xi - \zeta_L > 0$. Some algebra then shows that the expression above has the same sign as $(1-\alpha)(1-\zeta_L) + \alpha(1-\zeta_H)\xi$ and is always positive, completing the proof.

Proof of Proposition 8

It is easy to generalize the first-best allocation described in Section 2.2 for the binary model. Solving the planner problem for the extended model, the optimal output level in period 1 is $y_1^C(\theta, \tilde{\theta}, \zeta) = y_1^M(\theta, \tilde{\theta}, \zeta) = y_1^*(\theta)$ for all θ, $\tilde{\theta}$ and ζ, where $y_1^*(\theta)$ satisfies

$$\theta u'(y_1^*(\theta)) = v'(y_1^*(\theta)/\theta), \tag{27}$$

Moreover, optimal consumption in period 2 is $c_2^C(\theta, \tilde{\theta}, \zeta) = c_2^M(\theta, \tilde{\theta}, \zeta) = e_2$ for all $\theta, \tilde{\theta}$ and ζ.

Next, we prove that any unconstrained equilibrium achieves a first-best allocation. Since (4) holds as an equality for all θ, $\tilde{\theta}$ and ζ, both for credit and money households, it follows that $c_2^i(\theta, \tilde{\theta}, \zeta)$ is equal to a constant c_2 for all θ, $\tilde{\theta}$, ζ, and $i = C, M$. Then, market clearing requires $c_2 = e_2$. Substituting in (3) (for a consumer in island θ) and (6) (for a producer in island θ), and given that (3) holds as an equality, we obtain $c_1^C(\theta, \zeta) = c_1^M(\theta, \zeta) = c_1(\theta)$ and $n^C(\theta, \zeta) = n^M(\theta, \zeta) = n(\theta)$ for all θ and ζ, where

$$u'(c_1(\theta)) = \frac{p_1(\theta)}{p_2}U'(e_2) \text{ and } v'(n(\theta)) = \theta\frac{p_1(\theta)}{p_2}U'(e_2).$$

These two conditions, and market clearing in island θ, imply that $y_1^C(\theta,\zeta) = y_1^M(\theta,\zeta) = y_1^*(\theta)$ as defined by the planner optimality condition (27). Therefore, consumption levels in periods 1 and 2 achieve the first-best. Since any consumption allocation in period 3 is consistent with first-best efficiency, this completes the argument.

The proof that $\gamma = \beta$ is necessary for an unconstrained equilibrium to exist is the same as in the binary model. To prove sufficiency, when $\gamma = \beta$ we can construct an unconstrained equilibrium with prices

$$\begin{aligned} p_1^i(\theta) &= p_3 u'(y_1^*(\theta)) \text{ for all } \theta \in \Theta, i = C, M \\ p_2^i &= p_3 U'(e_2) \text{ for } i = C, M, \end{aligned}$$

for some $p_3 \in (0, \hat{p}_3]$, where

$$\hat{p}_3 \equiv \frac{1/(1-\phi)}{u'(y_1^*(\bar{\theta}))y_1^*(\bar{\theta}) + U'(e_2)\, e_2}.$$

From the argument above, consumption levels in periods 1 and 2 are at their first-best level. Substituting in the budget constraints the prices above and the first-best consumption levels in periods 1 and 2, we obtain

$$c_3^C(\theta,\tilde{\theta},\zeta) = c_3^M(\theta,\tilde{\theta},\zeta) = e_3 - u'(y_1^*(\tilde{\theta}))y_1^*(\tilde{\theta}) + u'(y_1^*(\theta))y_1^*(\theta).$$

Moreover, choosing any $p_3 \leq \hat{p}_3$ ensures that money holdings are non-negative. It is straightforward to check that this allocation satisfies market clearing and that it is individually optimal, completing the proof.

Finally, it is easy to show that $y_1^*(\theta)$ is increasing, by applying the implicit function theorem to the planner's optimality condition (27).

Preliminary results for Proposition 9

In order to prove Proposition 9, it is useful to prove several preliminary lemmas. These results will also be useful to prove Proposition 10.

The following lemmas establish that the system of functional equations (17)-(18) has a unique solution $\left(p_1^M(\cdot,\zeta), y_1^M(\cdot,\zeta)\right)$, for a given ζ. To do so, we define a fixed point problem for the function $x(\cdot,\zeta)$. Recall from the text that $x(\theta,\zeta) \equiv p_1^M(\theta,\zeta)y_1^M(\theta,\zeta)$. To save on notation, in the lemmas we fix ζ and refer to $p_1^M(\cdot,\zeta)$, $y_1^M(\cdot,\zeta)$, $x(\cdot,\zeta)$, and $F(\cdot,\zeta)$, as $p(\cdot)$, $y(\cdot)$, $x(\cdot)$, and $F(\cdot)$.

Notice that, in an island where $\theta = 0$, $x(0) = 0$. Moreover, non-negativity of consumption in period 2 requires that $x(\theta) \leq 1$ for all θ. Therefore, we restrict attention to the set of measurable, bounded functions $x : [0, \overline{\theta}] \to [0, 1]$ that satisfy $x(0) = 0$. We use X to denote this set.

Lemma 2 *Given $\theta > 0$ and a function $x \in X$, there exists a unique pair (p, y) which solves the system of equations*

$$u'(y) - p \int_0^{\overline{\theta}} U'\left(1 - py + x(\tilde{\theta})\right) dF(\tilde{\theta}) = 0, \tag{28}$$

$$v'(y/\theta) - \theta p \int_0^{\overline{\theta}} U'\left(1 - x(\tilde{\theta}) + py\right) dF(\tilde{\theta}) = 0. \tag{29}$$

The pair (p, y) satisfies $py \in [0, 1]$.

Proof. We proceed in two steps, first we prove existence, then uniqueness.

Step 1. Existence. For a given $p \in (0, \infty)$, it is easy to show that there is a unique y which solves (28) and a unique y which solves (29), which we denote, respectively, by $y^D(p)$ and $y^S(p)$. Finding a solution to (28)-(29), is equivalent to finding a p that solves

$$y^D(p) - y^S(p) = 0. \tag{30}$$

It is straightforward to prove that $y^D(p)$ and $y^S(p)$ are continuous on $(0, \infty)$. We now prove that they satisfy four properties: (a) $py^D(p) < 1$ for all $p \in (0, \infty)$, (b) $y^S(p) < \theta\overline{n}$ for all $p \in (0, \infty)$, (c) $\limsup_{p \to 0} y^D(p) = \infty$, and (d) $\limsup_{p \to \infty} py^S(p) = \infty$. Notice that $x(0) = 0$ with positive probability, so the Inada condition for U can be used to prove property (a). Similarly, to prove property (b), we can use the assumption $\lim_{n \to \overline{n}} v'(n) = \infty$. To prove (c) notice that (a) implies $\limsup_{p \to 0} py^D(p) \leq 1$. If $\limsup_{p \to 0} py^D(p) = 1$, then, we immediately have $\limsup_{p \to 0} y^D(p) = \infty$. If, instead, $\limsup_{p \to 0} py^D(p) < 1$, then there exists a $K \in (0, 1)$ and an $\epsilon > 0$ such that $py^D(p) < K$ for all $p \in (0, \epsilon)$. Since U' is decreasing, this implies that $U'(1 - py^D(p) + x(\tilde{\theta}))$ is bounded above by $U'(1 - K) < \infty$ for all $p \in (0, \epsilon)$, which implies

$$\lim_{p \to 0} p \int_0^{\overline{\theta}} U'(1 - py^D(p) + x(\tilde{\theta})) dF(\tilde{\theta}) = 0.$$

Using (28), this requires $\lim_{p\to 0} u'\left(y^D(p)\right) = 0$ and, hence, $\lim_{p\to 0} y^D(p) = \infty$. To prove property (d), suppose, by contradiction, that there exist a $K > 0$ and a $P > 0$, such that $py^S(p) \leq K$ for all $p \geq P$. Then $U'(1 - x(\tilde{\theta}) + py^S(p))$ is bounded below by $U'(1+K) > 0$ for all $p \in (P, \infty)$, which implies

$$\lim_{p\to\infty} p \int_0^{\overline{\theta}} U'(1 - x(\tilde{\theta}) + py^S(p)) dF(\tilde{\theta}) = \infty. \tag{31}$$

Moreover, since $0 \leq py^S(p) \leq K$ for all $p \geq P$, it follows that $\lim_{p\to\infty} y^S(p) = 0$ and thus

$$\lim_{p\to\infty} v'\left(y^S(p)/\theta\right) < \infty. \tag{32}$$

Using equation (29), conditions (31) and (32) lead to a contradiction, completing the proof of (d). Properties (a) and (d) immediately imply $\limsup_{p\to\infty}\left(py^S(p) - py^D(p)\right) = \infty$, while (b) and (c) imply $\limsup_{p\to 0}\left(y^D(p) - y^S(p)\right) = \infty$. It follows that there exists a pair (p', p''), with $p' < p''$, such that $y^D(p') - y^S(p') > 0$ and $y^D(p'') - y^S(p'') < 0$. By the intermediate value theorem there exists a p which solves (30). Property (a) immediately implies that $py \in [0,1]$, where $y = y^D(p) = y^S(p)$.

Step 2. Uniqueness. Let $\hat{p}$ be a zero of (30), and $\hat{y} = y^D(\hat{p}) = y^S(\hat{p})$. To show uniqueness, it is sufficient to show that $dy^D(p)/dp - dy^S(p)/dp < 0$ at $p = \hat{p}$. Applying the implicit function theorem gives

$$\left[\frac{dy^D(p)}{dp}\right]_{p=\hat{p}} = \frac{\int_0^{\overline{\theta}} U'\left(\tilde{c}_2^D\right) dF(\tilde{\theta}) - \hat{p}\hat{y}\int_0^{\overline{\theta}} U''\left(\tilde{c}_2^D\right) dF(\tilde{\theta})}{u''(\hat{y}) + \hat{p}^2\int_0^{\overline{\theta}} U''\left(\tilde{c}_2^D\right) dF(\tilde{\theta})},$$

where $\tilde{c}_2^D = 1 - \hat{p}\hat{y} + x(\tilde{\theta})$ and

$$\left[\frac{dy^S(p)}{dp}\right]_{p=\hat{p}} = \frac{\int_0^{\overline{\theta}} U'\left(\tilde{c}_2^S\right) dF(\tilde{\theta}) + \hat{p}\hat{y}\int_0^{\overline{\theta}} U''\left(\tilde{c}_2^S\right) dF(\tilde{\theta})}{v''\left(\hat{y}/\theta\right)/\theta^2 - \hat{p}^2\int_0^{\overline{\theta}} U''\left(\tilde{c}_2^S\right) dF(\tilde{\theta})}.$$

where $\tilde{c}_2^S = 1 - x(\tilde{\theta}) + \hat{p}\hat{y}$. Using (28)-(29), the required inequality can then be rewritten as

$$\frac{v''\left(\hat{y}/\theta\right)}{\theta^2}\left(\frac{u'(\hat{y})}{\hat{p}} - \hat{p}\hat{y}\int_0^{\overline{\theta}} U''\left(\tilde{c}_2^D\right) dF(\tilde{\theta})\right) - \frac{v'\left(\hat{y}/\theta\right)}{\theta\hat{p}}\left(u''(\hat{y}) + \hat{p}^2\int_0^{\overline{\theta}} U''\left(\tilde{c}_2^D\right) dF(\tilde{\theta})\right)$$
$$+\hat{p}\int_0^{\overline{\theta}} U''\left(\tilde{c}_2^S\right) dF(\tilde{\theta})\left(u'(\hat{y}) + \hat{y}u''(\hat{y})\right) > 0.$$

The first two terms on the left-hand side are positive. Assumption A2 implies that also the last term is positive, completing the argument. ■

Lemma 3 *Given a function $x \in X$, for any $\theta > 0$ let $(p(\theta), y(\theta))$ be the unique pair solving the system (28)-(29) and define $z(\theta) \equiv p(\theta) y(\theta)$. The function $z(\theta)$ is monotone increasing.*

Proof. Define the two functions

$$\begin{aligned} h_1(z, y; \theta) &\equiv u'(y)y - z\int_0^{\overline{\theta}} U'(1 - z + x(\tilde{\theta}))dF(\tilde{\theta}), \\ h_2(z, y; \theta) &\equiv v'(y/\theta)\, y/\theta - z\int_0^{\overline{\theta}} U'(1 - x(\tilde{\theta}) + z)dF(\tilde{\theta}), \end{aligned}$$

which correspond to the left-hand sides of (28) and (29) multiplied, respectively, by y and y/θ. Lemma 2 ensures that for each $\theta > 0$ there is a unique positive pair $(z(\theta), y(\theta))$ which satisfies

$$h_1(z(\theta), y(\theta); \theta) = 0 \text{ and } h_2(z(\theta), y(\theta); \theta) = 0.$$

Applying the implicit function theorem, gives

$$z'(\theta) = \frac{\frac{\partial h_1}{\partial y}\frac{\partial h_2}{\partial \theta} - \frac{\partial h_2}{\partial y}\frac{\partial h_1}{\partial \theta}}{\frac{\partial h_1}{\partial z}\frac{\partial h_2}{\partial y} - \frac{\partial h_2}{\partial z}\frac{\partial h_1}{\partial y}}. \tag{33}$$

To prove the lemma it is sufficient to show that $z'(\theta) > 0$ for all $\theta \in (0, \overline{\theta}]$. Using z and y as shorthand for $z(\theta)$ and $y(\theta)$, the numerator on the right-hand side of (33) can be written as

$$-\frac{y}{\theta^2}\left[v'(y/\theta) + v''(y/\theta)\, y/\theta\right]\left[u'(y) + u''(y)y\right],$$

and the denominator can be written, after some algebra, as

$$\begin{aligned} &\left[v'(y/\theta) + v''(y/\theta)\, y/\theta\right]\frac{z}{\theta}\int_0^{\overline{\theta}} U''\left(1 - z + x(\tilde{\theta})\right)dF(\tilde{\theta}) + \\ &+ \left[u'(y) + u''(y)y\right] z\int_0^{\overline{\theta}} U''\left(1 - x(\tilde{\theta}) + z\right)dF(\tilde{\theta}) + \frac{y^2}{z\theta^2}\left[u''(y)v'(y/\theta)\,\theta - u'(y)\, v''(y/\theta)\right]. \end{aligned} \tag{34}$$

Assumption A2 ensures that both numerator and denominator are negative, completing the proof. ■

We can now define a map T from the space X into itself.

Definition 2 *Given a function $x \in X$, for any $\theta > 0$ let $(p(\theta), y(\theta))$ be the unique pair solving the system (28)-(29). Define a map $T : X \to X$ as follows. Set $(Tx)(\theta) = p(\theta) y(\theta)$ if $\theta > 0$ and $(Tx)(\theta) = 0$ if $\theta = 0$.*

The following lemmas prove monotonicity and discounting for the map T. These properties will be used to find a fixed point of T. In turns, this fixed point will be used to construct the equilibrium in Proposition 9.

Lemma 4 *Take any $x^0, x^1 \in X$, with $x^1(\theta) \geq x^0(\theta)$ for all θ. Then $(Tx^1)(\theta) \geq (Tx^0)(\theta)$ for all θ.*

Proof. For each $\tilde{\theta} \in [0, \overline{\theta}]$ and any scalar $\lambda \in [0, 1]$, with a slight abuse of notation, we define $x(\tilde{\theta}, \lambda) \equiv x^0(\tilde{\theta}) + \lambda \Delta(\tilde{\theta})$, where $\Delta(\tilde{\theta}) \equiv x^1(\tilde{\theta}) - x^0(\tilde{\theta}) \geq 0$. Notice that $x(\tilde{\theta}, 0) = x^0(\tilde{\theta})$ and $x(\tilde{\theta}, 1) = x^1(\tilde{\theta})$. Fix a value for θ and define the two functions

$$\begin{aligned} h_1(z, y; \lambda) &\equiv y u'(y) - z \int_0^{\overline{\theta}} U'(1 - z + x(\tilde{\theta}, \lambda)) dF(\tilde{\theta}), \\ h_2(z, y; \lambda) &\equiv v'(y/\theta)\, y/\theta - z \int_0^{\overline{\theta}} U'(1 - x(\tilde{\theta}, \lambda) + z) dF(\tilde{\theta}). \end{aligned}$$

Applying Lemma 2, for each $\lambda \in [0, 1]$ we can find a unique positive pair $(z(\lambda), y(\lambda))$ that satisfies

$$h_1(z(\lambda), y(\lambda); \lambda) = 0 \text{ and } h_2(z(\lambda), y(\lambda); \lambda) = 0.$$

We are abusing notation in the definition of $h_1(\cdot, \cdot; \lambda)$, $h_2(\cdot, \cdot; \lambda)$, $z(\lambda)$, and $y(\lambda)$, given that the same symbols were used above to define functions of θ. Here we keep θ constant throughout the proof, so no confusion should arise. Notice that, by construction, $(Tx^0)(\theta) = z(0)$ and $(Tx^1)(\theta) = z(1)$. Therefore, to prove our statement it is sufficient to show that $z'(\lambda) \geq 0$ for all $\lambda \in [0, 1]$.

Applying the implicit function theorem yields

$$z'(\lambda) = \frac{\frac{\partial h_1}{\partial y}\frac{\partial h_2}{\partial \lambda} - \frac{\partial h_2}{\partial y}\frac{\partial h_1}{\partial \lambda}}{\frac{\partial h_1}{\partial z}\frac{\partial h_2}{\partial y} - \frac{\partial h_2}{\partial z}\frac{\partial h_1}{\partial y}}. \tag{35}$$

Using z and y as shorthand for $z(\lambda)$ and $y(\lambda)$, the numerator on the right-hand side of

(35) can be written as

$$[u'(y) + u''(y)y]\, z \int_0^{\overline{\theta}} U''(1 - x(\tilde{\theta}, \lambda) + z)\Delta(\tilde{\theta})dF(\tilde{\theta}) +$$
$$+\frac{z}{\theta}\left[v'(y/\theta) + v''(y/\theta)\, y/\theta\right] \int_0^{\overline{\theta}} U''(1 - z + x(\tilde{\theta}, \lambda))\Delta(\tilde{\theta})dF(\tilde{\theta}).$$

The denominator takes a form analogous to (34). Again, assumption A2 ensures that both the numerator and the denominator are negative, completing the argument. ■

Before proving the discounting property, it is convenient to restrict the space X to the space $\tilde{X}$ of functions bounded in $[0, \overline{z}]$ for an appropriate $\overline{z} < 1$. The following lemma shows that the map T maps $\tilde{X}$ into itself, and that any fixed point of T in X must lie in $\tilde{X}$.

Lemma 5 *There exists a $\overline{z} < 1$, such that if $x \in X$ then $(Tx)(\theta) \leq \overline{z}$ for all θ.*

Proof. Set $\overline{x}(0) = 0$ and $\overline{x}(\theta) = 1$ for all $\theta > 0$. Setting $x(.) = \overline{x}(.)$ and $\theta = \overline{\theta}$, equations (28)-(29) take the form

$$\begin{aligned} u'(y) &= p\left[F(0)\, U'(1 - py) + (1 - F(0))\, U'(2 - py)\right], \\ v'(y/\overline{\theta}) &= \overline{\theta} p\left[F(0)\, U'(1 + py) + (1 - F(0))\, U'(py)\right]. \end{aligned}$$

Let $(\hat{p}, \hat{y})$ denote the pair solving these equations, and let $\overline{z} \equiv \hat{p}\hat{y}$. Since $F(0) > 0$ and U satisfies the Inada condition, $\lim_{c \to 0} U'(c) = \infty$, inspecting the first equation shows that $\overline{z} < 1$. Now take any $x \in X$. Since $x(\theta) \leq \overline{x}(\theta)$ for all θ, Lemma 4 implies that $(Tx)(\theta) \leq (T\overline{x})(\theta)$. Moreover, Lemma 3 implies that $(T\overline{x})(\theta) \leq (T\overline{x})(\overline{\theta}) = \overline{z}$. Combining these inequalities we obtain $(Tx)(\theta) \leq \overline{z}$. ■

Lemma 6 *There exists a $\delta \in (0, 1)$ such that the map T satisfies the discounting property: for any $x^0, x^1 \in \tilde{X}$ such that $x^1(\theta) = x^0(\theta) + a$ for some $a > 0$, the follow inequality holds*

$$\left|\left(Tx^1\right)(\theta) - \left(Tx^0\right)(\theta)\right| \leq \delta a \text{ for all } \theta \in \left[0, \overline{\theta}\right].$$

Proof. Proceeding as in the proof of Lemma 4, define $x(\tilde{\theta}, \lambda) \equiv x^0(\tilde{\theta}) + \lambda\Delta(\tilde{\theta})$, where now $\Delta(\tilde{\theta}) = a$ for all $\tilde{\theta}$. After some algebra, we obtain

$$z'(\lambda) = \frac{\left(1 + \frac{yu''(y)}{u'(y)}\right) A + \left(1 + \frac{nv''(n)}{v'(n)}\right) B}{\left(1 + \frac{yu''(y)}{u'(y)}\right) A + \left(1 + \frac{nv''(n)}{v'(n)}\right) B + \frac{nv''(n)}{v'(n)} - \frac{yu''(y)}{u'(y)}} a, \tag{36}$$

where y and n are shorthand for $y(\lambda)$ and $y(\lambda)/\theta$ and

$$A = -\frac{z(\lambda)\int_0^{\overline{\theta}} U''\left(1-x(\tilde{\theta},\lambda)+z(\lambda)\right)dF(\tilde{\theta})}{\int_0^{\overline{\theta}} U'\left(1-x(\tilde{\theta},\lambda)+z(\lambda)\right)dF(\tilde{\theta})},$$

$$B = -\frac{z(\lambda)\int_0^{\overline{\theta}} U''\left(1-z(\lambda)+x(\tilde{\theta},\lambda)\right)dF(\tilde{\theta})}{\int_0^{\overline{\theta}} U'\left(1-z(\lambda)+x(\tilde{\theta},\lambda)\right)dF(\tilde{\theta})}.$$

Now, given that $z(\lambda)$ and $x(\tilde{\theta},\lambda)$ are both in $[0,\overline{z}]$ and $\overline{z} < e_2$, and given that U has continuous first and second derivatives on $(0,\infty)$, it follows that both A and B are bounded above. We can then find a uniform upper bound on both A and B, independent of λ and of the functions x^0 and x^1 chosen. Let C be this upper bound. Given that $u''(y) \leq 0$, then

$$\left(1+\frac{yu''(y)}{u'(y)}\right)A + \left(1+\frac{nv''(n)}{v'(n)}\right)B \leq \left(2+\frac{nv''(n)}{v'(n)}\right)C.$$

Therefore, (36) implies

$$z'(\lambda) \leq \left(1+\frac{nv''(n)/v'(n) - yu''(y)/u'(y)}{(2+nv''(n)/v'(n))C}\right)^{-1} a.$$

Recall that $\underline{\rho} > 0$ is a lower bound for $-yu''(y)/u'(y)$. Then

$$\frac{nv''(n)/v'(n) - yu''(y)/u'(y)}{(2+nv''(n)/v'(n))C} \geq \frac{-yu''(y)/u'(y)}{2C} \geq \frac{\underline{\rho}}{2C}.$$

Setting $\delta \equiv 1/[1+\underline{\rho}/(2C)] < 1$, it follows that $z'(\lambda) \leq \delta a$ for all $\lambda \in [0,1]$. Integrating both sides of the last inequality over $[0,1]$, gives $z(1) - z(0) \leq \delta a$. By construction $(Tx^1)(\theta) = z(1)$ and $(Tx^0)(\theta) = z(0)$, completing the proof. ■

Proof of Proposition 9

We first uniquely characterize prices and allocations in a fully constrained equilibrium. Next, we will use this characterization to prove our claim. The argument in the text and the preliminary results above show that if there exists an equilibrium with $m_2^M(\theta,\tilde{\theta},\zeta) = 0$ for all θ and $\tilde{\theta}$, then $p_1^M(\theta,\zeta)$ and $y_1^M(\theta,\zeta)$ must solve the functional equations (17)-(18) for any given ζ. To find the equilibrium pair $\left(p_1^M(\theta,\zeta), y_1^M(\theta,\zeta)\right)$ we first find a fixed point of the map T defined above (Definition 2). Lemmas 4 and 6 show that T is a map

from a space of bounded functions into itself and satisfies the assumptions of Blackwell's theorem. Therefore, a fixed point exists and is unique. Let x denote the fixed point, then Lemma 2 shows that we can find two functions $p_1^M(\theta,\zeta)$ and $y_1^M(\theta,\zeta)$ for a given ζ that satisfy (28)-(29). Since $x(\theta,\zeta)$ is a fixed point of T we have $x(\theta,\zeta) = p_1^M(\theta,\zeta)y_1^M(\theta,\zeta)$, and substituting in (28)-(29) shows that (17)-(18) are satisfied. Therefore, in a fully constrained equilibrium $p_1^M(\theta,\zeta)$ and $y_1^M(\theta,\zeta)$ are uniquely determined, and so is labor supply $n^M(\theta,\zeta) = y_1^M(\theta,\zeta)/\theta$. Moreover, from the budget constraint and the market clearing condition in period 2, consumption in period 2 is uniquely determined by (16). The price p_2^M is equal to 1, given the normalization in the text. From the consumer's budget constraint in period 3 we obtain $c_3^M = e_3$. Combining the Euler equations (3) and (5) and the envelope condition (7), p_3 is uniquely pinned down by

$$\frac{1}{p_3} = \beta\gamma^{-1}\mathbb{E}[U'(c_2^M(\theta,\tilde{\theta},\zeta))]. \tag{37}$$

Moreover, equilibrium money holdings are $m_1(\theta,\zeta) = 1 - p_1^M(\theta,\zeta)y_1^M(\theta,\zeta)$, $m_2(\theta,\tilde{\theta},\zeta) = 0$, and $m_3(\theta,\tilde{\theta},\zeta) = \gamma$. Define the cutoff

$$\hat{\gamma} \equiv \beta\frac{\mathbb{E}[U'(c_2^M(\theta,\tilde{\theta},\zeta))]}{\min_\zeta\{U'(c_2^M(\overline{\theta},\underline{\theta},\zeta))\}}.$$

The only optimality condition that remains to be checked is the Euler equation in period 2, that is, equation (4). Given the definition of $c_2^M(\theta,\tilde{\theta},\zeta)$, Lemma 3 implies that it is an increasing function of θ and a decreasing function of $\tilde{\theta}$. It follows that a necessary and sufficient condition for (4) to hold for all θ, $\tilde{\theta}$ and ζ is

$$\min_\zeta\{U'(c_2^M(\overline{\theta},\underline{\theta},\zeta))\} \geq \frac{1}{p_3}. \tag{38}$$

Substituting the expression (37) for $1/p_3$, this condition is equivalent to $\gamma \geq \hat{\gamma}$. Therefore, if a fully constrained equilibrium exists, $c_2^M(\theta,\tilde{\theta},\zeta)$ is uniquely determined and condition (38) implies that $\gamma \geq \hat{\gamma}$, proving necessity. Moreover, if $\gamma \geq \hat{\gamma}$, the previous steps show how to construct a fully constrained equilibrium, proving sufficiency.

Finally, the proof that nominal income $p_1^M(\theta,\zeta)\,y_1^M(\theta,\zeta)$ is monotone increasing in θ, for a given ζ, follows immediately from Lemma 3. To prove that also output $y_1^M(\theta,\zeta)$ is monotone increasing in θ, let us use the same functions $h_1(z,y;\theta)$ and $h_2(z,y;\theta)$ and the same notation as in the proof of Lemma 3. For a given ζ, apply the implicit function

theorem to get

$$y'(\theta) = \frac{\frac{\partial h_2}{\partial z}\frac{\partial h_1}{\partial \theta} - \frac{\partial h_1}{\partial z}\frac{\partial h_2}{\partial \theta}}{\frac{\partial h_1}{\partial z}\frac{\partial h_2}{\partial y} - \frac{\partial h_2}{\partial z}\frac{\partial h_1}{\partial y}}. \tag{39}$$

Then it is sufficient to show that $y'(\theta) > 0$ for all $\theta \in (0, \overline{\theta}]$. Using z and y as shorthand for $z(\theta)$ and $y(\theta)$, the numerator on the right-hand side of (39) can be written as

$$\frac{y}{\theta^2}\left[v'(y/\theta) + v''(y/\theta)\, y/\theta\right]\left[z\int_0^{\overline{\theta}} U''(1-z+x(\tilde{\theta}))dF(\tilde{\theta}) - \int_0^{\overline{\theta}} U'(1-z+x(\tilde{\theta}))dF(\tilde{\theta})\right],$$

and is negative. Finally, the denominator is equal to (34) and is negative thanks to assumption A2, as we have argued in the proof of Lemma 3. This completes the argument.

Proof of Proposition 10

The proof proceeds in three steps. The first two steps prove that, for each θ, the nominal income in island θ, $x(\theta, \zeta)$, is increasing with the aggregate shock ζ. Using this result, the third step shows that $y_1^M(\theta, \zeta)$ is increasing in ζ. Consider two values ζ^I and ζ^{II}, with $\zeta^{II} > \zeta^I$. Denote, respectively, by T_I and T_{II} the maps defined in Definition 2 under the distributions $F(\theta|\zeta_I)$ and $F(\theta|\zeta_{II})$. Let x^I and x^{II} be the fixed points of T_I and T_{II}, that is, $x^I(\theta) \equiv x(\theta, \zeta^I)$ and $x^{II}(\theta) \equiv x(\theta, \zeta^{II})$ for any θ. Again, to save on notation, we drop the period index for y_1.

Step 1. Let the function x^0 be defined as $x^0 = T_{II}x^I$. In this step, we want to prove that $x^0(\theta) > x^I(\theta)$ for all $\theta > 0$. We will prove it pointwise for each θ. Fix $\theta > 0$ and define the functions

$$\begin{aligned} h_1(z, y; \zeta) &\equiv yu'(y) - z\int_0^{\overline{\theta}} U'(1-z+x^I(\tilde{\theta}))dF(\tilde{\theta}|\zeta), \\ h_2(z, y; \zeta) &\equiv v'(y/\theta)\, y/\theta - z\int_0^{\overline{\theta}} U'(1-x^I(\tilde{\theta})+z)dF(\tilde{\theta}|\zeta), \end{aligned}$$

for $\zeta \in [\zeta^I, \zeta^{II}]$. Lemma 2 implies that we can find a unique pair $(z(\zeta), y(\zeta))$ that satisfies

$$h_1(z(\zeta), y(\zeta); \zeta) = 0 \text{ and } h_2(z(\zeta), y(\zeta); \zeta) = 0.$$

Once more, we are abusing notation in the definition of $h_1(\cdot, \cdot; \zeta)$, $h_2(\cdot, \cdot; \zeta)$, $z(\zeta)$, and $y(\zeta)$. However, as θ is kept constant, there is no room for confusion. Notice that $z(\zeta^I) = x^I(\theta)$, since x^I is a fixed point of T_I, and $z(\zeta^{II}) = x^0(\theta)$, by construction.

Therefore, to prove our statement we need to show that $z(\zeta^{II}) > z(\zeta^{I})$. It is sufficient to show that $z'(\zeta) > 0$ for all $\zeta \in [\zeta^{I}, \zeta^{II}]$. Applying the implicit function theorem gives

$$z'(\zeta) = \frac{\frac{\partial h_1}{\partial y}\frac{\partial h_2}{\partial \zeta} - \frac{\partial h_2}{\partial y}\frac{\partial h_1}{\partial \zeta}}{\frac{\partial h_1}{\partial z}\frac{\partial h_2}{\partial y} - \frac{\partial h_2}{\partial z}\frac{\partial h_1}{\partial y}}. \tag{40}$$

Notice that $x^{I}(\tilde{\theta})$ is monotone increasing in $\tilde{\theta}$, by Lemma 3, and U is strictly concave. Therefore, $U'(1 - z + x^{I}(\tilde{\theta}))$ is decreasing in $\tilde{\theta}$ and $U'(1 - x^{I}(\tilde{\theta}) + z)$ is increasing in $\tilde{\theta}$. By the properties of first-order stochastic dominance, $\int_0^{\overline{\theta}} U'(1 - z + x^{I}(\tilde{\theta}))dF(\tilde{\theta}|\zeta)$ is decreasing in ζ and $\int_0^{\overline{\theta}} U'(1 - x^{I}(\tilde{\theta}) + z)dF(\tilde{\theta}|\zeta)$ is increasing in ζ. This implies that $\partial h_1/\partial\zeta > 0$ and $\partial h_2/\partial\zeta < 0$. Using y as shorthand for $y(\zeta)$, the numerator on the right-hand side of (40) is, with the usual notation,

$$\left[u'(y) + yu''(y)\right]\frac{\partial h_2}{\partial \zeta} - \frac{1}{\theta}\left[v'(y/\theta) + v''(y/\theta)\, y/\theta\right]\frac{\partial h_1}{\partial \zeta}.$$

The denominator is the analogue of (34). Once more, assumption A2 ensures that both numerator and denominator are negative, completing the argument.

Step 2. Define the sequence of functions $(x^0, x^1, ...)$ in X, using the recursion $x^{j+1} = T_{II}x^{j}$. Since, by step 1, $x^0 \geq x^{I}$ (where by $x^0 \geq x^{I}$ we mean $x^0(\theta) \geq x^{I}(\theta)$ for all $\theta > 0$) and, by Lemma 4, T_{II} is a monotone operator, it follows that this sequence is monotone, with $x^{j+1} \geq x^{j}$. Moreover, T_{II} is a contraction by Lemmas 4 and 6, so this sequence has a limit point, which coincides with the fixed point x^{II}. This implies that $x^{II} \geq x^0$ and, together with the result in step 1, shows that $x^{II} > x^{I}$, as we wanted to prove.

Step 3. Fix $\theta > 0$ and, with the usual abuse of notation, define the functions

$$\begin{aligned} h_1(z, y; \zeta) &\equiv yu'(y) - z\int_0^{\overline{\theta}} U'(1 - z + x(\tilde{\theta}, \zeta))dF(\tilde{\theta}|\zeta), \\ h_2(z, y; \zeta) &\equiv v'(y/\theta)\, y/\theta - z\int_0^{\overline{\theta}} U'(1 - x(\tilde{\theta}, \zeta) + z)dF(\tilde{\theta}|\zeta). \end{aligned}$$

Notice the difference with the definitions of h_1 and h_2 in step 1, now $x(\tilde{\theta}, \zeta)$ replaces $x^{I}(\tilde{\theta})$. The functions $z(\zeta)$ and $y(\zeta)$ are defined in the usual way. Applying the implicit function theorem, we get

$$y'(\zeta) = \frac{\frac{\partial h_2}{\partial z}\frac{\partial h_1}{\partial \zeta} - \frac{\partial h_1}{\partial z}\frac{\partial h_2}{\partial \zeta}}{\frac{\partial h_1}{\partial z}\frac{\partial h_2}{\partial y} - \frac{\partial h_2}{\partial z}\frac{\partial h_1}{\partial y}}.$$

To evaluate the numerator, notice that

$$\begin{aligned}
\frac{\partial h_1}{\partial z} &= -\int_0^{\overline{\theta}} U'(1-z+x(\tilde{\theta},\zeta))dF(\tilde{\theta}|\zeta) + z\int_0^{\overline{\theta}} U''(1-z+x(\tilde{\theta},\zeta))dF(\tilde{\theta}|\zeta) < 0, \\
\frac{\partial h_2}{\partial z} &= -\int_0^{\overline{\theta}} U'(1-x(\tilde{\theta},\zeta)+z)dF(\tilde{\theta}|\zeta) - z\int_0^{\overline{\theta}} U''(1-x(\tilde{\theta},\zeta)+z)dF(\tilde{\theta}|\zeta) \leq \\
&\leq -\int_0^{\overline{\theta}} \left[U'(1-x(\tilde{\theta},\zeta)+z) + (1-x(\tilde{\theta},\zeta)+z)U''(1-x(\tilde{\theta},\zeta)+z)\right] dF(\tilde{\theta}|\zeta) \leq 0,
\end{aligned}$$

where the last inequality follows from assumption A1' (this is the only place where this assumption is used). Furthermore, notice that

$$\frac{\partial h_1}{\partial \zeta} = -z\int_0^{\overline{\theta}} U''(1-z+x(\tilde{\theta},\zeta))\frac{\partial x(\tilde{\theta},\zeta)}{\partial \zeta}dF(\tilde{\theta}|\zeta) - z\int_0^{\overline{\theta}} U'(1-z+x(\tilde{\theta},\zeta))\frac{\partial f(\tilde{\theta}|\zeta)}{\partial \zeta}d\tilde{\theta} > 0$$

where the first element is positive from steps 1 and 2, and the second element is positive because ζ leads to a first order stochastic increase in $\tilde{\theta}$ and $U'(1-z+x(\tilde{\theta},\zeta))$ is decreasing in $\tilde{\theta}$. A similar reasoning shows that

$$\frac{\partial h_2}{\partial \zeta} = z\int_0^{\overline{\theta}} U''(1-x(\tilde{\theta},\zeta)+z)\frac{\partial x(\tilde{\theta},\zeta)}{\partial \zeta}dF(\tilde{\theta}|\zeta) + z\int_0^{\overline{\theta}} U'(1-x(\tilde{\theta},\zeta)+z)\frac{\partial f(\tilde{\theta}|\zeta)}{\partial \zeta}d\tilde{\theta} < 0.$$

Putting together the four inequalities just derived shows that the numerator is negative. The denominator takes the usual form, analogous to (34), and is negative. This completes the proof.

Partially constrained equilibria

To compute the equilibria in Section 4, it is useful to characterize the equilibrium behavior for: (i) economies with $\phi \in [0,1)$ and $\gamma \in (\beta, \hat{\gamma})$ where the liquidity constraint in period 2 is non-binding for some pairs $(\theta, \tilde{\theta})$ of anonymous households, and (ii) economies where the assumption $F(0|\zeta) > 0$ is relaxed, allowing for a binding liquidity constraint in period 1 for some θ, again for anonymous households. In equilibrium, the credit households behave exactly as in the first-best. To characterize the behavior of the anonymous households, it is sufficient to find prices and quantities solving the system formed by (6), the market clearing condition $c_1(\theta,\zeta) = \theta n(\theta,\zeta)$ for all θ, ζ, and equations

$$u'\left(c_1^M(\theta,\zeta)\right) = \max\left\{u'\left(\frac{1}{p_1^M(\theta,\zeta)}\right), \frac{p_1^M(\theta,\zeta)}{p_2^M(\zeta)}\int_0^{\overline{\theta}} U'(c_2^M(\tilde{\theta},\theta,\zeta))dF(\tilde{\theta}|\zeta)\right\} \text{ for all } \theta,\zeta,$$

$$c_2^M(\theta,\tilde{\theta},\zeta)=\min\left\{\frac{1}{p_2^M(\zeta)}-\frac{p_1^M(\tilde{\theta},\zeta)}{p_2^M(\zeta)}c_1^M(\tilde{\theta},\zeta)+\frac{p_1^M(\theta,\zeta)}{p_2^M(\zeta)}c_1^M(\theta,\zeta),U'^{-1}\left(\frac{p_2^M(\zeta)}{p_3}\right)\right\}\text{ for all }\theta,\tilde{\theta},\zeta,$$

$$\int_0^{\overline{\theta}}\int_0^{\overline{\theta}}c_2^M(\theta,\tilde{\theta},\zeta)dF(\theta)dF(\tilde{\theta})=e_2\text{ for all }\zeta,$$

$$\frac{1}{p_3}=\beta\gamma^{-1}\int_{\underline{\zeta}}^{\overline{\zeta}}\int_0^{\overline{\theta}}\frac{u'(c_1^M(\theta,\zeta))}{p_1^M(\theta,\zeta)}dF(\theta|\zeta)dG(\zeta).$$

The system is written in general form, allowing for cases where the constraints $m_1^M(\theta,\zeta)\geq 0$ and $m_2^M(\theta,\tilde{\theta},\zeta)\geq 0$ are binding only for a subset of households.

Proof of Proposition 11

From expression (21) it follows that

$$\begin{aligned}\frac{\partial Y_1(\zeta,\xi)}{\partial\zeta} &= \sum_{i=C,M}\omega^i\int_0^{\overline{\theta}}y_1^i(\theta,\xi)\frac{\partial f(\theta|\zeta)}{\partial\zeta}d\theta,\\ \frac{\partial Y_1(\zeta,\xi)}{\partial\xi} &= \sum_{i=C,M}\omega^i\int_0^{\overline{\theta}}\frac{\partial y_1^i(\theta,\xi)}{\partial\xi}dF(\theta|\zeta).\end{aligned}$$

In the case of an unconstrained equilibrium, the analogue of Proposition 8 can be easily derived, showing that $\partial y_1^i(\theta,\xi)/\partial\xi=0$ and $\partial y_1^i(\theta,\xi)/\partial\theta>0$ for $i=C,M$. These properties imply that $\partial Y_1(\zeta,\xi)/\partial\zeta>0$ and $\partial Y_1(\zeta,\xi)/\partial\xi=0$.

Next, consider a fully constrained equilibrium, where $\phi=0$ and $\gamma\geq\hat{\gamma}$. For each value of ξ, the functions $p_1^M(\theta,\xi)$ and $y_1^M(\theta,\xi)$ can be derived solving the following system of functional equations, analogous to (17)-(18):

$$\begin{aligned}u'(y_1^M(\theta,\xi)) &= p_1^M(\theta,\xi)\int_0^{\overline{\theta}}U'\left(c_2^M(\tilde{\theta},\theta,\xi)\right)dF(\tilde{\theta}|\xi,\theta),\\ v'\left(y_1^M(\theta,\xi)/\theta\right) &= \theta p_1^M(\theta,\xi)\int_0^{\overline{\theta}}U'\left(c_2^M(\theta,\tilde{\theta},\xi)\right)dF(\tilde{\theta}|\xi,\theta),\end{aligned}$$

where

$$c_2^M(\tilde{\theta},\theta,\xi)=1-p_1^M(\theta,\xi)y_1^M(\theta,\xi)+p_1^M(\tilde{\theta},\xi)y_1^M(\tilde{\theta},\xi).$$

The only formal difference between these and (17)-(18) is that the distribution $F(\tilde{\theta}|\xi,\theta)$ depends also on θ. However, this does not affect any of the steps of Proposition 9 (there is only a minor difference in the proof of the analogue of Lemma 3, details available on

request). Therefore, this system has a unique solution for each ξ. Next, following the steps of Propositions 9 and 10, we can show that $y_1^M(\theta, \xi)$ is increasing in θ and ξ. This implies that $\partial Y_1(\zeta, \xi)/\partial\zeta > 0$ and $\partial Y_1(\zeta, \xi)/\partial\xi > 0$.

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