

A Search and Learning Model of Export Dynamics

(Preliminary and Incomplete)

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1 Introduction

A goal of policy in many developing countries is establishing new export markets for nontraditional exports. By finding new buyers abroad governments hope to create jobs, augment demand for their currency, and further industrial development. Well-known success stories from Latin America include Brazilian regional jets, Chilean wines, and Colombian cut flowers.

Despite widespread interest in the determinants of industry-specific export surges, economists have yet to develop an analytical framework that reliably explains this phenomenon. Traditional gravity models, which relate bilateral trade flows to the GDP's of trading partners and the distances between them, focus on long run determinants of aggregate bilateral export flows, and are poorly-suited for the analysis of short-term industry-specific export dynamics.¹ Sunk-cost hysteresis models, which emphasize the start-up costs that new exporters face, help us explain patterns of foreign market entry and exit by individual firms (Dixit, 1989; Baldwin and Krugman, 1989; Das, et al., 2007). But aside from exchange rate effects, they treat the export volumes of firms that successfully penetrate foreign markets as determined outside the model. They thus provide little guidance as to why some exporters are able to expand their foreign sales rapidly while others struggle.

This paper considers export booms as reflecting a process of search and learning in a foreign market. That is, producers who are interested in a particular foreign market devote resources to identifying potential buyers there. When they find one, they learn something (receive a noisy signal) about the appeal of their products in this market. They also learn

¹Recent contributions to the gravity literature include Helpman et al. (2008) and Anderson and van Wincoop (2003). Deardorff (1998) provides a survey of the earlier literature.

about the scope for profits by observing the experiences of rival sellers of similar products in the same foreign market. Taking stock of the available information, firms initially not selling there update their beliefs about the scope for export profits, and they adjust the intensity of their search efforts accordingly, attempting to maximize their net expected profit streams. Export booms take place when home-market firms receive positive early signals about the scope for profits, both from their own experiences and from the experiences of rivals, and they intensify their search and marketing efforts, adding quickly to their foreign client base.

While our immediate goal is a better understanding of the dynamics of export surges, another is to develop a theory that can confront data on individual export transactions. Over the last decade and a half trade economists confronted new datasets on the exports of individual firms. Dealing with these data forced them to develop new models of international trade consistent with basic features of these new data. More recently, data on the individual transactions of firms have started to become available for research, posing new challenges for trade theory. For one thing, these data are lumpy in that they report individual export shipments occurring at a particular date rather than cumulative export flows over a longer interval of time. For another, they require a dynamic framework to link sales at different dates to the same and to different buyers.

The direct motivation for this paper comes from a descriptive analysis of a decade's worth of individual merchandise shipments from Colombia to the United States. We begin by reviewing the stylized facts that come out of this analysis, including a number of findings that we have not reported in our work (Eaton, et al., 2008a, henceforth EEKT). Then we introduce our model, discuss its calibration, and demonstrate that, with the framework outlined in the

previous paragraph, we are able to explain the basic features of the shipments data.

2 Firm-Level Trade: Transaction Level Evidence

The empirical motivation for our model comes from two sources. The first is a comprehensive data set that describes all shipments from Colombia to the United States (and elsewhere) that passed through Colombian customs during the period 1996-2005. Each customs record includes a date, the U.S. dollar value of the product shipped, a 6-digit harmonized system product code (augmented by additional product information), a quantity index, a seller ID code, and the location of the buyer.² The second data base provides analogous information for the period 1992-2005. However it is based on U.S. Customs records, and it describes imports by buyers in the United States from Colombian exporters (as well as other sources). Critically, in addition to providing the information contained in the Colombian records, the U.S. customs data include ID codes for both sellers and buyers. It therefore allows us to identify the formation and dissolution of business relationships between individual buyers in the U.S. and sellers in Colombia.

²Because we use the same data that are used for official statistics, the merchandise exports in our data set aggregate to within one percent of total merchandise exports reported by the Colombian Bureau of Statistics (Departamento Administrativo Nacional de Estadística or DANE). The deviation is due to mistakes in the records of tax identifiers. Since following firms over time is central to our analysis, our database includes only records of transactions in which the tax identifier has the appropriate format. Not satisfying this requirement is a clear indication that the firm is not correctly identified in the record.

2.1 Evidence from Colombian Customs Records

Following Brooks (2006) and EEKT, Table 1 provides various annual measures of Colombian exports to the United States for the years 1996-2007.³ Each column follows an exporting cohort, i.e., a group of firms that began exporting in a particular year, after at least one year of no exporting, from the year of its appearance through time. (Since we don't know the history of firms before 1996, the 1996 "cohort" consists of all firms present that year regardless of when they began exporting.) The panels of the Table report number of exporters, total exports, and exports per firm, and transactions per firm, respectively.

Consider Panel A first. Firms can drop out of the U.S. market and later reappear, which is why the number of firms in a cohort occasionally rises. But usually each cohort's membership falls as it matures. Note that there is very high attrition the first year, with at least half and up to three-fourths of firms dropping out. Conditional on making it to the second year, the survival probability is much higher, however, with an average attrition rate around 20 percent. Thus, in terms of numbers, the most recent cohort is always larger than any previous one (except for the 1996 "cohort," which is a special case). Note that by the end of the period, firms that were around in 1996 constitute only about one in seven of firms exporting to the United States.

Panel B shows that, despite the rapid initial decline in its membership, the total sales of a cohort tends to rise over time, although quite unevenly. By the end of the period the 1996 cohort contributes about 76 percent of total sales, despite their relatively small number. The 2005 cohort contributes the second largest share.

³Similar tables for Colombian exports to all destinations combined appear in EEKT.

Table 1. Firms that exported to the United States by initial export year cohorts, 1996-2007

A. Number of firms													
First year of report between 1996 and 2007													
Year	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	Total
1996	3129	0	0	0	0	0	0	0	0	0	0	0	3129
1997	1390	1646	0	0	0	0	0	0	0	0	0	0	3036
1998	1034	257	978	0	0	0	0	0	0	0	0	0	2269
1999	896	169	309	834	0	0	0	0	0	0	0	0	2208
2000	848	159	238	317	991	0	0	0	0	0	0	0	2553
2001	769	151	216	253	356	1128	0	0	0	0	0	0	2873
2002	743	133	187	218	266	403	1253	0	0	0	0	0	3203
2003	744	135	170	191	246	335	419	1635	0	0	0	0	3875
2004	700	129	148	191	221	262	343	491	1894	0	0	0	4379
2005	672	114	112	142	180	219	239	316	416	2218	0	0	4628
2006	620	116	113	145	160	178	206	265	300	551	1727	0	4381
2007	572	102	87	130	148	152	170	211	246	413	369	1358	3958
B. Value of exports (US\$ Millions)													
First year of report between 1996 and 2007													
Year	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	Total
1996	4145	0	0	0	0	0	0	0	0	0	0	0	4145
1997	4115	147	0	0	0	0	0	0	0	0	0	0	4262
1998	3685	196	169	0	0	0	0	0	0	0	0	0	4049
1999	5083	299	135	98	0	0	0	0	0	0	0	0	5615
2000	5934	269	92	170	63	0	0	0	0	0	0	0	6527
2001	4637	211	87	133	117	69	0	0	0	0	0	0	5254
2002	4576	153	75	115	84	97	52	0	0	0	0	0	5151
2003	4813	296	88	101	124	99	90	167	0	0	0	0	5779
2004	5437	264	107	136	140	104	92	183	135	0	0	0	6598
2005	6529	206	122	155	191	163	115	177	240	581	0	0	8480
2006	7670	279	122	141	220	175	123	148	220	499	54	0	9650
2007	7883	233	107	153	255	177	119	167	208	560	45	466	10373
C. Exports per firm (US\$ Thousands)													
First year of report between 1996 and 2007													
Year	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	Total
1996	1325	0	0	0	0	0	0	0	0	0	0	0	1325
1997	2961	89	0	0	0	0	0	0	0	0	0	0	1404
1998	3563	761	173	0	0	0	0	0	0	0	0	0	1785
1999	5673	1768	438	117	0	0	0	0	0	0	0	0	2543
2000	6997	1691	386	536	64	0	0	0	0	0	0	0	2557
2001	6030	1395	402	527	330	61	0	0	0	0	0	0	1829
2002	6159	1149	399	529	316	240	41	0	0	0	0	0	1608
2003	6470	2189	520	530	502	296	216	102	0	0	0	0	1491
2004	7767	2048	722	710	633	398	270	373	71	0	0	0	1507
2005	9716	1805	1093	1095	1059	744	483	560	577	262	0	0	1832
2006	12370	2405	1083	970	1373	985	595	557	735	906	31	0	2203
2007	13782	2288	1235	1177	1723	1163	703	791	844	1357	121	343	2621
D. Number of Transactions													
First year of report between 1996 and 2007													
Year	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	Total
1996	54001	0	0	0	0	0	0	0	0	0	0	0	54001
1997	49243	5624	0	0	0	0	0	0	0	0	0	0	54867
1998	47759	6045	5335	0	0	0	0	0	0	0	0	0	59139
1999	48994	5614	5234	4604	0	0	0	0	0	0	0	0	64446
2000	50854	6284	4149	5760	4694	0	0	0	0	0	0	0	71741
2001	52701	6276	4207	5116	5309	6854	0	0	0	0	0	0	80463
2002	53439	4511	4146	4815	4569	8144	5140	0	0	0	0	0	84764
2003	82886	6364	5344	6762	6366	8024	7933	8774	0	0	0	0	132453
2004	79122	7254	4929	7264	7190	6807	8229	7736	8822	0	0	0	137353
2005	98822	11052	5148	8575	7196	8283	8995	9014	9750	15344	0	0	182179
2006	101616	12862	4547	8370	6251	6647	8966	7533	10171	15670	6563	0	189196
2007	104549	11993	4165	8458	5709	6848	8860	7144	8801	16574	6544	4999	194644

Notes: Colombian firms are classified according to the first year in which they reported exporting to the United States in our sample period (1996-2007). The 1996 "cohort" includes all firms exporting that year regardless of when they began exporting.

The decline in number of firms per cohort along with their increasing contribution to total sales means, of course, that sales per firm are growing substantially, as shown Panel C. In fact, export sales for young survivors in each cohort tend to grow far more rapidly than total export sales, so that cohorts' market shares tend to rise *despite* rapid attrition during their early years. Finally, note that cohort size and success (in terms of survival and sales) vary substantially across entry years. For example, the 2005 cohort appears very robust both in terms of number of exporters and exports per firm, with 2006 weak by comparison. This suggests that entry selection mechanisms vary over time in response to market-wide forces.

Together Panels A through C tell us about the role of the extensive and intensive margins in the growth of Colombian exports to the United States. Over the period total exports grew by a factor of about 2.5 but the number of exporters by a factor of only about 1.3, implying that exports per firm just about doubled. Hence much more of the action was in the amount a given exporter sold in the United States than in the number of sellers.

Finally Panel D reports the total number of export transactions, which grew by a factor of 3.6. Hence transaction sizes were shrinking over the period (by about 25 percent). Interestingly for the 1996 cohort the transaction size stayed about the same while the number of transactions per firm grew enormously. Across cohorts growth is mostly in the number of transactions rather than in transaction size.

2.2 Evidence from U.S. Customs Records

Individual buyers and sellers are identified in the transaction level data collected by the United States Census Bureau. Accordingly, this dataset allows us to keep track of how many buyers

each Colombian exporter is shipping to, and to see when buyers are dropped or added. We next use these data to characterize the buyer-seller matchings that took place during our sample period of 1992-2005. The Table below provides some summary statistics:

Table 2

	Colombian Exporters	U.S. Importers	Exporter-Importer Pairs
start	3,742	1,265	5,297
end	5,297	2,214	8,046

The number of Colombian exporters appearing in the sample grew from 3,742 in 1992 to 5,297 in 2005, a growth of 3.5 per cent per year, while the number of U.S. importing firms grew by 4.4 percent. The number of Colombian exporter-U.S. importer pairs (representing at least one transaction between them in a year) grew at an annual rate of 3.3 percent. A typical Colombian exporter was involved in around 1.4 relationships with U.S. firms while a U.S. buyer was involved in around 4 relationships with Colombian firms. Both figures declined slightly over the period.

Most relationships are very short-lived. Of the buyer-seller matches that existed at the beginning of the period, 47 percent didn't make it to 1993. But of those that survived into that year, almost 70 percent made it into the year after that. Similarly, of the relationships that existed in 2005, 48 percent started that year, but of those that started the previous year, 65 percent had been around at least 3 years before. Of the matches present in 1992, only 85 endure (are present every year) throughout the period.

2.2.1 Transition Probabilities

Table 4 reports the probability with which a Colombian firm participating in certain number of relationships with buyers transits into different number of relationships the following year. This table reports the annual average for 1992-1997 across all industries. Numbers for later periods are very similar. Thus, of firms not exporting to the United States at all in year t but that do export in year $t + 1$, 92.5 percent sell to only one U.S. firm, etc. Of those that sell to one U.S. buyer in a year, 63 percent don't export the next year, while only about 6 percent go on to establish a larger number of relationships. For firms with two relationships in a year, about 14 percent enter into a larger number of relationships, etc. Hence there is an enormous amount of churning at the lower end. Even for firms with a large number of relationships the most likely outcome is to have fewer the next year.

Table 4. Transition Probabilities: Number of Clients

t+1\ t	0	1	2	3	4	5	6-10	11-25
0	0.000	0.630	0.265	0.153	0.050	0.024	0.039	0.000
1	0.925	0.310	0.344	0.246	0.131	0.079	0.039	0.000
2	0.056	0.046	0.244	0.222	0.202	0.211	0.087	0.000
3	0.012	0.010	0.096	0.186	0.223	0.168	0.082	0.000
4	0.004	0.003	0.031	0.116	0.165	0.184	0.117	0.000
5	0.002	0.000	0.012	0.045	0.108	0.105	0.169	0.380
6-10	0.002	0.000	0.004	0.016	0.113	0.205	0.429	0.620
11-25	0.000	0.000	0.004	0.016	0.009	0.024	0.039	0.000
sum	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

We can ask what this pattern of entry and growth implies about the ergodic distribution of relationships. If we assume that the number of entrants in a year replace exiters to the extent that the overall number of firms rises by 3.5 percent a year, the ergodic distribution implied by this transition matrix is given by:

Table 4

	1	2	3	4	5	6 to 10	11 to 25
ergodic	.809	.109	.039	.019	.010	.013	.002
period average	.800	.114	.041	.020	.010	.012	.003

For purposes of comparison we report the average annual share of Colombian firms in each group over the period as well. Note that the ergodic distribution implied by the transition matrix is very close to the distribution in the data.

3 A Model of Exporting at the Transactions Level

The tables above describe exports from a very different perspective than the standard one. Rather than considering exports as a flow over some interval of time, we are observing individual buyer-seller relationships with intermittent activity. Existing trade models, including those emanating from the emerging literature on the export activity of individual firms, model sales as flows to specific markets rather than as a discrete set of business relationships. (Arkolakis, 2008, is an exception.)

We propose a model that is consistent with the patterns documented in the previous section, and that provides new micro foundations for export dynamics. It explains firm-specific export adjustments on three margins: clients (buyers) per destination market, per-period sales per client, and duration of the buyer-seller relationship. The model is consistent with several key patterns documented above: (1) many new exporting firms appear each period; (2) most new exporters sell tiny amounts and disappear from export markets in the following period;

(3) those exporters who survive expand their export volume very rapidly over the following period, often accumulating additional buyers; (4) firms that sell more initially are more likely to survive into the following period.⁴ It also explains (5) inter-temporal fluctuations in the size of the entering cohort, and (6) market-wide and relationship-specific fluctuations in per-period sales volumes.

The model builds on existing models of firm heterogeneity and exporting. As in Melitz (2003) and Bernard et al. (2003), firms are heterogeneous in terms of their underlying efficiency, with more efficient firms having a greater incentive to overcome trade costs to sell in foreign markets. As in Das et al. (2007) and Irarrazabal and Opromolla (2007), firms experience shocks to their efficiency that lead them to switch into or out of exporting. As in Arkolakis (2008), by incurring a larger fixed cost a firm can increase the number of buyers it can reach. Finally, as in Rauch and Watson (2003), firms are initially uncertain about how their product will be received in an export market.

What we add to these models is the following structure of decision making and learning. Suppose that before it enters an export market a firm is unsure of the appeal that its product has to buyers there. However, the firm can invest in activities that bring its product to the attention of individual buyers, such as advertising, participation in trade fairs, and maintenance of a foreign sales office. The more a firm spends on these activities, the more likely it will encounter a foreign buyer per unit of time. And when a match does occur, its sale not only generates a profit for the firm, it conveys information to the firm about its product's appeal in that market. On the basis of this information the firm updates its beliefs about its product's ultimate chances for success in that market. Good news means that future matches

⁴See EEKT (2008).

are likely to be more profitable, so it strengthens its efforts to encounter buyers, while bad news discourages the firm from putting in so much effort.⁵

3.1 Profits

To characterize the profit flow, consider firm j with an efficiency φ_{jt} (taking into account transport costs) at time t . This efficiency is known to the firm and evolves over time with idiosyncratic shocks. Given that it pays a wage (or more generally, unit input price) w_t it can produce at cost w_t/φ_{jt} in terms of local currency. If the exchange rate is e_t , its unit cost in the foreign market is $e_t w_t/\varphi_{jt}$. So assuming that all foreign buyers have Dixit-Stiglitz preferences with known demand elasticity η , seller j offers price:

$$p_{ijt} = \frac{\eta}{\eta - 1} \frac{e_t w_t}{\varphi_{jt}} \quad (1)$$

to any foreign buyer i with whom it matches.⁶

If potential buyer i is confronted with an opportunity to purchase firm j 's product, that is, if j matches with i , j 's period t sales to i are:

$$x_{ijt} = \exp(z_j + \epsilon_{ij}) \left(\frac{p_{ijt}}{P_t} \right)^{1-\eta} \bar{X}_t. \quad (2)$$

⁵Related to our work is Ruhl and Willis's (2008) recent model of export dynamics in which firms experience productivity shocks that may cause them to enter or to exit exporting. To capture the observation that exports tend to grow over time from an initially small level they introduce demand shifters that vary with years of export experience.

⁶For simplicity we assume that the firm makes a take-it-or-leave-it price offer. An alternative specification would introduce bilateral bargaining between buyer and seller, although the seller's uncertainty about the buyer's evaluation of the product renders this second approach substantially more complicated.

Here z_j is a product appeal index that is common to all foreign buyers but is initially unknown to the firm. Whatever its true product appeal we assume that every firm starts out with a prior that is distributed $N(0, \sigma_z^2)$. The term ϵ_{ij} is an idiosyncratic product appeal shock that is distributed i.i.d. $N(0, \sigma_\epsilon^2)$ across foreign buyers, \bar{X}_t is the average spending level among potential buyers, and P_t is the relevant price index for all competing products in the export market.⁷

The flow profit in exporter currency implied by (1) and (2) is:

$$\pi(P_t, P_{Ct}, e_t, z_j, \epsilon_{ij}, \varphi_{jt}) = \frac{1}{\eta} \frac{\bar{X}_t}{e_t P_t^C} \exp(z_j + \epsilon_{ij}) \left(\frac{e_t w_t \eta / (\eta - 1)}{\varphi_{jt} P_t} \right)^{1-\eta}, \quad (3)$$

where P_t^C is the price level in the home country (in our case Colombia). Or, combining all the aggregate variables and constants:

$$\pi(X_t, z_j, \epsilon_{ij}, \varphi_{jt}) = X_t \exp(z_j + \epsilon_{ij}) \varphi_{jt}^{\eta-1} \quad (4)$$

where:

$$X_t = \frac{1}{\eta} \frac{\bar{X}_t}{e_t P_t^C} \left(\frac{e_t w_t \eta / (\eta - 1)}{P_t} \right)^{1-\eta}.$$

The term X_t summarizes all the macroeconomic information about the export market (i.e., information that applies to any seller j from C when matched to any buyer i). We can thus characterize the aggregate state of demand in the export market with X_t . We assume that X and φ evolve over time according to a Markov process, so that given (X_t, φ_{jt}) in period t , the period $t + 1$ values have a joint distribution $G(X', \varphi' | X_t, \varphi_{jt})$. The difference between X

⁷Not all buyers necessarily face the same range of goods and hence the same aggregate price index P_t . We treat idiosyncratic components of the price index as P_t as reflected in ϵ_{ijt} .

and φ is that the first applies to all firms while the second is idiosyncratic to a specific firm. Hence the first generates behavior that is correlated across firms while the second can vary across firms.

For purposes of the dynamic optimization problem to be introduced below, it will be convenient to define $\tilde{\pi}(X_t, z_j, \varphi_{jt})$ as the expected present value of firm j 's entire profit stream associated with a match as perceived at time t . That is, $\tilde{\pi}(X_t, z_j, \varphi_{jt})$ is the discounted expected future value of the $\pi(X_t, z_j, \epsilon_{ij}, \varphi_{jt})$ trajectory, with expectations taken over ϵ_{ij} and the future trajectory of (X_t, φ_{jt}) . The value of $\tilde{\pi}(X_t, z_j, \varphi_{jt})$ depends on the firm's discount rate r and the hazard of separating from a particular buyer, which we treat as occurring at the exogenous rate δ . Thus $\tilde{\pi}(X_t, z_j, \varphi_{jt})$ solves:

$$\begin{aligned} \tilde{\pi}(X_t, z_j, \varphi_{jt}) &= X_t [\exp(z_j + \sigma_\epsilon^2/2)] \varphi_{jt}^{\eta-1} \\ &\quad + \frac{1-\delta}{1+r} \int_{X'} \int_{\varphi'} \tilde{\pi}(X', z_j, \varphi') dG(X', \varphi' | X_t, \varphi_{jt}). \end{aligned}$$

This expression gives the value to firm j of matching with a buyer at time t , conditioned on macro conditions, X_t , the firm's product appeal index, z_j , and productivity, φ_{jt} . Note that, having met this buyer and imputed $z_j + \epsilon_{ij}$ (as discussed below), there is nothing more for firm j to learn from her about product appeal since we assume that ϵ_{ij} is fixed over time. Nevertheless its sales to this buyer will continue to fluctuate in response to macroeconomic shocks X and shocks to the firm's efficiency φ .

3.2 Information about product appeal

Firm j knows neither z_j nor ϵ_{ij} individually, but each time the firm matches with another buyer it learns more about z_j . That is, knowing the macro state (X_t) , the price index (P_t)

and its own price (p_{jt}), firm j can compute the signal:

$$s_{ij} = z_j + \epsilon_{ij}$$

of its product appeal in foreign markets. Combining this signal with the signals it inferred from earlier matches, and with the prior beliefs $N(0, \sigma_z^2)$ it held about z_j before any matches occurred, firm j calculates the posterior distribution of z_j to be $N(\widehat{z}_j^n, \sigma_n^2)$, where:

$$\widehat{z}_j^n = z_j^0 \frac{\sigma_z^{-2}}{\sigma_z^{-2} + n\sigma_\epsilon^{-2}} + \bar{s}_j^n \frac{n\sigma_\epsilon^{-2}}{\sigma_z^{-2} + n\sigma_\epsilon^{-2}}, \quad (5)$$

and:

$$\sigma_n = (\sigma_z^{-2} + n\sigma_\epsilon^{-2})^{-1/2}, \quad (6)$$

n is the number of matches the firm has experienced, and $\bar{s}_j^n = n^{-1} \sum_{i=1}^n s_{ij}$.

This characterization of Bayesian learning generalizes to allow for correlation between a firm's product appeal in the foreign market and (1) its product appeal at home (d), or (2) the average appeal of rival (domestically-based) firms' products in the foreign market (y). To do the first, one can incorporate signals from domestic sales, $s_{jt}^d = \beta_d z_j + \epsilon_{jt}^d$, where $t = 1, \dots, T$ indexes the periods that the firm has been in operation. To do the second, one can incorporate signals from the foreign sales of rival firms in the same industry $s_{kt}^y = \beta_y z_j + \epsilon_{jkt}^y$, where $k = 1, \dots, n_{yt}$ indexes the number of such rival matches that the firm has observed as of time t . These modifications lead to straightforward generalizations of (5) and (6). Among other things, they generate learning spillovers that accelerate aggregate export responses to positive early experiences in new foreign markets.

3.3 Search intensity

As information accrues to a seller about foreign buyers' demand for its product, it adjusts the intensity with which it searches for new buyers. Let the firm experience new matches with hazard λ when it spends $c(\lambda)$ on search activities, where $c(\cdot)$ is increasing and convex.⁸ Then if the firm has received an average signal of \bar{s}_n after n encounters, the value of continued searching in the foreign market is $V(\hat{z}_j^n, n, X_t, \varphi_{jt})$, where:

$$\begin{aligned}
 & V(\hat{z}_j^n, n, X_t, \varphi_{jt}) \tag{7} \\
 = & \max_{\lambda} \left\{ -c(\lambda) + \lambda \int_z \tilde{\pi}(X_t, z, \varphi_{jt}) dF(z|\hat{z}_n, n) \right. \\
 & + \frac{1-\lambda}{1+r} \int_{X'} \int_{\varphi'} V(\hat{z}_j^n, n, X', \varphi') dG(X', \varphi'|X_t, \varphi_{jt}) \\
 & \left. + \frac{\lambda}{1+r} \int_{X'} \int_{\varphi'} \int_{\hat{z}'} V(\hat{z}', n+1, X', \varphi') d\Phi(\hat{z}'|\hat{z}_j^n) dG(X', \varphi'|X_t, \varphi_{jt}) \right\}. \tag{8}
 \end{aligned}$$

Here r is the discount rate, $F(z|\hat{z}_n, n)$ is the posterior distribution for z after the n^{th} match, and $\Phi(\hat{z}'|\hat{z}_j^n) = N(\hat{z}_j^n, \sigma_{n+1})$ is the posterior distribution for z that the firm expects to prevail after the $n+1^{\text{st}}$ match, given \hat{z}_n .

Three margins of firm-level export response are characterized by this value function. First, average sales per foreign transaction at time t (\bar{x}_{jt}) are determined by product appeal (z_j),

⁸Following Arkolakis (2008), if we think that the market has M potential buyers and sampling occurs without replacement we can generalize the hazard rate to be $\tilde{\lambda} = \lambda \cdot h(n)$ where $h(n)$ is decreasing in n , bounded on $[0,1]$, and $h(M) = 0$. For example, if the probability of a match is proportional to the pool of potential buyers who have not yet been visited, this function might take the form: $h(n) = \frac{M-n}{M}$. Working against this effect is the possibility that as matches accumulate, a firm's reputation grows, and it becomes *less* costly to reach new customers. Hence a general expression for $h(n)$ that does not impose a sign on its derivative may be the most appropriate formulation. If this function is identified, it provides a test of Arkolakis (2008).

productivity (φ_{jt}), and macro conditions (X_t). Second, buyers per firm are governed by the search intensity λ , which responds to sales history (x_{ijt-n}). (The case of zero buyers corresponds to non-participation in export markets, of course, but zero buyers need not imply $\lambda = 0$.)

3.4 Stationary State

We consider an environment where firms are buffeted by shocks to their macroeconomic environment and to their own productivity, and in which they start out ignorant of their product appeal abroad, but learn about it over time. Hence some key variables in our model are highly nonstationary, and we need to use numerical techniques to characterize their transition dynamics. Nevertheless it is useful to consider what happens in a stable environment in which all learning has taken place.

Say that $(X_t, \varphi_{jt}) = (X, \varphi_j)$ and $n \rightarrow \infty$ so that $\bar{s}_n \rightarrow z$ and new matches convey no further information. Asymptotically, the distinction between $V(\hat{z}_n^j, n, X, \varphi_j)$ and $V(\hat{z}', n+1, X, \varphi_j)$ disappears, and the problem becomes $rV(z) = \max_{\lambda} \{-c(\lambda) + \lambda \tilde{\pi}(X, z_j, \varphi_j)\}$. The solution is:

$$V(z_j) = \frac{-c(\lambda^*) + \lambda^* \tilde{\pi}(X, z_j, \varphi_j)}{r},$$

where λ^* solves $c'(\lambda^*) = \tilde{\pi}(X, z_j, \varphi_j)$. So, not surprisingly, steady state search efforts and the present value of participating in foreign markets are monotonically increasing in the payoff to a successful match. As in Arkolakis (2008), more efficient firms (with higher φ_j) undertake more search effort and encounter more buyers. However, in our model, firms learn about their product appeal as they acquire buyers. They adjust their search intensity accordingly, and

they lose buyers over time as matches go sour. In a stationary equilibrium, with no macro or idiosyncratic shocks, and after all learning has taken place, firms settle into constant search intensities. If firm j chooses match hazard λ_j^* in this stationary equilibrium, it sells to an average number of buyers $n(j)$ that satisfies $\delta n(j) = \lambda_j^*$.

3.5 Specification for Numerical Solution

To solve the problem numerically we parameterize the cost of matching as:

$$c(\lambda) = b \left(\frac{\lambda}{1-\lambda} \right)^2 + f \cdot 1[\lambda > 0], \quad \lambda \in [0, 1) \quad (9)$$

where f is the fixed cost of maintaining positive levels of search. Finally, we (arbitrarily) choose parameter values to be as indicated in Table 1. We treat shocks to efficiency and macroeconomic shocks as following independent first-order autoregressive processes, so that:

$$\varphi_{jt+1} = \psi \varphi_{jt} + \epsilon_t^\varphi \quad (10)$$

$$X_{t+1} = \phi X_t + \epsilon_t^X$$

where:

$$\ln \epsilon_t^\varphi \sim N(0, \sigma_\varphi^2) \quad (11)$$

$$\ln \epsilon_t^X \sim N(0, \sigma_X^2).$$

The model thus has four sources of variance: each firm's underlying true product appeal z , which is drawn from $N(0, \sigma_z^2)$, the noise around true product appeal associated with each match, which is drawn from $N(0, \sigma_\epsilon^2)$, the shocks to productivity, ϵ_t^φ , and the macroeconomic shock, ϵ_t^X , both given in (11).

The model is fully described by the expression for profit (3), from which we can calculate, using (10), the expected value of a relationship (??), the equation for updating beliefs about product appeal (5), the value function (7) and the cost function (9). The complete set of parameters of the model are given in the following Table:

Table: Parameters for Simulations				
<i>Parameter</i>		<i>base value</i>	<i>alternative calibration</i>	
rate of time preference	r	0.05	0.05	
rate of separation	δ	0.25	0.20	
search cost function scale parameter	b	0.20	0.20	
profit function scale parameter	c	0.00001	0.05	
fixed cost of searching	f	0.01	0.02	
standard deviation of noise in signal	σ_ϵ^2	1.67	0.30	
standard deviation of product appeal	σ_z^2	1.76	0.30	
root of efficiency process	ϕ	0.90	0.90	
root of macro process	φ_X	0.80	0.80	
standard deviation of efficiency innovation	σ_ϕ^2	0.10	0.10	
standard deviation of macro innovation	σ_X^2	0.40	0.16	

where we indicate the values we place on them in our baseline simulation.

3.6 Calibration

To calibrate our simulations, we draw parameter values from a variety of sources. The separation rate δ is based on match-specific survival rates among all Colombian exporters to the U.S. market. The root φ_X and innovation variance σ_X^2 for the market-wide shock are based on a simple AR1 fit to the Colombian real exchange rate. The root ϕ and variance σ_ϕ^2 are based on plant-level panel-based estimates of productivity processes among Colombian firms. The cross-buyer variance in product appeal for a given seller, σ_ϵ^2 , is estimated using the subsample of matches corresponding to sellers with more than one buyer, and is calculated as the residual variation in match value after controlling for buyer fixed effects. The variation in

product appeal effects, σ_z^2 , is calculated as the variation in match value (using the full sample of matches) less σ_ϵ^2 . The profit function scaling parameter is chosen to equate average match value to the average match value in our sample. Finally, for this draft, the cost function parameters b and f are chosen to generate dynamics that qualitatively replicate those reviewed in section 2 above.

Clearly, this calibration is very crude and preliminary. Among other problems, the estimates of σ_z^2 and σ_ϵ^2 are subject to selection bias, and they presume that firms do not distinguish sub-markets for particular types of products from each other. For example, an exporter of men’s shirts is presumed to face markets with the same characteristics as an exporter of flat-rolled steel. We therefore experiment with an alternative calibration in which we cut both variances by roughly a factor of 5. (This adjustment required an associated adjustment in the profit function scaler, c .) In future work, the model will be econometrically estimated.

3.7 Numerical Procedure

Our numerical solution proceeds in four steps:

3.7.1 Markov transition matrices for signals

With our parameterization learning occurs very fast. After encountering twenty buyers the seller’s uncertainty about its product’s true appeal has dwindled, according to (6), to having a standard deviation of .07. Hence we limit the number of encounters that convey information about product appeal to twenty, and treat the seller as fully informed about its own product appeal thereafter. For transitions across the first twenty periods we discretize the space of noisy signals into 50 signals evenly spaced across 3 standard deviations, and calculate the

Markov transition matrices across each possible pair, using (6).

3.7.2 Markov transition matrices for macroeconomic and efficiency processes

We follow a similar procedure of discretizing efficiency levels into 20 possible values and the macroeconomic states into 8 possible values. We then calculate the Markov transition probabilities across them. For purposes of our simulations we assume that all macro uncertainty comes from the exchange rate, and we fit an AR1 process to the dollar-peso exchange rate to impute transition probabilities.

3.7.3 Discretization of Effort

We allow for 50 possible effort levels across $[0, 1]$.

3.7.4 Value Function Iteration

We solve for the value function V and associated policy function for λ that solves (7) for different numbers of meetings n , profit signals i , efficiencies k , and macroeconomic shocks m .

3.8 Policy functions

The first panel of figure 1 shows the value of access to foreign buyers that firms perceive after one signal, as a function of the signal they have received. Not surprisingly, there is a positive relationship, and firms that receive better signals choose to search more intensively, so that in the panel below the probability of a match increases with a signal. Note that this probability falls to zero for any signal slightly above zero and below, indicating that the firm has given up.

The second panel of this figure shows how values and search intensities have changed after 5 signals have accrued. (The horizontal axis is the posterior mean after 5 signals, \hat{z}^5 .) Note that the value of search has fallen relative to its value after one signal for those firms with low average signals because these signals become increasingly precise as experience accumulates. (When five buyers tell you they don't care for your product, there is a good chance that your product has poor market potential.) The last panel of figure 1 Again, below some threshold signal, the return to search is less than the associated fixed cost (f), and so no search takes place. If f were to increase, this cutoff would shift to the right (not pictured).

Figure 2 shows how the policy function characterized in figure 1 translates into behavior for a simulated set of 1,000 firms. Here the horizontal axis is true z value rather than signal. The top panel describes match hazards for a new cohort of firms, none of which has received any signals yet. Since all firms share the same priors at this point there is no relationship between z values and search intensity. The variation in the match hazard arises from differences in their efficiency. Some firms don't search at all because their current productivity level is low. Turning to the bottom panel, after 5 periods, a relationship between z and search intensity emerges, but heterogeneity in behavior remains, given z . Thus firms know enough about their true z 's that it is reflected in their search intensity, but differences in the idiosyncratic features of the buyers (ϵ 's) that the exporters have randomly matched with still matter, as does their productivity.

Figure 3 depicts match probabilities as functions of exporters' productivity levels and the macro state. Not surprisingly, improvements in either encourage search. The shape of the function changes over time, however, as seen in figures 1 and 2. In particular, after 5 matches

have accrued, those firms with relatively low productivity have been convinced to stop looking for buyers, and the truncation point is particularly high when the macro state (i.e., the real exchange rate) is poor. Thus macro conditions may explain the cross-cohort variation in export market participation seen in Table 1 above.

3.9 Export Trajectories

To link this model back to export flows, we need simply keep track of match patterns, random productivity shocks, and random separation patterns for a simulated set of firms. For these simulations we use the actual real dollar-peso exchange rate, but we retain the assumption that firms base their beliefs about future exchange rate realizations on the AR1 process we have estimated. (That is, firms are assumed to have rational expectations, not perfect foresight.)

Figure 4 shows clients per exporter and total exports for 1,000 firms exploring a new market that appears in year 0. (Imagine a Caribbean island dismantling prohibitive trade barriers, and the population of Colombian producers commencing to learn about the islanders' demand for their products.) Although the number of exporters falls over time (not pictured), clients per surviving exporter and total exporters both rise. They climb particularly rapidly in the early years because the firms which have received positive signals tend to (1) be exporting relatively more, and (2) intensify their search as these signals accrue. Eventually firms learn their true z values and settle into a stable search intensity. This translates into a stable number of clients per firm. Total exports fluctuate much more than the number of clients due to exchange rate changes. Note that the pattern implies that for a new cohort expansion in total exports is very much on the extensive margin, but as the cohort matures fluctuations in

sales are much more on the intensive margin.

Figure 5 shows the five year transition densities for numbers of clients, providing a simulated analog to Table 3 above. The top panel describes transitions between year 1 and year 5, when firms are in their early learning stages. Because they have not yet built their clientele, most of the action involves movement between 0, 1 and 2 clients. After 5 periods have elapsed, many firms that have low product appeal have dropped out, and the remaining firms have built larger client bases (bottom panel).

The export aggregates associated with this new market exploration are depicted in the panels of figure 6. Each of these graphs also includes the simulated exchange rate series, which appears as the lower line.⁹ The first panel indicates that during the early years, exchange rate effects are dominated by learning effects, but eventually, exports start to track the exchange rate, both because shipment values depend upon the exchange rate and because search intensities increase when the Colombian peso depreciates (second panel and fourth panel). The number of exporters is not very sensitive to the exchange rate because matches last for multiple periods, and exporters have no incentive to terminate clients when appreciation occurs. These features of the model induce a kind of hysteresis in trade flows. However, instead of attributing irreversibilities to one-time market entry costs, as in the existing literature, we attribute irreversibilities to the fact that devaluation induces learning but appreciation does not induce forgetting.

Table 1 above defined the year t cohort to consist of all firms who exported in period t but did not export in period $t - 1$. Applying this definition to our simulated data, we obtain figure

⁹Note that the units differ across graphs. The exchange rate process used in this experiment is obtained by fitting a simple AR1 to real Colombian peso-dollar rate (1982-2007).

7. The four panels correspond to the cohorts that enter in years 2, 3, 4 and 5. Consistent with table 1, our model shows that membership in each cohort drops off rapidly after its first year. (Refer to the lower line each graph.) However, total exports go through a growth period as those cohort members who survive add to their client base, and thus exports per surviving cohort member climb rapidly in the early years of each cohort's existence. This, too, matches up well to the actual data.

4 Econometric estimation (to come)

5 Conclusions (to come)

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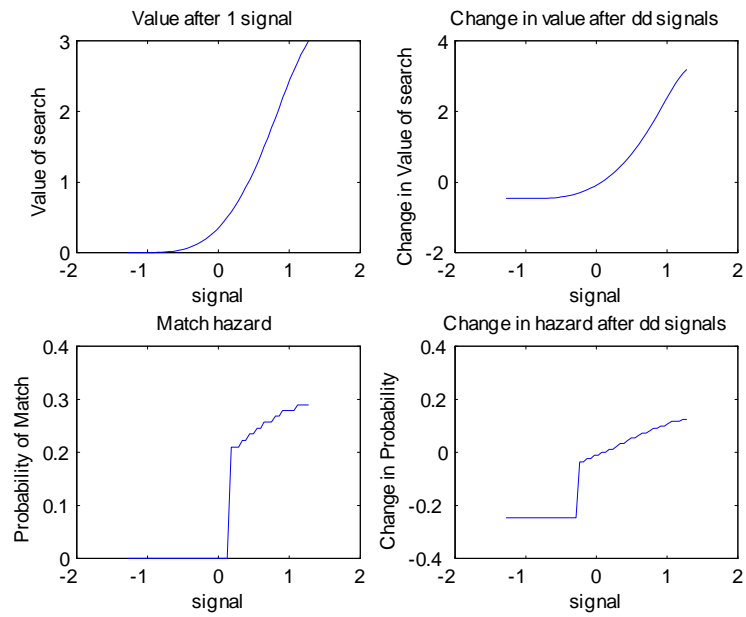
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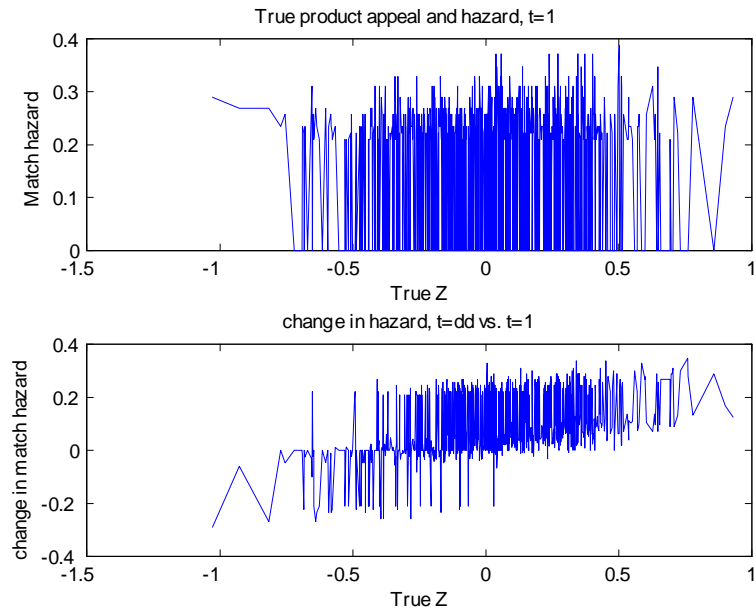


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Figure 1:

Signal, value, match hazard, and learning

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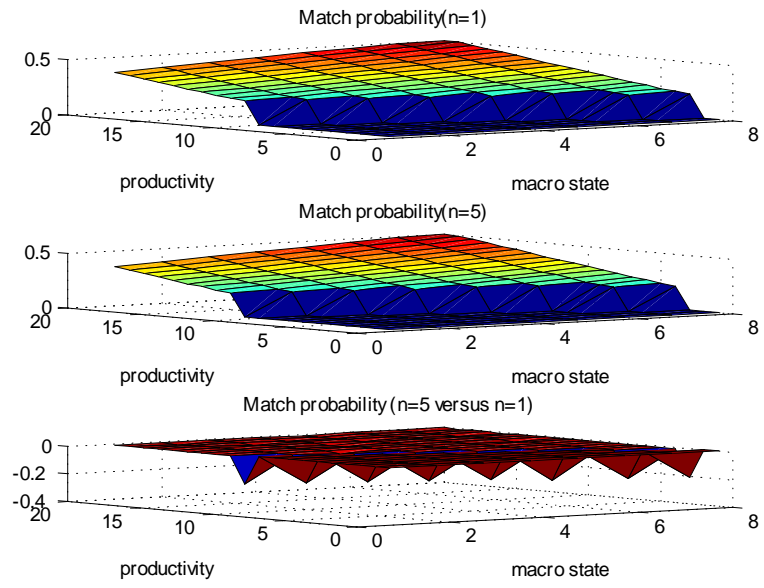


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Figure 2:

True product appeal and match hazard: initial and change after 5 signals

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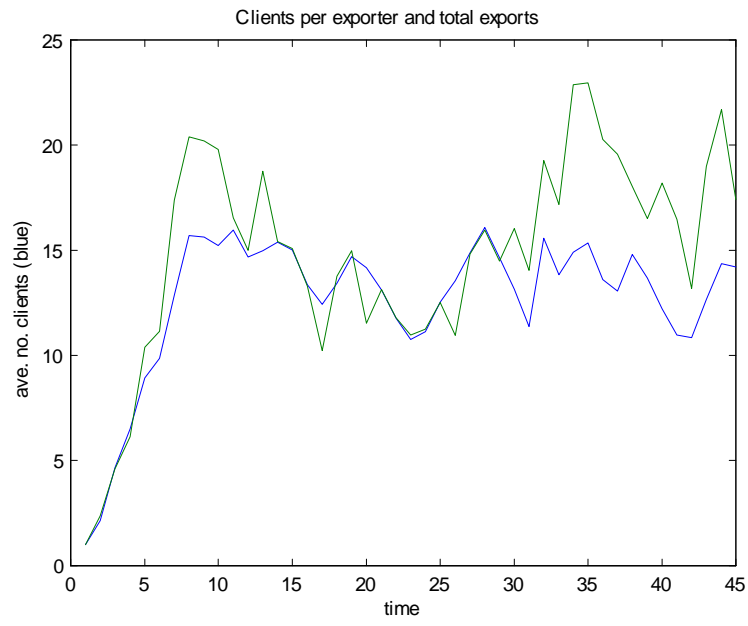


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Figure 3:

Match Probabilities, firm productivity, and macro state

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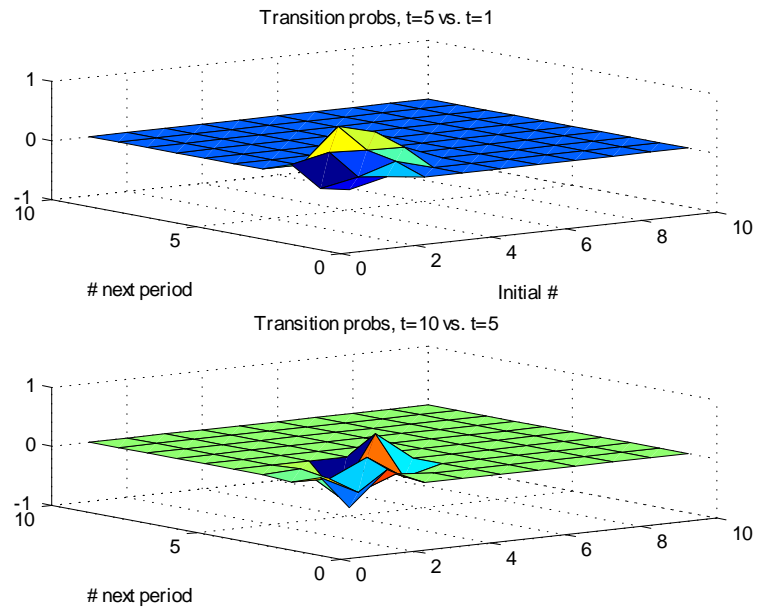


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Figure 4:

Clients per exporter and total exports: new cohort through time

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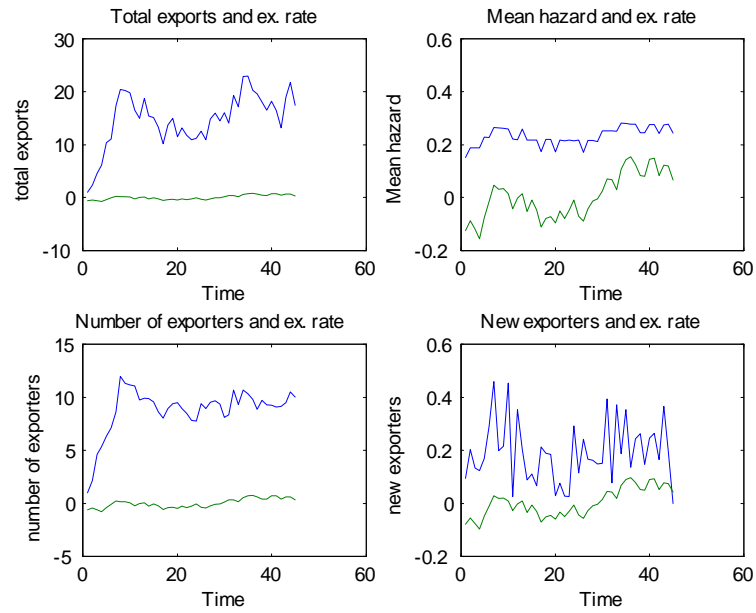


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Figure 5:

5-period transition probabilities (conditional densities)

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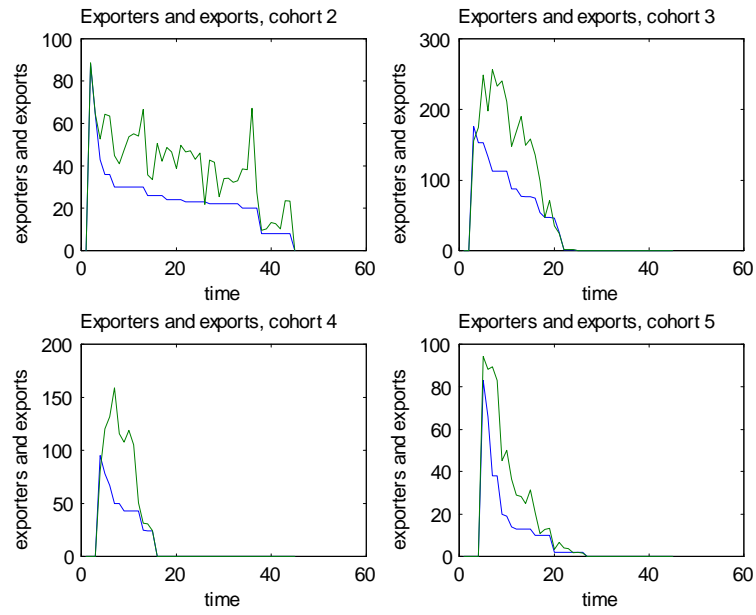


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Figure 6:

Export aggregates and the real exchange rate (new cohort)

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Figure 7: Cohort-specific exports and exporters