Discussion of Gonzalez and Shi
"An Equilibrium Theory of Learning, Search, and Wages"

Robert Shimer

November 19, 2009

Partial Equilibrium Models

- existence of a reservation wage
  - Dirichlet: Rothschild
  - Gaussian: DeGroot
  - general conditions: Burdett-Vishwanath

Partial Equilibrium Models

- existence of a reservation wage
  - Dirichlet: Rothschild
  - Gaussian: DeGroot
  - general conditions: Burdett-Vishwanath
- results:
  - reservation wage declines with duration
  - individual-specific job finding rate increases
  - job finding rate conditional on duration may or may not fall

Partial Equilibrium Models

- existence of a reservation wage
  - Dirichlet: Rothschild
  - Gaussian: DeGroot
  - general conditions: Burdett-Vishwanath
- results:
  - reservation wage declines with duration
  - individual-specific job finding rate increases
  - job finding rate conditional on duration may or may not fall
- simple case: search intensity model

Search Intensity Model

- a worker contacts the market at an unknown rate, a_h or a_l
  - prior of a_h is $\mu \in (0, 1)$
  - $\alpha(\mu) = a_h\mu + a_l(1 - \mu)$
- the worker chooses search intensity at those moments, θ, at cost θc
  - succeeds in finding a job with probability $f(\theta)$
- she cannot distinguish why a job was not found
- a job generates flow income y

Search Intensity Model

- a worker contacts the market at an unknown rate, a_h or a_l
  - prior of a_h is $\mu \in (0,1)$
  - $\alpha(\mu) = a_h \mu + a_l (1-\mu)$
- the worker chooses search intensity at those moments, θ, at cost θc
  - succeeds in finding a job with probability $f(\theta)$
- she cannot distinguish why a job was not found
- a job generates flow income y
- Bellman equation:

$$rV(\mu) = \max_{\theta} \Big(\alpha(\mu)\big(f(\theta)\big(\tfrac{y}{r} - V(\mu)\big) - \theta c\big) - V'(\mu)(a_h - a_l) f(\theta)\mu(1-\mu) \Big)$$

Results

- search intensity falls with unemployment duration (i.e. with μ)
- change in $\alpha(\mu)f(\theta(\mu))$ is ambiguous
- no change in wages
  - obviously, but in contrast to reservation wage models

Results

- search intensity falls with unemployment duration (i.e. with μ)
- change in $\alpha(\mu)f(\theta(\mu))$ is ambiguous
- no change in wages
    - obviously, but in contrast to reservation wage models
- proof: see Gonzalez and Shi

Continuous Time Model

- workers: $rV(\mu) = \max_\theta \left(\alpha(\mu) f(\theta)(J_e(w(\theta)) - V(\mu)) + V'(\mu)\phi(\mu,\theta) \right)$
  - $\alpha(\mu) = a_h \mu + a_l(1-\mu)$
  - $\phi(\mu,\theta) = -(a_h - a_l) f(\theta)\mu(1-\mu)$
  - $J_e(w) = w/r$

Continuous Time Model

- workers: $rV(\mu) = \max_\theta \Big(\alpha(\mu)f(\theta)(J_e(w(\theta)) - V(\mu)) + V'(\mu)\phi(\mu,\theta)\Big)$
  - $\alpha(\mu) = a_h\mu + a_l(1-\mu)$
  - $\phi(\mu,\theta) = -(a_h - a_l)f(\theta)\mu(1-\mu)$
  - $J_e(w) = w/r$
- firms: $c = \frac{f(\theta)}{\theta}J_f(w(\theta))$
  - $J_f(w) = (y-w)/r$

Continuous Time Model

- workers: $rV(\mu) = \max_\theta \Big(\alpha(\mu) f(\theta)(J_e(w(\theta)) - V(\mu)) + V'(\mu)\phi(\mu,\theta)\Big)$
  - $\alpha(\mu) = a_h\mu + a_l(1-\mu)$
  - $\phi(\mu,\theta) = -(a_h - a_l) f(\theta)\mu(1-\mu)$
  - $J_e(w) = w/r$
- firms: $c = \dfrac{f(\theta)}{\theta} J_f(w(\theta))$
  - $J_f(w) = (y-w)/r$
- solve for $w(\theta)$ using the firms' problem:

$$rV(\mu) = \max_\theta \Big(\alpha(\mu)\big(f(\theta)\big(\tfrac{y}{r} - V(\mu)\big) - \theta c\big) - V'(\mu)(a_h - a_l) f(\theta)\mu(1-\mu)\Big)$$

- identical to the single-agent decision problem

Block Recursivity

- note that it was not necessary to keep track of the belief distribution
  - but in steady state, this is not really a big deal
- we can therefore study a standard search model

Standard Search Model

- workers: $rV(\mu) = \alpha(\mu)f(\theta)(J_e(\phi_e(\mu)) - V(\mu)) + V'(\mu)\phi_u(\mu))$
  - $\alpha(\mu) = a_h\mu + a_l(1-\mu)$
  - $\phi_u(\mu) = -(a_h - a_l)f(\theta)\mu(1-\mu)$
  - $\phi_e(\mu) = \frac{a_h\mu}{a_h\mu + a_l(1-\mu)}$
  - $J_e(\mu) = w(\mu)/r$
- firms: $c = \frac{f(\theta)}{\theta}\int J_f(g(\mu))dF(\mu)$
  - $J_f(\mu) = (y - w(\mu))/r$
- Nash bargaining: $J_f(\mu) = J_e(\mu) - V(\mu)$
- $F(\mu)$ is the appropriate stationary distribution

Results

- μ falls during an unemployment spell
- V is increasing in μ
- w is increasing in V
- summary:
  - reemployment wage is decreasing in duration
  - job finding probability is decreasing in duration

Summary

- learning in search is a neglected and likely important topic
- Gonzalez and Shi's analysis is clever and very clean
- but other frameworks are also useful for addressing these questions
  - partial equilibrium
  - search and bargaining
- value-added of competitive search may be clearer out of steady state