Retirement in a Life Cycle Model With Home Production

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Background/Motivation

This paper constitutes a first step toward understanding retirement in the context of optimal life cycle labor supply.

Two motivations:

1. Need a theory of retirement to assess changes in social security or medicare

2. May influence inference about important preference parameters

Retirement and Preference Parameters

- A large literature uses life cycle data to estimate the IES for labor supply
- Standard approach is to focus on labor supply during prime age years (prominent exception is French (2005))
- Common conclusion is that the IES is small
- If retirement is taken as exogenous then the retirement decision conveys no information about preference parameters.
- But if retirement is an endogenous decision then it would presumably also convey information about preference parameters.

Question: What does the retirement decision imply about the IES?

What We Do

We consider retirement in three models:

1. Standard life cycle model

2. Life cycle model with a nonconvexity

3. Life cycle model with a nonconvexity and home production

What We Find

In each case we find tensions in reconciling the model's predictions with various "consensus" estimates of key labor supply parameters.

- In standard model it is very hard to generate retirement
- In non-convex model it is hard to reconcile retirement with low IES and reasonable extent of nonconvexity
- In home production model it is hard to reconcile change in home production time at retirement with moderate substitution between time and goods

Retirement in a Standard Life Cycle Model

Individual solves:

$$Max \sum_{t=0}^{T} [\log(c_t) + \alpha_t v(1 - h_t)]$$

$$\text{s.t. } \sum_{t=0}^{T} c_t = \sum_{t=0}^{T} w_t h_t + Y$$

FOC for interior solution for h_t:

$$\alpha_t v'(1 - h_t) = \mu w_t$$

Generating Retirement

I will use the term “retirement” to describe a situation in which hours of work change from full time work to zero.

Assuming $h_t > 0$, the optimal solution for $h_{t+1} > 0$ iff:

$$v'(1) < v'(1 - h_t) \frac{\alpha_t}{\alpha_{t+1}} \frac{w_{t+1}}{w_t}$$

Equivalently, $h_{t+1} = 0$ iff:

$$v'(1) \geq v'(1 - h_t) R_{t+1}$$

where

$$R_{t+1} = \frac{\alpha_t}{\alpha_{t+1}} \frac{w_{t+1}}{w_t}$$

Simple Quantitative Exercise

Functional Form:

$$v(1-h) = \frac{A}{1-\frac{1}{\gamma}}(1-h)^{1-\frac{1}{\gamma}}$$

Assume full time work corresponds to $h_t = .45$

Question: What value of R_{t+1} is required for $h_{t+1} = 0$ to be optimal.

Table 1

Value of R_{t+1} to Induce Retirement

$IES=2$	$IES=1$	$IES=.75$	$IES=.50$	$IES=.25$	$IES=.10$	$IES=.05$
.61	.48	.38	.23	.05	.001	.000

Table 2

Value of R_{t+1} to Induce Transition from Full-Time to Part-Time

$IES=2$	$IES=1$	$IES=.75$	$IES=.50$	$IES=.25$	$IES=.10$	$IES=.05$
.80	.63	.54	.40	.16	.01	.000

Table 3

Value of R_{t+1} to Induce Retirement from Part-Time

$IES=2$	$IES=1$	$IES=.75$	$IES=.50$	$IES=.25$	$IES=.10$	$IES=.05$
.88	.76	.70	.58	.34	.07	.01

Model With Fixed Costs of Working

Model of Prescott et al (2009)

Individual solves:

$$\max \int_0^1 [\log(c(t)) + \alpha(t)v(1 - h(t))]dt$$

$$\text{s.t.} \int_0^1 c(t)dt = \int_0^1 w(t)\max[0, h(t) - \bar{h}]dt + Y$$

Symmetry implies that we can rewrite the problem as:

$$\max_{e,h} \log[e(h - \bar{h})w + Y] + ev(1 - h) + (1 - e)v(1)$$

Assuming interior solutions for both e and h the FOCs are:

$$\frac{(h-\bar{h})w}{e(h-\bar{h})w+Y} = v(1) - v(1-h)$$

$$\frac{w}{e(h-\bar{h})w+Y} = v'(1-h)$$

Divide these two equations by each other to obtain:

$$h-\bar{h} = \frac{v(1)-v(1-h)}{v'(1-h)}$$

Assume as before:

$$v(1-h) = \frac{A}{1-\frac{1}{\gamma}}(1-h)^{1-\frac{1}{\gamma}}$$

Previous equation becomes:

$$h-\bar{h} = \frac{1}{1-\frac{1}{\gamma}}\left[1-(1-h)^{1-\frac{1}{\gamma}}\right](1-h)^{\frac{1}{\gamma}}$$

This equation must hold if the solution for e is interior.

Note that the value of e does not enter this equation.

Numerical Exercise

Similar to earlier exercise, we set $h = .45$

We now ask what value of $\bar{h}$ is required to induce an interior solution for e, i.e., retirement.

Note that one does not have to specify a value for e to compute the required value of $\bar{h}$

Table 4

Value of $\bar{h}$ Required for Retirement

$IES=2$	$IES=1$	$IES=.75$	$IES=.50$	$IES=.25$	$IES=.10$	$IES=.05$
$.08$	$.14$	$.18$	$.23$	$.32$	$.40$	$.43$

Comparison with French (2005)

Alternative Form of Nonconvexity

Assume that wage rate is increasing in hours worked:

$$w(h) = w_0 h^{\theta}$$

Individual problem becomes:

$$\max_{e,h} \log(e w_0 h^{1+\theta} + Y) + e v(1-h) + (1-e) v(1)$$

Repeating the same steps as before, we arrive at the expression:

$$\frac{h}{1+\theta} = \frac{1}{1-\frac{1}{\gamma}}[1-(1-h)^{1-\frac{1}{\gamma}}](1-h)^{\frac{1}{\gamma}}$$

Numerical Results

Table 5

Value of θ Required for Retirement

IES=2	IES=1	IES=.75	IES=.50	IES=.25	IES=.10	IES=.05
.22	.46	.64	1.04	2.53	8.19	18.2

Combining the two nonconvexities:

$$\frac{h-\bar{h}}{1+\theta} = \frac{1}{1-\frac{1}{\gamma}}[1-(1-h)^{1-\frac{1}{\gamma}}](1-h)^{\frac{1}{\gamma}}$$

Table 6

Value of θ Required for Retirement When $\bar{h} = .1h$

$IES=2$	$IES=1$	$IES=.75$	$IES=.50$	$IES=.25$	$IES=.10$	$IES=.05$
.09	.31	.48	.84	2.17	7.27	16.3

Model With Home Production

Preferences:

$$\int_0^1 [\log c(t) + v(1 - h_m(t) - h_n(t))]dt$$

where:

$$c(t) = [ag(t)^{\varepsilon} + (1 - a)h_n(t)^{\varepsilon}]^{1/\varepsilon}.$$

Budget equation:

$$\int_0^1 g(t)dt = \int_0^1 w_0[h_m(t) - \bar{h}]^{1+\theta}dt$$

Solution to the problem can be summarized by the following values:

- fraction of life in market employment, e
- hours of market work when working, h
- consumption of goods when working and retired, g_w and g_r
- home production time when working and retired, h_w and h_r

Numerical Results

Take elasticity parameters γ and ε as given

Set $\bar{h} = .045$

Normalize $w_0 = 1$

Choose values of a, A and θ so that $h = .45$, $h_w = .10$, $e = 2/3$

Results

Table 8

Values of θ for Home Production Model

	$\varepsilon = 0$	$\varepsilon = .20$	$\varepsilon = .40$	w/o HP
$IES = 1.00$	$.18$	$.17$	$.16$	$.31$
$IES = .50$	$.41$	$.38$	$.34$	$.84$
$IES = .25$	$.70$	$.63$	$.52$	2.17
$IES = .10$	1.03	$.86$	$.70$	7.27

Table 9

Values of h_r and g_r/g_w in the Home Production Model

	h_r			g_r/g_w		
	$\varepsilon = 0$	$\varepsilon = .2$	$\varepsilon = .4$	$\varepsilon = 0$	$\varepsilon = .2$	$\varepsilon = .4$
$IES = 1.00$	.23	.23	.24	1.00	.96	.90
$IES = .50$	.24	.26	.29	1.00	.93	.83
$IES = .25$	.35	.37	.39	1.00	.89	.76
$IES = .10$	.46	.47	.48	1.00	.86	.70

ATUS Data

Time Use By Age: Men and Women

Age	MW	HP	SH	LE	ED	PC
56	23.4	16.7	5.9	35.5	9.1	65.5
58	23.6	15.5	5.6	37.3	8.5	65.2
60	19.6	16.0	6.0	36.6	10.2	66.3
62	16.3	16.6	5.9	39.5	9.8	66.3
64	13.3	18.4	6.2	43.9	9.4	64.9
66	7.55	18.2	6.2	44.0	10.1	68.2
68	8.25	17.4	5.4	46.8	9.9	66.6
70	3.78	17.4	6.3	49.7	10.5	67.5
72	5.67	18.5	6.0	47.6	10.4	67.1

Table 11

Estimated Time Use Effects–Total

	MW	HP	SH	ED	LE	PC
a	-1.5(.1)	.16(.04)	.02(.02)	.09(.02)	1.0(.06)	.17(.04)
h	-	-.12(.02)	-.01(.01)	-.06(.01)	-.65(.04)	-.12(.02)

HP: home production

SH: shopping

ED: eating and drinking

LE: leisure time

PC: personal care

Table 12

Estimated Time Use Effects–Men

	MW	HP	SH	ED	LE	PC
a	-1.7(.2)	.01(.01)	.08(.03)	.12(.03)	1.2(.1)	.17(.04)
h	-	-.03(.03)	-.05(.02)	-.07(.02)	-.7(.04)	-.11(.02)

Conclusion

We have considered models in which utility from leisure is strictly concave, implying that all else equal, individuals prefer smooth leisure over time.

Retirement generates a very dramatic change in hours of market work

In a model in which leisure and work are mirror images of each other, this is hard to reconcile with low values of the IES.

In a model with home production, it is hard to reconcile the small increase in home production time with moderate elasticity of substitution between time and goods and low values of the IES