Competitive Search: A Test of Direction and Efficiency

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Introduction

- Search & Matching: Important framework for labor market analysis
  - Equilibrium unemployment
  - Wage dispersion
- Key properties of competitive search from Moen(1997)

(1) Sub-markets: Workers search in markets with different wages
(2) Direction: Workers wait longer for higher wages
(3) Posting: Firms choose and commit to paying a wage

Questions

(1) Do the competitive search "restrictions" hold?

(2) What are the implications for the search literature?

Overview of Tests and Findings

Competitive search assumes several conditions

(1) Sub-markets - job finding rates and wage pairs

(2) Direction - higher wages $\leftrightarrow$ lower job finding rate

(3) Posting - Surplus is split efficiently, i.e., Hosios (1990)

Findings

(1) Sub-markets - fail to reject in all industries

(2) Direction - fail to reject in all industries

(3) Efficiency - reject in three of six industries

Model

- Time is continuous
- Workers:
  - Homogeneous, risk neutral and discount at rate r
  - Employed (E) or unemployed (U)
  - Unemployment utility flow: z
  - Poisson job arrival rate: $p(\theta_i)$, $\theta \equiv \frac{V}{U}$
- Firms:
  - Free-entry of firms, risk neutral and discount at rate r
  - Firm is vacant (V) or filled (J)
  - Vacancy costs: γ
  - Job filling rate: $q(\theta_i)$
  - Fixed cost to draw a productivity $y_1, ..., y_n$ with probability $f_1, ..., f_n$
  - Idiosyncratic job destruction at rate s

Flow Bellman Equations

- Workers

$$\begin{aligned} rU_i &= z + p(\theta_i)(E_i - U_i) \\ rE_i &= w_i - s(E_i - U_i) \end{aligned}$$

- Firms

$$\begin{aligned} rV(y_i, w, \theta) &= -\gamma + q(\theta)[J(y_i, w) - V(y_i, w, \theta)] \\ rJ(y_i, w) &= y_i - w - sJ(y_i, w) \end{aligned}$$

Equilibrium

(1) Steady-state flows and identity

- flow into and out of each sub-market is equal
- sum of the unemployed and employed equals one

(2) Workers direct their search to best sub-market

(3) Firms post wages that maximize profits

Key Equilibrium Conditions

(1) Sub-market flows

$$u_i p(\theta_i) = f_i es$$

(2) Workers' indifference across sub-markets

$$p(\theta_i) = \frac{rU - z}{w_i - rU}(r + s)$$

(3) Surplus splitting rule

$$w_i = \eta_i y_i + (1 - \eta_i) rU$$

where η_i = elasticity of $q(\theta_i)$

Data

- Current Population Survey (March 2006)
  - duration of unemployment in weeks - t
  - hourly wages - w
  - unemployment rate - u
- Job Openings and Labor Turnover Survey (March 2006)
  - vacancies - v
- IRS - SOI on Corporate Returns - Table 7 (2006)
  - (labor earnings) / (labor earnings + firm earnings) - π

Construction of Likelihood Function

(1) Employed Observations

$$f(w|e,i) = \frac{1}{\sqrt{2\pi}} exp^{-\frac{1}{2}\left(\frac{w-\mu_i}{\sigma_w}\right)^2}, \text{ and } f(w,e) = e\sum_{i=1}^{n} f_i f(w|e,i)$$

(2) Unemployed observations

$$f(t|u,i) = p_i exp^{-p_i t}, \text{ and } f(t,u) = u\sum_{i=1}^{n} \frac{u_i}{u} f(t|u,i)$$

where $p_i = p(\theta_i)$

Complete Likelihood Function

$$ln\ L(\psi) = \sum_{j \in e} ln(f(w_j, e_j)) + \sum_{j \in u} ln(f(t_j, u_j))$$

subject to

- $e + \sum_{i=1}^{n} u_i = 1$,
- $\sum_{i=1}^{n} u_i p_i = es$, and
- $\sum_{i=1}^{n} f_i = 1$

where $\psi = \{u_1, ..., u_n, p_1, ..., p_n, f_1, ..., f_n, \mu_1, ..., \mu_n, \sigma_w, s, e\}$

Additional Estimators

- To identify the elasticity of the matching function
  - Matching function is Cobb-Douglas:

$$p(\theta_i) = x(u_i, v_i)/u_i = u_i^{1-\eta} v_i^{\eta}/u_i$$

  - The estimator is

$$\hat{v} = \sum_{i=1}^{n} v_i = \sum_{i=1}^{n} p_i^{1/\eta} u_i,$$

- Use JOLTS data for $\hat{v}$

...Estimators

- To identify posting and efficiency
  - Wage-splitting rule is

$$w = \beta y + (1 - \beta) rU$$

  - The estimator is

$$\hat{\pi} = \frac{\sum_{i=1}^{n} f_i \mu_i}{\sum_{i=1}^{n} f_i y_i} = \frac{\sum_{i=1}^{n} f_i \mu_i}{\sum_{i=1}^{n} f_i \frac{\mu_i - (1-\beta) rU}{\beta}},$$

  - $\beta = \eta$ implies efficiency
- Use IRS data for $\hat{\pi}$

Identification and Estimation

- Likelihood and estimators identify
  - Wages $(w_1, \ldots, w_n)$ that occur with probability $(f_1, \ldots, f_n)$ and measurement error σ_w
  - Arrival rates $(p_1, \ldots, p_n)$ that occur with probability $(u_1, \ldots, u_n)$
  - Elasticity parameter η
  - Wage splitting parameter β
  - job destruction rate s

Tests of Competitive Search

- Specification A - No equilibrium conditions are imposed
- Specification B - Sub-markets

$$\begin{aligned} H_o &: p_i u_i = f_i es \text{ for all } i \\ H_a &: p_i u_i \neq f_i es \text{ for one } i \end{aligned}$$

- Specification C - Direction

$$\begin{aligned} H_o &: p_i = (r+s)\frac{rU-z}{w_i - rU} \text{ for all } i \\ H_o &: p_i \neq (r+s)\frac{rU-z}{w_i - rU} \text{ for one } i \end{aligned}$$

- Specification D - Efficiency

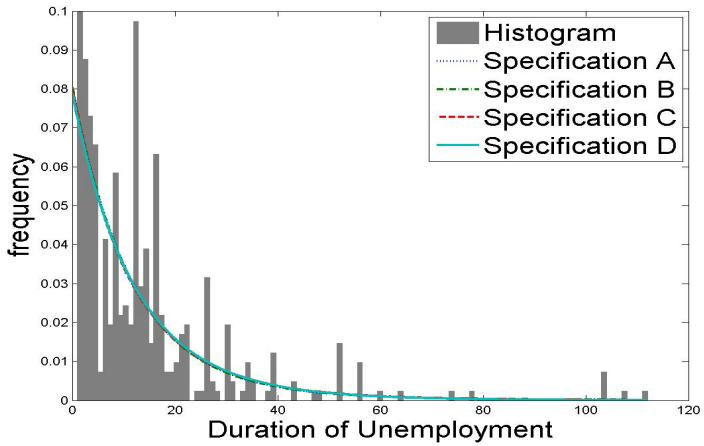
$$\begin{aligned} H_o &: \eta = \beta \\ H_a &: \eta \neq \beta \end{aligned}$$

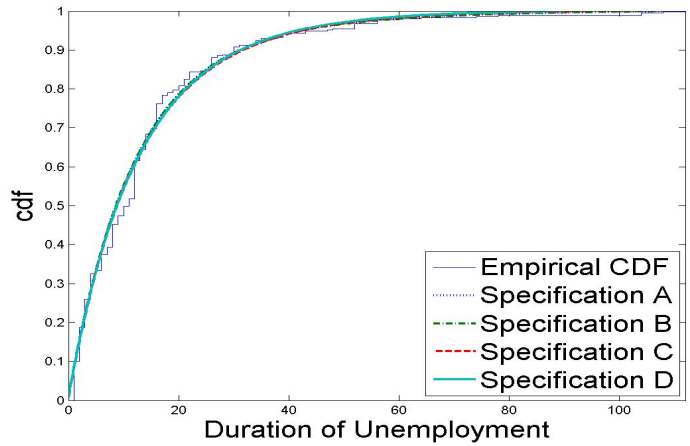
...Data

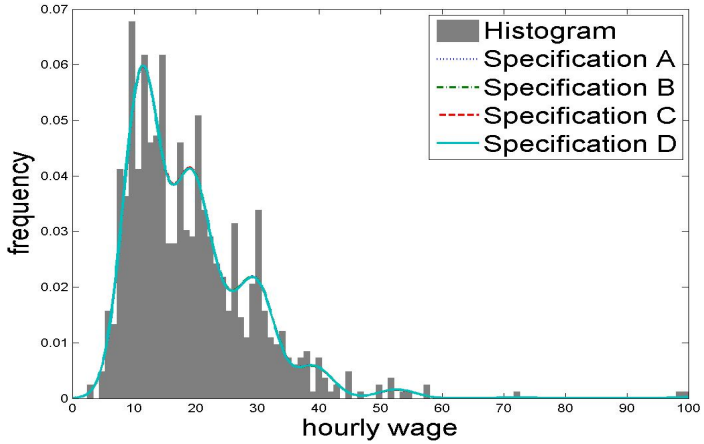
	Construction	Education & Health Services
	CPS data	
w	19.03	18.22
t	13.91	18.35
$\hat{u}$	0.075	0.023
	JOLTS data	
$\hat{v}$	0.015	0.023
	IRS data	
$\hat{\pi}$	0.474	0.765

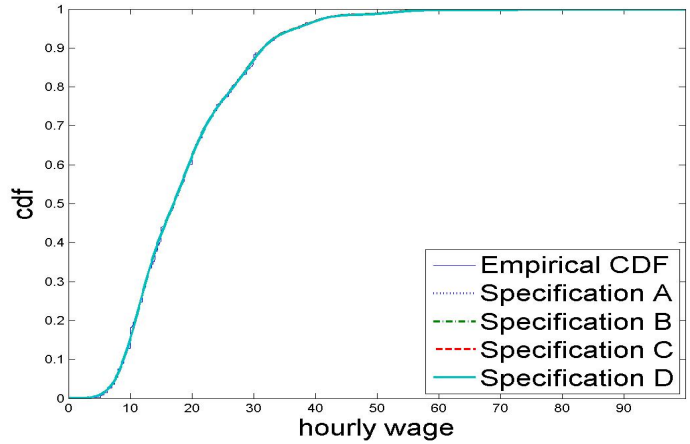
▸ All Industries

Estimates: Construction



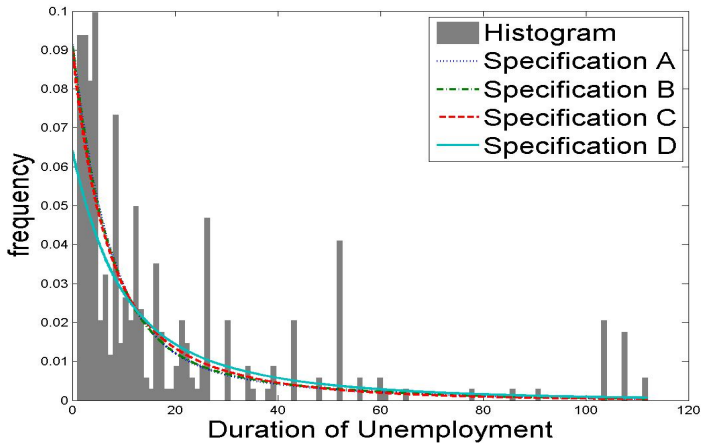


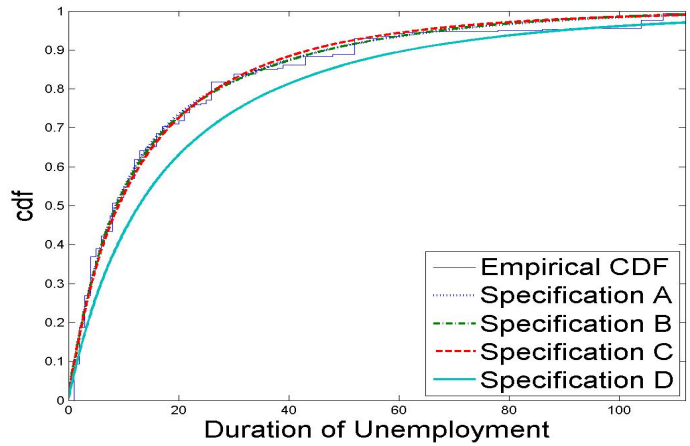


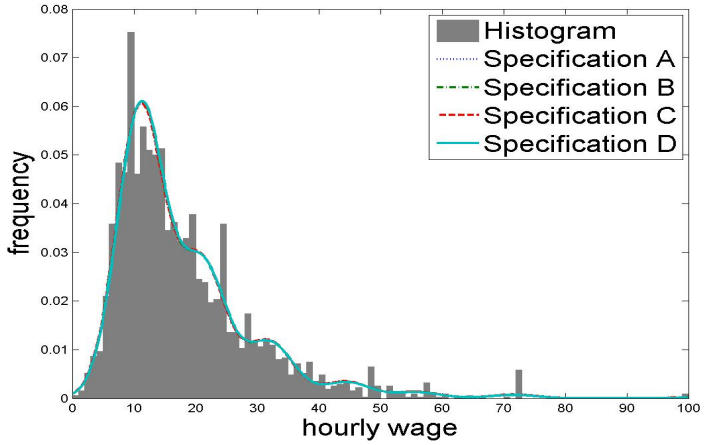


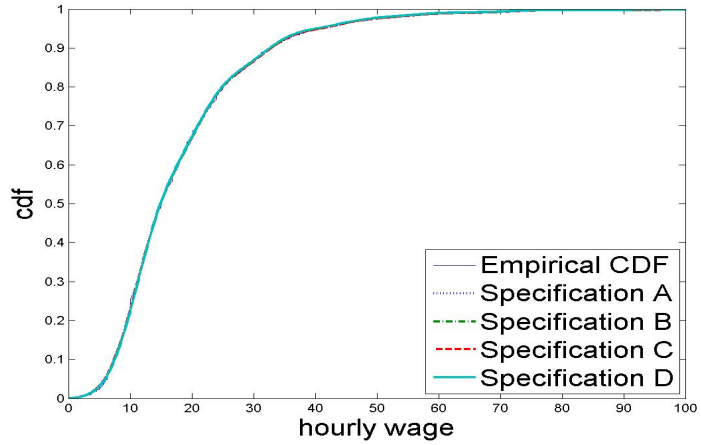
▸ Estimates

Estimates: Education & Health Services









▸ Estimates

Specification Tests

	Construction	Education & Health Services
ln L under Specification		
A - base	-4,465.58	-12,578.8
B - submarkets	-4,465.64	-12,578.91
C - direction	-4,466.91	-12,581.45
D - efficiency	-4,467.08	-12,592.31
Test of submarkets (A vs. B)		
LR test	0.1	0.23
p-value	≈ 1	≈ 1
Test of directed search (B vs. C)		
LR test	2.54	5.07
p-value	0.86	0.54
Test of efficiency (C vs. D)		
LR test	0.34	21.73
p-value	0.56	0

▸ All Industries

Conclusions & Future Avenues

- Current Conclusions
  - Fail to reject submarkets and workers direct search
  - Reject efficiency in a subset of industries
- Future additions
  - Alternative matching functions
  - Estimation by occupations, education, etc.
  - Use of panel data (NLSY or matched CPS)

Complete Descriptive Statistics

	Aggregate	Construction	Manufacturing	Trade, Transportation, & Utilities	Professional & Business Services	Education & Health Services	Leisure & Hospitality
				CPS data			
w	18.15	19.03	19.8	15.77	21.59	18.22	11.79
	(12.12)	(9.91)	(12.09)	(10.61)	(15.33)	(11.68)	(9)
t	17.06	13.91	20.38	16.18	18.49	18.35	15.54
	(21.79)	(16.33)	(23.91)	(21.99)	(22.74)	(24.73)	(20.6)
$\hat{u}$	0.043	0.075	0.041	0.045	0.056	0.023	0.073
				JOLTS data			
$\hat{v}$	0.031	0.015	0.023	0.025	0.053	0.023	0.045
				IRS data			
$\hat{\pi}$	0.515	0.474	0.46	0.644	0.61	0.765	0.714

Note: Standard deviations are in parenthesis, unemployment duration is weekly, wages are hourly.

Back to the data

Parameter Estimates: Construction

Table: Specification C

Markets	i = 1	i = 2	i = 3	i = 4	i = 5	i = 6	i = 7	i = 8
u_i/u	0.317	0.285	0.083	0.206	0.074	0.026	0.004	0.005
p_i	0.112	0.08	0.067	0.058	0.046	0.036	0.027	0.02
f_i	0.442	0.283	0.069	0.148	0.043	0.012	0.001	0.001
μ_i	11.29	19.2	24.44	29.92	39.19	52.66	72.13	100

Other Estimates

σ_w	β	η	s	rU	z
3.015	0.566	0.598	0.007	-8.488	-274.7

Table: Specification D

Markets	i = 1	i = 2	i = 3	i = 4	i = 5	i = 6	i = 7	i = 8
u_i/u	0.336	0.283	0.08	0.199	0.07	0.025	0.003	0.005
p_i	0.104	0.079	0.068	0.059	0.049	0.039	0.03	0.023
f_i	0.443	0.282	0.069	0.149	0.043	0.012	0.001	0.001
μ_i	11.29	19.2	24.32	29.89	39.18	52.67	72.15	100.03

Other Estimates

σ_w	$\eta = \beta$	s	rU	z
3.016	0.604	0.006	-13.2	- 315.8

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Parameter Estimates: Education & Health Services

Table: Specification C

Markets	i = 1	i = 2	i = 3	i = 4	i = 5	i = 6	i = 7	i = 8
u_i/u	0.045	0.228	0.317	0.221	0.095	0.048	0.036	0.009
p_i	0.197	0.187	0.075	0.045	0.031	0.024	0.018	0.013
f_i	0.098	0.473	0.263	0.11	0.033	0.013	0.007	0.001
μ_i	10.75	11.1	21.02	31.94	44.35	55.87	71.62	99.01

Other Estimates

σ_w	β	η	s	rU	z
3.816	0.711	0.605	0.002	4.472	-411.1

Table: Specification D

Markets	i = 1	i = 2	i = 3	i = 4	i = 5	i = 6	i = 7	i = 8
u_i/u	0.336	0.283	0.08	0.199	0.07	0.025	0.003	0.005
p_i	0.104	0.079	0.068	0.059	0.049	0.039	0.03	0.023
f_i	0.443	0.282	0.069	0.149	0.043	0.012	0.001	0.001
μ_i	11.29	19.2	24.32	29.89	39.18	52.67	72.15	100.03

Other Estimates

σ_w	$\eta = \beta$	s	rU	z
3.016	0.604	0.006	-13.2	-107.2

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Complete Specification Tests

	Aggregate	Construction	Manufacturing	Trade, Transportation, & Utilities	Professional & Business Services	Education & Health Services	Leisure & Hospitality
ln L under Specification							
A	-60,518.4	-4,465.58	-7,379.08	-11,401.91	-5,987.13	-12,578.8	-5,248.58
B	-60,518.52	-4,465.64	-7,379.09	-11,401.96	-5,987.14	-12,578.91	-5,248.66
C	-60,529.59	-4,466.91	-7,380.22	-11,406.93	-5,988.11	-12,581.45	-5,249.97
D	-60,602.44	-4,467.08	-7,384.74	-11,406.96	-5,999.45	-12,592.31	-5,250.56
Test of submarkets (A vs. B)							
LR test	0.24	0.1	0.03	0.09	0.02	0.23	0.15
p-value	1	1	1	1	1	1	1
Test of Homogeneous workers (B vs. C)							
LR test	22.12	2.54	2.25	9.95	1.94	5.07	2.62
p-value	0	0.86	0.9	0.13	0.92	0.54	0.85
Test of efficiency with homogeneous workers (C vs. D)							
LR test	145.71	0.34	9.05	0.05	22.69	21.73	1.19
p-value	0	0.56	0	0.83	0	0	0.28

Back to the Results

Generalized Cobb-Douglas: $Au^{1-\eta}v^{\eta}$

- One approach is to assume industries have same η and A, or

$$\hat{v}^c = \sum_{i=1}^{n} v_i^c = A^{-1/\eta} \sum_{i=1}^{n} (p_i^c)^{1/\eta} u_i^c,$$

$$\hat{v}^e = \sum_{i=1}^{n} v_i^e = A^{-1/\eta} \sum_{i=1}^{n} (p_i^e)^{1/\eta} u_i^e,$$

- Identification
  - Ratios solve for η
  - Levels solve for A

Negative z with Random Matching and a Productivity Distribution

$$\begin{aligned} rU &= z + p\int_{rU} \frac{w - rU}{r+s} dF(w) \\ &\approx z + \frac{p}{r+s}(\bar{w} - rU) \end{aligned}$$

- Likely parameters
  - $rU = 5$
  - $p = 5$, $r = .05$, $s = .4$
- This implies
  - if $\bar{w} = 7$, then $z = -17.2$
  - if $\bar{w} = 12$, then $z = -72.7$